A New Maneuver for Escape Trajectories

Robert B. Adams, Ph.D.

Advanced Space Propulsion Workshop
Pasadena, CA

October 28-30, 2008

- **Genesis**

- **Hypothesis**

- **Derivation**
  - Discussion
  - Physical Explanation
  - Analogy
  - Optimization

- **Implications**
  - L1 Maneuver
  - Semi-Tangential Transfer
  - Optimization of thrust direction

- **Summary**

- This maneuver came from my work on my doctoral dissertation
- The theme of the dissertation was to find an analytical approximation for medium thrust trajectories
- To that end I did considerable research into high and low thrust trajectories so as to give a reference for whatever medium thrust solution I generated
- In most low thrust derivations the idea that escape velocity is best achieved by accelerating along the velocity vector
  - Reason given is that change in specific orbital energy is a function of velocity and acceleration

$$\frac{dE}{dt} = \vec{V} \cdot \vec{a}$$

  - However Levin, 1952 suggested that while this is a locally optimal solution it might not be a globally optimal one.
  - Turning acceleration inward would drop periapse giving a higher velocity later in the trajectory. Acceleration at that point would be dotted against a higher magnitude V giving a greater rate of change of mechanical energy

- I gave that a lot of thought and formed a hypothesis. Could I decelerate from my initial orbit and then accelerate at periapse to gain a greater specific orbital energy? My hypothesis was that I would not see a gain. I was wrong.
- The diagram for the maneuver is shown below
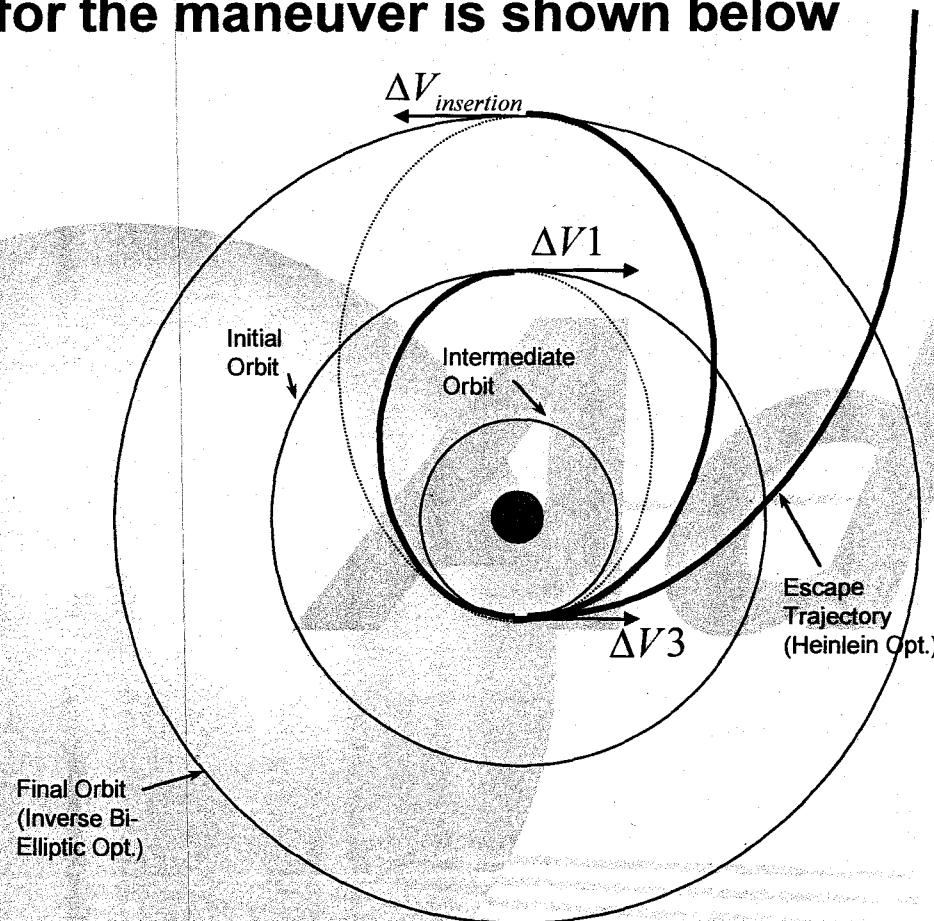
- **Consider the derivation for the specific orbital energy for the maneuver. Position 3 is the status of the vehicle after the burn at periapse**

$$\xi_3 = \frac{(V_2 + \Delta V_3)^2}{2} - \frac{\mu}{r_3}$$

- **Position 2 is the status of the vehicle before the burn. Orbital radius at position 2 and 3 are equal as the burn is considered impulsive**

$$r_2 = 2a_1 - r_1$$

$$V_2 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_2}}$$

- **Position 1 is the status of the vehicle after the burn from the initial orbit. Radius is same as at position 0, initial position of the vehicle**

$$a_1 = -\frac{\mu}{2\xi_1}$$

$$\xi_1 = \frac{(V_0 + \Delta V_1)^2}{2} - \frac{\mu}{r_1}$$

- **Combine all of the above to get**

$$\xi_3 = \frac{\left(\sqrt{\dfrac{-2\mu}{\dfrac{\mu}{\sqrt{\dfrac{V_1^2}{2}-\dfrac{\mu}{r_1}}}+r_1}+2\sqrt{\dfrac{V_1^2}{2}-\dfrac{\mu}{r_1}}+\Delta V_3}\right)^2}{2} + \frac{\mu}{\dfrac{\mu}{\sqrt{\dfrac{V_1^2}{2}-\dfrac{\mu}{r_1}}}+r_1}$$

- **Some simple algebra gives**

$$\xi_3 = \frac{\left(\sqrt{\dfrac{4\mu^2+r_1^2 V_1^4 - 4\mu r_1 V_1^2}{r_1^2 V_1^2}}+\Delta V_3\right)^2}{2} + \frac{\mu r_1 V_1 - 2\mu^2}{r_1^2 V_1^2}$$

6

- **Noting that the initial orbital velocity is**

$$V_0 = \sqrt{\frac{\mu}{r_0}}$$

- **Then including it and the first ΔV maneuver yields**

$$\xi_3 = \frac{\left(\sqrt{\dfrac{4\mu^2 + r_0^2\left(\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1\right)^4 - 4\mu r_0\left(\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1\right)^2}{r_0^2\left(\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1\right)^2}} + \Delta V_3 \right)^2}{2} + \frac{\mu r_0\left(\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1\right) - 2\mu^2}{r_0^2\left(\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1\right)^2}$$

