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VOUGHT-SIKORSKY AIRCRAFT DIVISION

RADIATOR DESIGN AND INSTALLATION - II

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RADIATOR DESIGN AND INSTALLATION - II

By Arthur W. Tifford

SUMMARY

A mathematical analysis of radiator design has been made. The volume of the radiator using least total power has been expressed in a single formula which shows that the optimum radiator volume is independent of the shape of the radiator and which makes possible the construction of design tables that give the optimum radiator volume per 100-horsepower heat dissipation as a function of the speed, of the altitude, and of one parameter involving characteristics of the airplane.

Although, for a given set of conditions, the radiator volume using the least total power is fixed, the frontal area, or the length, of the radiator needs to be separately specified in order to satisfy certain other requirements such as the ability to cool with the pressure drop available while the airplane is climbing. In order to simplify the specification for the shape of the radiator and in order to reduce the labor involved in calculating the detailed performance of radiators, generalized design curves have been developed for determining the pressure drop, the mass flow of air, and the power expended in overcoming the cooling drag of a radiator from the physical dimensions of the radiator. In addition, a table is derived from these curves, which directly gives the square root of the pressure drop required for ground cooling as a function of the radiator dimensions, of the heat dissipation, and of the available temperature difference. Typical calculations using the tables of optimum radiator volume and the design curves are given.

The jet power that can be derived from the heated air is proportional to the heat dissipation and is approximately proportional to the square of the airplane speed, and to the reciprocal of the absolute temperature of the atmosphere. A table of jet power per 100 horsepower of heat dissipation at various airplane speeds and altitudes is presented.

INTRODUCTION

The mathematical analysis of radiator design presented in this paper shows that the volume of the radiator using least power is practically independent of the shape of the radiator and is given by a single expression involving the design conditions. The present analysis, then unpublished, suggested the possibility of constructing the generalized design chart for radiators given in reference 1. The generalized radiator chart gives the complete picture of all radiator designs. This paper, however, presents tables and charts that permit a more rapid determination of the design requirements of radiator installations with the tube diameters available today. Similar tables and charts can be worked up for any other tube diameters that may be used in the future.

SYMBOLS

The following symbols are used in the report and are listed alphabetically for ready reference. Any units may be used in connection with these symbols as long as the basic equations are dimensionally satisfied.

- A total radiator frontal area, square feet
- A_o open frontal area of radiator, square feet
- c_1, c_2 dimensionless constants
- c_p specific heat at constant pressure, Btu per pound per $^{\circ}\text{F}$
- C_D drag coefficient of wing
- C_L lift coefficient of wing
- D hydraulic diameter, feet
- f free-area ratio, ratio of open frontal area to total frontal area
- g acceleration of gravity, 32.2 feet per second per second

H	quantity of heat dissipated, Btu per second or horsepower
k	thermal conductivity, Btu per square foot per $^{\circ}\text{F}$ per foot per second
L	tube length, feet
M	mass flow of fluid per unit time, pounds per second
N	nondimensional quantity
p	static pressure, pounds per square foot
p_0	atmospheric static pressure, pounds per square foot
p_1	absolute static pressure in entrance of radiator, pounds per square foot
Δp	static-pressure difference, pounds per square foot
Δp_e	pressure drop due to exit loss, pounds per square foot
Δp_f	pressure drop due to skin friction, pounds per square foot
Δp_{fem}	pressure drop due to skin friction, exit loss, and momentum loss, pounds per square foot
P	power, foot-pounds per second or horsepower
P_{jet}	jet power, foot-pounds per second or horsepower
P_t	total power expenditure chargeable to radiator, foot-pounds per second or horsepower
P_D	power required to force air through radiator, foot-pounds per second or horsepower
P_W	power required to support and propel weight of radiator, foot-pounds per second or horsepower
q	dynamic pressure ($\frac{1}{2} \rho V^2$), pounds per square foot
q_1	dynamic pressure in entrance of radiator, pounds per square foot

R Reynolds number $(\rho V D / \mu)$

T_0 absolute temperature of free air stream, $^{\circ}\text{F}$

T_1 absolute temperature of air in entrance of radiator, $^{\circ}\text{F}$

T_2 absolute temperature of air in exit of radiator, $^{\circ}\text{F}$

T_w average absolute temperature of tube wall, $^{\circ}\text{F}$

V velocity, feet per second

V_0 free stream velocity, feet per second

V_1 average velocity of air in entrance of radiator,
feet per second

V_2 average velocity of air in exit of radiator, feet
per second

V_3 average velocity of air in exit of duct, feet per
second

W mechanical power, foot-pounds per second

W_r weight of radiator, pounds

y nondimensional quantity
$$\left[\frac{1 - e^{-4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D}}}{4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D}} \right]$$

α exit-loss factor $\left(\frac{\Delta P_e}{\rho_2 V_2^2} \right)$

β heating factor $\left[1 + \frac{T_w - T_1}{T_1} \left(1 - \frac{f^2}{2} \right) \right]$

γ ratio of specific heat at constant pressure to specific heat at constant volume for air

ϵ dimensionless factor by which to multiply radiator weight to account for additional required airplane structure (For calculations, ϵ is taken as 1.5.)

η_p pump efficiency of duct with radiator installed

η_t heat-transfer efficiency $\left(\frac{T_2 - T_1}{T_w - T_1} \right)$

μ coefficient of viscosity, slugs per foot per second

μ_1 coefficient of viscosity of air in radiator entrance, slugs per foot per second

ρ air density, slugs per cubic foot

ρ_0 density of air in free stream, slugs per cubic foot

ρ_1 density of air in radiator entrance, slugs per cubic foot

ρ_2 density of air in radiator exit, slugs per cubic foot

ρ_r density of radiator based on open volume (W_r/LA_0) , pounds per cubic foot

Generalized parameters:

$$A' = \frac{A_0 D}{LA_0}$$

$$M' = \frac{M}{H} \frac{T_w - T_1}{T_w - T_1}$$

$$\Delta p' = \frac{\sqrt{\frac{\rho_1 \Delta p}{\beta D^2}}}{H} \frac{LA_0 (T_w - T_1)}{LA_0 (T_w - T_1)}$$

$$P_D' = \frac{\left[P_D \left(\frac{\rho_1 LA_0}{D} \right)^2 \right]^{1/3}}{H} \frac{T_w - T_1}{T_w - T_1}$$

RADIATOR DESIGN

The power chargeable to a radiator installation in either a wing or an engine nacelle is composed of two parts: the power required to force the cooling air through the radiator and the duct system and the power required to carry the radiator and its supports. The relations existing between the power expenditure, the radiator dimensions, and the heat dissipation have been analyzed in appendix A. This analysis shows that the radiator volume requiring least power expenditure is:

$$\text{Optimum } LA=N \frac{H}{2gc_p(T_w-T_1)f} \left(\frac{5D^3}{\eta_p \epsilon \frac{C_D}{C_L} V_o \rho_r \rho_1^2 \mu_1^{1/2} c_2^{5/2}} \right)^{2/7}$$

where

$$N = 1.15 (1 + 1.70 \alpha)^{2/7}$$

Tables I through IV have been developed from this equation for the range of operating conditions encountered in practice. Each table gives the optimum radiator volume per 100 horsepower of heat dissipation for ethylene-glycol honeycomb-radiator installations for a given airplane speed as a function of the altitude and the product $\eta_p \epsilon \frac{C_D}{C_L}$. The assumed design conditions follow:

Circular tubes of 0.25-inch diameter with a tube-wall thickness of 0.005 inch are used. The minimum liquid passageway is 0.028 inch, and the ratio of the open to the total frontal area f is 0.68. The exit-loss factor is 0.1 in this case. (See reference 2.) The radiator weight density ρ_r is 98 pounds per cubic foot of open volume. The ethylene-glycol temperature has been taken as 52° F below the boiling point of a 97-percent glycol solution at each altitude and is given in the following table:

Altitude (ft)	Ethylene-glycol temperature °F
Sea level	290
5,000	284
10,000	274
15,000	264
20,000	253
25,000	242
30,000	231
35,000	220
40,000	209

The standard atmosphere has been used for the altitude calculations, and adiabatic effects have been included.

When the cooling system is scaled in order to raise the pressure and the operating temperature of the coolant, the optimum radiator volumes are obtained by reducing the values in tables I to IV by the ratio of the temperature differences available for cooling in the nonscaled and scaled systems.

Typical calculation of optimum radiator volume.— The calculation of the ethylene-glycol radiator installation for a 400-mile-per-hour airplane with a 1000-horsepower engine serves as a typical example of how tables I through IV are used. The airplane is supercharged to 20,000 feet; the assumed climbing speed at sea level is 140 miles per hour; and the pumping efficiency of the duct system is

0.90. The value of $\epsilon \frac{C_D}{C_L}$ is 0.15 and the value of $\eta_p \epsilon \frac{C_D}{C_L}$ is 0.90 (0.15) or 0.135.

From table III at $\eta_p \epsilon \frac{C_D}{C_L} = 0.10$ for an altitude of 20,000 feet, 0.459 cubic foot of radiator is required for each 100-horsepower dissipation; for $\eta_p \epsilon \frac{C_D}{C_L} = 0.15$ at 20,000 feet, 0.409 cubic foot per 100 horsepower is required. The optimum radiator for $\eta_p \epsilon \frac{C_D}{C_L} = 0.135$ at 20,000 feet is found by interpolation to be 0.424 cubic foot per 100 horsepower. The radiator installation will be designed for a heat dissipation of approximately one-half the rated power of the engine or 500 horsepower. The

required radiator volume, therefore, is 5×0.424 or 2.12 cubic feet.

Calculation of frontal area of optimum radiator volume.

The optimum radiator volume or surface can now be placed in any position desired. In the following example, the radiator surface will be placed in such a position that it meets the cooling requirements of the engine at sea level. At the climbing speed of 140 miles per hour, 50 pounds per square foot is assumed available for cooling.

In appendix B the pressure drop has been expressed as a function of the dimensions of the radiator. The derived relations have been plotted in figure 1, where the pressure-drop ordinate

$$\frac{\sqrt{\frac{\rho_1 \Delta p}{\beta D^5}}}{H} \cdot LA_0 (T_w - T_1)$$

is evaluated for sea-level conditions as

$$\frac{\sqrt{\frac{(0.002378)(50)}{1.343 \left(\frac{1}{48}\right)^5}}}{500} = \frac{14.27}{1.50} = 9.5$$

$$0.68(2.12)(231)$$

The radiator has an exit-loss factor of 0.1 and, from figure 1, the L/D of the tubing is therefore 61, or the tube length is 15 inches. The frontal area of the radiator is $\frac{2.12}{15/12}$ or 1.70 square feet.

Calculations of total power expenditure and mass flow of air.— After the radiator installation is determined, the power expended and the mass flow of air required for cooling at 20,000 feet can be calculated. The operating conditions are:

Heat dissipation, H, horsepower	500
Free stream velocity, V_o , miles per hour	400
Average absolute temperature of tube wall, T_w (52° F below boiling point of a 97-percent glycol solution at 20,000 feet), °F absolute	713
Absolute temperature of free air stream, at 20,000 feet, T_o , °F absolute	447.7
Adiabatic temperature rise at 400 miles per hour, °F	28.5
Temperature of air at entry, T_1 , °F absolute	476
Available temperature difference, $T_w - T_1$, °F	237
Air density at 20,000 feet, ρ_o , slugs per cubic foot	0.001267
Density of air at radiator entrance after adia- batic compression, ρ_1 , slugs per cubic foot	0.001469

The part of the total power expended in pushing air through the radiator will be calculated first:

$$A = 1.70 \text{ square feet}$$

$$\frac{L}{D} = 61$$

From figure 1, $P_D' = 0.298$

$$P_D' = \frac{\left[P_D \left(\frac{\rho_1 L A_o}{D} \right)^2 \right]^{1/3}}{H / (T_w - T_1)}$$

Therefore

$$0.298 = \frac{\left\{ P_D \left[\frac{(0.001469) (0.68) (2.12)}{\frac{1}{48}} \right]^2 \right\}^{1/3}}{\frac{500}{237}}$$

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or

$$P_D = \frac{(0.628)^3}{(0.1018)^2}$$
$$= 23.9 \text{ horsepower}$$

The power required to carry the weight of the radiator is:

$$P_W = \epsilon \frac{C_D}{C_L} V_o W_r = (0.15) (586) (143)$$
$$= 12,500 \text{ foot-pounds per second}$$
$$= 22.8 \text{ horsepower}$$

The total power expenditure is therefore

$$P_t = \frac{23.9}{0.9} + 22.8 = 26.6 + 22.8 = 49.4 \text{ horsepower}$$

If the space is available, the optimum radiator volume can be shaped with a bigger frontal area and shorter tubes. The power expenditure and the pressure drop required for cooling will be reduced because of the smaller L/D of the tubes (fig. 1).

The mass flow of air required for cooling at 20,000 feet is:

From figure 1,

$$M' = 5.16$$

where

$$M' = \frac{M}{H/(T_w - T_1)}$$

therefore,

$$M = 5.16 \left(\frac{500}{237} \right)$$

$$= 10.9 \text{ pounds per second}$$

General use of design curves of figure 1.— The design curves of figure 1 make possible a quick determination of

the required pressure drop, of the required mass flow of cooling air, and of the power required to push the cooling air through the radiator for any radiator whether it has the optimum volume or not. These curves should, therefore, be of great value to designers of cooling installations who are often presented with the problem of choosing a radiator from several radiators suggested for a given installation by various radiator manufacturers. The optimum design, of course, serves as a general guide in the selection of the radiator. It does not, however, in itself specify the best radiator of a group including radiators of different lengths and frontal areas. Such a specification can come only from the calculation of the detailed performance of the various radiators. The curves of figure 1 reduce the labor involved in such calculations.

Jet power.-- The thrust power that results from the heat added to the air by the radiator is treated in appendix C. This thrust, or jet, power is proportional to the heat dissipation and to the square of the airplane speed. The jet power varies with the altitude inversely as the atmospheric temperature. The jet power per 100 horsepower of heat dissipation for various airplane speeds and various altitudes is given in table V. At high airplane speeds the radiator installation exerts a net thrust.

