NASA/TP—2008–215580

On the Period-Amplitude and Amplitude-Period Relationships

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November 2008

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Space Administration

Marshall Space Flight Center • MSFC, Alabama 35812

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NOMENCLATURE

12-mma	12-month moving averages
ASC	ascent duration (the elapsed time from *Rm* to *RM*)
cl	confidence level of the inferred regression
DES	descent duration (the elapsed time from *RM* to the next *Rm*)
Δ	residual = (observed parametric value – predicted parametric value)/predicted parametric value
EM	epoch of maximum amplitude occurrence
Em	epoch of minimum amplitude occurrence
n	sunspot cycle number
P	probability
PER	minimum-to-minimum period: sunspot cycle length in months between occurrences of minimum amplitudes (minimum values of the 12-mma of monthly mean sunspot number) for two successive sunspot cycles
R	monthly mean sunspot number
r	the coefficient of regression
r^2	the coefficient of determination
RM	maximum amplitude (the maximum value of the 12-mma of *R*)
Rm	minimum amplitude (the minimum value of the 12-mma of *R*)
se	the standard error of estimate
x	the independent variable in the regression equation
y_{all}	the inferred regression equation using all available cycles
y'	the inferred regression equation excluding certain cycles deemed statistical outliers

TECHNICAL PUBLICATION

ON THE PERIOD-AMPLITUDE AND AMPLITUDE-PERIOD RELATIONSHIPS

1. INTRODUCTION

Sunspot cycles are generally described using the 12-mo moving average (12-mma) of the monthly mean sunspot number (*R)* (also sometimes called the 13-mo running mean).[1–3] The minimum value of the 12-mma of *R* is called minimum amplitude (*Rm*), while the maximum value of the 12-mma of *R* is called maximum amplitude (*RM*). The interval between successive occurrences of *Rm* is called the sunspot cycle length or period (*PER*), and is comprised of two components: the ascent duration (*ASC*), which is the interval from *Rm* occurrence to *RM* occurrence in the same sunspot cycle, and the descent duration (*DES*), which is the interval from *RM* occurrence of a sunspot cycle to the *Rm* occurrence for the succeeding cycle.

Long ago, Waldmeier[4,5] noted that the shape of the curve describing a sunspot cycle is primarily determined by the height or maximum amplitude of the cycle, with larger cycles attaining maximum amplitude more quickly than smaller cycles. This relationship between *RM* and *ASC* is often called the "Waldmeier effect."[6–8] Other "effects" have also been noted, including the "maximum-minimum effect" (correlating the maximum amplitude to the minimum amplitude for the same cycle), the "amplitude-period effect" (correlating the maximum amplitude of the following cycle to the period of the preceding cycle), the "even-odd effect" (also called the "Gnevyshev-Ohl Rule," correlating the odd-following cycle's maximum amplitude to the even-leading cycle's maximum amplitude in even-odd cycle pairs), and the "three-cycle periodicity scheme" (describing cycles as strings of three cycles each, varying in relative size from low to higher to highest),[7–14] although the reality of some of these effects is highly debatable. For example, simple statistical testing of maximum amplitudes based on annual averages and using the binomial formula suggests that for cycles –4 to 23 the probability of getting 3 of 9 three-cycle groupings, consisting of low to higher to highest maximum amplitude cycles, by chance is 16.4 percent, not a statistically significant result, whereas the probability of getting 10 of 14 two-cycle even-odd groupings, with the odd cycle being the larger, by chance is 6.1 percent, a marginally significant result.

In this Technical Publication, the Amplitude-Period effect is examined in order to determine the expected *RM* for cycle 24, the next sunspot cycle. Also, another effect, called the "Period-Amplitude effect," is examined in order to determine the expected *PER* for cycle 23, the current ongoing sunspot cycle.

2. RESULTS AND DISCUSSION

Figure 1(a) depicts the cyclic variation of *RM* for cycles 0–23 and figure 1(b) depicts the cyclic variation of *PER* for cycles 1–22 (the horizontal lines running through both figures depicts the medians). Runs-testing[15] indicates that both parameters can be regarded as varying randomly. For example, presuming the accuracy of the *RM* values, one determines 12 values above the median (114.9 mo) and 12 values below the median in 10 runs, inferring a random distribution of *RM* at the 5-percent level of significance. Likewise, presuming the accuracy of the *PER* values, one determines 11 values above the median (130.5 mo) and 11 values below the median in 8 runs, inferring (although just barely) a random distribution of *PER* at the 5-percent level of significance. (For convenience, table 1 identifies the actual values of *RM* and *PER* for cycles 0–23 that are plotted in figure 1 and the epochs of occurrences for *Rm* and *RM*.)

