To: Mr. Thos. J. McCarthy

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ACCELERATIONS IN LANDING WITH A TRICYCLE-TYPE LANDING GEAR

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In connection with the application of stable tricycle-type landing gears to transport airplanes, the question arises as to whether certain of the passengers may not experience relatively great accelerations in an emergency landing. Since the main landing wheels are behind the center of gravity in this type of gear, a hard-braked landing will cause immediate nosing down of the airplane and, when this motion is stopped due to the front wheel striking the ground, there will be some tendency for the rearmost passengers to be thrown out of their seats. The following rough calculations are designed to show the magnitudes of the various reactions experienced in a severe landing under these circumstances.

DEFINITIONS OF SYMBOLS

(See figure 1)

Axes.- The airplane axes are fixed at the center of gravity and are chosen parallel and perpendicular to the ground at the instant of landing as shown in figure 1.

θ, angle of pitch of axes relative to their initial position on landing (positive when airplane is nosing up)

$$q = D\,\theta = \frac{d\theta}{dt}$$

U_0, forward velocity of axes on landing (along X axis)

u, change in forward velocity during landing

w, vertical descent velocity of axes during landing (along Z axis)

m, mass of airplane in slugs $= \frac{W}{g}$

mk_Y^2, moment of inertia of airplane in pitching

M, pitching moment about c.g. of airplane

$M_\theta = \frac{\partial M}{\partial \theta}/mk_Y^2$

$M_q = \frac{\partial M}{\partial q}/mk_Y^2$

b, wing span

S, wing area

c, wing chord

f, distance from airplane c.g. to tail post

$X_{F.W.}$, distance from airplane c.g. to front wheel

$X_{R.P.}$, distance from airplane c.g. to rearmost passenger

h, height of c.g. above ground (shock absorber extended)

μ, coefficient of sliding friction of tires on landing surface

ASSUMED LANDING CONDITIONS

a) The airplane strikes the ground at a vertical descent velocity of w feet per second and with a forward velocity of U_o feet per second. The wheels are considered to be locked and the landing attitude as in figure 1. The maximum vertical acceleration encountered is Dw_{max} and the maximum braking acceleration is $Du_{max} = \mu\ (Dw - g)_{max}$ both applied at the points of contact of the the main landing wheels with the ground.

b) The motion of the airplane after striking the ground will consist of combined pitching and vertical movement. The vertical movement of the c.g. is considered to be predetermined by the action of the main shock absorber and to consist of a practically constant deceleration of the vertical descent velocity w.

Owing to the deflection of the tires and imperfection in the absorber device there will, however, be some delay in the attainment of the full deceleration after landing. This delay is assumed to be represented by a simple formula, viz:

$$(Dw - g) = (Dw - g)_{max} \left(1 - e^{-nt}\right) \tag{1}$$

If the airplane remained in the attitude depicted in figure 1 throughout the interval of absorption of the shock after landing, every part of the machine would experience the same deceleration Dw as the center of gravity. However, since the braking load mDu is applied some distance below the c.g., the airplane will also experience an angular acceleration on striking the ground equal to

$$Du \frac{mh}{mk_Y{}^2}$$

neglecting the variation in h due to the deflection of the shock absorber. This acceleration will cause a rotation in pitch, resulting in parts of the airplane behind the c.g. being accelerated upward relative to the c.g. and consequently receiving a greater shock during the landing. In addition to this, the pitching angular velocity acquired during the period of absorption of the shock and before the front wheel strikes the ground must be reduced to zero by a shock absorber attached to the front wheel. This later deceleration in pitching, subsequent to the main landing shock, tends to lift the rear passengers from their seats and on this account should not be allowed to exceed 32.2 feet per $(\text{second})^2$.