CONCLUSIONS

A formula has been developed which shows that the optimum radiator volume is essentially a function of only four parameters; namely, the heat dissipation, the airplane speed, the altitude, and a parameter involving characteristics of the airplane. It is useful to note that the volume of the most efficient radiator is independent of the frontal area of the radiator. For a given set of conditions the optimum radiator volume is fixed; the frontal area of the radiator can then be separately specified in order to satisfy certain other requirements, such as the ability to cool with the pressure drop available while the airplane is climbing. The larger the radiator frontal area, the smaller are the power expenditure and the pressure drop required for cooling. If, therefore, the size of the duct permits the use of a larger radiator frontal area than the frontal area of the radiator using all the pressure drop available while the airplane is climbing, the radiator with larger frontal area should be used.

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APPENDIX A

THEORETICAL ANALYSIS OF RADIATOR DESIGN

The power chargeable to an installation of a radiator in a wing or in an engine nacelle is composed of two parts: the power required to force the cooling air through the radiator and duct system P_D/η_p and the power required to carry the radiator and its supports P_W . The power required to circulate the coolant is negligible.

$$P_t = \frac{P_D}{\eta_p} + P_W = \frac{A_o V_1 \Delta p}{\eta_p} + \epsilon \frac{C_D}{C_L} V_o \rho_r L A_o \quad (1)$$

The pressure drop across a radiator has been given in reference 2 as

Δp_{fen} = momentum loss + friction loss + exit loss

$$= \left(1 - \frac{f^2}{2}\right) \left(\frac{T_2}{T_1} - 1\right) \rho_1 V_1^2 + \left(\frac{\Delta p_f}{q_1}\right) q_{mean} + \alpha \rho_2 V_2^2$$

where

$$q_{mean} = \frac{\rho_1 V_1^2 + \rho_2 V_2^2}{4}$$

This relation will be put into a more convenient form.

Heat-transfer and friction data (for example, figs. 2 and 3 taken from reference 2) show that, in the turbulent region, Nusselt's number is proportional to the eight-tenths power of Reynolds number and the friction factor is inversely proportional to the two-tenths power of Reynolds number. The heat-transfer efficiency η_t ,

which is defined as the ratio $\frac{T_2 - T_1}{T_w - T_1}$, is therefore given by the equation

$$\eta_t = \frac{T_2 - T_1}{T_w - T_1} = 1 - 0.4 c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D} \quad (2)$$

(See reference 2.)

In addition, the friction can be correlated with the heat-transfer efficiency as in equation (3)

$$\frac{\Delta p_f}{4q_1} = c_2 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D} = \frac{c_2}{4c_1} \log_e \frac{1}{1 - \eta_t} \quad (3)$$

Radiators have straight frictional passages on the air side and, in such cases, the coefficient c_2 is twice the coefficient c_1 .

If these relations, the equation of continuity $\rho_1 V_1 = \rho_2 V_2$, and Charles' gas law $\frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}$ are used, the equation for the pressure drop becomes

$$\Delta p_{fem} = \rho_1 V_1^2 \left[\alpha \left(1 + \frac{T_w - T_1}{T_1} \eta_t \right) + \left(1 - \frac{f^2}{2} \right) \left(\frac{T_w - T_1}{T_1} \eta_t \right) + \left(\frac{1}{2} \log_e \frac{1}{1 - \eta_t} \right) \left(2 + \frac{T_w - T_1}{T_1} \eta_t \right) \right] \quad (4)$$

The heat dissipation of a radiator is given by the heat-balance equation (5).

$$H = M c_p (T_2 - T_1) = g \rho_1 V_1 A_o c_p (T_w - T_1) \eta_t \quad (5)$$

For a given heat dissipation H at a given available temperature difference $T_w - T_1$, there is a wide range of radiators using various values of Δp , P_t , η_t , V_1 , A_o , and L . Four equations, equations (1), (2), (4), and (5), relate these six quantities. Thus, only two independent variables exist. The total power expenditure therefore describes a unique surface in a three-dimensional plot against two such independent variables as the radiator frontal area and the mass flow of cooling air. Each point on this surface represents a specific radiator design with a definite pressure drop. Because a radiator is, as far as the physical phenomena occurring are concerned, a simplified intercooler, an absolute minimum to the power expenditure exists just as it does in intercooler design

(reference 3) when the mass flow of cooling air and the frontal area are very large. In practice, an infinite mass flow of cooling air would not be ideal because of the high duct losses that would exist and because the duct size limits the amount of frontal area that a radiator can profitably use.

The best radiator of a given frontal area is found in the analysis that follows. The effects of the losses due to heating on the optimum volume have been found to be negligible and are omitted for simplicity.

The total power expenditure when heating effects are neglected is

$$P_t = \frac{A_o V_1}{\eta_p} \left(\frac{\rho_1 V_1^2}{2} \right) \frac{\Delta P_f}{\frac{\rho_1 V_1^2}{2}} + \epsilon \frac{C_D}{C_L} V_o \rho_r L A_o$$

When equation (3) is substituted for $\frac{\Delta P_f}{\frac{\rho_1 V_1^2}{2}}$, the total

power expenditure is obtained as a function of the variables LA_o , A_o , and V_1 . The velocity of the air in the tubes V_1 can be expressed as a function of LA_o and A_o by means of the equation for the heat dissipation, equation (5).

$$H = g \rho_1 V_1 A_o c_p (T_w - T_1) \eta_t$$

$$= g \rho_1 V_1 A_o c_p (T_w - T_1) 4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D} y$$

where

$$y = \frac{1 - e^{-4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D}}}{4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \frac{L}{D}}$$

The variation of y with the air velocity is about half the variation of η_t with the air velocity, shown in figure 4, and is therefore negligible.

With V_1 eliminated, the total power expenditure becomes

$$P_t = \frac{\rho_1 H^{3.5}}{\eta_p \left[4c_1 \frac{LA_o}{D} \left(\frac{\mu_1}{\rho_1 D} \right)^{0.2} \right]^{2.5} \left[y c_p (T_w - T_1) g \rho_1 \right]^{3.5} + \epsilon \frac{C_D}{C_L} V_o \rho_r LA_o} \quad (6)$$

Equation (6) is differentiated with respect to the radiator volume, the frontal area in the y term being held constant, and the derivative is set equal to zero in order to obtain the conditions required for minimum power expenditure. In this way equation (7) is obtained:

$$\left[\frac{LA_o}{H} \frac{1}{c_p (T_w - T_1) g \rho_1} \right]^{3.5} \epsilon \frac{C_D}{C_L} \rho_r V_o \eta_p \left[\frac{4c_1}{D} \left(\frac{\mu_1}{\rho_1 D} \right)^{0.2} \right]^{2.5} = \frac{5}{2} \rho_1 \quad (7)$$

$$= \frac{y + 1.4 (1 - \eta_t - y)}{y^{4.5}}$$

or optimum LA_o

$$= N \frac{H}{2g c_p (T_w - T_1)} \left(\frac{5D^3}{\epsilon \frac{C_D}{C_L} V_o \rho_r \eta_p \rho_1^2 \mu_1^{0.5} c_2^{2.5}} \right)^{2/7} \quad (8)$$

where

$$N = \left[\frac{y + 1.4 (1 - \eta_t - y)}{y^{4.5}} \right]^{2/7}$$

The magnitude of N is determined by the frontal area and is unity for the "ideal" radiator of infinite frontal area. The quantity N can therefore be thought of as the ratio of the volume of the most efficient radiator of a given frontal area to the volume of the ideal radiator. Figure 5 shows how N varies with the heat-transfer efficiency. In practice η_t may lie between 0.3 and 0.6. The corresponding values of N are 1.11 and 1.22.