It is important to remember that the sunspot cycle record, as reconstructed by Wolf, is of non-uniform quality, being considered of poor quality prior to 1749 (prior to cycle 0), questionable quality between 1749 and 1817 (spanning about cycles 0–6), good quality between 1818 and 1847 (spanning about cycles 6–9) and reliable quality from 1848 (cycle 9 onward).[1,16] Comparison of Wolf's relative sunspot number against Hoyt and Schatten's group sunspot number and Schwabe's "cluster of spots" observations, however, has revealed the distinct possibility that the early record (actually prior to about cycle 12) may not be as reliable as is often believed.[17–20] (Using the group sunspot number, Hathaway, Wilson and Reichmann[14] have also shown that there is a long-term secular increase in the maximum amplitudes extending from the Maunder Minimum to the present, an interval of about 300 years, with variations occurring about the trend line.)

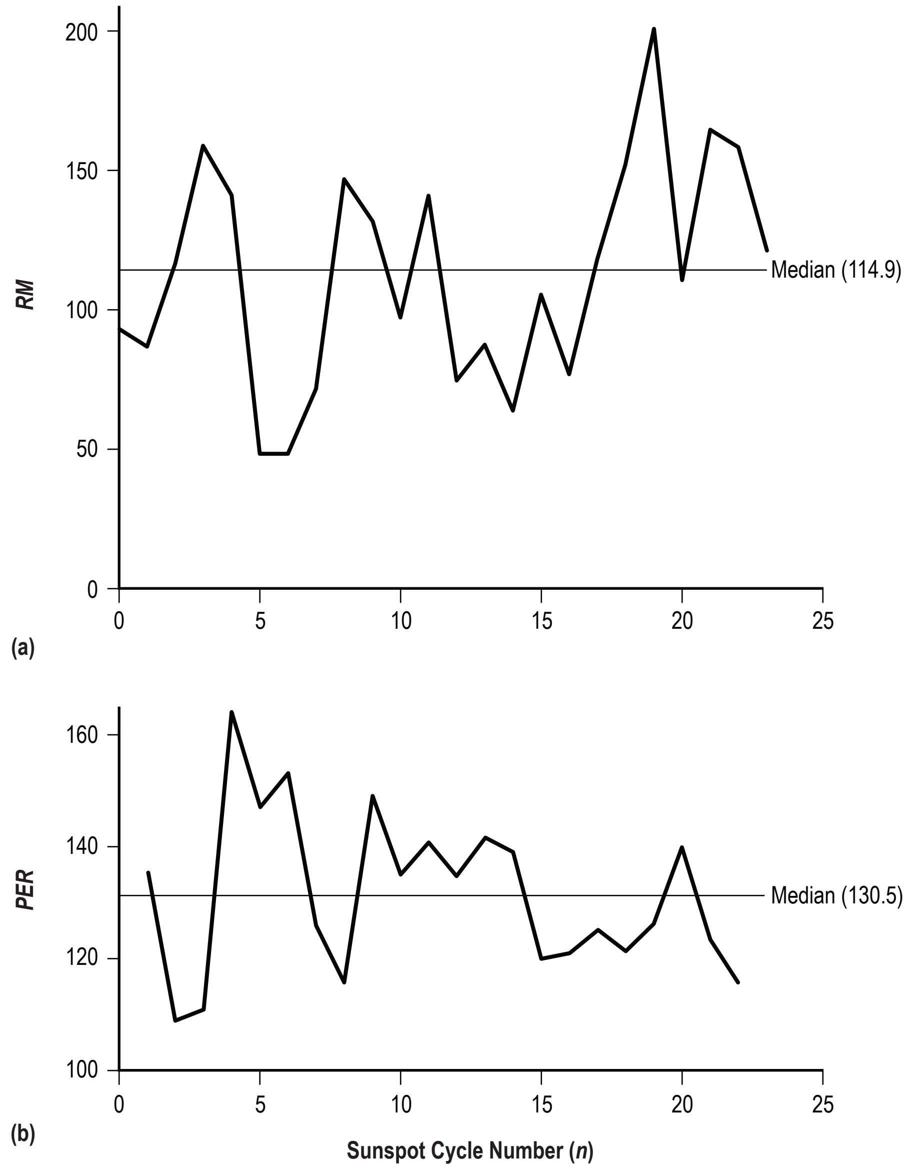

Figure 1. (a) Cyclic variation of the maximum amplitude (RM) for cycles 0–23; (b) cyclic variation of minimum-to-minimum period (PER) for cycles 1–22.