- **Using the convenience variable**

$$V_1 = \sqrt{\frac{\mu}{r_0}} - \Delta V_1$$

- **After considerable algebraic reduction the equation for specific mechanical energy becomes**

$$\xi_3 = \left(V_1^2 - 2\frac{\mu}{r_0}\right) \cdot \left(\frac{1}{2} - \frac{\Delta V_3}{V_1}\right) + \frac{\Delta V_3^2}{2}$$

- **For a direct burn out of the initial orbit the final specific mechanical energy becomes**

$$\xi_3 = \frac{\left(\sqrt{\frac{\mu}{r_0}} + \Delta V_1 + \Delta V_3\right)^2}{2} - \frac{\mu}{r_0}$$

- **Setting the two equal gives**

$$\frac{\left(\sqrt{\frac{\mu}{r_0}} + \Delta V_1 + \Delta V_3\right)^2}{2} - \frac{\mu}{r_0} = \left(V_1^2 - 2\frac{\mu}{r_0}\right) \cdot \left(\frac{1}{2} - \frac{\Delta V_3}{V_1}\right) + \frac{\Delta V_3^2}{2}$$

- **Solving for the squared term and squaring yields**

$$V_1^2 - 2V_1\Delta V_3 + \frac{4\mu\Delta V_3}{V_1 r_0} = \frac{\mu}{r_0} + 2\Delta V_1\sqrt{\frac{\mu}{r_0}} + \Delta V_1^2 + 2\Delta V_1\Delta V_3$$

- **After substituting for V_1 and going through considerable algebra the equation becomes**

$$\Delta V_1 + \Delta V_3 = \sqrt{\frac{\mu}{r_0}}$$

- **Thus this new maneuver outperforms a direct burn when the overall ΔV budget exceeds the initial orbital velocity**

- **Literature review**
  - I could not find any reference to this concept in the scientific literature.
  - The closest reference I could find was concerning the Oberth Effect, the effect that in a parabolic or hyperbolic trajectory the maximum gain in specific mechanical energy is gained when the burn is conducted at periapse
  - The only reference I could find was in a 1952 Heinlein juvenile novel called "The Rolling Stones" (note this predates the band of the same name)
  - In the story Heinlein dedicates a paragraph on how the family Stone leaves their home on the moon to travel to Mars. In so they decelerate to a close flyby of earth and then perform a burn at perigee to achieve their TMI burn
  - Since Heinlein came up with it first, I call this the Heinlein maneuver

- **Confirmation of proof**
  - I haven't gone through the derivation at this level of detail with anyone until now. This proof comprises a portion of my dissertation and is under review by my committee.
  - I used several methods to confirm I was getting good results
    - Built spreadsheets to calculate direct and Heinlein specific orbital energies both from first principles and derived equations above
    - Made sure that trivial cases gave correct answers
      - $\Leftarrow \Delta V_1 = 0$ for Heinlein maneuver collapses to solution for direct
      - $\Leftarrow \Delta V_3 = 0$ for Heinlein collapses to first half of Hohmann transfer

- I struggled a great deal with this concept. It seems like you're getting something for nothing.
- After a week of chewing on this I finally came up with a physical explanation
- The direct maneuver leaves the propellant mass orbiting the central body at some orbit inside the initial orbit with apoapse at the same radius of the initial orbit
- The Heinlein maneuver leaves the propellant mass of the first maneuver outside the initial orbit with periapse at the same radius
- The second mass is left in a considerably lower orbit
- At the breakeven point given above, the additional specific mechanical energy given to the spacecraft by the Heinlein maneuver is balanced by the additional specific mechanical energy extracted from the propellant at position 3 minus the additional specific mechanical energy lost to the propellant at position 1

- **Consider a car on the interstate with a rocket in the trunk.**
  - The rocket gives a set constant acceleration
  - Starting at zero turn the rocket on until it reaches 10 mph
  - The change in kinetic energy for the car (assuming negligible propellant mass used) is

$$\Delta KE = \frac{V_f^2}{2} - \frac{V_i^2}{2} = \frac{10^2}{2} - \frac{0^2}{2} = 50mph^2$$

  - Consider the same car moving at 50 mph that accelerates to 60 mph

$$\Delta KE = \frac{V_f^2}{2} - \frac{V_i^2}{2} = \frac{60^2}{2} - \frac{50^2}{2} = 55mph^2$$

  - Where did the extra energy come from? The propellant used had a greater total energy. It had the same latent energy as the car at 0 mph but also had the initial kinetic energy that comes from moving at 50 mph
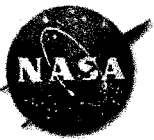
- **If the Heinlein maneuver outperforms the direct option in particular situations then where does it give the maximum benefit?**
  - The total DV is given as
    $$\Delta V_1 + \Delta V_3 = \Delta V_t$$
  - Holding the total ΔV constant what is the optimal split between the first and third maneuvers? Note that the derivative final specific mechanical energy can be found from the chain rule

    $$\frac{d\xi}{d\Delta V_1} = \frac{\partial \xi}{\partial \Delta V_1}\frac{\partial \Delta V_1}{\partial \Delta V_1} + \frac{\partial \xi}{\partial \Delta V_3}\frac{\partial \Delta V_3}{\partial \Delta V_1}$$
  - Taking the appropriate derivatives yields

    $$\frac{\partial \xi_3}{\partial \Delta V_1} = \left(2V_1(-r_0)\right)\left(\frac{1}{2r_0} - \frac{\Delta V_3}{V_1 r_0}\right) + \left(r_0 V_1^2 - 2\mu\right)\frac{\Delta V_3}{V_1^2 r_0^{3/2}}\left(-\sqrt{r_0}\right)$$

    $$\frac{\partial \xi_3}{\partial \Delta V_3} = \frac{-\left(V^2 - 2\mu\right)}{a'\sqrt{r_0}} + \Delta V_3$$

    $$\frac{\partial \Delta V_1}{\partial \Delta V_1} = 1 \qquad \frac{\partial \Delta V_3}{\partial \Delta V_1} = -1$$

13

- **Substituting yields**

$$\frac{d\xi_3}{d\Delta V_1} = \left(-r_0 V_1^{3} + \Delta V_3 r_0 V_1^{2} - 2\mu\Delta V_3\right)\frac{1}{r_0 V_1^{2}}(1) + \left(\frac{-\left(r_0 V_1^{2} - 2\mu\right)}{r_0 V_1} + \Delta V_3\right)(-1)$$

- **The equation reduces, after considerable algebra to**

$$\frac{d\xi_3}{d\Delta V_1} = \frac{2\mu}{r_0 V_1^{2}}\Delta V_3 - \frac{2\mu}{r_0 V_1} = 0$$

