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Analysis and Design of Launch Vehicle Flight Control Systems

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This paper describes the fundamental principles of launch vehicle flight control analysis and design. In particular, the classical concept of "drift-minimum" and "load-minimum" control principles is re-examined and its performance and stability robustness with respect to modeling uncertainties and a gimbal angle constraint is discussed. It is shown that an additional feedback of angle-of-attack or lateral acceleration can significantly improve the overall performance and robustness, especially in the presence of unexpected large wind disturbance. Non-minimum-phase structural filtering of "unstably interacting" bending modes of large flexible launch vehicles is also shown to be effective and robust.

I. Introduction

Note to Session Organizer/Reviewers: This draft manuscript summarizes very preliminary results obtained during an early phase of a project for the launch vehicle flight control systems analysis and design as applied to Ares-I Crew Launch Vehicle. During the next several months, a more detailed, rigorous study will be conducted in the areas of drift-minimum vs load-minimum control, flexible-body stabilization and analysis, gain scheduling vs. adaptive control, etc. A companion paper on dynamic modeling of large flexible launch vehicles is also being submitted to this Space Exploration and Transportation GNC session.

II. Rigid-Body Control Analysis

Consider a simplified linear dynamical model of a launch vehicle [15], as illustrated in Fig. 2, as follows:

$$\ddot{\theta} = M_\alpha \alpha + M_\delta \delta \tag{1}$$

$$\ddot{Z} = -\frac{F}{m}\theta - \frac{N_\alpha}{m}\alpha + \frac{T}{m}\delta \tag{2}$$

$$\alpha = \theta + \frac{\dot{Z}}{V} + \alpha_w \tag{3}$$

$$F = T_o + T - D \tag{4}$$

where θ is the pitch attitude, α the angle of attack, Z the inertial Z-axis drift position of the center-of-mass, $\dot{Z}$ the inertial drift velocity, m the vehicle mass, T_o the ungimbaled sustainer thrust, T the gimbaled thrust, $N = N_\alpha \alpha$ the aerodynamic normal (lift) force acting on the center-of-pressure, D the aerodynamic axial (drag) force, F the total

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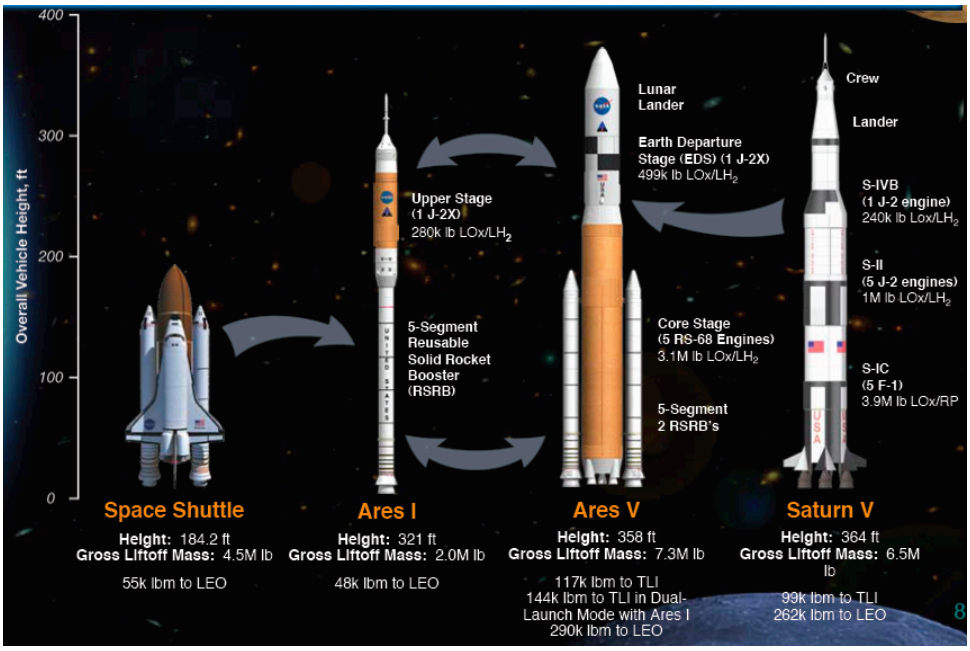

Figure 1. Comparison of Space Shuttle, Ares I, Ares V, and Saturn V Launch Vehicles [1].

x-axis force, δ the gimbal deflection angle, V the vehicle velocity, $\alpha_w = V_w/V$ the wind-induced angle of attack, V_w the wind disturbance velocity, and

$$M_\alpha = x_{cp} N_\alpha / I_y \tag{5}$$

$$M_\delta = x_{cg} T / I_y \tag{6}$$

$$N_\alpha = \frac{1}{2}\rho V^2 S C_{N_\alpha} \tag{7}$$

where I_y is the pitch moment of inertia. For an effective thrust vector control of a launch vehicle, we need

$$M_\delta \delta_{\max} > M_\alpha \alpha_{\max} \tag{8}$$

where $\delta_{\max}$ is the gimbal angle constraint and $\alpha_{\max}$ is the maximum wind-induced angle of attack.

