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AN APPROXIMATE METHOD OF CALCULATION OF RELATIVE HUMIDITY REQUIRED TO PREVENT FROSTING ON INSIDE OF AIRCRAFT PRESSURE CABIN WINDOWS

By Alun R. Jones

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INTRODUCTION

This report has been prepared in response to a request received from an aircraft company for information. A typical example was selected for the presentation of an approximate method of calculation of the relative humidity required to prevent frosting on the inside of a plastic window in a pressure-type cabin on a high-speed airplane. The conditions assumed for this example are as follows:

Window	Single pane of transparent plastic 5/8 inches thick (k for plastic = 0.1827 kg. cal./m $^{\circ}$C hr.)
Outside temperature	-40°F
Cabin temperature	40°F
Air speed	400 m.p.h.
Altitude	25,000 ft.
Cabin pressure	10.9 lb./sq. in.

The solution for this particular set of conditions is 15 per cent relative humidity. The method used to obtain this solution follows.

METHOD

It is assumed that the velocity of the air across the window is equal to the air speed, 400 m.p.h. The temperatures involved are shown in figure 1.

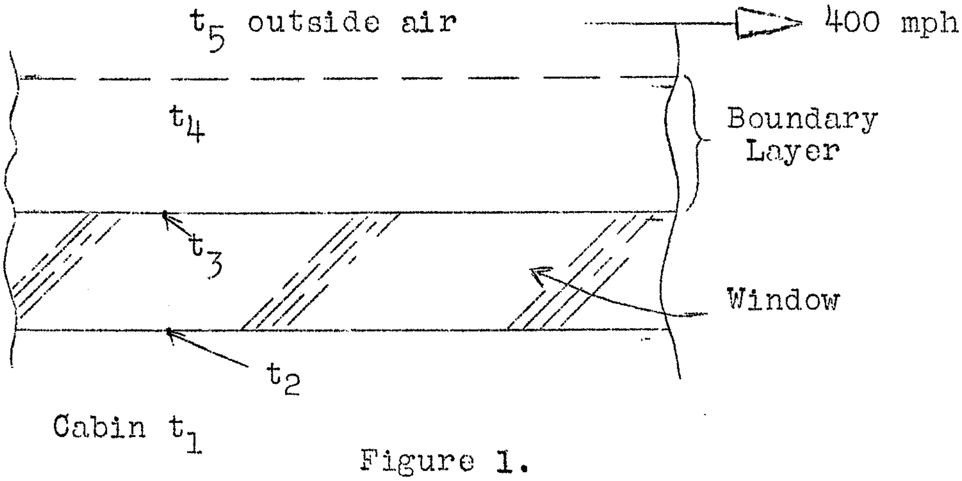

Figure 1.

Since all temperatures involved will be low, the factor of radiation may be neglected and the heat transmission process will be free convection, t_1 to t_2, conduction through the plastic, t_2 to t_3, and forced convection into the boundary layer, t_3 to t_4.

The equations of the heat flow at steady state (from reference 1) give:

$$h_1 A(t_1 - t_2) = \frac{k A(t_2 - t_3)}{l} = h_2 A(t_3 - t_4) \quad (1)$$

where

h_1 = transfer coefficient, cabin air to window
A = window area
t_1, t_2, t_3, t_4, t_5 = as shown in figure 1
k = conductivity of plastic
l = window thickness
h_2 = transfer coefficient, window to outside air

The unknowns are h_1, t_2, t_3, t_4, and h_2. Since only two equations are available we must evaluate three of these in some manner.

An approximation for h_1 is obtained from equation (29), page 244, reference 1. Noting the remark on page 245 that h is further effected by pressure, we write:

$$h_1 = 0.3(t_1 - t_2)^{0.25} \sqrt{\frac{p}{14.7}} \tag{2}$$

The value of t_4 (temperature of air in boundary layer) can be calculated from equation (7), reference 2.

$$t_5 - t_4 = - 0.45\, V^2 \times 10^{-3}$$

This equation is for V in m/sec and $t_5 - t_4$ in degrees Centigrade. Applying $V = 400$ m.p.h. we obtain $t_5 - t_4 = 25°$ Fahrenheit, hence $t_4 = -15°$ Fahrenheit.