- **Recalling that**

$$V_1 = \sqrt{\frac{\mu}{r_0}} - \Delta V_1$$

- **Substituting above and reducing yields**

$$\Delta V_1 + \Delta V_3 = \sqrt{\frac{\mu}{r_0}}$$

- Since this is the same as the breakeven point it didn't make a lot of sense to me. However I did a second derivative to figure out whether it was a maxima or minima

- Note that

$$\frac{d\xi}{d\Delta V_1} = f\left(\Delta V_1, \Delta V_3\right)$$

- So the chain rule gives me

$$\frac{d^2\xi}{d\Delta V_1^2} = \frac{df}{d\Delta V_1} = \frac{\partial f}{\partial \Delta V_1}\frac{\partial \Delta V_1}{\partial \Delta V_1} + \frac{\partial f}{\partial \Delta V_3}\frac{\partial \Delta V_3}{\partial \Delta V_1}$$

- Taking the appropriate derivatives again and substituting yields

$$\frac{df}{d\Delta V_1} = \left(\frac{4\mu}{V_1^3 r_0}\Delta V_3 - \frac{2\mu}{V_1^2 r_0}\right)(1) + \left(\frac{2\mu}{V_1^2 r_0}\right)(-1)$$

- Reducing gives

$$\frac{df}{d\Delta V_1} = \frac{4\mu}{V_1^3 r_0}\Delta V_3 - \frac{4\mu}{V_1^2 r_0}$$

- **Substituting back for V_1 and reducing gives**

$$\frac{df}{d\Delta V_1} = \frac{d^2\xi}{d\Delta V_1^{2}} = \frac{4}{V_1^{2}}\frac{\mu}{r_0}\left(\frac{\Delta V_1 + \Delta V_3 - \sqrt{\dfrac{\mu}{r_0}}}{\sqrt{\dfrac{\mu}{r_0}} - \Delta V_1}\right)$$

- **The numerator above is the key. At the breakeven point the numerator is zero. If the total ΔV exceeds the initial orbital velocity then the numerator is positive suggesting concave up and the extrema point is a minima. Conversely if the ΔV is less than the initial orbital velocity then the extrema point is a maxima. Thus the extrema point is actually a saddle point**

- Given the incomplete result from derivative analysis I turned to a graphical analysis. Below is a graph of specific mechanical energy vs. ΔV_1. The lines represent increasing total ΔV. The graph is for a spacecraft in initial orbit around the sun at one AU

- The asymptotic line is at 29.784 km/sec, the initial orbital velocity
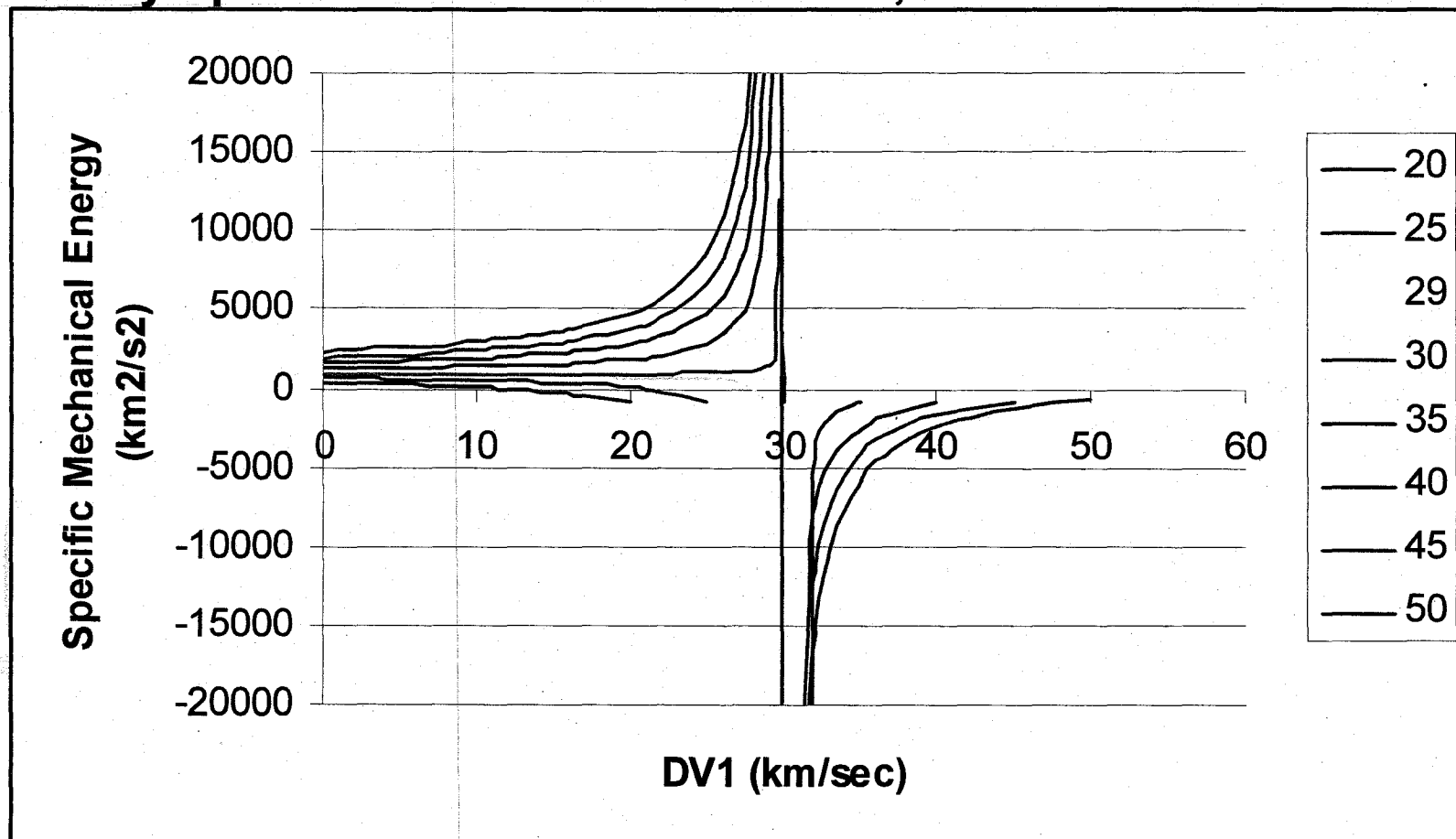
- **I also ratio'ed Heinlein V_∞ vs. Direct V_∞ for the same conditions**
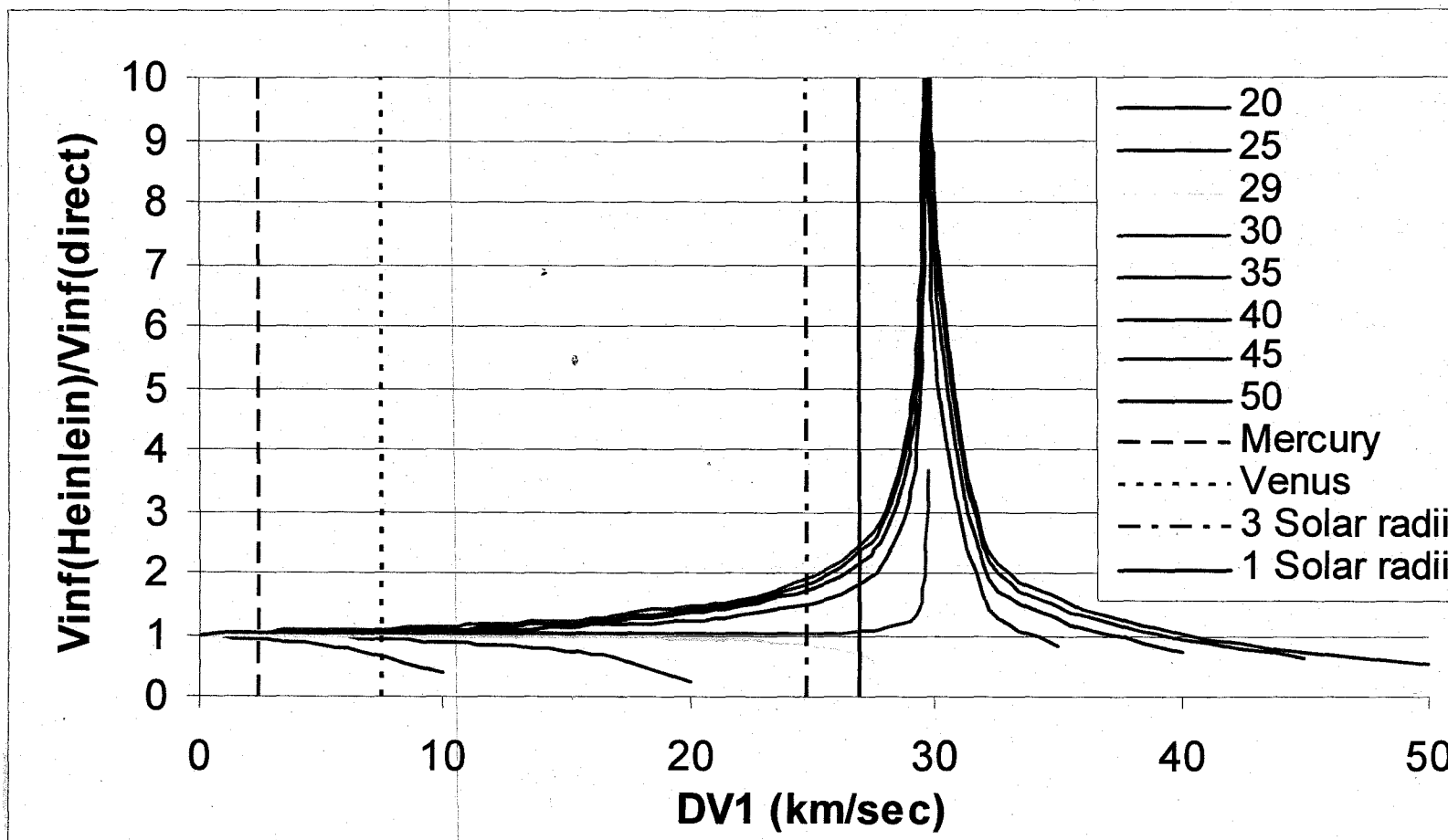
- This maneuver appears to have a top limit of doubling the V_∞ from starting at an Earth like orbit. Starting from a higher orbit (like Mars) would gain a larger proportional ratio