The open-loop transfer functions from the control input $\delta(s)$ can then be obtained as

$$\frac{\theta(s)}{\delta(s)} = \frac{s}{\Delta(s)}\left[M_\delta\left(s + \frac{N_\alpha}{mV}\right) + \frac{M_\alpha T}{mV}\right] \tag{9}$$

$$\frac{Z(s)}{\delta(s)} = \frac{1}{\Delta(s)}\left[\frac{T}{m}\left(s^2 - M_\alpha\right) - \frac{M_\alpha(F + N_\alpha)}{m}\right] \tag{10}$$

$$\frac{\alpha(s)}{\delta(s)} = \frac{s}{\Delta(s)}\left[\frac{T}{mV}s^2 - M_\delta s + \frac{M_\delta F}{mV}\right] \tag{11}$$

where

$$\Delta(s) = s\left[s^3 + \frac{N_\alpha}{mV}s^2 - M_\alpha s + \frac{M_\alpha F}{mV}\right] \tag{12}$$

Consequently, the 4th-order system described by Eq. (1) - (3) is completely controllable by δ and is observable by Z; however, the system is not observable by θ and α.

In 1959, Hoelkner introduced the "drift-minimum" and "load-minimum" control concepts as applied to the launch vehicle flight control system [6]. The concepts have been further investigated in [7-14]. Basically, Hoelkner's controller utilizes a full-state feedback control of the form

$$\delta = -K_1\theta - K_2\dot{\theta} - K_3\alpha \tag{13}$$

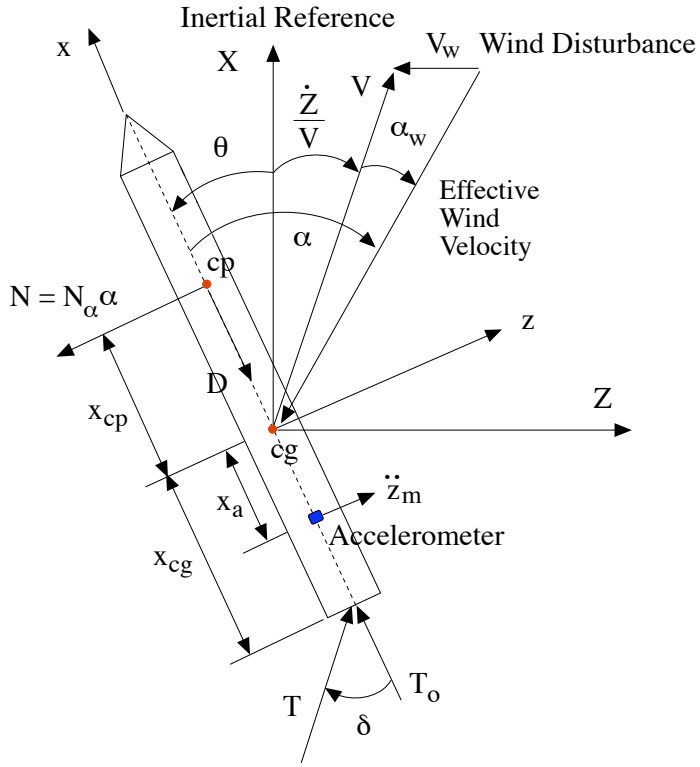

Figure 2. Simplified pitch-axis model of a launch vehicle.

for a 3rd-order dynamical model of the form

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ \alpha\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & M_\alpha\\ -F/(mV) & 1 & -N_\alpha/(mV)\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \alpha\end{bmatrix} + \begin{bmatrix}0\\ M_\delta\\ T/(mV)\end{bmatrix}\delta + \begin{bmatrix}0\\ 0\\ \dot{\alpha}_w\end{bmatrix} \tag{14}$$

This 3rd-order system is observable by θ or α. The feedback gains are to be properly selected to minimize the lateral drift velocity $\dot{Z} = V(\alpha - \theta - \alpha_w)$ or the bending moment caused by the angle of attack. Note that

$$\frac{\dot{Z}}{V} \equiv \gamma = \alpha - \theta - \alpha_w \tag{15}$$

where γ is often called the flight-path angle.

Instead of measuring the angle-of-attack, we may employ a body-mounted accelerometer, as illustrated in Fig. 2, as follows:

$$\begin{aligned}\delta = & -K_1\theta - K_2\dot{\theta} + K_a\ddot{z}_m\\ = & -K_1\theta - K_2\dot{\theta} + K_a\left(-\frac{N_\alpha}{m}\alpha + \frac{T}{m}\delta + \frac{x_a}{m}\ddot{\theta}\right)\\ = & -K_1\theta - K_2\dot{\theta} + K_a\frac{x_a}{m}\ddot{\theta} - K_a\frac{N_\alpha}{m}\alpha + K_a\frac{T}{m}\delta\end{aligned}$$

Because the resulting effect of $\ddot{z}_m$ feedback is basically the same as the α feedback, we consider here only the control logic described by Eq. (13).