The value of N can therefore be assumed to be 1.15 and the most efficient radiator volume can be directly calculated, within a few percent, from the existing physical conditions by means of equation (8).

The exit loss of the radiator may be considered as a ducting loss and may be included in the duct pumping efficiency. Because a knowledge of the magnitude of the exit loss of the radiator is more readily available, however, to the radiator designer than to the duct designer, it is convenient to consider the exit loss of the radiator separately from the losses existing in the duct. When the exit loss is considered separately in the analysis, a new term arises in the evaluation of N . The following table shows this new term.

η_t	N
0	$1 (1 + \infty \alpha)^{2/7}$
.095	$1.033 (1 + 2.62\alpha)^{2/7}$
.450	$1.167 (1 + 1.61\alpha)^{2/7}$
.632	$1.23 (1 + 1.495\alpha)^{2/7}$
.865	$0.911 (1 + 5.5\alpha)^{2/7}$

Figure 6 plots the coefficient of the exit-loss term as a function of the heat-transfer efficiency. In practice, η_t lies between 0.3 and 0.6 and, therefore, the exit-loss coefficient may be taken as 1.70 with very little error. The quantity N , is then, for practical purposes,

$$N = 1.15 (1 + 1.70\alpha)^{2/7} \quad (9)$$

The minimum power expenditure is obtained by substituting the formula for the optimum radiator volume (equation (8)) into the equation for power expenditure; then,

$$P_t = \frac{0.4P_W}{(Ny)^{3.5}} + P_W$$

or

$$\frac{P_t}{P_w} = \frac{0.4}{(Ny)^{3.5}} + 1$$

The ratio P_t/P_w is plotted as a function of the frontal area in figure 7. As the frontal-area coefficient decreases below unity, which corresponds, for the conventional size of tubing of about 1/4-inch diameter, to an L/D of about 70, the power expenditure of the radiator increases very rapidly.

APPENDIX B

DERIVATION OF CURVES OF FIGURE 1

The pressure drop across a radiator has been given in appendix A as

$$\Delta p_{fem} = \rho_1 V_1^2 \left[\alpha \left(1 + \frac{T_w - T_1}{T_1} \eta_t \right) + \left(1 - \frac{f^2}{2} \right) \left(\frac{T_w - T_1}{T_1} \eta_t \right) + \left(\frac{1}{2} \log_e \frac{1}{1 - \eta_t} \right) \left(2 + \frac{T_w - T_1}{T_1} \eta_t \right) \right] \quad (4)$$

When V_1 is eliminated by means of the heat-balance equation and the equation is simplified, the pressure drop is obtained as

$$\Delta p_{fem} = \left[\frac{H}{c_p (T_w - T_1) A_o g} \right]^2$$

$$\frac{\left(\alpha + \log_e \frac{1}{1 - \eta_t} \right) \left(1 + \frac{T_w - T_1}{T_1} \eta_t \frac{\alpha + 1 - \frac{f^2}{2} + \frac{1}{2} \log_e \frac{1}{1 - \eta_t}}{\alpha + \log_e \frac{1}{1 - \eta_t}} \right)}{\rho_1 \eta_t^2}$$

The heating term $\frac{T_w - T_1}{T_1} \eta_t \frac{\left(\alpha + 1 - \frac{f^2}{2} + \frac{1}{2} \log_e \frac{1}{1 - \eta_t} \right)}{\alpha + \log_e \frac{1}{1 - \eta_t}}$

can be simplified by neglecting the exit-loss term in both numerator and denominator. When η_t is expanded in a power series, the term further simplifies for practical purposes to $\frac{T_w - T_1}{T_1} \left(1 - \frac{f^2}{2}\right)$ and the pressure drop is given by the equation

$$\Delta P_{fem} = \left[\frac{H}{c_p (T_w - T_1) A_o g} \right]^2 \frac{\left(\alpha + \log_e \frac{1}{1 - \eta_t} \right) \left[1 + \frac{T_w - T_1}{T_1} \left(1 - \frac{f^2}{2} \right) \right]}{\rho_1 \eta_t^2} \quad (10)$$

When the terms are transposed and simplified, this relation is put in the form

$$\Delta P' = \frac{\sqrt{\frac{\Delta P_{fem} \rho_1}{1 + \frac{T_w - T_1}{T_1} \left(1 - \frac{f^2}{2} \right)}}}{\frac{HD}{c_p (T_w - T_1) g L A_o}} = \frac{\frac{L}{D} \left(\log_e \frac{1}{1 - \eta_t} \right)^{1/2} \left(1 + \frac{\alpha}{\log_e \frac{1}{1 - \eta_t}} \right)^{1/2}}{\eta_t} \quad (11)$$

$$= \frac{\frac{L}{D} \left(\log_e \frac{1}{1 - \eta_t} \right)^{1/2} \left(1 + \frac{\alpha}{2 \log_e \frac{1}{1 - \eta_t}} + \dots \right)}{\eta_t}$$

The ratio L/D and the heat-transfer efficiency η_t appear on the right-hand side of the equation. But the heat-transfer efficiency is primarily a function of L/D and depends on the operating Reynolds number only to a slight extent, as shown in figure 4. By the use of an average value of 18,000 for the operating Reynolds number, the heat-transfer efficiency can be found, within a few percent, from the physical dimensions of the radiator. This value has been used in the plot of equation (11) in figure 1. The effect of the exit loss on the pressure-drop ordinate is directly proportional to the magnitude of the exit loss. The pressure-drop curves in figure 1 are therefore applicable to all radiators.

Table VI has been computed from the design curves of figure 1 for an atmospheric temperature of 100° F at sea level. This table gives the square root of the pressure drop in pounds per square foot for each 100-horsepower dissipation per square foot of open frontal area for a 100° F temperature difference for various values of L/D of conventional honeycomb radiators. By means of this table the pressure drop required across any radiator to give adequate cooling at sea level is easily and quickly determined.