It will be necessary to calculate the angular velocity and acceleration in pitching acquired during the period of absorption of the main shock in order to find the reactions experienced by the passengers in the rear seats. This pitching motion is at first caused by the braking load applied at the wheels but, as the motion commences, is modified by several secondary factors due to the motion itself, as follows:

a) A pitching moment due to change in angle of pitch after landing arising partly from the static stability of the airplane in pitch and partly from the displacement of the c.g. of the airplane away from the point of support. The part of this secondary moment arising from the aerodynamic effect is calculated as:

$$M_1 = \theta \frac{dCm}{d\alpha} S \frac{\rho}{2} U_o^2 c \tag{2}$$

or

$$M_{\theta_1} = \frac{dCm}{d\alpha} \frac{S \frac{\rho}{2} U_o^2}{m} \frac{c}{k_Y^2}$$

where the change in angle of attack α is assumed to be equal to the change in pitch angle θ since the airplane is sensibly constrained to travel along the ground after landing. The part of the moment arising from the displacement of the c.g. is:

$$M_2 = - m(Dw - g)h \sin \theta \tag{3}$$

The variation of $(Dw - g)$ with time, given by equation (1) will be approximated in computing this secondary effect by simply taking 90 percent of the maximum value. The difference between $\sin \theta$ and θ will also be neglected, resulting in

$$M_{\theta_2} = - 0.9 \ (Dw - g)_{max} \frac{mh}{mk_Y^2} \tag{4}$$

Actually the length h will be decreased somewhat due to the deflection of the shock absorber and the above estimate of M_{θ_2} may be considered conservative. The total secondary effect due to rotation in pitch is:

$$M_\theta = M_{\theta_1} + M_{\theta_2} \tag{5}$$

b) A pitching moment due to the aerodynamic damping of the pitching motion. This moment is calculated by using the customary aerodynamic formula:

$$M_{(damping)} = - \frac{qf}{U_o} \frac{dC_{Lt}}{d\alpha_t} S_t \frac{\rho}{2} U_o^2 f \tag{6}$$

where the subscripts t refer to the horizontal tail plane.

$$M_q = -\frac{f^2}{k_Y^{\,2}} \frac{dC_{L_t}}{d\alpha_t} \frac{S_t \frac{\rho}{2} U_o}{m} \tag{7}$$

With the aid of these factors defining the secondary effects due to motion the pitching motion due to any torque applied by braking at the wheels may be calculated. The equation of motion is:

$$D^2\theta - M_q D\theta - M_\theta \theta = M_o \tag{8}$$

where $mk_Y^{\,2} \times M_o$ is the applied torque and is assumed to be known in terms of the time. Following the assumed law of variation of shock-absorber reaction (equation (1)) and considering the applied torque as being due to the breaking reaction at the wheels we have:

$$\begin{aligned} M_o &= Du \frac{mh}{mk_Y^{\,2}} = Du_{max} \frac{mh}{mk_Y^{\,2}} (1 - e^{-nt}) \\ &= M_{o_{max}} (1 - e^{-nt}) \end{aligned} \tag{9}$$

GENERAL SOLUTION OF EQUATION OF MOTION

Equation (8) may be integrated in general terms and actual numerical values for the various quantities may be substituted into the general solution later.

The easiest way to integrate this equation is to make use of the Heaviside expansion theorem. The first step is to obtain an algebraic resolution for θ, viz,

$$\theta = \frac{1}{D^2 - M_q D - M_\theta} M_o = \frac{1}{F(D)} M_o \tag{10}$$

According to equation (9) M_o is given by two terms. The distribution of the operator $\frac{1}{F(D)}$ over these terms results in:

$$\frac{1}{F(D)}\, M_o = M_{o_{max}} \left(\frac{1}{F(D)} - \frac{1}{F(D)}\, e^{-nt} \right) \tag{11}$$

The expansion of the first term is

$$\frac{1}{F(D)} = \frac{1}{F(0)} + \sum_{\lambda} \frac{e^{\lambda t}}{\lambda\, F'(\lambda)} \tag{12}$$

and of the second term

$$\frac{1}{F(D)}\, e^{-nt} = \frac{e^{-nt}}{F(-n)} + \sum_{\lambda} \frac{e^{\lambda t}}{(\lambda + n)\, F'(\lambda)} \tag{13}$$

where the λ's are the roots of the auxiliary equation $F(\lambda) = 0$.