Substituting Eq. (13) into Eq. (1) - (2) or Eq. (14), we obtain the closed-loop transfer function from the wind disturbance $\alpha_w(s)$ to the drift velocity $\dot{Z}(s)$ as

$$\frac{\dot{Z}}{\alpha_w V} = -\frac{A_2 s^2 + A_1 s + A_o}{s^3 + B_2 s^2 + B_1 s + B_o} \tag{16}$$

where

$$B_2 = M_\delta K_2 + \frac{T}{mV}\left(K_3 + \frac{N_\alpha}{T}\right)$$
$$B_1 = M_\delta(K_1 + K_3) - M_\alpha + \frac{K_2 T}{mV}\left(M_\alpha + \frac{M_\delta N_\alpha}{T}\right)$$
$$B_o = \frac{TK_1}{mV}\left(M_\alpha + \frac{M_\delta N_\alpha}{T}\right) - \frac{F}{mV}\left(M_\delta K_3 - M_\alpha\right)$$
$$A_2 = \frac{T}{mV}\left(K_3 + \frac{N_\alpha}{T}\right)$$
$$A_1 = \frac{K_2 T}{mV}\left(M_\alpha + \frac{M_\delta N_\alpha}{T}\right)$$
$$A_o = B_o$$

For a unit-step wind disturbance of $\alpha_w(s) = 1/s$, the steady-state value of $\dot{Z}$ can be found as

$$\frac{\dot{Z}_{ss}}{V} = \lim_{s\to 0} \frac{-(A_2 s^2 + A_1 s + A_o)}{s^3 + B_2 s^2 + B_1 s + B_o} = \frac{-A_o}{B_o} = -1 \tag{17}$$

The launch vehicle drifts along the wind direction with $\dot{Z}_{ss} = -V_w$ and also with $\theta = \dot{\theta} = \alpha = \delta = 0$ as $t \to \infty$. It is interesting to notice that the steady-state drift velocity (or the flight path angle) is independent of feedback gains provided an asymptotically stable closed-loop system with $B_o \neq 0$.

If we choose the control gains such that $B_o = 0$ (i.e., one of the closed-loop system roots is placed at $s = 0$), the steady-state value of $\dot{Z}$ becomes

$$\frac{\dot{Z}_{ss}}{V} = \lim_{s\to 0} \frac{-(A_2 s + A_1)}{s^2 + B_2 s + B_1} = \frac{-A_1}{B_1} = \frac{-1}{1 + C} \tag{18}$$

where

$$C = \frac{mV[M_\delta(K_1 + K_3) - M_\alpha]}{M_\alpha K_2 T + M_\delta N_\alpha / T} \tag{19}$$

For a stable closed-loop system with $M_\delta(K_1 + K_3) - M_\alpha > 0$, we have $C > 1$ and

$$|\dot{Z}_{ss}| < V_w \tag{20}$$

when $B_o = 0$. The drift-minimum condition, $B_o = 0$, can be rewritten as

$$\frac{M_\delta K_3 - M_\alpha}{M_\delta K_1} = \frac{N_\alpha}{F}\left(1 + \frac{x_{cp}}{x_{cg}}\right) \tag{21}$$

Consider the following closed-loop transfer functions:

$$\frac{\alpha}{\alpha_w} = -\frac{s(s^2 + M_\delta K_2 s + M_\delta K_1)}{s^3 + B_2 s^2 + B_1 s + B_o} \tag{22}$$

$$\frac{\delta}{\alpha_w} = -\frac{s(K_3 s^2 + M_\alpha K_2 s + M_\alpha K_1)}{s^3 + B_2 s^2 + B_1 s + B_o} \tag{23}$$

For a unit-step wind disturbance of $\alpha_w(s) = 1/s$, we have $\alpha = \delta = 0$ as $t \to \infty$. However, for a unit-ramp wind disturbance of $\alpha_w(s) = 1/s^2$, we have

$$\lim_{t\to\infty} \alpha(t) = M_\delta K_1$$
$$\lim_{t\to\infty} \delta(t) = M_\alpha K_1$$

Consequently, the bending moment induced by α and δ can be minimized by choosing $K_1 = 0$, which is the "load-minimum" condition introduced by Hoelkner [6]. The closed-loop system with $K_1 = 0$ is unstable because

$$B_o = -\frac{F}{mV}\left(M_\delta K_3 - M_\alpha\right) < 0 \tag{24}$$

However, the load-minimum control for short durations has been known to be acceptable provided a deviation from the nominal flight trajectory is permissible.

A set of full-state feedback control gains, (K_1, K_2, K_3), can be found by using a pole-placement approach or the linear-quadratic-regulator (LQR) control method [21-22], as follows:

$$\min_{\delta} \int_0^{\infty} (x^T Q x + \delta^2) dt \tag{25}$$

subject to $\dot{x} = Ax + B\delta$ and $\delta = -Kx$ where $x = [\theta \;\; \dot{\theta} \;\; \alpha]^T$ and $K = [K_1 \;\; K_2 \;\; K_3]$.