Table 1. Cyclic values of minimum (Rm) and maximum (RM) amplitudes, epochs of occurrence (Em and EM, respectively), ascent (ASC) and descent (DES) durations, and the minimum-to-minimum periods (PER) for cycles 0–23.

Cycle	*Rm*	*Em*	*RM*	*EM*	*ASC*	*DES*	*PER*
00	–	–	92.6	04-1750	–	59	–
01	8.4	03-1755	86.5	06-1761	75	60	135
02	11.2	06-1766	115.8	09-1769	39	69	108
03	7.2	06-1775	158.5	05-1778	35	76	111
04	9.5	09-1784	141.2	02-1788	41	123	164
05	3.2	05-1798	49.2	02-1805	81	66	147
06	0.0	08-1810	48.7	05-1816	69	84	153
07	0.1	05-1823	71.7	11-1829	78	48	126
08	7.3	11-1833	146.9	03-1837	40	76	116
09	10.5	07-1843	131.6	02-1848	55	94	149
10	3.2	12-1855	97.9	02-1860	50	85	135
11	5.2	03-1867	140.5	08-1870	41	100	141
12	2.2	12-1878	74.6	12-1883	60	75	135
13	5.0	03-1890	87.9	01-1894	46	96	142
14	2.6	01-1902	64.2	02-1906	49	90	139
15	1.5	08-1913	105.4	08-1917	48	72	120
16	5.6	08-1923	78.1	04-1928	56	65	121
17	3.4	09-1933	119.2	04-1937	43	82	125
18	7.7	02-1944	151.8	05-1947	39	83	122
19	3.4	04-1954	201.3	03-1958	47	79	126
20	9.6	10-1964	110.6	11-1968	49	91	140
21	12.2	06-1976	164.5	12-1979	42	81	123
22	12.3	09-1986	158.5	07-1989	34	82	116
23	8.0	05-1996	120.8	04-2000	47	–	–

Figure 2(a) displays scatter plots of the Amplitude-Period relationship which compares RM for a sunspot cycle against PER for the preceding sunspot cycle and figure 2(b) displays the Period-Amplitude relationship which compares PER against RM for the same sunspot cycle. In both plots two diagonal lines are drawn. The heavier diagonal (y_{all}) is the inferred regression line using all available sunspot cycles, while the dashed diagonal (y') is the inferred regression line having removed certain cycles considered to be statistical outliers. The vertical and horizontal thin lines are the medians and the number beside each dot identifies the sunspot cycle numbers (n). The dots inside boxes are those cycles considered here to be statistical outliers, being the cycles with the largest deviations from the inferred all-inclusive regression lines. The results of the linear regression analyses appear to the right, giving the inferred regression equations y_{all} and y', the coefficients of correlation (r) and determination (r^2) (which is a measure of the variance explained by the inferred linear regression), the standard error of estimate (se) and the confidence level (cl) of the inferred regression. Also given are the results of Fisher's exact tests[21] for 2×2 contingency tables for both relationships.