- The ΔV requirement to meet the criteria for the Heinlein maneuver does not necessarily need to come from on board propellant. For instance a solar escape trajectory could be achieved by first doing a gravity assist around Jupiter to lower perihelion and then conduct a ΔV_3 burn to escape. I have not yet analyzed this option

- If there is water on the moon as hoped then there are strong implications for any mission requiring Earth escape. Lunar propellant has the advantage of having a high specific mechanical energy relative to the earth. The scenario and analysis follows.

- Semi-tangential missions for orbit raising can also benefit from this maneuver. More work needs to be done but the analysis to date follows.

- I've also investigated whether ΔV_1 burns at directions other than 180 deg are effective. Unfortunately they are not. Analysis follows

- **It may be that the Pluto-Kuiper Express mission could have benefited from this maneuver. I have not yet analyzed this option**

- **The proposed 1000 AU Interstellar Precursor mission benefits strongly from this maneuver.**
  - Main criteria for missions like this is they can be completed in one persons career lifetime, generally 50 years
  - Direct maneuver, requires $\Delta V = 74$ km/sec
  - Heinlein maneuver (with 3 solar radii flyby) $\Delta V = 40$ km/sec

- **The advantage of the Heinlein Maneuver is strongly dependant on the allowable periapse radius.**
  - Thus for eventual use where Sol is the central body may require advanced thermal protection systems and a detailed understanding of the radiation environment near the sun
  - For Earth centered maneuvers the ability to measure and predict the atmosphere at Earth's highest altitudes may prove critical to maximize the effect of the maneuver. More research into the effects of high speed impact of atomic oxygen on spacecraft systems may be warranted too.

- **Compare a mission that conducts a direct burn to escape to one that parks at L1, docks with fuel raised from the moon, and conducts a Heinlein maneuver to escape**
  - Direct option
    - Two stage vehicle that delivers a payload to a given V_∞. Both stages burn instantaneously and one immediately after the other
  - Heinlein option w/lander
    - First option is that mass in LEO contains same payload as above, a stage to insert said payload to L1 and a separate stage that contains a lander and propellant to land on the moon. Lander is sized to carry enough fuel on launch from moon to rendezvous with payload at L1 and execute Heinlein maneuver
  - L1 option w/lander
    - Same as above, but burn is conducted to take spacecraft directly out of L1 to escape
  - Heinlein option w/o lander
    - Same as Heinlein option, but Hohmann transfer stage carries payload and empty stage for escape maneuver. Lander is considered part of infrastructure and will carry full tank up to mate with payload, and carry empty stage down for next maneuver
  - L1 option w/o lander
   - Same as above but direct burn to escape