III. Rigid-Body Control Example

Consider a launch vehicle control design example discussed by Greensite in [15]. Its basic parameters are given as in [15]

$$\begin{array}{lll} I_y = 2.43E6 \;\; \text{slug-ft}^2, & m = 5830 \;\; \text{slug}, & T = 341,000 \;\; \text{lb} \\ F = 375,000 \;\; \text{lb}, & x_{cp} = 38 \;\; \text{ft}, & x_{cg} = 32.3 \;\; \text{ft} \\ V = 1320 \;\; \text{ft/sec}, & V_w = 132 \;\; \text{ft/sec}, & \alpha_w = 5.73 \;\; \text{deg} \\ N_\alpha = 240,000 \;\; \text{lb/rad}, & M_\alpha = 3.75 \;\; \text{s}^{-2}, & M_\delta = 4.54 \;\; \text{s}^{-2} \end{array} \tag{26}$$

The open-loop poles of this example vehicle are: -1.9767, 0.0488, 1.8967

Note that the wind-induced angle of attack of 5.73 deg considered for this example in [15] is somewhat unrealistic because it will require a maximum gimbal deflection angle of

$$\delta_{\max} > \frac{M_\alpha}{M_\delta} \alpha_w = 4.73 \text{ deg}$$

Most practical thrust vector control systems have a maximum gimbal angle constraint of about ± 5 deg. In this paper, we also assume a second-order gimbal actuator dynamics of the form

$$\delta(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \delta_c(s) \tag{27}$$

where $\zeta = 1$ and $\omega_n = 50$ rad/s.

Table 1. Summary of rigid-body control analysis and design

Case No.	Controller Type	Feedback Gains (K_1, K_2, K_3)	Closed-Loop Poles
1	$(\theta, \dot{\theta})$-Feedback Control [15]	(2, 0.8, 0)	-1.7488±1.3934j, -0.1596
2	Drift-Minimum Control [15]	(2, 0.8, 3.614)	-1.9087±4.2774j, 0.0
3	Load-Minimum Control [15]	(0, 0.8, 3.614)	-1.9323±3.0533j, 0.0471
4	LQR Control (Q = 0)	(0.6852, 0.8491, 0.9542)	-1.9767, -1.8967, -0.0488
5	Drift-Minimum Control	(0.3220, 0.8352, 1.2765)	-1.9767, -1.8967, 0.0
6	Load-Minimum Control	(0, 0.8352, 1.2765)	-3.1323, -0.7816, 0.0405

IV. Flexible-Body Control Analysis

More detailed control and stability analysis results for Figs. 10 and 11 will be included in the final manuscript.