The power expenditure of an installation is the sum of the power required to carry the weight of the radiator and its supports and the power required to push the cooling air through the radiator and the duct system. The power required to carry the radiator weight is easily calculated:

$$P_W = \epsilon \frac{C_D}{C_L} V_o W_R$$

The power required to push the cooling air through the radiator is

$$P_D = A_o V_1 \Delta p$$

The pressure drop that is used in calculating the power expenditure of a radiator does not include the pressure drop due to the added momentum of the heated air because this part of the total power expended is returned almost completely as jet power at the duct exit when the airplane is cruising. When V_1 and Δp are eliminated by means of the heat-balance and pressure-drop equations, equations (5) and (10), respectively) the power expended in pushing air through the radiator becomes

$$P_D = \left[\frac{H}{c_p (T_w - T_1) g} \right]^{3/4} \frac{1 + \log_e \frac{1}{1 - \eta_t}}{A_o^2 \rho_1^2 \eta_t^3}$$

When these terms are transposed and simplified, this equation is put in the form

$$\begin{aligned}
 P_D' &= \frac{\sqrt[3]{P_D \left(\frac{\rho_1 L A_o}{D} \right)^2}}{\frac{H}{c_p (T_w - T_1) \eta_t}} = \frac{\left(1 + \frac{\alpha}{\log_e \frac{1}{1-\eta_t}} \right)^{1/3} \left(\frac{L}{D} \right)^{2/3} \left(\log_e \frac{1}{1-\eta_t} \right)^{1/3}}{\eta_t} \\
 &= \frac{\left(1 + \frac{1}{3} \frac{\alpha}{\log_e \frac{1}{1-\eta_t}} + \dots \right) \left(\frac{L}{D} \right)^{2/3} \left(\log_e \frac{1}{1-\eta_t} \right)^{1/3}}{\eta_t} \quad (12)
 \end{aligned}$$

Equation (12) was used in plotting the cooling power curves in figure 1. Again the effect of exit loss upon the ordinate is directly proportional to the magnitude of the exit loss and therefore the curves are easily applied to the calculations for any radiator.

The mass flow of air required for cooling can be determined in terms of the amount of heat dissipation and the available temperature difference from the L/D of the radiator tubing. For this purpose the equation for the heat dissipation of a radiator, $H = Mc_p(T_w - T_1) \eta_t$, is put in the form

$$M' = \frac{M}{\frac{H}{(T_w - T_1)}} = \frac{1}{c_p \eta_t}$$

Figure 1 shows how the mass-flow factor depends upon the L/D of the radiator tubing.

APPENDIX C

JET EFFECT

Some of the heat added to the air in the radiator can be converted into thrust power. In reference 2 it is shown that the mechanical power which can theoretically be recovered is

$$W = 778H \cdot \left(1 - \frac{T_o}{T_1} \right) = 778H \left[1 - \left(\frac{p_o}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (13)$$

The temperature of the air in the entrance of the radiator T_1 is greater than the atmospheric temperature T_0 by the amount of temperature rise caused by the adiabatic compression of the air ahead of the radiator.

$$T_1 = T_0 + \frac{1}{2c_p} \frac{V_0^2}{778g}$$

If this relation is substituted in equation (13), the mechanical power that can theoretically be recovered becomes

$$\frac{W}{778H} = \frac{\frac{V_0^2}{1556gc_p}}{T_0 + \frac{V_0^2}{1556gc_p}} = \frac{\frac{V_0^2}{1556gc_p T_0}}{1 + \frac{V_0^2}{1556gc_p T_0}} \quad (14)$$

At a speed of 500 miles per hour the adiabatic rise of temperature $\frac{V_0^2}{1556gc_p}$ is 44.6° F . Even at this high

speed $\frac{V_0^2}{1556gc_p T_0}$ is less than 0.1. The second term in the denominator of equation (14) is consequently always very small as compared with unity, and W/H is practically proportional to V_0^2/T_0 .

The efficiency of conversion of recoverable mechanical power to thrust power is $\frac{2V_0}{V_0 + V_3}$ or $\frac{2}{1 + \frac{V_3}{V_0}}$. The ratio $\frac{V_3}{V_0}$ is equal to $\sqrt{\frac{T_2}{T_1}}$. The jet efficiency therefore is

$$\frac{2V_0}{V_0 + V_3} = \frac{2}{1 + \sqrt{\frac{T_2}{T_1}}} = \frac{1}{1 + \frac{1}{4} \frac{T_w - T_1}{T_1} \eta_t} \quad (15)$$

By a combination of equations (14) and (15), the power derivable from the jet is obtained as

$$P_{\text{jet}} = \frac{\frac{V_o^2 H}{2gc_p T_o}}{\left(1 + \frac{1}{4} \frac{T_w - T_1}{T_1} \eta_t\right) \left(1 + \frac{V_o^2}{1556gc_p T_o}\right)} \quad (16)$$

The radiator design and the magnitude of the jet power are connected only through the η_t term of the jet efficiency. The jet efficiency is practically 100 percent and varies only a few percent in practice. No matter what radiator is installed to dissipate the required amount of heat H , the jet power obtained will be the same. The jet power, therefore, does not affect the optimum radiator design.

Because the jet efficiency is practically constant, the jet power is directly proportional to the heat dissipation and to the square of the speed of the airplane. Also the jet power is greater at altitude because of the lowered atmospheric temperature.

V Table (VI) gives the jet power per 100 horsepower of heat dissipation as a function of the altitude and of the speed of the airplane. (The standard atmosphere has been assumed.) At high airplane speeds the jet power exceeds the power required to carry the radiator weight and to push the air through the radiator and the duct system. In other words, the radiator installation becomes a source of net thrust.

REFERENCES

1. Brevoort, M. J.: Radiator Design. NACA A.C.R., July 1941.
2. Brevoort, M. J., and Leifer, M.: Radiator Design and Installation. NACA A.C.R., May 1939.
3. Joyner, Upshur T.: Mathematical Analysis of Aircraft Intercooler Design. T.N. No. 781, NACA, 1940.

TABLE I - RADIATOR VOLUME IN CUBIC FEET FOR EACH
100-HORSEPOWER DISSIPATION
($V_o = 200$ mph)

Altitude (ft)	$\eta_p \epsilon \frac{C_D}{C_L}$			
	0.05	0.10	0.15	0.20
Sea level	0.545	0.447	0.398	0.367
5,000	.564	.463	.413	.380
10,000	.597	.489	.435	.401
15,000	.635	.521	.464	.427
20,000	.682	.560	.499	.459
25,000	.732	.601	.535	.494
30,000	.794	.652	.580	.535
35,000	.860	.705	.628	.578
40,000	1.024	.839	.747	.688

TABLE II - RADIATOR VOLUME IN CUBIC FEET FOR EACH
 100-HORSEPOWER DISSIPATION
 ($V_o = 300$ mph)

Altitude (ft)	$\eta_p \epsilon \frac{C_D}{C_L}$			
	0.05	0.10	0.15	0.20
Sea level	0.484	0.397	0.354	0.326
5,000	.500	.410	.365	.336
10,000	.526	.432	.385	.354
15,000	.558	.457	.408	.375
20,000	.598	.490	.437	.403
25,000	.642	.526	.469	.432
30,000	.692	.568	.506	.466
35,000	.751	.616	.548	.505
40,000	.894	.732	.652	.601

TABLE III - RADIATOR VOLUME IN CUBIC FEET FOR EACH
100-HORSEPOWER DISSIPATION
($V_0 = 400$ mph)