- **Assumptions/Calculations**
  - Three propulsive options considered
    - 320 sec Isp, 0.1 inert mass fraction (GOx/GH2)
    - 485 sec Isp, 0.12 inert mass fraction (Lox/LH2)
    - 850 sec Isp, 0.2 inert mass fraction (Nuclear thermal)
  - Hohmann transfer to L1
    - 3.082 km/sec first burn (also used for TLI burn)
    - 0.828 km/sec second burn
  - Lunar operations
    - 2.376 km/sec launch from Moon
    - 0.89+2.03 km/sec land on Moon
  - Other
    - No additional mass accounted for lander structure
    - Hohmann transfers are assumed to be impulsive
    - Launch DV does not account for gravitational losses or Lunar rotation
  - Options Considered but not shown here
    - All mass to lunar surface, Heinlein maneuver after launch from moon
    - HOH stage carries payload and lander to L1; payload and lander break apart there
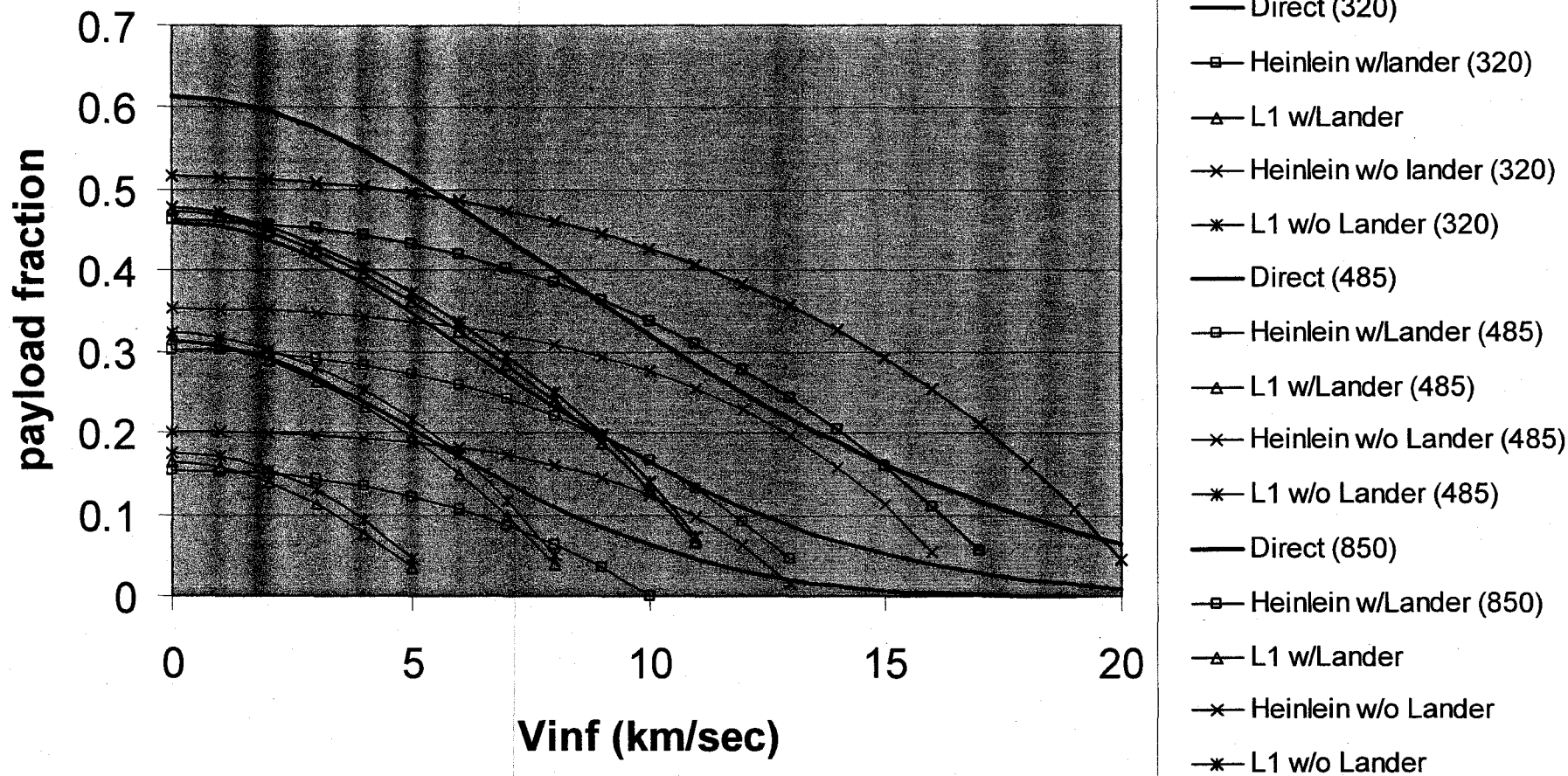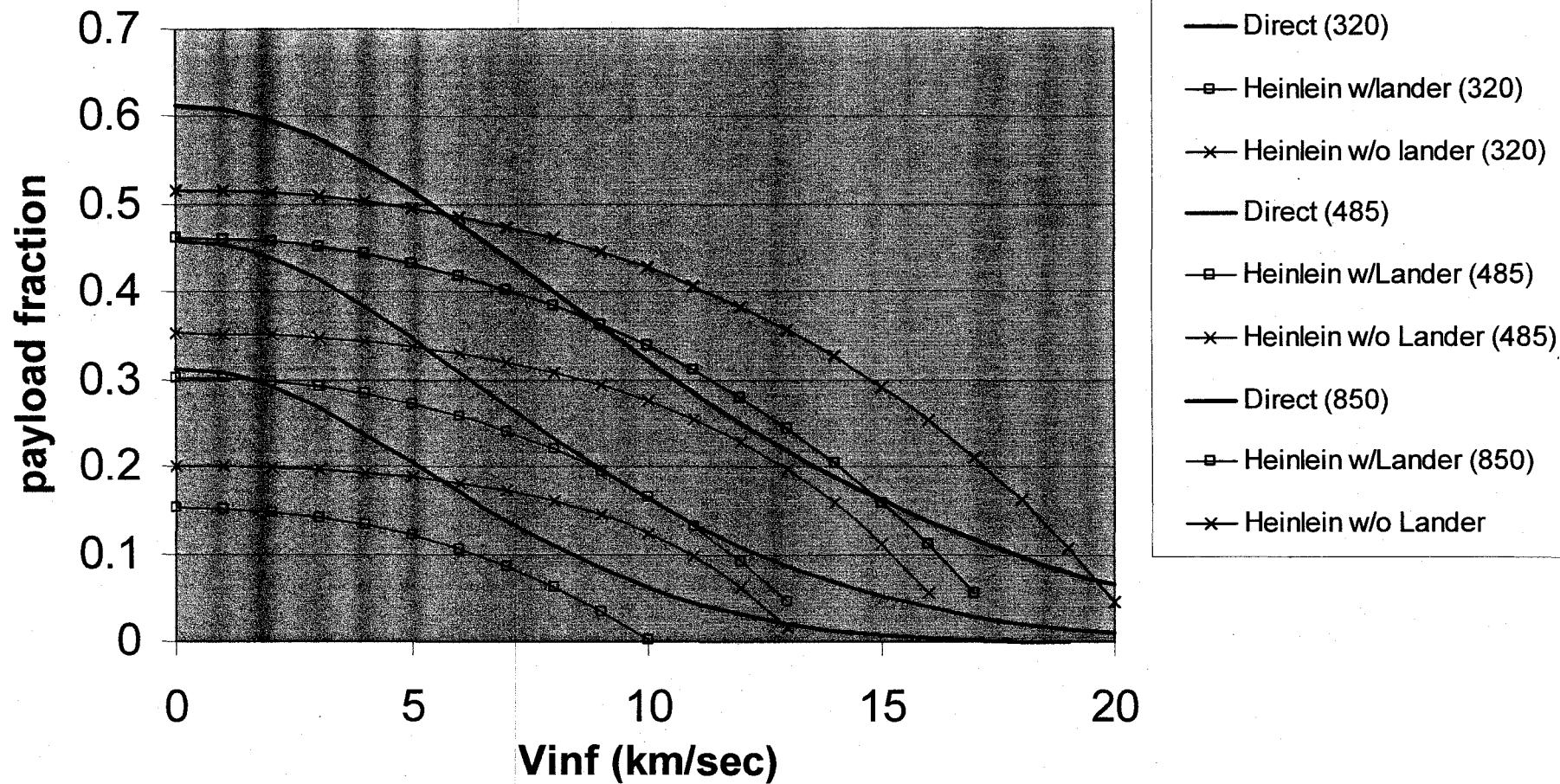
- **Discussion**
  - The Heinlein maneuver offers a lot of advantage for missions with V_∞ in the 5 to 20 km/sec range
    - The Heinlein maneuver can double the V_∞ achieved for a given payload ratio in this velocity range
  - The V_∞ range is within that for many missions of interest
    - Short term stay missions to Mars require V_∞ of 5-6 km/sec
    - Hohmann transfer to Jupiter requires V_∞ of 8-9 km/sec
    - Pluto-Kuiper Express launched in 2006 achieved a V_∞ of 16.21 km/sec
  - This advantage only happens if propellant is found on the moon
  - A similar advantage may exist for an Earth-Sun L1 rendezvous given propellant mined from an NEO; I haven't investigated that yet
  - Particular missions
    - It may be that the best use of the Heinlein maneuver for round trip missions to Mars is to lower trip time. The flat nature of the curve suggests a gain in V_∞ from the usual 5-6 km/sec range can be had for a moderate reduction in payload fraction
    - This maneuver combined with a solar Heinlein maneuver could be powerful for solar escape missions, Pluto-Kuiper-Oort cloud missions and orbit raising to the gas giants