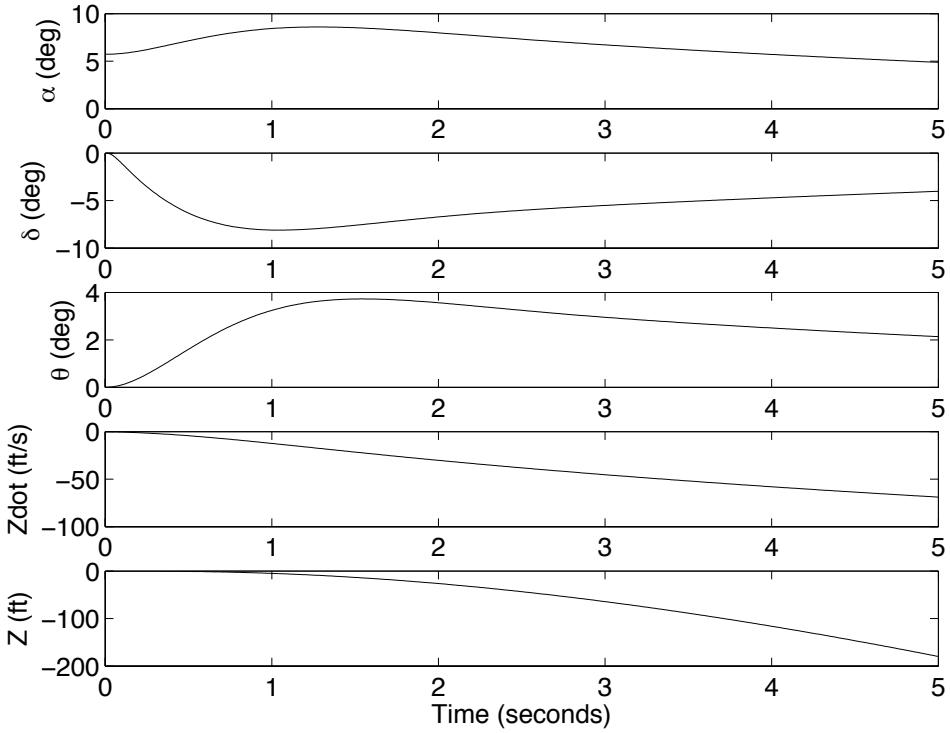

Figure 3. $(\theta, \dot{\theta})$**-feedback control (Case 1).**

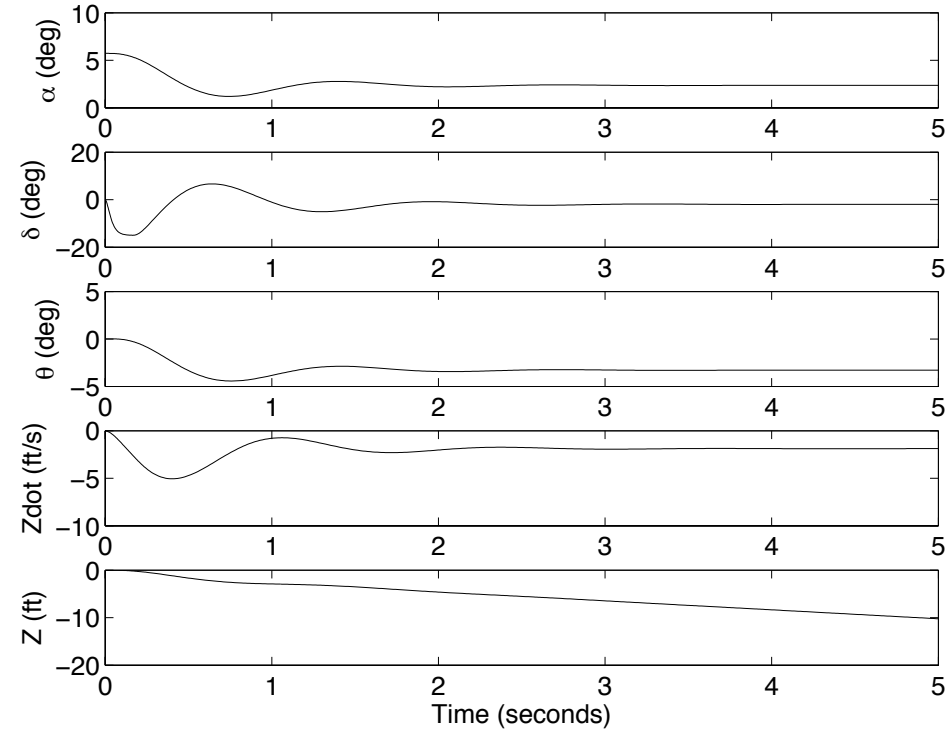

Figure 4. Drift-minimum control (Case 2).

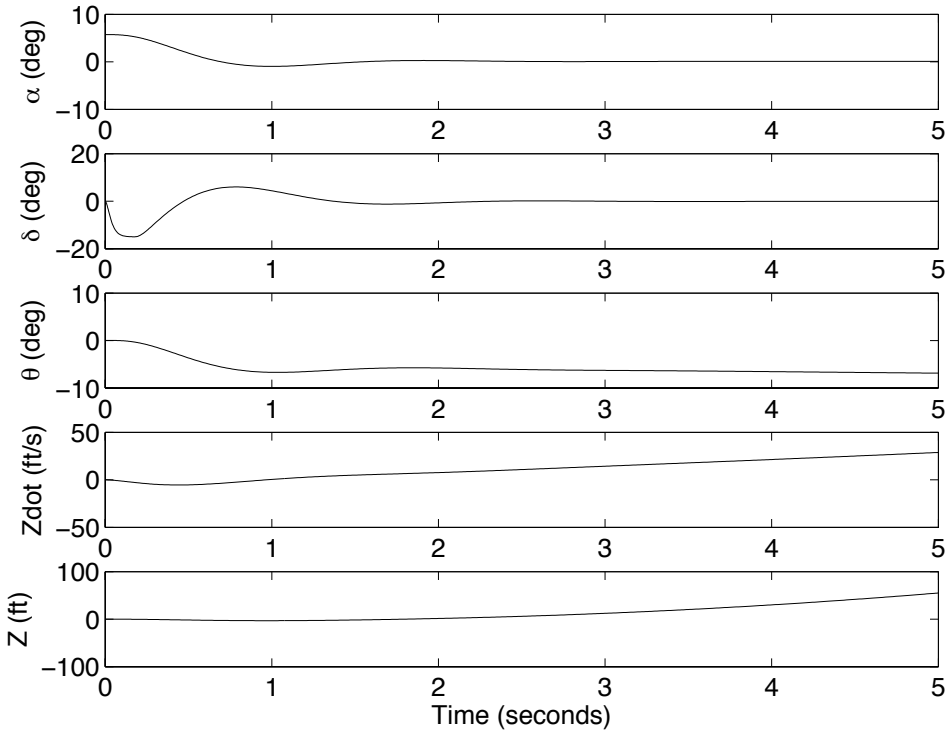

Figure 5. Load-minimum control (Case 3).