In the present problem

$$\lambda_1 = \frac{1}{2} \left(M_q + \sqrt{M_q^{2} + 4M_\theta} \right) \tag{14}$$

$$\lambda_2 = \frac{1}{2} \left(M_q - \sqrt{M_q^{2} + 4M_\theta} \right)$$

The substitution of these roots into the expansions (12) and (13) results in:

$$\frac{\theta_1}{M_{o_{max}}} = \frac{1}{F(D)} = -\frac{1}{M} + \frac{e^{\lambda_1 t}}{\lambda_1 (2\lambda_1 - M_q)} + \frac{e^{\lambda_2 t}}{\lambda_2 (2\lambda_2 - M_q)} \tag{15}$$

$$\frac{\theta_2}{M_{o_{max}}} = \frac{1}{F(D)} e^{-nt} = \frac{e^{-nt}}{n^2 + M_q\ n - M_\theta} + \frac{e^{\lambda_1 t}}{(\lambda_1+n)(2\lambda_1 - M_q)} + \frac{e^{\lambda_2 t}}{(\lambda_2+n)(2\lambda_1 - M_q)} \tag{16}$$

where θ_1 is the angle of pitch at any instant under the condition of a sudden impact, that is, under the condition that $M_{o_{max}}$ reaches its full value instantaneously, and θ_2 represents the amount to be subtracted from θ_1 to account for the delay in the full action of the shock absorber:

$$\theta = \theta_1 - \theta_2 \tag{17}$$

The motion represented by the foregoing equations is to be continued only so long as is required for the landing gear to absorb the impact of the landing. If w_o is the vertical descent velocity on landing, this time is found from

$$w_o = - \int_0^{t_1} Dw\ dt \tag{18}$$

The total vertical stroke of the shock absorber necessary to absorb the vertical velocity w_o with the specified acceleration is:

$$\Delta h = \int_0^{t_1} \left[\int_{t_1}^{t} Dw\ dt \right] dt \tag{19}$$

Subsequent to the time (t_1) of absorption of the vertical descent velocity w_o, the loads at the wheels will be much reduced and a new equation of motion with different values of M_θ and M_o will be in force. The angular velocities and displacements calculated from the original equations at the time $t = t_1$ should be taken as initial conditions in the new equation. Presumably, the time of absorption of the landing shock will not be sufficient for the front wheel to strike the ground and the conditions under the new equations of motion will be extended to that time. In the numerical calculations that follow it was found that the equations for the second phase of the motion after landing represented a continued rotation in pitch at a practically constant angular velocity, the aerodynamic stability and damping effects nearly counterbalancing the reduced braking and nosing-over moments. For this reason it was assumed that the angular velocity in pitch at the instant the front wheel struck the ground was the same as that acquired during the shock-absorption period.

CALCULATIONS FOR AIRPLANE A

1) Assumed specifications of airplane:

$$w = 18{,}000 \text{ lb.}$$

$$b = 80 \text{ ft.}$$

$$S = 939 \text{ sq. ft.}$$

$$c = 11.86 \text{ ft.}$$

$$f = 36.6 \text{ ft.}$$

$$X_{F.W.} = 14 \text{ ft.}$$

$$X_{R.P.} = 14 \text{ ft.}$$

$$h = 8.4 \text{ ft.}$$

$$k_Y = 11.75 \text{ ft.}$$

$$\frac{dCm}{d\alpha} = -0.658$$

$$S_t = 154.6 \text{ sq. ft.}$$

2) At a lift coefficient of approximately 2.0 the landing speed will be 60 miles per hour.

