

Interaction of Aircraft Wakes from Laterally Spaced Aircraft

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Large Eddy Simulations are used to examine wake interactions from aircraft on closely spaced parallel paths. Two sets of experiments are conducted, with the first set examining wake interactions out of ground effect (OGE) and the second set for in ground effect (IGE). The initial wake field for each aircraft represents a rolled-up wake vortex pair generated by a B-747. Parametric sets include wake interactions from aircraft pairs with lateral separations of 400, 500, 600, and 750 ft. The simulation of a wake from a single aircraft is used as baseline. The study shows that wake vortices from either a pair or a formation of B-747's that fly with very close lateral spacing, last longer than those from an isolated B-747. For OGE, the inner vortices between the pair of aircraft, ascend, link and quickly dissipate, leaving the outer vortices to decay and descend slowly. For the IGE scenario, the inner vortices ascend and last longer, while the outer vortices decay from ground interaction at a rate similar to that expected from an isolated aircraft. Both OGE and IGE scenarios produce longer-lasting wakes for aircraft with separations less than 600 ft. The results are significant because concepts to increase airport capacity have been proposed that assume either aircraft formations and/or aircraft pairs landing on very closely spaced runways.

Nomenclature

B	= aircraft wingspan
b_0	= initial separation distance between co-rotating vortices, $\pi B/4$
r	= radial distance from vortex center
r_c	= initial vortex core radius
T	= nondimensional time, tV_0 / b_0
t	= time
u, v, w	= velocities in x, y , and z directions, respectively
V_0	= initial wake vortex descent velocity, $\Gamma_0 / (2\pi b_0)$
x, y, z	= longitudinal, lateral, vertical space coordinate
Γ	= vortex circulation
Γ_0	= initial circulation
Δx	= grid size in longitudinal direction
Δy	= grid size in lateral direction
Δz	= grid size in vertical direction
OGE	out of ground effect (wakes located away from the influence of the ground)
IGE	in ground effect (wakes located below an altitude equivalent to B)

I. Introduction

ANTICIPATED growth in the demand for air traffic services has led to concerns on how to more efficiently utilize the air transportation system. Demand already exceeds capacity at five major U.S. airports, and other airports are forecast to join this group in the near future.¹ In order to meet the anticipated demand, new concepts are being developed and evaluated for enabling airport and airspace capacity enhancements. Safety considerations must be fully addressed prior to the implementation of any new concept.

Several concepts with the aim of improving capacity are those that consider more efficient use of closely spaced parallel runways (CSPR). Currently in the U.S., wake vortex restrictions may apply to departures from CSPR that

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have runways separated by less than 2500 *ft* (762 *m*).² In some situations depending upon the mix of aircraft and weather conditions, current rules may require CSRP runways to function as one runway. These restrictions are applied due to the possibility of wake vortices being horizontally transported into the path of a parallel departure. New concepts are being developed that will ease these restrictions and allow a larger number of airport departures.^{3,4} When weather conditions allow airport visual meteorological conditions (VMC), simultaneous paired approaches can be made to parallel runways that are as close as 700 *ft*.⁵ However, if the airport is under instrument meteorological conditions (IMC) and the runways are separated by less than 4300 *ft* (3000 *ft* with special radar), additional restrictions may apply to parallel approaches. Capacity is further limited if the parallel runways are less than 2500 *ft*. In this case, the rules require that the approaches be sequenced as if to one runway.⁶ Consequently, capacity under IMC may drop below 50%.⁷ Concepts to mitigate capacity loss for CSRP approaches attempt to take advantage of available new technologies, and address aircraft sequencing and glide path management.^{3,7,8} A concept by Rossow^{9,10} proposes precision approaches that follow individual compact flight corridors. In this concept, a wake-zone fast-time model¹¹ is used to manage the wake vortex risk,¹² and the corridors are navigated with systems that utilize global positioning satellites (GPS). Rossow's concept also allows for new runways to be paved between existing ones, thus further improving capacity by adding new runways on existing airport property (Fig. 1). The Terminal Area Capacity Enhancement Concept (TACEC) expands on Rossow's ideas by considering new technologies that allow the system to work in IMC while minimizing blunders and collision risks.^{13,14,15,16} The TACEC system will allow up to four commercial aircraft to fly in close formation during final approach and land on very closely spaced parallel runways[†] (VCSRP) while on autopilot.¹⁷ The TACEC system is under development with a goal for implementation in 2022.¹⁸

A concern that can impact the benefit of these concepts is the interaction of wake vortices from closely paired aircraft. In TACEC, capacity benefits are gained by the use of closely spaced parallel approaches, while maintaining current in-trail separation standards between the pairs (or formations) of aircraft. If the interaction between vortices from paired aircraft on parallel approach (or departure) results in shorter-lived wakes, the in-trail spacing's could be reduced and more capacity could be realized from the concept. However, if the wakes from paired aircraft have increased lifetimes, capacity could be lost (or the wake encounter risk increased), due to a need for longer in-trail spacing. Also, application of current fast-time models to VCSRP concepts might be inadequate, since the current models assume wake interactions are independent of other aircraft.