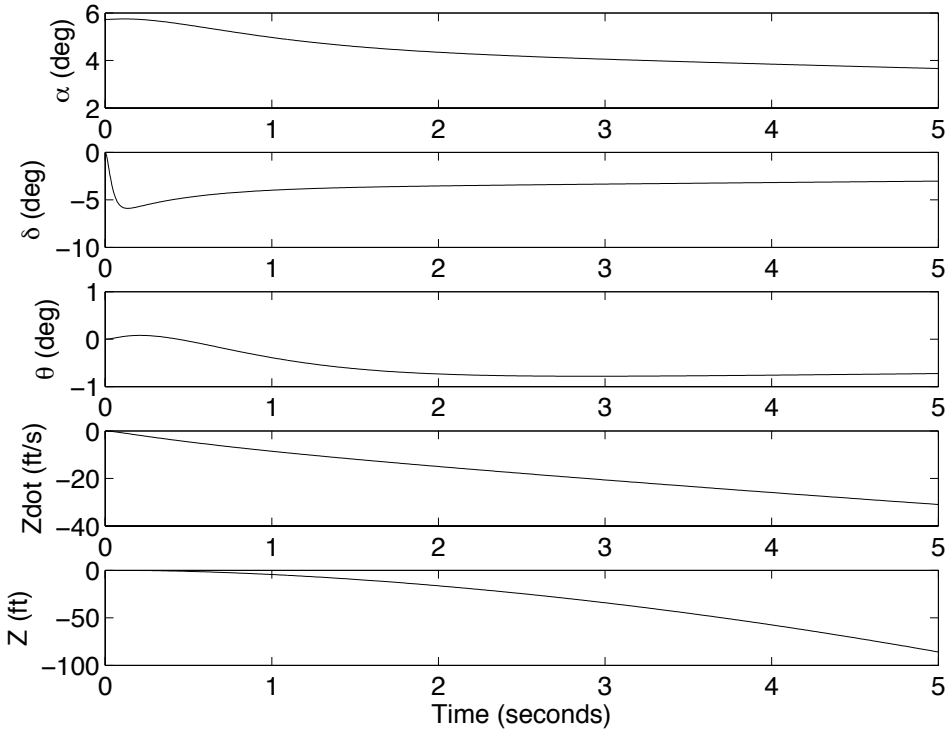

Figure 6. LQR control (Case 4).

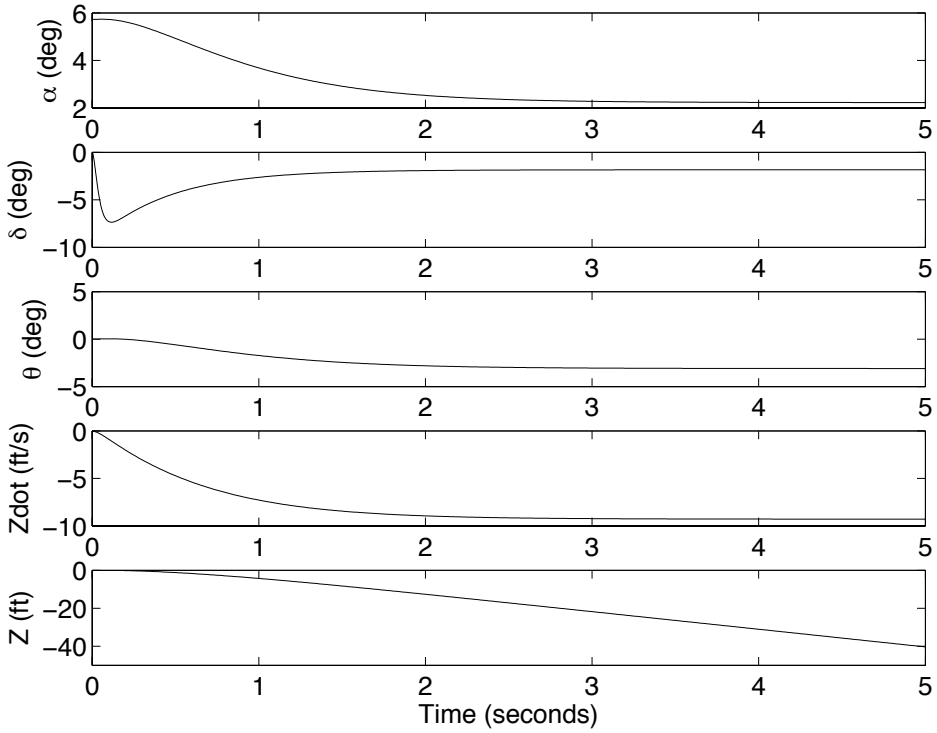

Figure 7. Drift-minimum control (Case 5).