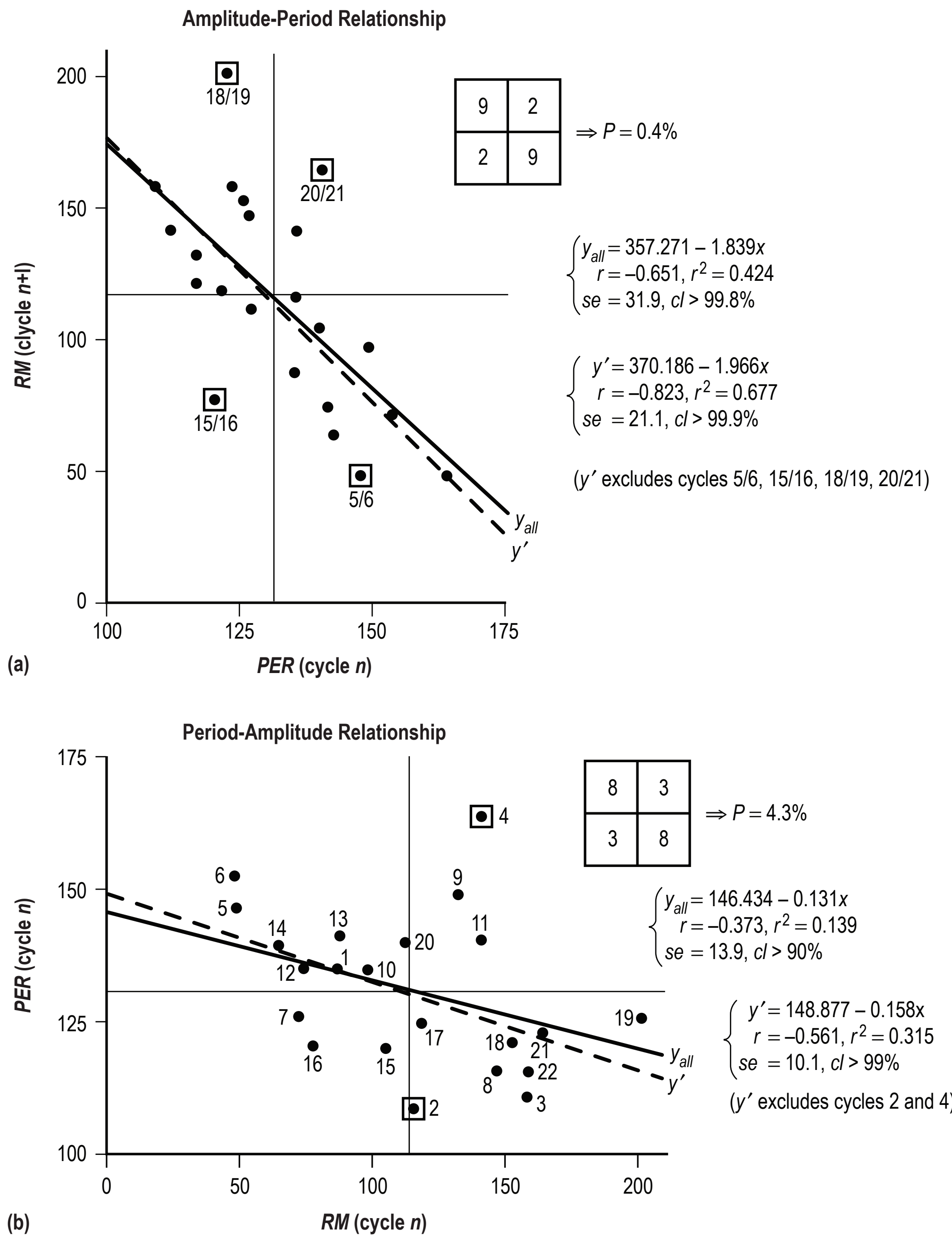

Figure 2. (a) The Amplitude-Period Relationship: Scatter plot of *RM* (cycle $n + 1$) versus *PER* (cycle n); (b) The Period-Amplitude Relationship: Scatter plot of *PER* (cycle n) versus *RM* (cycle n).

Concerning the Period-Amplitude relationship, based on Fisher's exact test for the 2×2 contingency table, one finds that the probability (P) of obtaining the observed result, or one more suggestive of a departure from independence (chance), is 4.3 percent. Hence, given a cycle's *RM*, one determines that the cycle in question is either above or below the median and, dependent upon whether it is above or below the median, can venture an educated guess as to the likelihood that the cycle will have a *PER* shorter or longer than median, since the inferred relationship more often associates higher (lower) than median *RM* with shorter (longer) than median *PER*, true for 16 of 22 sunspot cycles, failing for cycles 4, 7, 9, 11, 15, and 16. Following such an approach, one probably would expect cycle 23's *PER* to be shorter than the median because its known *RM* (=120.8) is greater than the median, unless, of course, cycle 23 is a statistical outlier. Because cycle 23 has already persisted, at least, 142 mo (May 1996 through February 2008), plainly, its *PER* is found to run counter to the inferred behavior for the majority of sunspot cycles.