Altitude (ft)	$\eta_p \propto \frac{C_D}{C_L}$			
	0.05	0.10	0.15	0.20
Sea level	0.459	0.377	0.336	0.309
5,000	.472	.387	.345	.318
10,000	.497	.408	.363	.334
15,000	.525	.430	.384	.352
20,000	.560	.459	.409	.376
25,000	.601	.494	.440	.406
30,000	.645	.529	.471	.434
35,000	.696	.571	.509	.468
40,000	.830	.681	.606	.558

TABLE IV - RADIATOR VOLUME IN CUBIC FEET FOR EACH
100-HORSEPOWER DISSIPATION
($V_0 = 500$ mph)

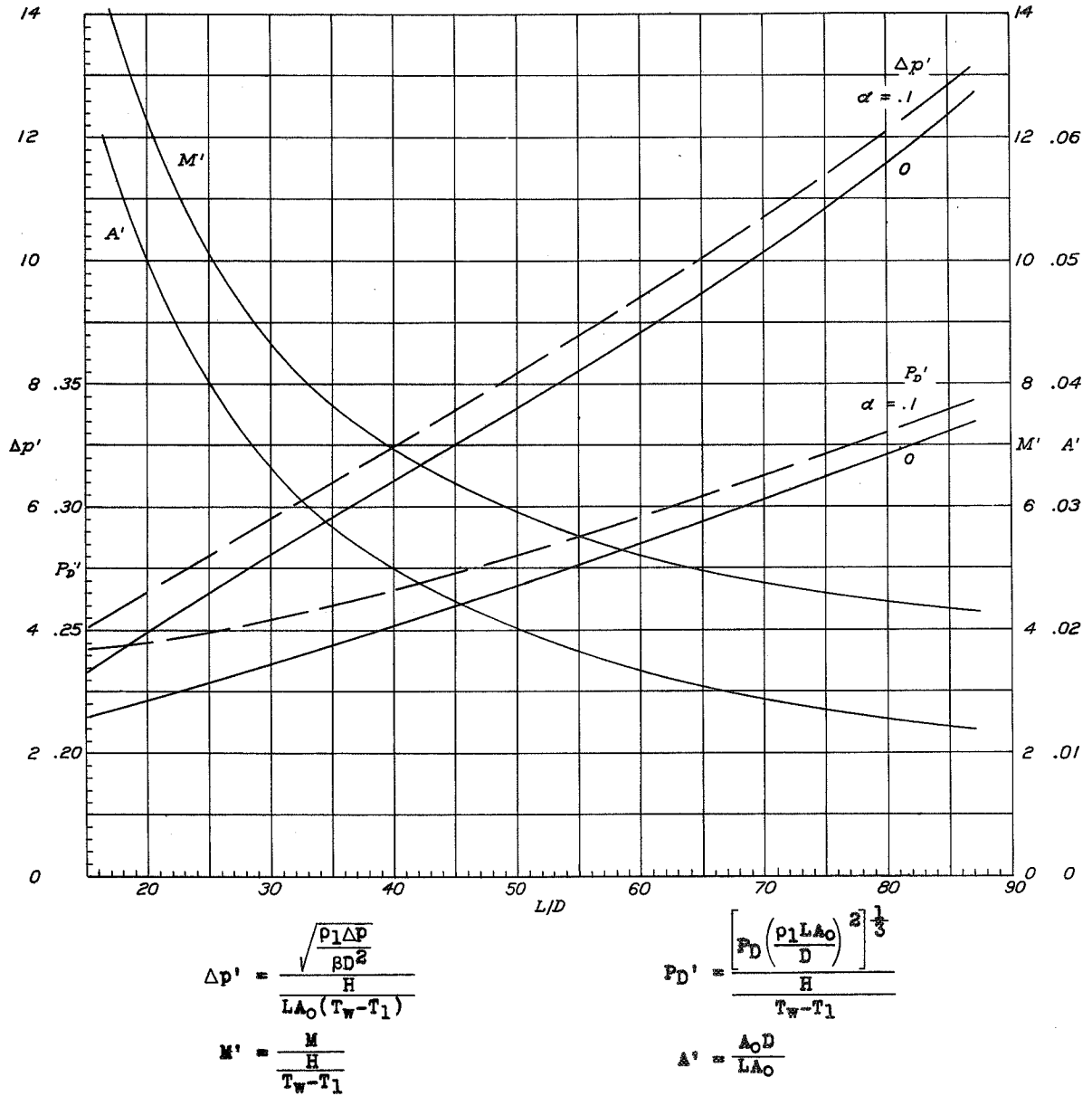
Altitude (ft)	$\eta_p \epsilon \frac{C_D}{C_L}$			
	0.05	0.10	0.15	0.20
Sea level	0.452	0.371	0.330	0.304
5,000	.461	.378	.337	.310
10,000	.483	.397	.353	.326
15,000	.508	.417	.371	.343
20,000	.540	.444	.395	.364
25,000	.577	.473	.422	.389
30,000	.617	.506	.451	.415
35,000	.667	.547	.487	.449
40,000	.798	.654	.582	.537

TABLE V - JET POWER IN HORSEPOWER FOR EACH
100-HORSEPOWER HEAT DISSIPATION

Altitude (ft)	Jet power per 100-horsepower heat dissipation (hp)			
	$V_o = 200$ mph	$V_o = 300$ mph	$V_o = 400$ mph	$V_o = 500$ mph
Sea level	1.36	2.99	5.21	7.91
5,000	1.41	3.10	5.39	8.17
10,000	1.46	3.21	5.58	8.45
15,000	1.51	3.32	5.76	8.73
20,000	1.57	3.45	5.98	9.04
25,000	1.63	3.59	6.22	9.39
30,000	1.70	3.74	6.47	9.76
35,000	1.78	3.90	6.75	10.16
40,000	1.79	3.91	6.77	10.19

TABLE VI - PRESSURE DROP ACROSS RADIATOR AT SEA LEVEL

$\frac{L}{D}$	$\sqrt{\Delta p_{fem}}$ lb/sq ft per 100-horsepower dissipation per square foot of open frontal area for 100° F available temperature difference
30	4.51
40	4.08
50	3.83
60	3.68
70	3.59
80	3.54

Figure 1.- Radiator design chart. H and P_D are in horsepower.