In this study we will use a Large Eddy Simulation (LES) to examine the wake interaction from parallel aircraft with very close lateral spacing, and attempt to determine the lateral separations that could have an impact on wake in-trail separations.

II. LES Model

This study uses the Terminal Area Simulation System (TASS) which is a LES model for simulating meteorological phenomena. The model has an initialization package that allows for the simulation of atmospheric wake vortices and has been used in the examination of wake vortex interactions with the environment and ground surface. Details of the model are described in the appendix.

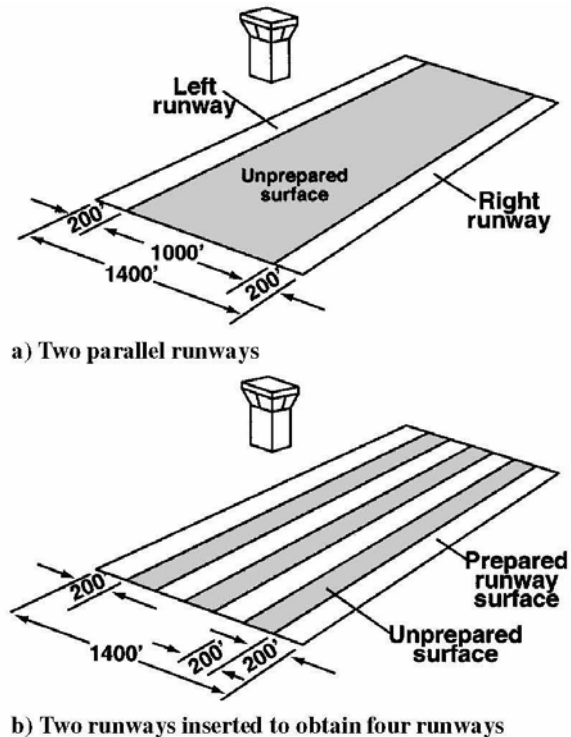


Figure 1. Proposed addition of new runways between existing parallel runways from Rossow [9].

[†] Very closely spaced parallel runways have a lateral separation of 750 *ft* (229 *m*) or less.¹⁷

In previous studies, the TASS model has been used to develop deterministic fast-time models, such as the TASS Driven Algorithm for Wake Prediction (TDAWP).^{11,19} Also it has been used to characterize the onset of Crow²⁰ instability.^{21,22} Simulations with TASS agree with Sarpkaya²³ and Crow and Bate's²⁴ analytical predictions that show the time of vortex linking is dependent upon atmospheric turbulence via the nondimensional eddy dissipation rate (Fig. 2).

In the current study, simulations with TASS are conducted for both in ground effect (IGE) and out of ground effect (OGE). The simulations for OGE assume periodic boundary conditions on all boundaries. Experiments for wake vortices that are IGE use the surface ground option in TASS. All simulations presented in this paper assume a dry atmosphere with neutral stratification, and a calm mean wind.

Initialization of the wake vortices assumes a rolled-up counter-rotating vortex pair for each aircraft. Each wake vortex is specified by its: initial circulation, lateral vortex separation, and core radius; as well as its position within the domain. The initial wake vortex for all cases is representative of a B-747. The initial separation between co-rotating vortices is, $b_0 = 50\text{ m}$, and the initial circulation is, $\Gamma_0 = 565\text{ m}^2\text{ s}^{-1}$. The wake vortices from the aircraft pairs (Fig. 3) are initialized at the same altitude and assume that the aircraft are directly abeam each other. The initial wakes are injected within an environment having an isotropic and homogeneous turbulent wind field.

III. Out of Ground Effect Study

A set of parametric runs are examined assuming different lateral spacing between aircraft. The physical domain is approximately $21\ b_0$ in the longitudinal direction, $15\ b_0$ in the lateral, and $12.75\ b_0$ in the vertical. The domain is resolved by $292 \times 292 \times 245$ grid points (approximately 20 million points), with a grid size of $\Delta x = 3.6\text{ m}$, and $\Delta y = \Delta z = 2.6\text{ m}$. All of the OGE experiments use the same very light turbulence, which is characterized by an eddy dissipation rate of $2 \times 10^{-6}\text{ m}^2\text{ s}^{-3}$. The environment should promote long-lived wake vortices and provide a conservative estimate of any wake interactions.

The OGE parametric study consists of the following cases:

- Baseline, representing a single B-747 generator
- Paired aircraft cases for B-747
 - Two aircraft separated laterally by 400 ft
 - Two aircraft separated laterally by 500 ft
 - Two aircraft separated laterally by 600 ft
 - Two aircraft separated laterally by 750 ft
- Formation aircraft case for B-747
 - Three aircraft each separated laterally by 400 ft.

