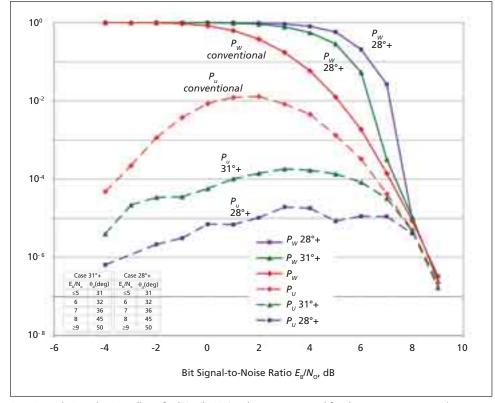# Bounded-Angle Iterative Decoding of LDPC Codes

Maximum undetected-error rates are reduced greatly; overall error rates are increased negligibly (at low error rates).

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Bounded-angle iterative decoding is a modified version of conventional iterative decoding, conceived as a means of reducing undetected-error rates for short low-density parity-check (LDPC) codes. For a given code, bounded-angle iterative decoding can be implemented by means of a simple modification of the decoder algorithm, without redesigning the code.

Iterative decoders for well-designed LDPC codes are inherently capable of detecting as well as correcting errors. In a given instance, an error is detected if a code word is not found at the conclusion of iterations: that is, an error is detected if the estimated code symbols at the end of iterations do not constitute a valid code word. However, when this approach is followed without modification, the maximum undetected-error rates obtained for short LDPC codes are typically too high for many applications.

Bounded-angle iterative decoding is based on a representation of received words and code words as vectors in an n-dimensional Euclidean space (where n is an integer). In bounded-angle iterative decoding as in conventional iterative decoding, the estimates of the decoder are rejected (an error is detected) if a code word is not found before the maximum allowed number of iterations. Conversely, iterations may be stopped as soon as a code word is detected. The difference between bounded-angle and conventional iterative decoding manifests itself once a code word has been detected. At this point, the decoding algorithm computes the angle in the n-dimensional Euclidean space between the received word and the detected code word. If this angle is less than an arbitrarily specified threshold angle, θ_d, then the detected code word is accepted; if this angle exceeds θ_d, then the detected code word is rejected.

The undetected-error rate in bounded-angle iterative decoding is necessarily less than that in conventional iterative decoding because the modified decoding algorithm rejects at least the same code words as does the conventional decoding algorithm. By rejecting more code words, the modified algorithm reduces the undetected-error rate while increasing the overall error rate. Through judicious choice of θ_d, optimized as a function of signal-to-noise ratio E_b/N_0 (see figure), one can reduce the decoder's maximum undetected-error rate by orders of magnitude while increasing its overall error rate by amounts that are negligible for all values of E_b/N_0 high enough to have produced a low error rate using the unmodified decoder. The main value of the modified algorithm lies in the possibility of making this favorable tradeoff between the two error rates, while not redesigning the code and only trivially modifying the decoder.

Case 31°+		Case 28°+	
E_b/N_0	θ_a(deg)	E_b/N_0	θ_a(deg)
≤5	31	≤5	31
6	32	6	32
7	36	7	36
8	45	8	45
≥9	50	≥9	50

Error Rates in Iterative Decoding of a (32,16) LPDC code were computed for three test cases: one using a conventional decoding algorithm and two using the bounded-angle version of this algorithm with threshold angles of 28° and higher, and 31° and higher, respectively, varying with E_b/N_0 according to the table.