Rather than employing the 2×2 contingency table, one can examine the inferred linear correlation between the two parameters to estimate the length of cycle 23. Based on the all-inclusive regression (y_{all}), one finds the correlation ($r=-0.373$) between the two parameters is only of marginal statistical significance (cl >90 percent). Cycle 23's *RM*, being equal to 120.8, suggests *PER* equal to about 131±24 mo (the 90-percent prediction interval), based on the all-inclusive regression fit. Because cycle 23 has already persisted, at least 142 mo, one infers only about a 5-percent chance that it will persist longer than another 13 mo; that is, there is a 95-percent chance that cycle 23 will end before March 2009.

Instead, if one ignores cycles 2 and 4 (the extremes in terms of *PER*, 108 and 164 mo, respectively), the resultant inferred regression (y', having $r=-0.561$) is highly statistically significant (cl >99 percent). Now, cycle 23's *RM* (=120.8) suggests that its *PER* will equal about 130±18 mo (the 90-percent prediction interval), inferring that there is only about a 5-percent chance that it will persist longer than another 6 mo; that is, there is a 95-percent chance that cycle 23 will end before September 2008.[22–25]

Concerning the Amplitude-Period relationship, based on Fisher's exact test for a 2×2 contingency table, one finds the probability P of obtaining the observed result, or one more suggestive of a departure from independence (chance), is 0.4 percent. Hence, given a cycle's *PER*, one can venture an educated guess as to the expected size (*RM*) of the following sunspot cycle (that is, cycle 24). Since it is firmly established that cycle 23 is a cycle of longer than median *PER*, unless it (that is, the cycle pair 23/24) is a statistical outlier, one expects cycle 24 to have *RM* below its median value (114.9) because of the inferred preferential association between longer (shorter) *PER* cycles and lower (higher) *RM* following cycles, true for 18 of 22 sunspot cycles, with only cycle pairs 10/11, 15/16, 19/20, and 20/21 failing to conform to the inferred preferential association.

Instead, based on the all-inclusive inferred linear regression (y_{all}, having $r=-0.651$), which is highly statistically significant (cl >99.8 percent), one infers cycle 24's $RM \leq 96.1 \pm 55.0$ (the 90-percent prediction interval) using $PER \geq 142$ mo, suggesting only about a 5-percent chance that cycle 24's *RM* will exceed 151. Using *PER*=148 mo (that is, cycle 23 ending in August 2008) yields *RM* for cycle 24 to be even smaller, about 85±55, or only about a 5-percent chance that its *RM* will exceed 140. Hence, the longer cycle 23 persists, the smaller cycle 24's *RM* is expected to be, a result that seems to run contrary to the prediction of Dikpati, de Toma and Gilman,[26] who suggest that cycle 24 is a much larger than average size cycle and one that starts late (that is, following a longer than average length sunspot cycle; see also Hathaway and Wilson[27]). Using the slightly improved ($r=-0.823$, $cl=99.9$ percent) regression y', which ignores cycle pairs 5/6, 15/16, 18/19, and 20/21 (statistical outliers), one infers cycle 24's *RM* to be about ≤91.0±36.8, or having only about a 5-percent chance of exceeding 128 (see also Wilson and Hathaway[28]).

Figure 3 shows the cyclic variation of the residuals (or deviations (Δ)), computed as observed value minus predicted value (using the y_{all} regressions) divided by the predicted value. The residuals for the Amplitude-Period relationship are shown in figure 3(a) and the residuals for the Period-Amplitude relationship are shown in figure 3(b).