- **Since the Heinlein Maneuver has potential advantages for escape from a central body the next question is does it have application in orbit raising maneuvers.**

    - Escape velocity
        - Note that the escape velocity is calculated as

        $$V_{esc} = \sqrt{\frac{2\mu}{r_0}}$$

        - And note that the criteria for the Heinlein maneuver to be effective is

        $$\Delta V_{tot} \geq \sqrt{\frac{\mu}{r_0}}$$

        - Thus the ΔV budget for the Heinlein maneuver will always be much higher than escape velocity. The Heinlein maneuver cannot be used to improve either Hohmann or bi-elliptic maneuvers. The plot earlier of an "inverse bi-elliptic" is not tractable.
        - In fact, combining the above, the Heinlein maneuver can be stated as effective only when the desired V_∞ exeeds 1.41 times the initial velocity, or

        $$V_\infty \geq \sqrt{2\frac{\mu}{r_0}}$$

- However, a mission requiring orbit raising that places a premium on total mission time may benefit from the Heinlein Maneuver

- Consider the following: could a Heinlein maneuver, whipping around the sun get from an initial circular orbit to a final one in less time than going directly?

- First guess might be no, but that isn't necessarily true

- The equation for orbital period is

$$P = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}}$$

- Imagine a Hohmann transfer from r_0 to r_5. Compare that to a Heinlein maneuver that decelerates to a periapse to r_3 and then performs a Heinlein maneuver to r_5. When do these two maneuvers give the same total mission time?

$$\frac{\pi}{\sqrt{\mu}}\left(\frac{r_0 + r_5}{2}\right)^{\frac{3}{2}} = \frac{\pi}{\sqrt{\mu}}\left(\frac{r_0 + r_3}{2}\right)^{\frac{3}{2}} + \frac{\pi}{\sqrt{\mu}}\left(\frac{r_3 + r_5}{2}\right)^{\frac{3}{2}}$$

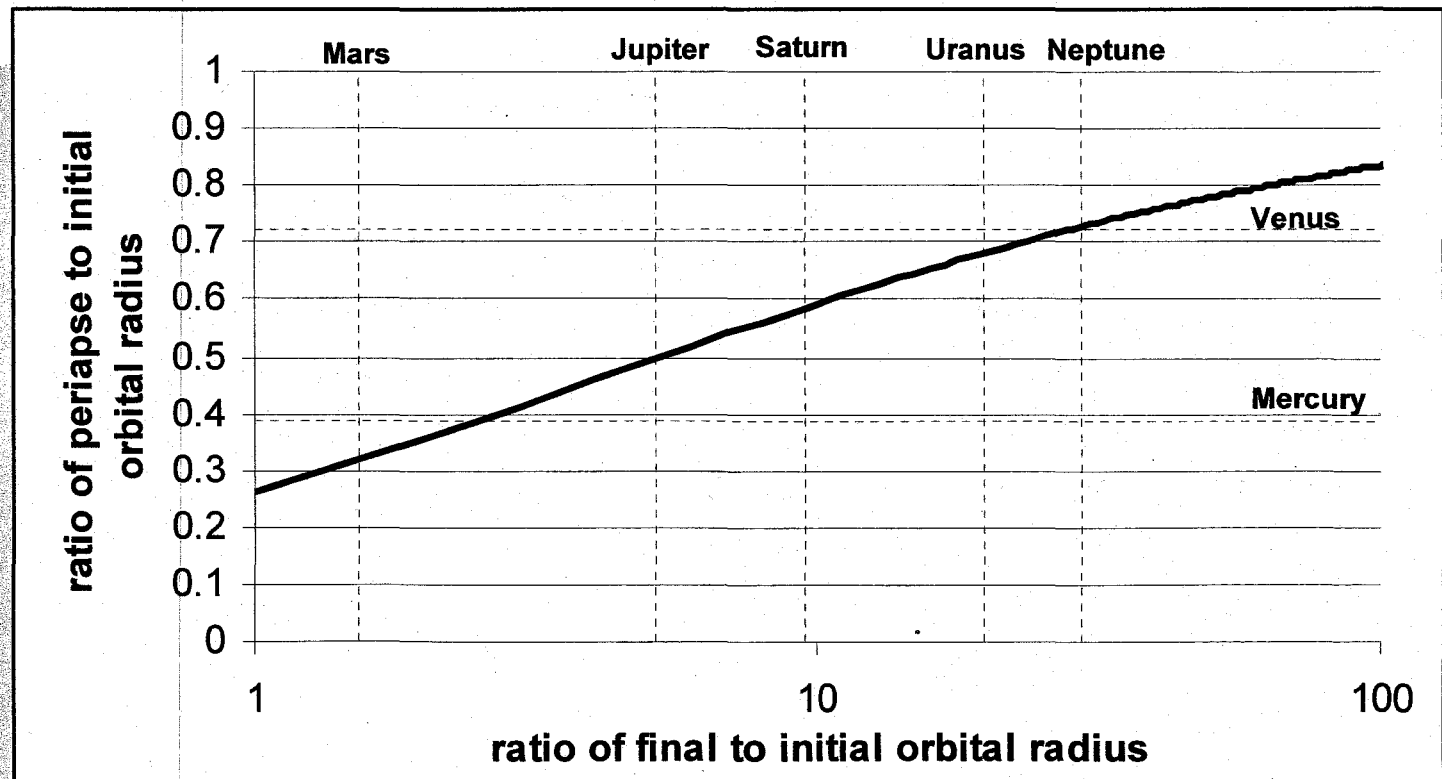- Where the term on the left is direct and on right is Heinlein

