Bayes Analysis and Reliability Implications of Stress-Rupture Testing a Kevlar/Epoxy COPV using Temperature and Pressure Acceleration

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ABSTRACT. Composite Overwrapped Pressure Vessel (COPVs) that have survived a long service time under pressure generally must be recertified before service is extended. Sometimes lifetime testing is performed on an actual COPV in service in an effort to validate the reliability model that is the basis for certifying the continued flight worthiness of its sisters. Currently, testing of such a Kevlar49®/epoxy COPV is nearing completion. The present paper focuses on a Bayesian statistical approach to analyze the possible failure time results of this test and to assess the implications in choosing between possible model parameter values that in the past have had significant uncertainty. The key uncertain parameters in this case are the actual fiber stress ratio at operating pressure, and the Weibull shape parameter for lifetime; the former has been uncertain due to ambiguities in interpreting the original and a duplicate burst test. The latter has been uncertain due to major differences between COPVs in the data base and the actual COPVs in service. Any information obtained that clarifies and eliminates uncertainty in these parameters will have a major effect on the predicted reliability of the service COPVs going forward. The key result is that the longer the vessel survives, the more likely the more optimistic stress ratio is correct. At the time of writing, the resulting effect on predicted future reliability is dramatic, increasing it by about one "nine", that is, reducing the probability of failure by an order of magnitude. However, testing one vessel does not change the uncertainty on the Weibull shape parameter for lifetime since testing several would be necessary..

I. Introduction

We study the effects of various test parameter choices on possible outcomes from the accelerated stress-rupture testing of a single, 40-inch diameter, Kevlar49®/epoxy composite-overwrapped pressure vessel (COPV) with a titanium liner, called SN007. In particular we focus on the implications of particular test survival times (or the failure time) on reliability predictions for multiple such vessels in future missions cycles of given time durations. The context is that much prior stress rupture test data is available on the stress-rupture performance of Kevlar49®/epoxy strands and laboratory scale vessels, thus allowing the prediction of reliability for a given stress ratio and time in service, but the details in terms of the yarn denier, the epoxy and the wrap pattern differ significantly. Furthermore, the exact stress ratio (stress level in service divided by maximum stress level from a burst test) is uncertain since (i) only two burst tests had originally been performed but with conflicting results, (ii) quality control measurements in terms of permanent delta volume growth during the proof test (autofrettage) differed by a factor of two across the various production and qualification test units, and (iii) stress analysis based on

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instrumentation of the burst tests and the design configuration led to conflicting stress ratio predictions for the vessels in service. This has resulted in considerable uncertainty in their actual reliability making predictions necessarily more conservative. In addition there was uncertainty in the amount of variability in the lifetime distribution as expressed in terms of the Weibull shape parameter for lifetime. These past analyses and data, discussed in two white papers [1,2], have pointed to two possible values of the stress ratio and two possible values of the Weibull shape parameter for lifetime, but with ambiguity on which pair actually applies to the 40-inch COPVs in question. The purpose of the single stress-rupture test on SN007 was to provide definitive information on which pair of key parameter values best describes the behavior of 40-inch of Kevlar49®/49 COPVs in question, since the predicted lifetime for the two pairs of parameter values differs by more than an order of magnitude.

Since an extensive and fairly consistent data base is available on both ambient and elevated temperature performance of Kevlar49®/epoxy materials and small test vessels, the test was also accelerated in time by a factor of about 40 (under the pessimistic parameter assumptions) using a higher steady temperature than occurs in service (though not higher than has typically occurred in the past during the pressurization phase) but at the maximum operating pressure used in service. This strategy was designed to provide the necessary test information in a few months rather than the 200,000 hours (28.5 years) that would be required under standard service conditions. In fact, two temperature levels were selected to be run in sequence: the first at 130°F was to be applied until the time corresponding to mean reliability of 0.9986 was reached under the most pessimistic stress ratio and Weibull lifetime parameter values. At that point the temperature was to be increased to 160°F and the test continued until the vessel either fails in stress rupture or survives a pre-set time at which a third stress level would be contemplated.