A. Aircraft pair with 400 ft separation

As shown by schematic in Fig. 3, two sets of wake vortices are generated by the laterally paired aircraft. Results from the simulation show that the vortex on either flank of the paired aircraft is longer lived and slower to descend than the wake from an aircraft on a single path. Specifically, the inner pair of vortices (composed of starboard vortex of the aircraft on the left and the port vortex of the aircraft on the right) interact and induce sinusoidal instabilities. This is

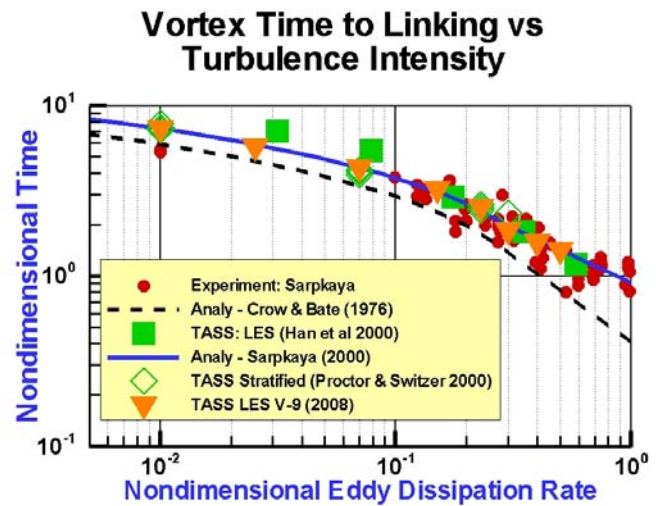


Figure 2. TASS predictions of linking time compared with Sarpkaya and Crow and Bate.

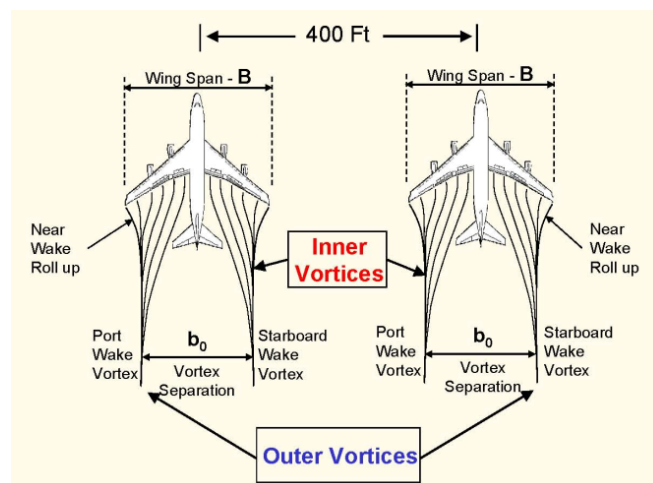


Figure 3. Schematic of wake vortices generated from very closely spaced aircraft.

illustrated in the schematic in Fig. 4. The kinematic contribution of the circulations causes the inner vortices not to descend as fast as the outer vortices. Once the inner vortices become displaced at a slightly higher altitude, the kinematic contribution from each of the vortices causes the lateral separation between the inner vortices to decrease. This accelerates the onset of Crow instability²⁰ causing the inner vortices to link and quickly decay, leaving the outer vortices with a much larger separation than b_0 . Since the outer vortices are widely separated, their descent due to mutual induction and their decay rate are greatly reduced. Also, because of the relatively wide separation between the two surviving outer vortices, the time scales for crow instability are greatly increased and linking may be either suppressed or greatly delayed. In the absence of Crow instability, the outer vortices do not undergo accelerated decay. Their rate of decay is gradual, due only to the direct effects of turbulence diffusion.

The vortex altitude vs time is shown in Fig. 5. For the paired aircraft case, the inner vortices begin to rise after one minute, while the outer vortices descend at a slower rate than the wake from the baseline, single-path aircraft. At two minutes, the wake from the single aircraft has descended slightly greater than 200 m. The outer vortices generated by the aircraft pair take an additional two minutes to descend to the same level.

The longevity of outer vortices produced by the paired aircraft is shown in Fig. 6. While the inner vortices decay within one minute, the outer vortices decay slowly and still persist after 16 min! It takes an additional 8 min to decay to $250 \text{ m}^2 \text{ s}^{-1}$ and 9.5 min to $150 \text{ m}^2 \text{ s}^{-1}$.

B. Sensitivity to Lateral Separation

The sensitivity of the wake vortex altitude to the lateral spacing between the generating aircraft pairs is shown in Fig. 7 and circulation in Fig. 9. Note that at two minutes, wakes from paired aircraft with less than 600 ft separation descend slower than baseline. Similarly, the wake circulation only is noticeably prolonged, for lateral aircraft separations less than 600 ft.

Not shown are simulation results from a formation of three aircraft with each having a lateral separation of 400 ft. Results from this case are similar to that of the paired aircraft. The major difference is that all of the inner vortices link and rapidly dissipate, leaving the two flanking vortices.