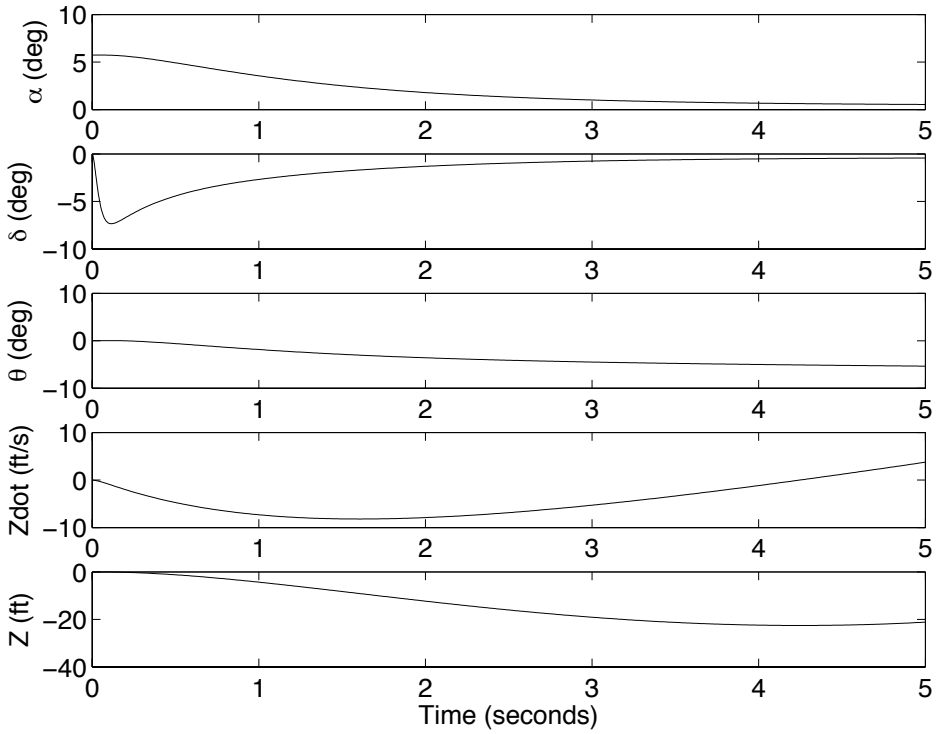

Figure 8. Load-minimum control (Case 6).

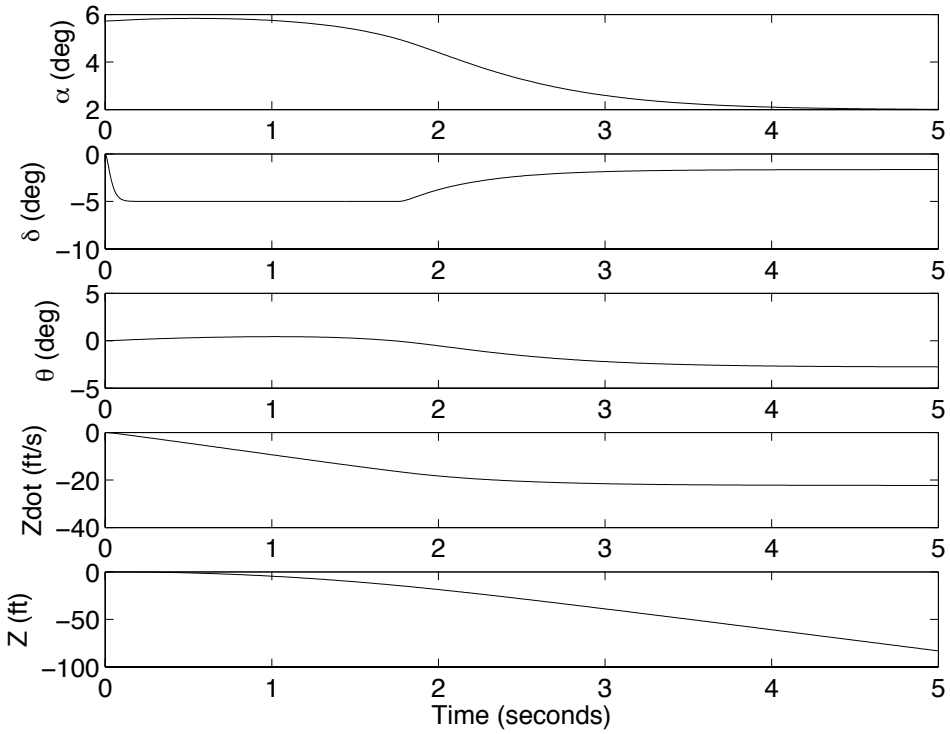

Figure 9. Case 5 (drift-minimum control) with $\delta_{\max} = \pm 5$ deg.

