Truncation Depth Rule-of-Thumb for Convolutional Codes

The new rule is more accurate and tight at high signal-to-noise ratios.

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In this innovation, it is shown that a commonly used rule of thumb (that the truncation depth of a convolutional code should be five times the memory length, m, of the code) is accurate only for rate 1/2 codes. In fact, the truncation depth should be 2.5 $m/(1 - r)$, where r is the code rate. The accuracy of this new rule is demonstrated by tabulating the distance properties of a large set of known codes. This new rule was derived by bounding the losses due to truncation as a function of the code rate.

The bound derives from a result on random trellis codes in G.D. Forney, Jr.'s "Convolutional codes II: Maximum likelihood decoding," *Information and Control*, vol. 25:222-266 (1974). An (M, ν) trellis is a trellis corresponding to a shift register of length ν where each register contains a M-vector and the input is an M-ary sequence (the corresponding trellis contains M^ν states). An (M, ν, n) trellis code augments an (M, ν) trellis by assigning n channel symbols to each edge. The rate of the

code is $r = \log_2 (M)/n$ bits/symbol. A random trellis code is an (M, ν, n) trellis in which each channel symbol on each edge is chosen randomly and independently according to some distribution p. When $M = q^k$ the (M, ν, n) trellis corresponds to a rate $\log_2(q) k/n$ nonsystematic convolutional code over $GF(q)$ with k equal constraint lengths $\nu_i = \nu$, $1 < i < k$. The memory of this code is $m = \max_i \nu_i = \nu$. It is presumed that the code is decoded via the Viterbi algorithm with decisions on edges of the trellis made after a delay of T trellis stages. A truncation error occurs when an incorrect edge is chosen that would not have been chosen with an infinite truncation depth.

In the case of punctured codes, the truncation depth on the mother code trellis should be increased as the rate increases. Punctured code can be created by forming (q^k, ν, n) code by puncturing a (q^k, ν_1, n_1) mother code, where k_1 divides k and $\nu = \nu_1 k_1/k$. This resulting code is the daughter code. The two

codes are represented with the same number of states, with k/k_1 stages of the mother code corresponding to 1 stage of the daughter code. The required truncation depth on the daughter code trellis corresponds to a truncation depth on the mother code trellis of

$$1 > \nu_1/(1 - r)$$

i.e., the truncation depth on the mother code goes as the memory of the mother code scaled by one minus the rate of the punctured code.

With regard to particular codes, a good indicator of the required truncation depth is the path length at which all paths that diverge from a particular path have accumulated the minimum distance of the code. It is shown that the new rule of thumb provides an accurate prediction of this depth for codes of varying rates.

This work was done by Bruce Moision of Caltech for NASA's Jet Propulsion Laboratory. Further information is contained in a TSP (see page 1). NPO-45009

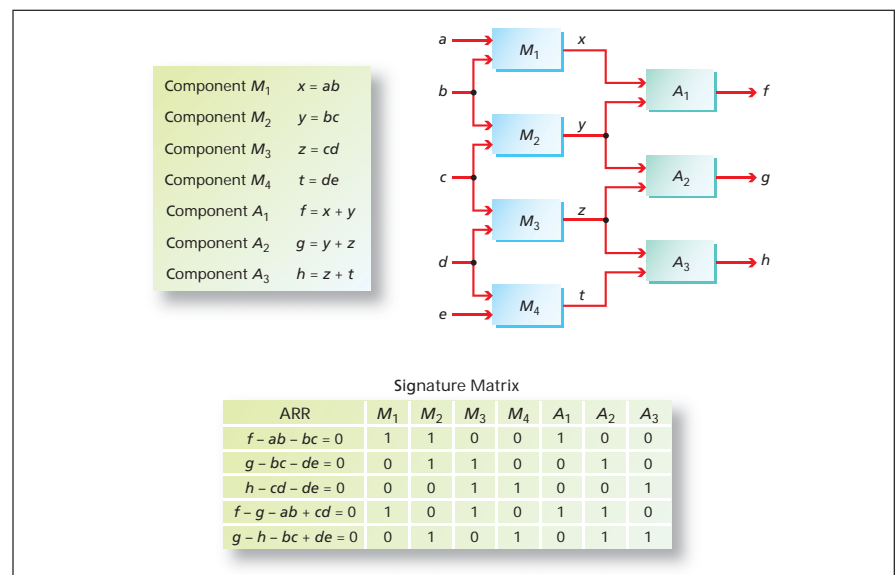# Efficient Method for Optimizing Placement of Sensors

This systematic method supplants *ad hoc* placement and exhaustive-search optimization methods.

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A computationally efficient method has been developed to enable optimization of the placement of sensors for the purpose of diagnosis of a complex engineering system (e.g., an aircraft or spacecraft). The method can be used both in (1) designing a sensor system in which the number and positions of sensors are initially not known and must be determined and (2) adding sensors to a pre-existing system to increase the diagnostic capability.

The optimal-sensor-placement problem can be summarized as involving the following concepts, issues, and subproblems:

- *Degree of Diagnosability* — This is a concept for characterizing the set of faults that can be discriminated by use of a given set of sensors.
- *Minimal Sensor Set* — The idea is one of finding a minimal set of sensors that guarantees a specific degree of diagnosability.
- *Minimal-Cost Sensors* — In a case in which different sensors are assigned with dif-

Component M_1	$x = ab$
Component M_2	$y = bc$
Component M_3	$z = cd$
Component M_4	$t = de$
Component A_1	$f = x + y$
Component A_2	$g = y + z$
Component A_3	$h = z + t$

Signature Matrix

ARR	M_1	M_2	M_3	M_4	A_1	A_2	A_3
$f - ab - bc = 0$	1	1	0	0	1	0	0
$g - bc - de = 0$	0	1	1	0	0	1	0
$h - cd - de = 0$	0	0	1	1	0	0	1
$f - g - ab + cd = 0$	1	0	1	0	1	1	0
$g - h - bc + de = 0$	0	1	0	1	0	1	1

A **System of Four Multiplier and Three Adder Gates** serves as an example for illustrating the concept of ARRs and a signature matrix. In this example, there are three sensors that measure the variables f, g, and h. Each element of the matrix is 1 or 0 if the ARR listed in the row containing that element is or is not, respectively, affected by a fault in the gate listed in the column containing that element.