- **Reducing yields**

$$\sqrt{(1+R)^3} = \sqrt{(1+\rho)^3} + \sqrt{(\rho+R)^3}$$

- **Where** $R = \dfrac{r_5}{r_0}; \rho = \dfrac{r_3}{r_0}$

- **Plotting this equation gives**

Note that the dotted lines show ratios relative to Earth

- Clearly the Heinlein maneuver is faster than a Hohmann when the ratios plot under the line in the figure above. Which relates back to a close approach to the central body, which the Heinlein maneuver benefits from anyways.

- Incorporating the ΔV maneuver to insert into the final orbit I calculated the total ΔV requirement vs trip time for a couple of cases. I first attempted to derive the relation but the algebra defied my attempts to reduce it

- First is an Earth – Mars orbit raising

- Hohmann outperforms no matter what perihelion radii is allowed

- Assumes starting in Heliocentric orbit at 1 AU
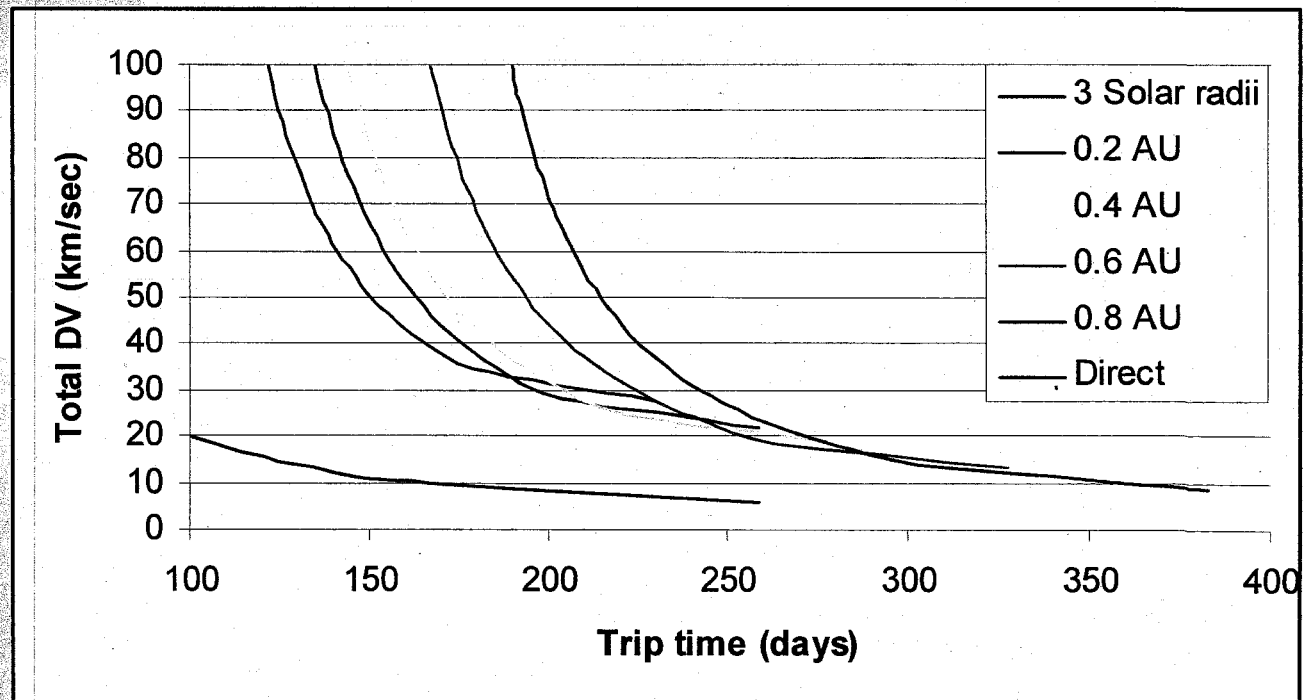
- **Here is the same graph but now for transfers from 1 to 100 AU**
- **There is a small ΔV savings at the lower trip times**
- **Achieving that savings requires a very close approach to the sun**