Results of the OGE simulations indicate that a lateral spacing for B-747's of less than 600 ft would not be recommended due to slower vortex descent and longer vortex life time. Otherwise, in trail

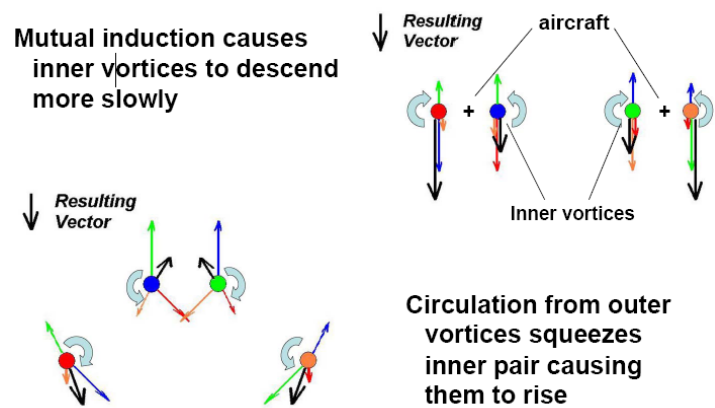


Figure 4. Schematic of wake vortex interaction from a pair of aircraft with very close lateral spacing.

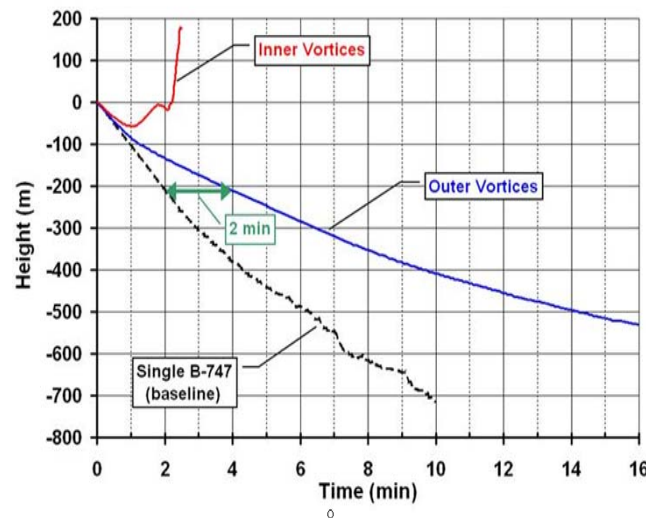


Figure 5. Vortex altitude vs time for B-747 pair with 400 ft lateral separation. Baseline case shown with dash line.

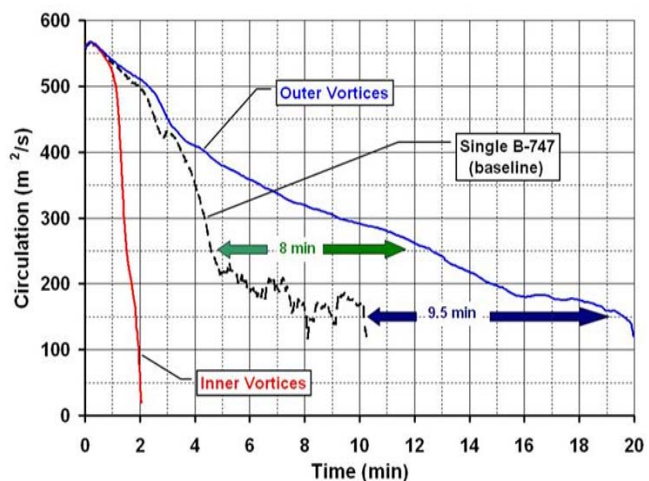


Figure 6. Vortex circulation vs time for B-747 pair with 400 ft lateral separation. Baseline case shown with dash line.

separations between the paired aircraft may need to be increased.

IV. In Ground Effect Study

Similar to the previous set, parametric runs are conducted with different lateral spacing between aircraft. The physical domain size for these simulations is approximately $25 b_0$ in the longitudinal direction, $18.6 b_0$ in the lateral, and $7.75 b_0$ in the vertical. The domain is resolved by $420 \times 372 \times 155$ grid points (approximately 24 million points), with a grid size of $\Delta x = 3.0 m$, and $\Delta y = \Delta z = 2.5 m$. All simulations in this set assume the same light turbulence, characterized by an eddy dissipation rate of $10^{-4} m^2 s^{-3}$.

The IGE parametric set consists of the following cases:

- Baseline, representing a single B-747 generator;
- Paired aircraft cases for B-747: with the two aircraft separated laterally by 400, 500, 600, 750, and 1000 ft.

All of the cases are initialized at an altitude of $50 m$ (b_0) above the ground.

As true with the previous set, the results show long-lived wake vortices for aircraft with very close lateral separation. However, unlike the OGE set, the inner vortex pair tends to be longer lasting. Results are shown in Figs. 9-11.