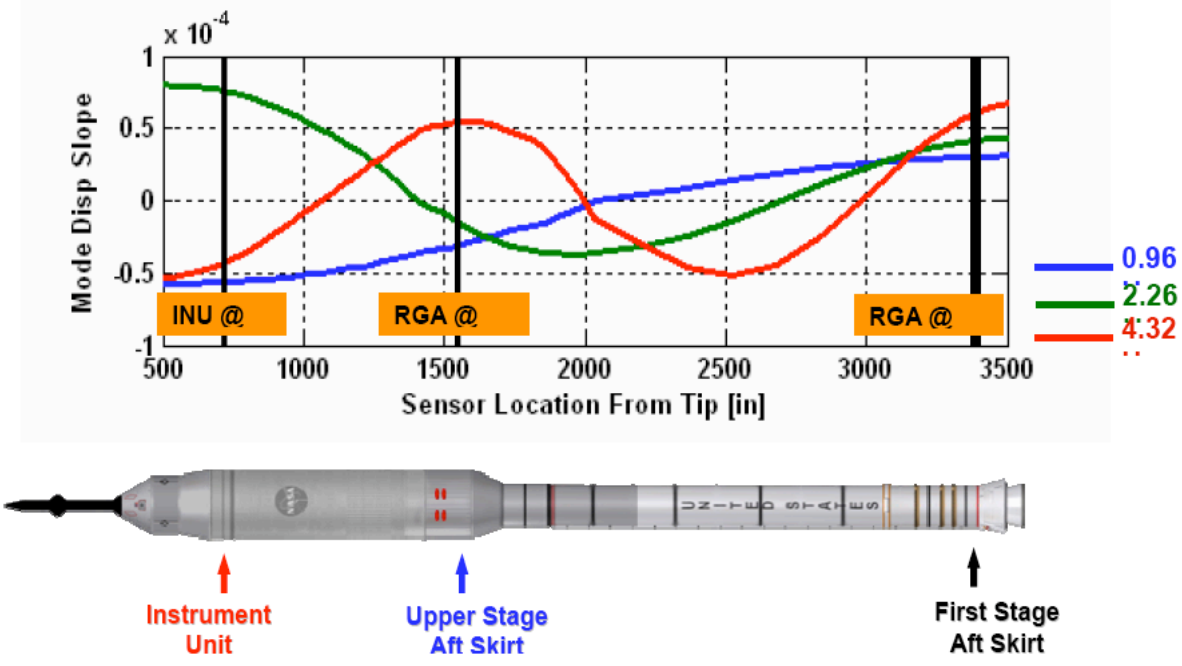

Figure 10. Illustrations of dominant bending modes and sensor locations (Ref. 2).

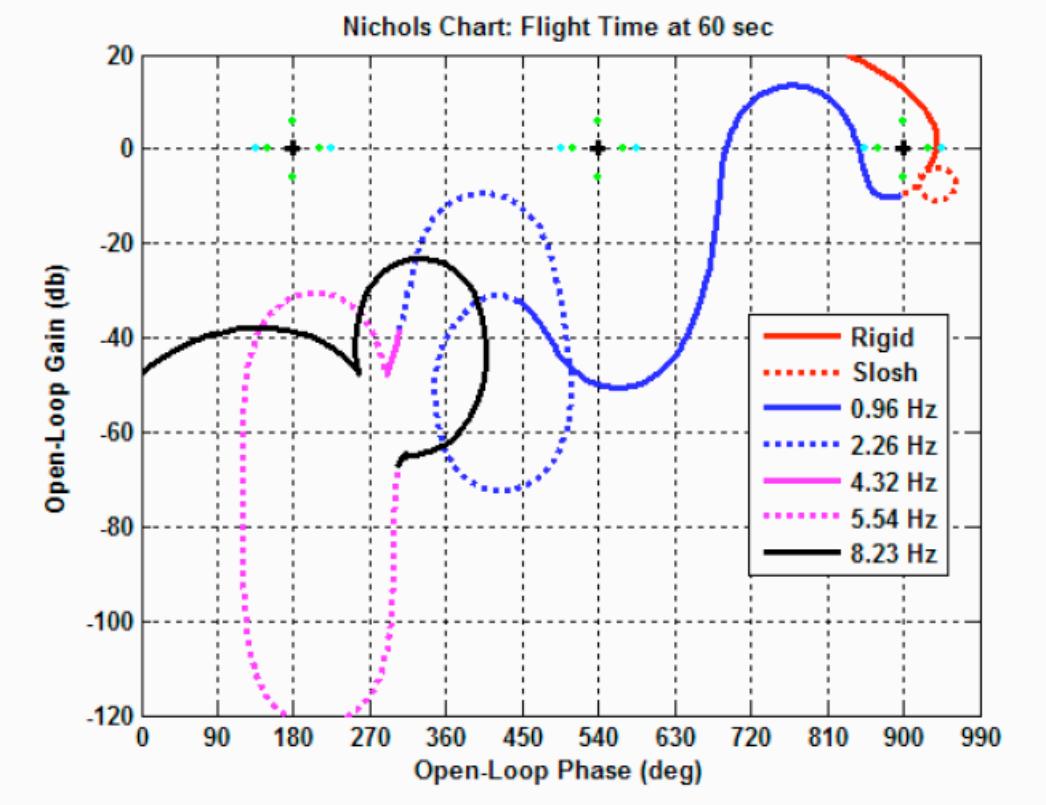

Figure 11. Nichols plot for a baseline pitch-axis flight control system (Ref. 2).

V. Conclusions

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