3) Assume a vertical descent velocity of $w_o = 15$ feet per second at the instant of landing and assume that the maximum vertical acceleration encountered after landing is

$$Dw = -3\ g \tag{20}$$

4) Assume that the law expressing the building up of

acceleration after the instant of contact with the ground is

$$(Dw - g) = (Dw - g)_{max} (1 - e^{-23t}) \qquad (21)$$

(See figure 2)

This formula is taken to represent roughly the conditions obtained with an ordinary oleo landing gear and was devised after examination of some experimental records obtained in drop tests of military airplanes.

5) Calculation of M_θ and M_q:

a) M_θ (See equations 2 and 4.)

$$M_{\theta_1} = -0.658 \times 939 \times 9.2 \times \frac{11.86}{77200} = -0.87$$

$$M_{\theta_2} = 4 \times \frac{18000 \times 8.4}{77200} \times 95 \text{ percent} \qquad (22)$$

Total $M_\theta = 6.5$

b) M_q (See equation 7.)

Assume that the slope of the tail-plane lift curve with angle of attack is 4.0.

$$M_q = \frac{-36.6^2 \times 4 \times 154.6 \times 0012 \times 88}{77200} = -1.13 \qquad (23)$$

6) If the landing is made with the brakes locked and the coefficient of sliding friction of the wheels on the landing surface is one-half, the applied angular acceleration due to the braking is:

$$M_o = -2 g \times \frac{8.4}{138} (1 - e^{-23t})$$

$$= -3.92 (1 - e^{-23t}) \text{ (See equation 9.)} \qquad (24)$$

(It is to be noted that the value of the coefficient of friction assumed may be left undetermined up to the point of the final solution.)

7) The equation of motion with the foregoing quantities substituted for M_θ, M_q, M_o is:

$$(D^2 + 1.13\ D - 6.5)\theta = -3.92\ (1 - e^{-23t})$$

or (25)

$$F(D)\theta = 3.92\ (1 - e^{-23t})$$

The roots of $F(D) = 0$, as required by equations (15) and (16), are

$$\lambda = \frac{-1.13 \pm \sqrt{1.13^2 + 4 \times 6.5}}{2}$$

or (26)

$$\lambda_1 = -3.16$$

$$\lambda_2 = 2.03$$

8) The final solution, consisting of two parts as in equations (15) and (16), is

$$\theta_1 = \frac{3.92}{6.5} - \frac{3.92}{16.4}\ e^{-3.16t} - \frac{3.92}{10.53}\ e^{2.03t}$$

or (27)

$$\theta_1 = 0.603 - 0.239\ e^{-3.16t} - 0.372\ e^{2.03t}$$

where θ_1 is the angle of pitch attained at the time t if the landing shock is instantaneous. θ_2, or the angle to be subtracted from θ_1 to account for the delay in the full shock-absorber action is:

$$\theta_2 = 0.0079\ e^{-23t} - 0.0380\ e^{-3.16t} + 0.0301\ e^{2.03t} \quad (28)$$

The complete solution follows as

$$\theta = 0.603 + 0.0079\ e^{-23t} - 0.277\ e^{-3.16t} - 0.342\ e^{2.03t}$$

(29)

The angular velocity and acceleration at any instant are found by differentiating this equation:

$$q = -0.182\ e^{-23t} + 0.874\ e^{-3.16t} - 0.694\ e^{2.03t}$$
$$Dq = 4.19\ e^{-23t} - 2.76\ e^{-3.16t} - 1.41\ e^{2.03t} \qquad (30)$$

At the time $t = 0$ $e^{Kt} = 1$, so that the assumed initial conditions on landing

$$\theta = q = Dq = 0 \qquad (31)$$

should be given by the sums of the coefficients of e in the above solutions. Thus:

$$\theta_{(t=o)} = 0.603 + 0.0079 - 0.277 - 0.342 = -0.008$$
$$q_{(t=o)} = -0.182 + 0.874 - 0.694 = -0.002 \qquad (32)$$
$$Dq_{(t=o)} = 4.19 - 2.76 - 1.41 = -0.02,$$

which is a check on the solution of the equations.