The wake interaction in ground effect can be described as follows. As each aircraft's co-rotating vortex pair descends and spreads laterally due to the impenetrable ground, the separation between the inner vortices is reduced. This causes the inner vortices to rise and link to form vortex rings. The outer vortices, on the other hand, remain laterally-separated and close to the ground. Since the outer

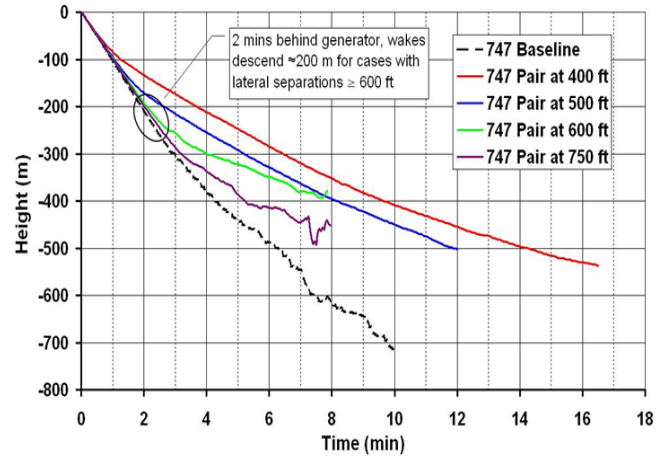


Figure 7. Comparison for different aircraft lateral separations for OGE. Altitude of outer wake vortices.

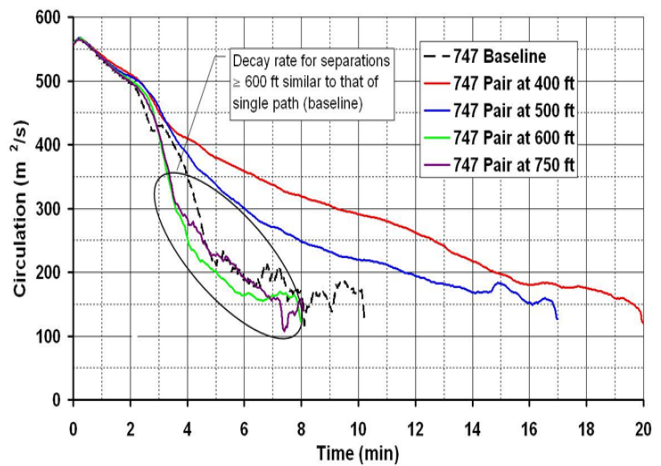


Figure 8. Comparison of circulation from paired aircraft for OGE. Circulation is for outer wake vortices.

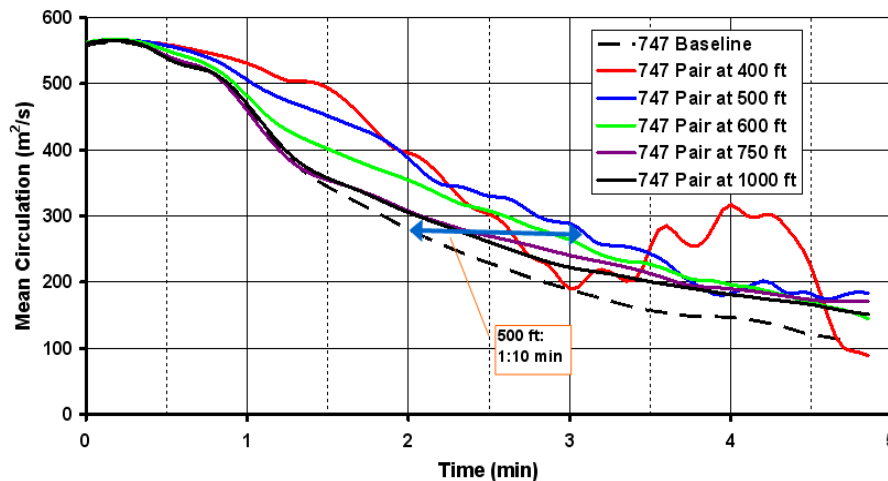


Figure 9. Comparison of circulation from paired aircraft for IGE. Circulation of inner wake vortices.

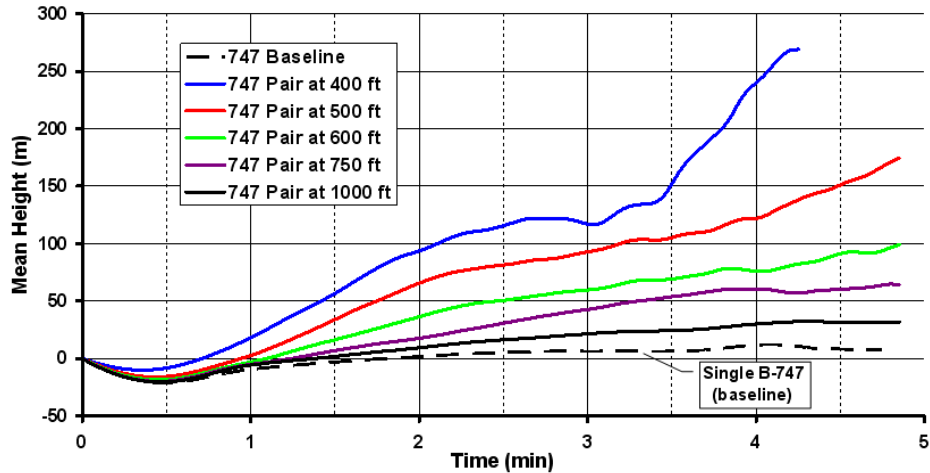


Figure 10. Comparison of altitude of inner vortices from paired aircraft for IGE.

vortices maintain their position close to the ground, frictional interaction enhances their decay, and at a faster rate than the inner vortices which are ascending away from the ground.