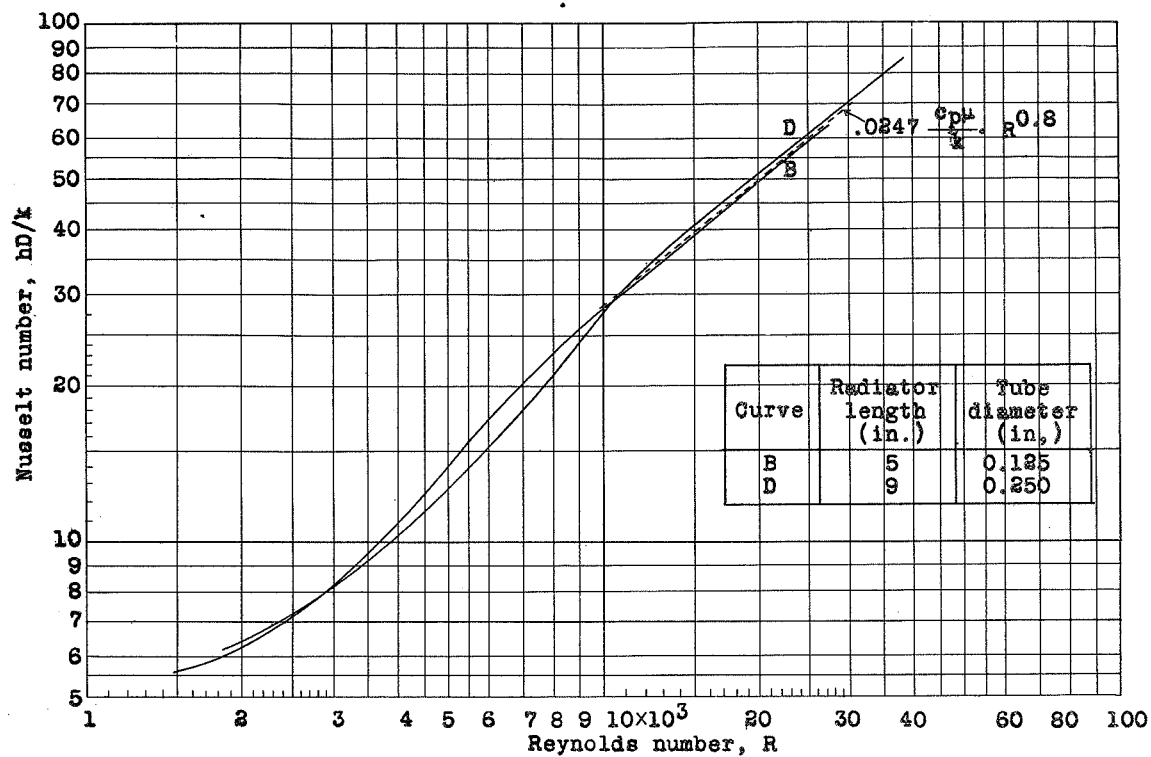


Figure 2.- Heat-transfer data taken from reference 2.

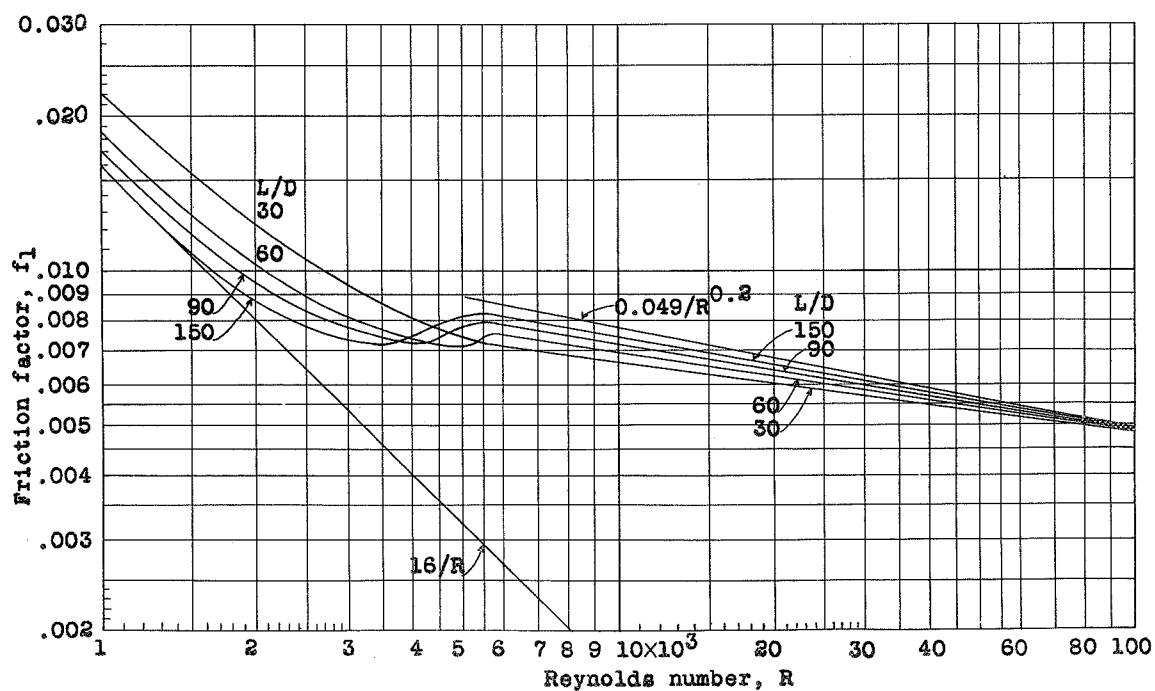


Figure 3.- Friction data taken from reference 2.

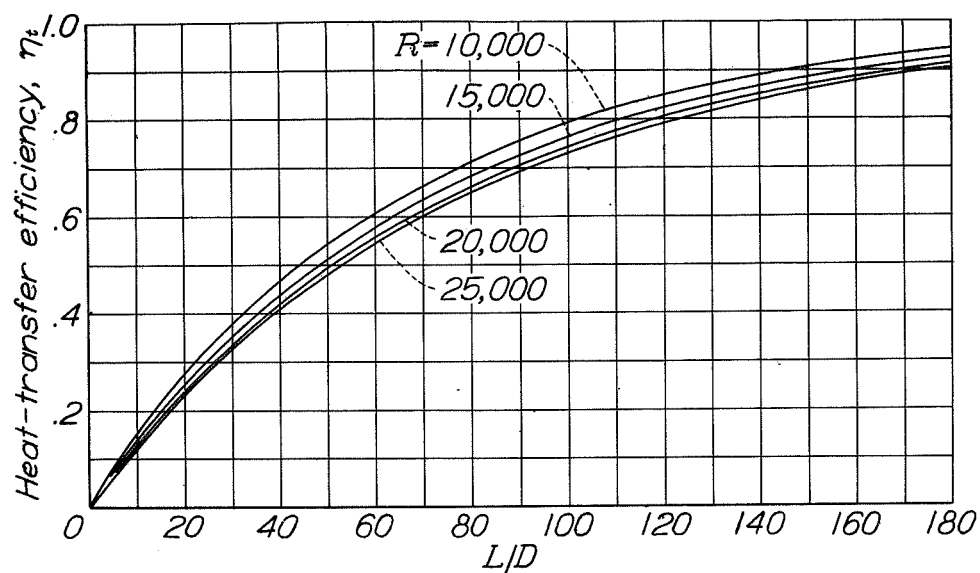


Figure 4.- The heat-transfer efficiency η_t as a function of L/D .