The value of h_2 is difficult to state since very few data are available at such high speed. However, application of equations from reference 1 and experience with transfer coefficients at lower velocities, lead to the conclusion h_2 must be approximately equal to 50 and would certainly be no less.

The problem now resolves to a trial and error method, but certain reasoning will lead to a quick result. An inspection of equation (2) indicates that **h_1** is not much affected by changes in $t_1 - t_2$, and for values of t_2 which seem within reason h_1 must be approximately 1.0.

Therefore, assume $h_1 = 0.8$, $h_2 = 50$ and equation (1) becomes

$$0.8\,(40 - t_2) = \frac{0.123\,(t_2 - t_3)\;8 \times 12}{5} = 50(t_3 + 15)$$

which gives $t_2 = -0.6^\circ$ Fahrenheit and $t_3 = -14.4^\circ$ Fahrenheit.

Substituting this value of t_2 in equation (2) results in $h_1 = 0.7$ which is close enough to the assumed value of 0.8. Notice that the effect of assuming higher values of h_2 will merely result in reducing $t_3 - t_4$, and for all practical purposes t_3 could have been assumed equal to t_4.

Condensation of moisture upon the inside of the window will occur when the water vapor present in the cabin air has a saturation temperature equal to or greater than the temperature of the inside surface of the window. Assume the water vapor has an actual vapor pressure of P_V and is at cabin temperature of 40° Fahrenheit. As this vapor is cooled it will maintain its same pressure as long as it is a vapor. In order for no condensation to occur until -0.6° Fahrenheit is reached, the value of P_V must be the saturation pressure corresponding to -0.6° Fahrenheit.

$$\text{Thus, relative humidity} = \frac{\text{saturation pressure at } -0.6^\circ\text{ F.}}{\text{saturation pressure at } 40^\circ\text{ F.}} \times 100$$

$$= \frac{0.92\text{ mm Hg}}{6.37\text{ mm Hg}} \times 100 = 15 \text{ per cent}$$

Air in cabin at 40° Fahrenheit and pressure of 10.9 pounds per one square inch must have relative humidity less than 15 per cent in order to prevent frosting of the windows.

From the problem above it can be seen that frost can be prevented by (1) reducing the humidity or (2) heating the window. As regards (1), a humidity of 15 per cent is considerably below the comfort limit of 30 per cent. If the cabin humidity is raised, extremely dry air would have to be introduced into the cabin air near the windows to prevent condensation.

Method (2) appears to be the best solution, and a simple calculation indicates that a window inner-surface temperature of 15° Fahrenheit would prevent frosting when the cabin air was at 40° Fahrenheit and relative humidity of 30 per cent. Since this temperature is not much above the -0.6° Fahrenheit from the problem just solved, probably an inner secondary window about 1/16-inch thick and separated from the outer window by a 1/16-inch air gap would be above 15° Fahrenheit.

If this should not be sufficient, the hot air method described in reference 3 was extremely successful in tests, and the cabin heating air could be admitted between double panes. This is the best method concerning which we have information, although any means that will raise the glass temperature to the required value will be equally successful.

The value of the heat-transfer coefficient depends upon a number of complex variables such as velocity of fluid flow, nature and shape of contact surfaces, and physical properties of the fluid. The coefficient is greatly influenced by type of flow, whether laminar or turbulent. An increase in velocity usually tends to decrease the boundary layer thus increasing the transfer coefficient.

The value of this heat-transfer coefficient has been established for a great many specific conditions and interested parties are directed to references 1 and 2, and also the volume of the Durand series concerned with heat flow, reference 4.

REFERENCES.

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3. Rodert, Lewis A., An Investigation of the Prevention of Ice on the Airplane Windshield, NACA T.N. 754.

4. Dryden, Hugh L., Aerodynamics of Cooling, Division T, Vol. VI, Aerodynamic Theory, Durand Series - Julius Springer, Berlin, 1936.