Vortex linking did not occur for the aircraft pairs separated by more than 750 *ft*. A comparison of circulation shows that the longevity of the inner vortices is increased with decreasing lateral separation (Fig. 9). This appears to be true even though the inner vortices link sooner for aircraft with smaller lateral separation. The outer vortices decay at similar rate as the base line (not shown), thus the ascending inner vortices may pose the greater hazard. A comparison of the altitudes for the inner vortices (Fig. 10) shows that the upward rebound of the inner vortices is

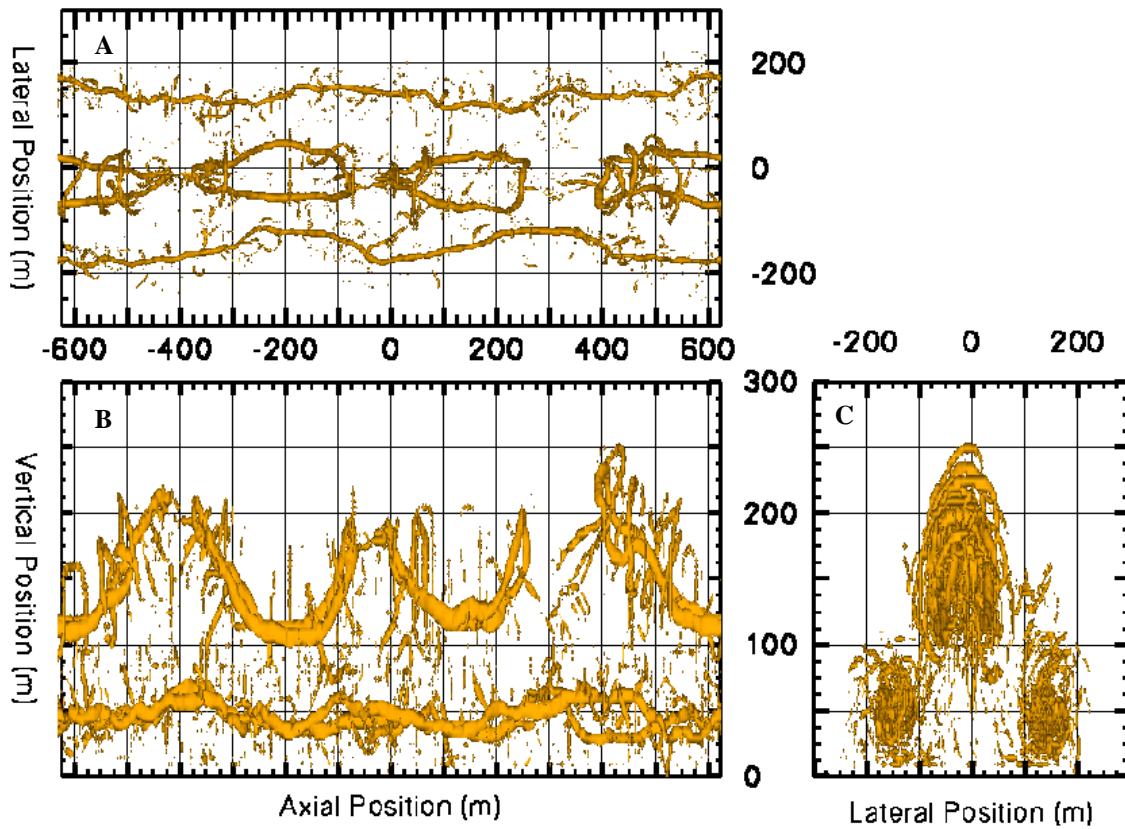


Figure 11. Three dimensional visualization of wake vortices from aircraft pair with 500 *ft* lateral separation for IGE case. Top view A), side view B), and end view C) are shown at 2.5 mins.

amplified with decreasing aircraft lateral separations. A visualization of the wake vortices from a pair of closely spaced aircraft are shown in Fig. 11.

The IGE numerical simulations indicate that paired B-747 aircraft with lateral separations less than or equal to 750 *ft* may have longer lasting wakes than a single B-747. Also ring vortices from interacting wakes generated by closely spaced aircraft are more likely to ascend upward into the flight path of trailing aircraft.

V. Discussion

The two sets of numerical simulations suggest that in-trail separations may need to be revisited for paired B-747 aircraft with lateral separations less than or equal to 750 *ft*. Another implication from this study is that the distance between VCSPR may need to be limited in order to avoid modification of current in trail separation standards.