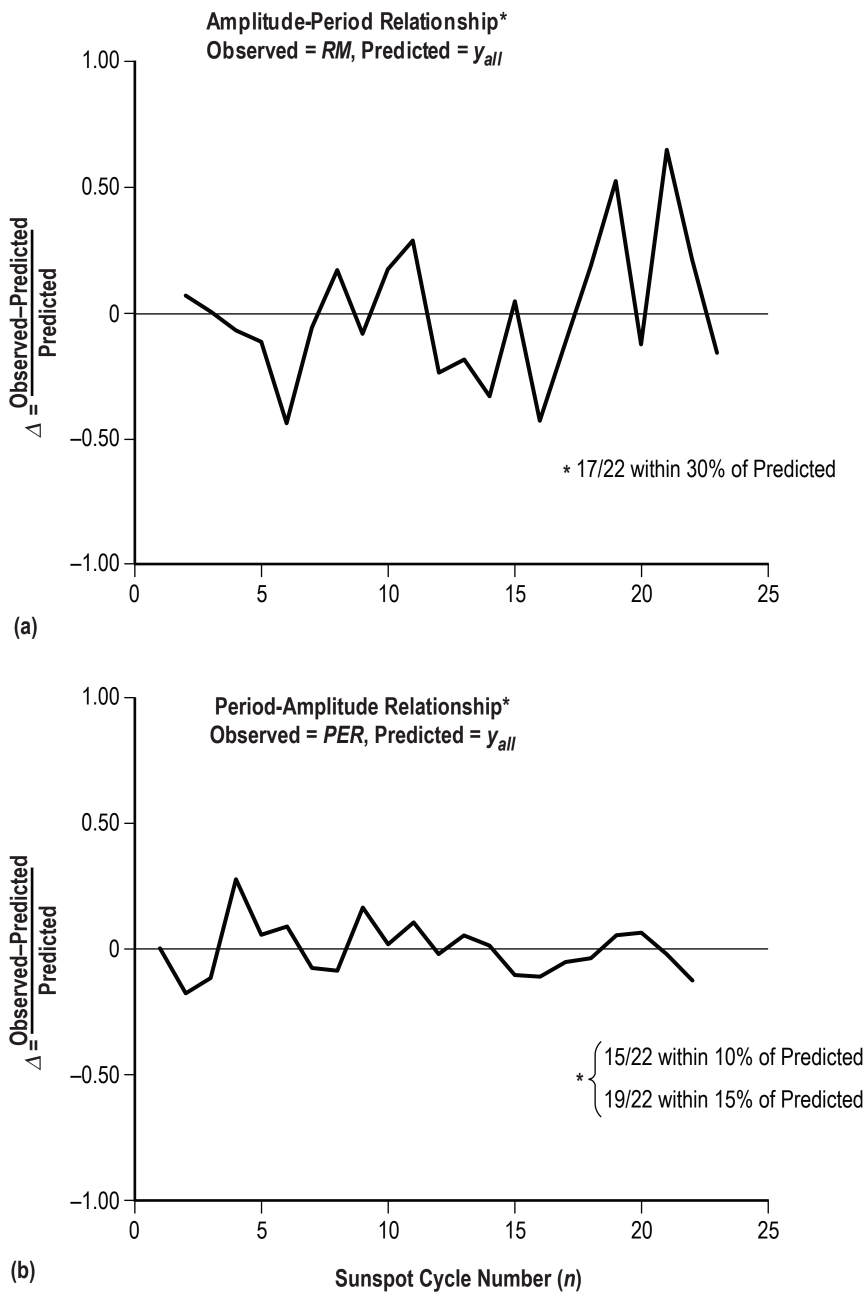

Figure 3. (a) Cyclic variation of the residual Δ = (Observed – Predicted)/Predicted for the Amplitude-Period relationship; (b) Cyclic variation of the residual Δ = (Observed – Predicted)/Predicted for the Period-Amplitude relationship.

In regards to the Period-Amplitude relationship, as stated above, cycle 23's *PER* has been estimated to be about 131 ± 24 mo. Figure 3(b) indicates that of the past 22 sunspot cycles, the estimated *PER* has fallen within 10 percent of the predicted value 68 percent of the time (15 of 22 cycles) and within 15 percent 86 percent of the time (19 of 22 cycles). Hence, one feels fairly confident that cycle 23's *PER* will be about 131 ± 13 mo (68 percent accuracy) or about 131 ± 20 mo (86 percent accuracy). Using the former estimate, cycle 23 would be expected to end before May 2008, while using the latter estimate it would be expected to end before the end of the year.

In regards to the Amplitude-Period relationship, as stated above, cycle 23's *PER* is now known to be at least 142 mo (through February 2008), inferring that cycle 24's *RM* should be expected to be about ≤96.1 ± 55.0. Figure 3(a) indicates that of the past 22 sunspot cycles, the following cycle's estimated *RM* has fallen within 30 percent of the predicted value 77 percent of the time (17 of 22 cycles). Hence, one feels fairly confident that cycle 24's *RM* will be about ≤96.1 ± 28.8 (77 percent accuracy). Thus, cycle 24 will likely (an 88.5-percent chance) be smaller than about 125, unless cycle pair 23/24 truly is a statistical outlier.[29]