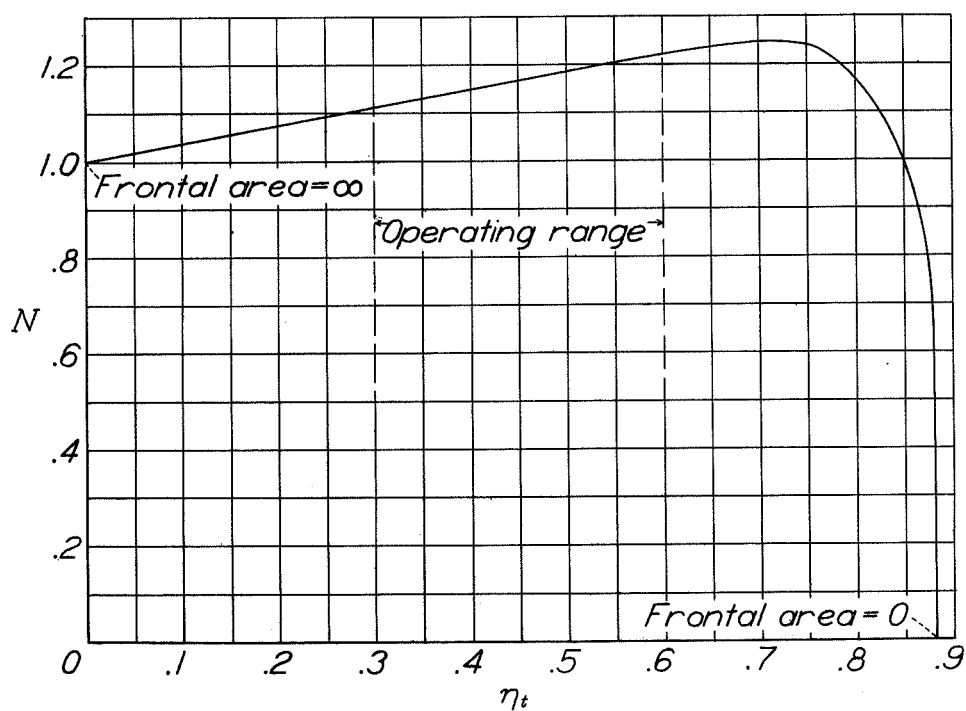


Figure 5.- The quantity N as a function of η_t .

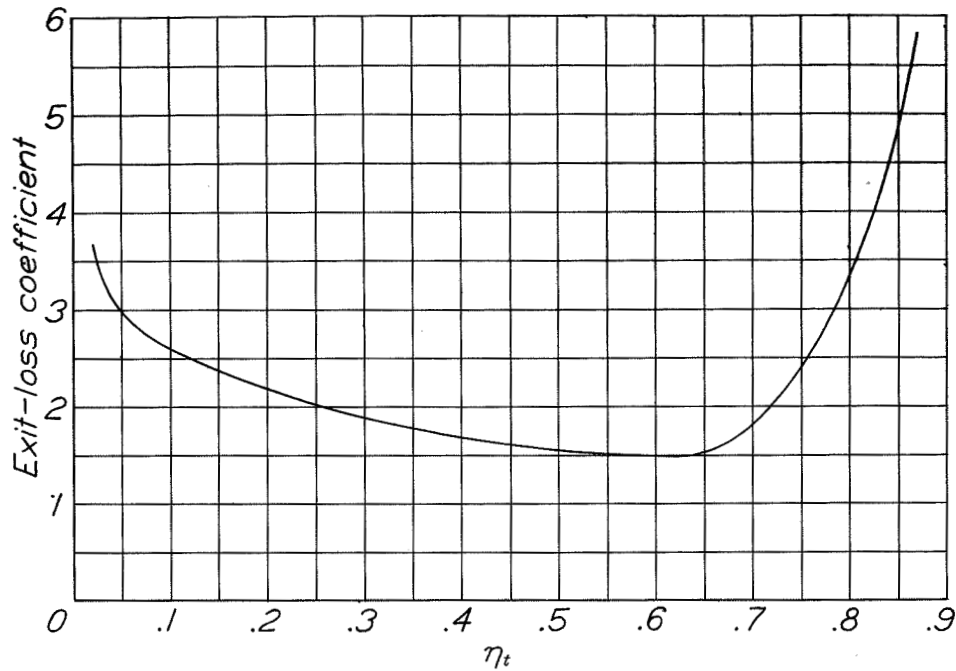


Figure 6.- The coefficient of the exit-loss term of N as a function of η_t .

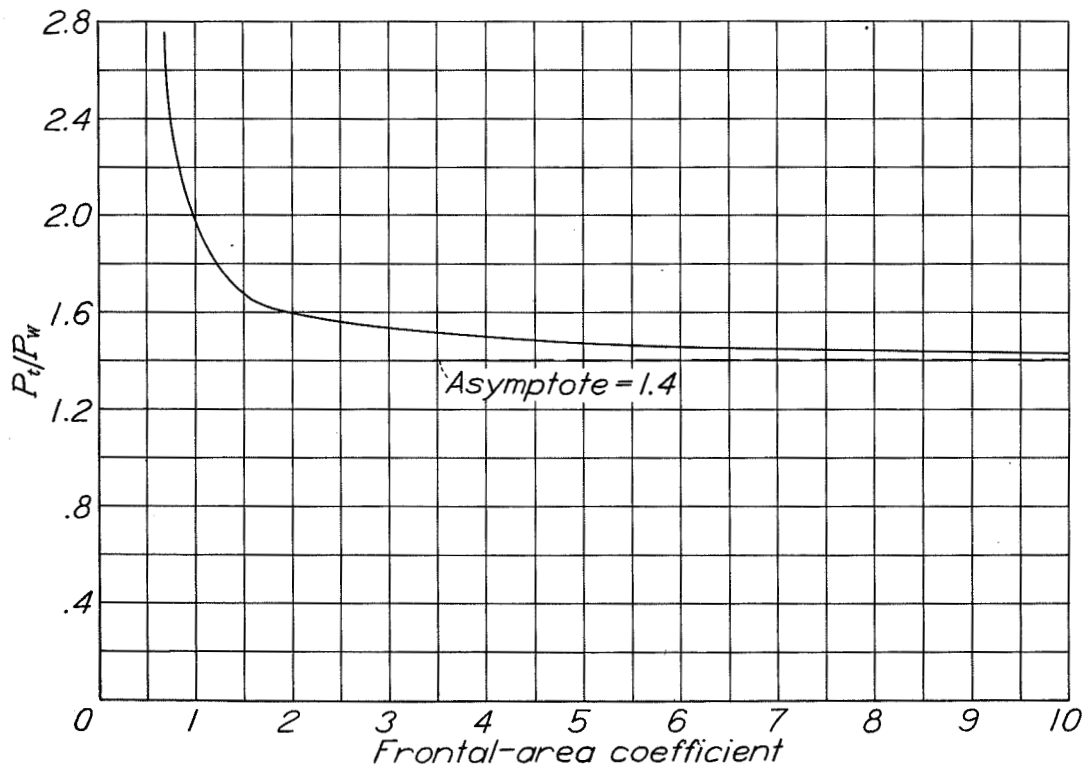


Figure 7.- The power expenditure of the most efficient radiators as a function of the frontal-area coefficient,

$$\frac{A_0 D}{\text{Optimum } (LA_0) \left[4c_1 \left(\frac{\mu_1}{\rho_1 V_1 D} \right)^{0.2} \right]}$$