The motion represented by the foregoing equations is to be continued only so long as is required for the landing gear to absorb the impact of the airplane. This time is found from

$$w_o = 15\ \text{ft./sec.} = \int_0^{t_1} 3g\ (1 - e^{-23t})\ dt$$

or (33)

$$4.18 - 96\ t_1 - 4.18\ e^{-23\,t_1} = -15$$

$$t_1 = 0.2\ \text{second, very nearly}$$

Having found the time of absorption of the shock, the total vertical stroke of the shock absorber may be found by integration; thus

$$\Delta h = \int_0^{0.2} \left[\int_{0.2}^{t} 3\ g\ (1 - e^{-23t})\ dt \right] dt$$

or, (34)

$$\Delta h = 1.76\ \text{feet, approximately}$$

It will be necessary to calculate θ, q, and Dq at

0.2 second from the equations given: (See equations (29) and (30).)

$$\theta_{0.2} = 0.603 + 0.0079\, e^{-4.6} - 0.277\, e^{-0.632} - 0.342\, e^{0.406}$$

$$= -\,3.24^{\circ} \quad \text{(degrees)}$$

$$q_{0.2} = -0.182\, e^{-4.6} + 0.874\, e^{-0.632} - 0.694\, e^{0.406} \qquad (35)$$

$$= -\,0.577 \text{ radian/second}$$

$$Dq_{0.2} = 4.19\, e^{-4.6} - 2.76\, e^{-0.632} - 1.41\, e^{0.406}$$

$$= -\,3.54 \text{ radians/second/second}$$

On substituting these initial conditions into the new equation of motion it is found that the aerodynamic stability and damping moment nearly balance the now much-reduced braking and nosing-over moments. For this reason it will be sufficiently accurate to neglect any changing conditions thereafter and to assume that the above angular velocity persists until the front wheel strikes the ground at $\theta = -\,15^{\circ}$. This angular velocity (of 0.577 radian/second) produces an upward velocity of

$$0.577 \times 14 \text{ ft.} = 8.1 \text{ ft./sec.} \qquad (36)$$

at the rearmost passenger's seat. If stopped suddenly at this speed the seat would cause the passenger to rise 1.02 feet. At constant deceleration, the front-wheel shock absorber must travel approximately $X_{F.W.}/X_{R.P.}$ of this distance to prevent the passenger from leaving the seat; hence, it is concluded that the front-wheel shock absorber should have a travel of slightly over one foot (including tire deflection) in order to accomodate the above conditions with a safe allowance for inefficiency of operation (that is, deviation from constant acceleration).

Assuming that the law expressing the rise of shock-absorber load with time is the same for the front wheel as for the rear, the more accurate value for the travel of the front-wheel system is: (choosing a new origin for the time scale)

$$\Delta h_f = \int_0^{t_1} \left[\int_{t_1}^{t} - 32.2 \left(1 - e^{-23t}\right) dt \right] dt \qquad (37)$$

The limit of integration t_1 is found from the known velocity by performing an additional integration, thus

$$-8.1 = \int_0^{t_1} -32.2\,(1 - e^{-23t})\,dt, \tag{38}$$

whence $t_1 = 0.295$ second, and the distance

$$\int_0^{0.295} \left[\int_{0.295}^{t} -32.2\,(1 - e^{-23t})\,dt \right] dt = 1.3 \text{ ft.}, \tag{39}$$

approximately.

CALCULATIONS FOR AIRPLANE B

1) Assumed specifications of airplane:

$$w = 54{,}000 \text{ lb.}$$

$$b = 139 \text{ ft.}$$

etc.