3. SUMMARY

In this Technical Publication, it has been shown that both the Amplitude-Period (the preferential association between the maximum amplitude of the following cycle and the period of the preceding sunspot cycle) and Period-Amplitude (the preferential association between the period of an ongoing sunspot cycle and the size of its maximum amplitude) relationships are statistically meaningful. As applied to cycle 23, the current sunspot cycle, while Fisher's exact test ($P = 4.3$ percent) associates larger (smaller) than median *RM* cycles with shorter (longer) than median *PER* cycles, true for 16 of 22 sunspot cycles, the all-inclusive regression y_{all} for the Period-Amplitude relationship ($r = -0.373$, $cl > 90$ percent) suggests cycle 23's $PER = 131 \pm 24$ mo (the 90-percent prediction interval), inferring the end of cycle 23 before March 2009. Ignoring cycles 2 and 4, the extremes in terms of *PER* (108 and 164 mo, respectively), yields the improved inferred regression y' ($r = -0.561$, $cl > 99$ percent) that suggests cycle 23's $PER = 130 \pm 18$ mo, or that cycle 23 very probably will end before September 2008.

As applied to cycle 24, the next sunspot cycle, while Fisher's exact test ($P = 0.4$ percent) associates longer (shorter) than median *PER* cycles with smaller (larger) than median *RM* cycles, true for 18 of 22 sunspot cycles, the all-inclusive regression y_{all} for the Amplitude-Period relationship ($r = -0.651$, $cl > 99.8$ percent) suggests cycle 24's $RM \leq 96.1 \pm 55.0$ (the 90-percent prediction interval, using $PER \geq 142$ mo), inferring that cycle 24's *RM* will measure less than about 151. Ignoring certain statistical outliers yields the improved inferred regression y' ($r = -0.823$, $cl > 99.9$ percent) that suggests cycle 24's $RM \leq 91.0 \pm 36.8$, or that cycle 24 very probably will be smaller than about 128.

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REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)* 01-11-2008	**2. REPORT TYPE** Technical Publication	**3. DATES COVERED** *(From - To)*

4. TITLE AND SUBTITLE On the Period-Amplitude and Amplitude-Period Relationships	**5a. CONTRACT NUMBER** **5b. GRANT NUMBER** **5c. PROGRAM ELEMENT NUMBER**
6. AUTHOR(S) Robert M. Wilson and David H. Hathaway	**5d. PROJECT NUMBER** **5e. TASK NUMBER** **5f. WORK UNIT NUMBER**
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) George C. Marshall Space Flight Center Marshall Space Flight Center, AL 35812	**8. PERFORMING ORGANIZATION REPORT NUMBER** M–1245
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546–0001	**10. SPONSORING/MONITOR'S ACRONYM(S)** NASA **11. SPONSORING/MONITORING REPORT NUMBER** NASA/TP—2008–215580

12. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified-Unlimited
Subject Category 92
Availability: NASA CASI 443–757–5802

13. SUPPLEMENTARY NOTES

Prepared for the Science and Exploration Vehicle Office, Science and Mission Systems Office

14. ABSTRACT

Examined are Period-Amplitude and Amplitude-Period relationships based on the cyclic behavior of the 12-month moving averages of monthly mean sunspot numbers for cycles 0–23, both in terms of Fisher's exact tests for 2×2 contingency tables and linear regression analyses. Concerning the Period-Amplitude relationship (same cycle), because cycle 23's maximum amplitude is known to be 120.8, the inferred regressions (90-percent prediction intervals) suggest that its period will be 131±24 months (using all cycles) or 131±18 months (ignoring cycles 2 and 4, which have the extremes of period, 108 and 164 months, respectively). Because cycle 23 has already persisted for 142 months (May 1996 through February 2008), based on the latter prediction, it should end before September 2008. Concerning the Amplitude-Period relationship (following cycle maximum amplitude versus preceding cycle period), because cycle 23's period is known to be at least 142 months, the inferred regressions (90-percent predition intervals) suggest that cycle 24's maximum amplitude will be about ≤ 96.1±55.0 (using all cycle pairs) or ≤ 91.0±36.7 (ignoring statistical outlier cycle pairs). Hence, cycle 24's maximum amplitude is expected to be less than 151, perhaps even less than 128, unless cycle pair 23/24 proves to be a statistical outlier.

15. SUBJECT TERMS

Sun, sunspot cycle, sunspot cycle prediction, sunspot cycle periods

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