A critical threshold of about 750 *ft* was found for paired B-747s. Values more appropriate for other sized aircraft are shown in Table 1. These critical lateral separations were determined by scaling with other aircraft: i.e., $\lambda_{\text{critical}} = 750 \text{ ft} (b_o / 50 \text{ m})$.

Table 1. Critical lateral separation scaled to other aircraft

Aircraft Pair	Lateral Separation	Aircraft Pair	Lateral Separation
B-747-400	750 ft	B-757-200	450 ft
A-340-300	725 ft	B-737-800	400 ft
B-777-200	725 ft	A-320	400 ft
B-767-300	575 ft	B-737-200	350 ft
A-300	550 ft		

The data shown in Table 1 implies current in-trail wake separation standards may be insufficient for aircraft with very close lateral spacing, if the lateral separation is less than or equal to 750 *ft* for heavy category aircraft, and less than 450 *ft* for large category aircraft.

Also since current fast-time models ignore interactions of wakes from other aircraft, their range of applicability may be limited when applied to closely spaced approaches or departures.

VI. Conclusion

Concepts for very closely spaced runways are under investigation in hopes of achieving capacity gains within the air transportation system. This study demonstrates that issues regarding wake vortex should be investigated fully before implementation of a new system. Potential in-trail wake issues were uncovered for paired aircraft with close lateral spacing.

Future work is needed to validate this study via special field measurement and laboratory investigations.

Appendix: Description of TASS

The numerical model used in this study is a three-dimensional Large Eddy Simulation (LES) model called the Terminal Area Simulation System²⁵ (TASS). The model has been applied to the simulation of a range of local weather phenomena that can affect aviation safety, such as convectively induced turbulence²⁶, thunderstorms²⁷, microbursts²⁸, and atmospheric boundary layer turbulence²⁹, as well as to the transport and decay of aircraft wake vortices in the atmosphere.^{30,31} The numerical model is: 1) essentially free of numerical diffusion,³² 2) has a meteorological framework, 3) has a realistic surface-stress formulation, and 4) has a subgrid turbulence-closure formulation with rotational damping of turbulence.

A. Model Equations

The TASS model contains a prognostic equation set for momentum, temperature and pressure, and employs a compressible time-split³³ formulation. Omitting coriolis terms (which are an option but not used in these simulations), the TASS equation set in standard tensor notation is as follows:

Momentum

$$\frac{\partial u_i}{\partial t} + \frac{H}{\rho_o} \frac{\partial p}{\partial x_i} = - \frac{\partial u_i u_j}{\partial x_j} + u_i \frac{\partial u_j}{\partial x_j} + g (H - l) \delta_{i3} + \frac{1}{\rho_o} \frac{\partial}{\partial x_j} \rho_o K_M \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]$$

Buoyancy Term

$$H = \left(\frac{\theta}{\theta_o} - \frac{p C_v}{P_o C_p} \right) [1 + 0.61(Q_v - Q_{vo}) - Q_T]$$

Pressure Deviation

$$\frac{\partial p}{\partial t} + \frac{C_p P_o}{C_v \rho_o} \frac{\partial \rho_o u_j}{\partial x_j} = 0$$

Thermodynamic Equation (Potential Temperature)

$$\frac{\partial \theta}{\partial t} = - \frac{1}{\rho_o} \frac{\partial \theta \rho_o u_j}{\partial x_j} + \frac{\theta}{\rho_o} \frac{\partial \rho_o u_j}{\partial x_j} + \frac{1}{\rho_o} \frac{\partial}{\partial x_j} [\rho_o K_H \frac{\partial \theta}{\partial x_j}] + \frac{L \theta}{\tau C_p} S$$

Potential Temperature

$$\theta = \tau \left(\frac{P_{oo}}{P} \right)^{\frac{R_d}{C_p}}$$

In the above equations, u_i is the tensor component of velocity, t is time, p is deviation from atmospheric pressure P , τ is atmospheric temperature, ρ is the air density, C_p and C_v are the specific heats of air at constant pressure and volume, g is the earth's gravitational acceleration, R_d is the gas constant for dry air, P_{oo} is a constant equivalent to 1000 millibars (10^5 pascals) of pressure. Q_v is the mixing ratio for water vapor, Q_T is sum of the mixing ratios for liquid and ice substances, L is the latent heat, and S is a water substance source term. Environmental state variables, e.g., ρ_o , Q_{vo} , P_o , and θ_o , are defined from the initial input sounding and are functions of height only.

Conservation of Scalar Variables (e.g., water vapor, cloud droplet water, etc.):

$$\frac{\partial Q}{\partial t} = - \frac{1}{\rho_o} \frac{\partial Q \rho_o u_j}{\partial x_j} + \frac{Q}{\rho_o} \frac{\partial \rho_o u_j}{\partial x_j} + \frac{1}{\rho_o} \frac{\partial}{\partial x_j} [\rho_o K_H \frac{\partial Q}{\partial x_j}] + S$$

Precipitation variables, such as water mixing ratios for rain, snow, and graupel, have an additional term to account for fall out.