- **Clearly more work needs to be done to quantify when this maneuver is most effective**
- **Also I believe that if I incorporate Earth escape into this analysis, where the initial velocity vector is allowed to float, that this analysis will improve considerably**
- **I also believe that this phenomena describes why VariTOP will drive to the lowest allowable perihelion in short term Mars missions.**
- **It also accounts for the third burn at periapse inherent in those trajectories**
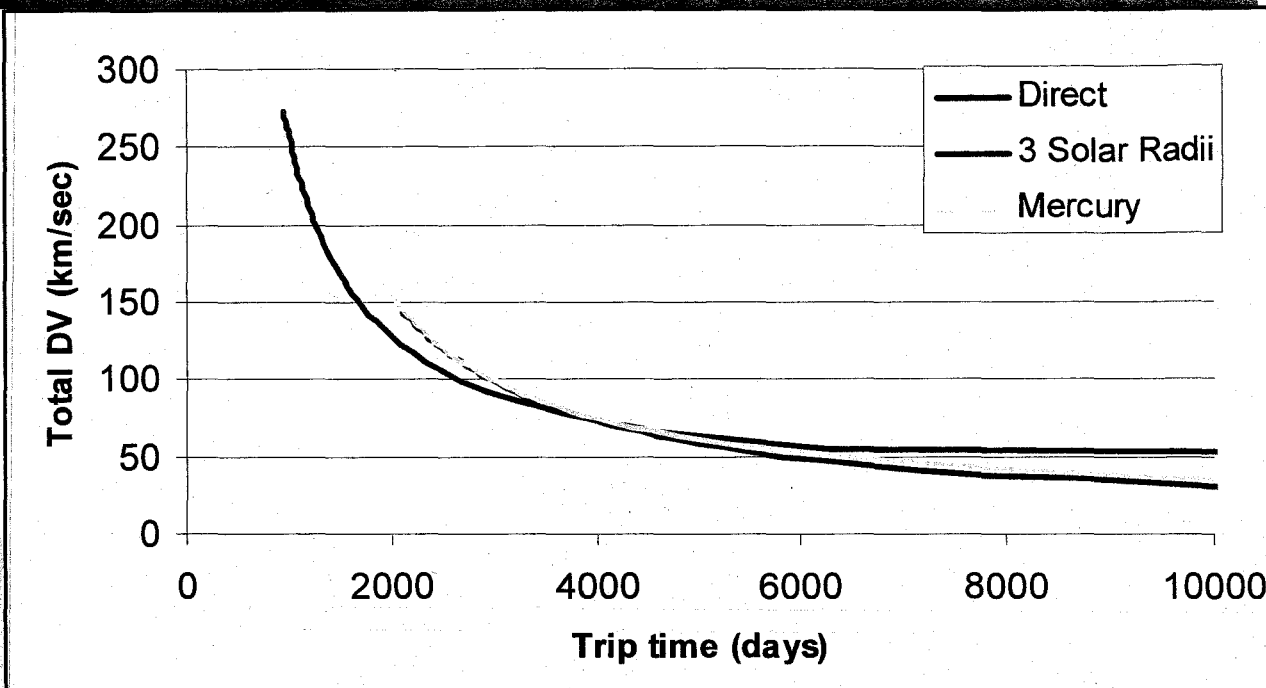
- **I also wondered if there was a better way to do the Heinlein maneuver.**
  - The first burn destroys specific mechanical energy.
  - Burning normal to the orbit would increase eccentricity while not adding or subtracting to specific mechanical energy. Increasing eccentricity in this case means a lower periapse. But would it be enough?
  - This analysis also defied algebraic reduction
  - So here is the graphical result
  - The ordinate is the ratio against a direct maneuver
  - Clearly 0 deg is best for a direct and 180 is best for Heinlein
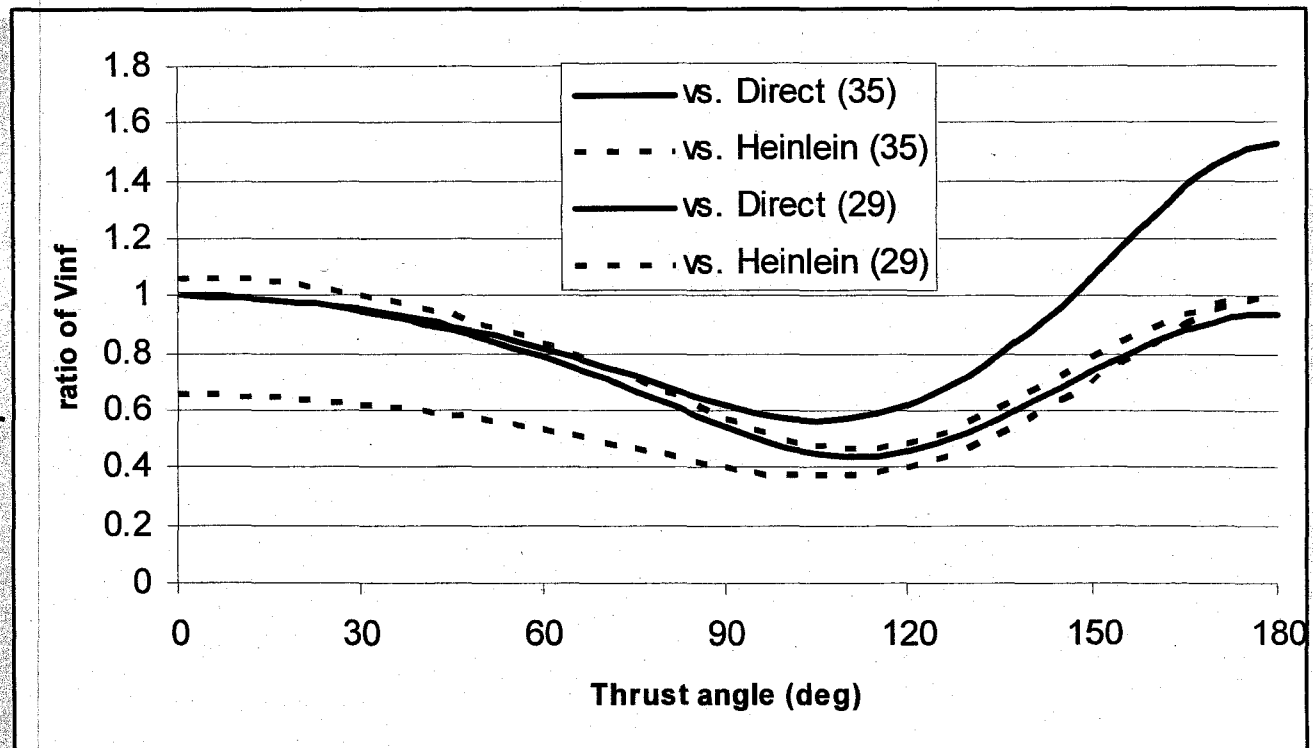
- **Uniqueness**
  - I've checked my results and I feel pretty confident that this maneuver is real
  - I've checked the literature and cannot find a reference to this maneuver (except Heinlein's juvenile novel) and a couple websites dedicated to his novel
  - I personally have never heard anyone mention anything like this before in the studies I've participated in
- **Confirmation**
  - If you wish to challenge or discuss these results my door is open. I think independent confirmation is important
  - As mentioned before this is under review by my committee
  - I've tried to give enough detail to allow independent confirmation without burying you with pages and pages of algebra
- **Implications**
  - This maneuver appears effective for several missions that are under consideration now or in the near future
  - Assuming my results are confirmed I'm trying to figure out the best way to disseminate this information to parties that would be interested. Suggestions on how to do that are welcome.