Airplane B is similar to airplane A except that the weight and wing area are increased by the factor 3. The moment of inertia is increased by $3 \times 3 = 9$, resulting in the following modifications to the quantities occurring in the equations of motion:

$$M_{\theta_B} = \frac{1}{\sqrt{3}} M_{\theta_A}$$

$$M_{q_B} = M_{q_A} \tag{40}$$

$$M_{o_B} = \frac{1}{\sqrt{3}} M_{o_A}$$

The equation of motion becomes:

$$D^2\theta = D\theta M_g + \theta M_\theta - 2.27\,(1 - e^{-23t}) \tag{41}$$

The solution of this equation, obtained in the same way as for airplane A, is:

$$\begin{aligned}
\theta &= 0.602 + 0.0046\, e^{-23t} - 0.2456\, e^{-2.58t} - 0.366\, e^{1.45t} \\
q &= -0.106\, e^{-23t} + 0.633\, e^{-2.58t} - 0.531\, e^{1.45t} \\
Dq &= 2.44\, e^{-23t} - 1.63\, e^{-2.58t} - 0.77\, e^{1.45t}
\end{aligned} \qquad (42)$$

Since the landing conditions are the same as before, the required stroke of the shock absorber is the same and the time taken to absorb the impact is likewise 0.2 second. The following table lists the important results of the calculations in the two cases.

	Airplane A 18,000 lb.	Airplane B 54,000 lb.
1) Maximum angular acceleration $[Dq_{(t = 0.2)}]$	-3.54	-1.97
2) Maximum angular velocity $[q_{(t = 0.2)}]$	- .577	- .336
3) [1] Maximum vertical reaction at rear seat	-5.54 g	-5.48 g
4) Maximum rising velocity at rear seat	8.1 ft./sec.	8.1 ft./sec
5) Travel of main shock absorber (including tire)	1.72 ft.	1.72 ft.
6) Travel of front shock absorber (including tire) necessary to prevent rear passenger from leaving seat	1.32 ft.	1.32 ft.

[1] Note: The value given here is for

$$(Dw - g) + X_{R.P.}\, Dq$$

DISCUSSION AND CONCLUSIONS

The foregoing calculations show that in landing under the assumed conditions the rearmost passenger receives a somewhat greater primary landing reaction than would be expected in the case of the conventional unstable type landing gear under the same conditions (5.5 g at the rear seat compared with 4 g at the center of gravity). In the calculations no account was taken of seat cushioning, nor of any flexibility of the structure of the airplane and, since this acceleration reaches its maximum and disappears within two-tenths of a second, it may be concluded that it would not result in injury or great discomfort to the passenger. The conditions assumed correspond to a severe landing that should be encountered only in an emergency and it may be assumed that the additional acceleration due to this landing-gear design would not be noticeable under ordinary circumstances.

The calculations further showed that a certain minimum shock-absorber travel for the front wheel was necessary to preclude the possibility of lifting the rearmost passenger from his seat. This necessary travel depends on the placing of the front wheel relative to the c.g. and relative to the backward distance of the rearmost passenger from the c.g., and is lessened when both distances are made shorter. It may be concluded, however, that in most cases the stroke required of the front shock absorber will be less than that of the rear ones.

The possibility of the rearmost passengers being thrown upward when the front wheel strikes the ground depends on the existence of a large braking effort at the main landing wheels. This braking effort also tends to cause the passenger to slide forward out of the seat. Supposedly the passenger is restrained both from sliding forward and from rising upward by a safety belt. It is to be noted that the braking deceleration assumed in the present analysis was of such magnitude that the passengers were more likely to be thrown forward and injured on this account directly than to be thrown upward and injured as a result of the subsequent checking of the nosing-over motions. It may be supposed that this braking deceleration will be present under similar conditions with any design of landing gear and, since the secondary vertical reaction that arises when the front wheel strikes the ground in the case of the stable-type gear is ordinarily much less than the braking reaction, it is concluded that there should be no difficulty with the stable tricycle landing gear on this account.