A modified Smagorinsky first-order closure is used for the subgrid eddy viscosity as:

$$K_M = l_s^2 \sqrt{\frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left(\frac{\partial u_k}{\partial x_k} \right)^2} \cdot \sqrt{1 - \alpha_1 Ri_s - \alpha_2 Ri_r}$$

The subgrid eddy viscosity for momentum, K_M , is modified by the Richardson numbers for both stratification, Ri_s , and for flow rotation, Ri_r .

The subgrid turbulence length scale, l_s , is determined from the grid volume and is matched to the appropriate length scale where the flow is under-resolved near the ground. That is:

$$l_s = \begin{cases} \alpha \Delta & z \geq \alpha \Delta / k \\ \frac{\alpha \Delta [1 + (\alpha \Delta / k z)^{m-1}]}{1 + (\alpha \Delta / k z)^m} & \alpha \Delta / k > z > \Delta z / 2 \\ k z & z \leq \Delta z / 2 \end{cases}$$

where k is von Karman's constant, and where m and α are invariant constants with values defined as $m = 3$ and $\alpha = 0.16$. The filter width is based on the minimal resolvable scale:

$$\Delta = [2\Delta x 2\Delta y 2\Delta z]^{1/3}$$

For simulations with fully periodic domains (i.e. no ground plane is assumed), ground matching is not applied.

B. Boundary Conditions

For out of ground effect (OGE) simulations all boundary conditions are assumed to be periodic. For in ground effect (IGE) simulations, the top boundary is impermeable and free slip, and a sponge and filter are applied to the top three levels to damp reflective waves from top boundary. The bottom boundary for IGE simulations represents the ground surface. It is assumed to be impermeable with nonslip velocity specifications. Monin-Obukhov similarity relations are used at the surface with a surface roughness length, $z_0 = 0.1 \text{ m}$. With this formulation ground stresses are determined locally from the wind speed, and thermal stratification. Details of the surface formulation are in Proctor and Han.³⁴

C. Turbulence Initialization

Prior to vortex initialization, an initial field of resolved-scale turbulence is allowed to develop under an artificial external forcing at low wavenumbers.³⁵ For OGE simulations, the approach is similar to studies with TASS, where wake vortex decay is examined within a Kolmogorov³⁶ spectrum of homogeneous turbulence (Han *et al.*^{21,30}). In domains used for IGE wake simulations the method is modified due to the inclusion of the ground. In this case since periodic boundary conditions are assumed only at the horizontal boundaries, the turbulence forcing is applied only to horizontal velocity over each horizontal plane. In this approach, the influence of the horizontal two-dimensional forcing spreads quickly to the vertical direction as well as to the vertical velocity through mass continuity. Both approaches generate resolved-scale turbulence fields that are nearly isotropic and have Kolmogorov subranges extending downscale from the largest resolved scales.

The turbulence field is considered well-developed once the turbulence statistics become nearly steady (about 18 eddy turn-over times). Once a turbulence field is grown, its velocity field can be rescaled to represent different turbulence intensities and used as an initial fields for parametric wake vortex simulations.³⁷ The turbulence kinetic energy dissipation rate is estimated by fitting Kolmogorov's theoretical spectrum in the inertial subrange to the simulated spectra.

D. Numerical Approximations

Time-derivative approximations for momentum and pressure are time-split explicit³³ for computational efficiency. The prognostic equations are approximated using 4th-order energy-conserving central space differencing and 2nd order time differencing.³² Only light numerical filtering is applied using a 6th-order filter. Potential temperature and water substances equations are approximated with third-order accurate time and space differences with upstream-biased quadratic interpolation.^{38,39} The horizontal derivatives in TASS are approximated on an Arakawa-C grid.⁴⁰ The numerical formulation for TASS is stable for long-term integrations and is essentially free from numerical diffusion.³²

E. Wake Initialization

The initial vortex system is representative of the post roll-up, wake-vortex velocity field and consists of a pair of counter-rotating vortices that have no initial variation in the axial direction. The velocity distribution for each vortex is based on field observations of several wake vortices measured early in their evolution.⁴¹ The vortex tangential velocity, V , is a function of radius, r , from the center of the vortex as

$$V(r) = (\Gamma_{\infty}/2\pi r) \{1 - \exp[-10(r/B)^{0.75}]\}$$

where Γ_{∞} is the vortex initial circulation. The dependency on wingspan B , rather than the vortex core radius r_c , is desirable since B is easily determined from aircraft type while r_c is difficult to accurately measure. The above equation is only applied for $r > 1.4 r_c$. For $r \leq 1.4 r_c$, the model is matched with the Lamb model,⁴² i.e.

$$V(r) = (\Gamma_{\infty}/2\pi r) 1.0939 \{1 - \exp[-10(1.4 r_c/B)^{0.75}]\} \{1 - \exp[-1.2527(r/r_c)^2]\}$$

Solutions for image vortices positioned outside of the domain are applied to guarantee consistency with boundary conditions.

Acknowledgments

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