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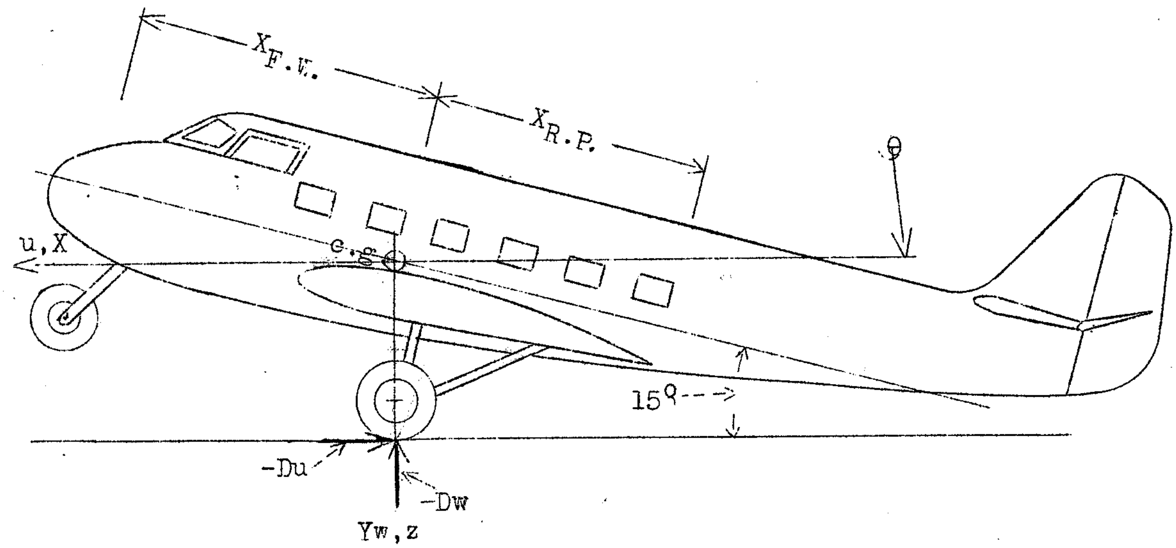

Figure 1.- Initial landing conditions : Definitions of symbols and axes.

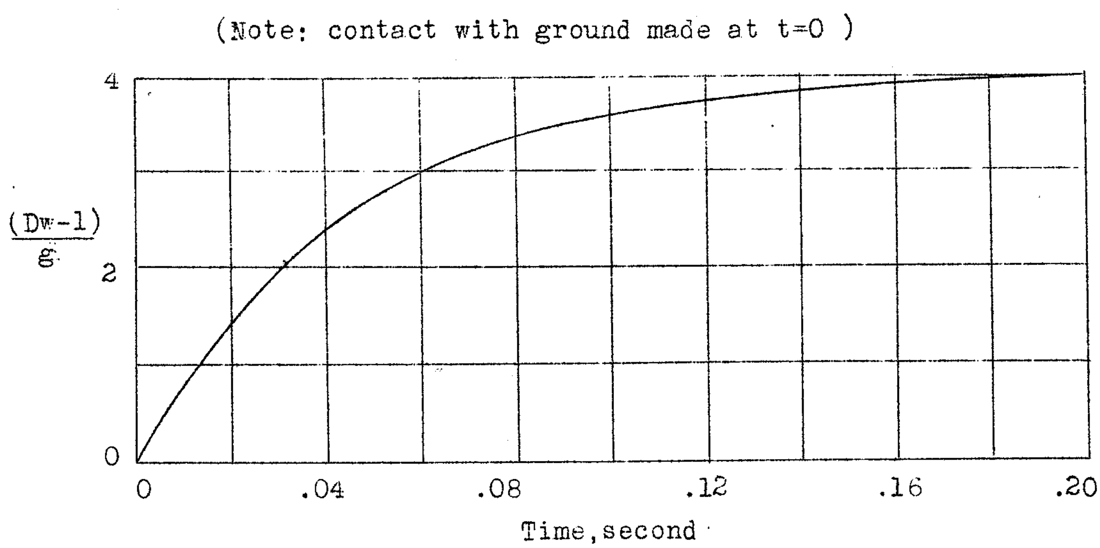

Figure 2.- Representation of shock-absorbing lag by formula: $(Dw-g)=-4g\ (1-e^{-23t})$