Given the significant prior history and knowledge regarding several of the key model parameters, the problem could be cast in a Bayesian statistical framework and model with uncertainty distributions on all the material stress-rupture parameters (based on statistical analysis of the extensive prior material data sets) as well as discrete Bernoulli uncertainty distributions on the two possible pairs of stress ratios and Weibull lifetime shape parameter values. Specifically, the result of the test are to permit resolution in choosing between (i) two competing stress ratio models (see ref. [2]), an optimistic Model 2, and a pessimistic Model 4 (equivalent to the currentl model) and (ii) two competing values for the Weibull lifetime shape parameter, a value 1.625 based on the Lawrence Livermore National Laboratories (LLNL) data base on small Kevlar49®/epoxy COPVs, and 2.45 based on the authors' study of the NASA-JSC Fleet Leader vessel data, which is more relevant to the Kevlar49®/epoxy 40-inch vessels in question (see ref. [1]. (Originally there were two other models, Model 1 and Model 3, but these proved irrelevant and were drooped from consideration [2].) Uncertainties regarding these two basic parameter pairs are the major driver of the relatively low predicted reliability of these vessels and any shrinking of this uncertainty would yield major benefits in terms of predicted reliability. Available data prior to the test data allows calculation of what is called a Bayesian 'prior' joint uncertainty distribution on all the parameter values as well as the reliability overall. Depending on the lifetime of SN007 observed from the test, Bayes theory allows calculation of a revised 'posterior' uncertainty distribution from which updated reliability predictions can be made for the 40-inch COPVs in question. In essence, the longer the vessel lasts the better the future reliability.

The results of the Bayes analysis calculations show that the longer vessel SN007 survives in the stress rupture test, the more likely it becomes (and dramatically so) that the more optimistic Model 2 for the stress ratio is correct. Beginning the test with probability 0.5 (even 50-50 chance) that Model 2 is correct rather than Model 4, the posterior probability Model 2 is correct rises even higher to 0.95. The ultimate effect on the predicted reliability of the 40-inch COPV in question is dramatic, being of the order of one 'nine' (reduction in the predicted probability of failure by an order of magnitude). The test turns out to be much less effective in choosing between the two competing lifetime shape parameter values, though there is a slight shift in favor of the more optimistic value, as the vessel survives longer and longer. Regarding the two competing shape parameter values, the case of putting three identical vessels on test is considered, and if they all fail fairly close together in time, a prior probability of 0.5 that the optimistic Weibull shape parameter value is correct rises to 0.76. However, to be as effective as in choosing between the two stress ratios at least 6 identical vessels would be required.

II. Reliability Model with Various Uncertain Parameters

For the stress-rupture testing, we consider the application of many sequential future mission cycles each of duration t_{mc} given that the vessel has survived the equivalent of m such mission cycles in the past. These cycles may actually involve pressurization and depressurization at the beginning and end of each cycle or simply be convenient divisions of time under steady pressure. (The overwrap lifetime has been shown to depend primarily on the cumulative time under pressure.) The model for the probability of survival of n such mission cycles is given by

$$
\begin{aligned}
\vec{R}_{007}\left(n \mid m\right) &= \exp\left\{-\left(\frac{\vec{s}_{007}}{\vec{\lambda}_{\text{LLNL}}}\right)^{\vec{\rho}\vec{\beta}}\left[\left(\frac{t_n+t_m}{\vec{t}_{\text{ref}}}\right)^{\vec{\beta}}-\left(\frac{t_m}{\vec{t}_{\text{ref}}}\right)^{\vec{\beta}}\right]\right\} \\
&= \exp\left\{-\left(\frac{\vec{s}_{007}}{\vec{\lambda}_{\text{LLNL}}}\right)^{\vec{\rho}\vec{\beta}}\left(\frac{t_{\text{mc}}}{\vec{t}_{\text{ref}}}\right)^{\vec{\beta}}\left[\left(n+m\right)^{\vec{\beta}}-\left(m\right)^{\vec{\beta}}\right]\right\}
\end{aligned}
\tag{1}
$$

where $t_n = nt_{\text{mc}}$ and $t_m = mt_{\text{mc}}$ and various other parameters in (1) are viewed as random variables with uncertainty distributions based on previous data and the stress ratio and Weibull lifetime shape parameter values. In particular

$\vec{s}_{007} =$ stress ratio on 40-inch vessel SN007,

$\vec{\rho} =$ power-law exponent relating stress ratio to the lifetime scale parameter measured from the LLNL data base,

$\vec{\beta} =$ Weibull shape parameter for lifetime relevant to 40-inch vessels and determined from the LLNL data base and test results from NASA Fleet Leaders,

$\vec{t}_{\text{ref}} =$ characteristic lifetime at stress ratio unity estimated from the LLNL data base,

$\vec{\lambda}_{\text{LLNL}} =$ a random variable reflecting uncertainty in the stress ratios used to generate the LLNL data base as a result of a limited number of burst tests.

Regarding notation, placement of a double-arrow over a parameter, θ, to give, $\vec{\theta}$, means that the parameter is treated as a random variable having an uncertainty distribution that may be one of several types: normal, log-normal, Weibull, Beta or Bernoulli. Each has parameters reflecting central tendency, $\hat{\theta}$, and variability, $\hat{\omega}$. For instance, for a log-normal uncertainty distribution on a parameter, $\ln\vec{\theta} = Normal\left(\ln\hat{\theta},\hat{\omega}\right)$ means $\ln\vec{\theta}$ has a normal uncertainty distribution, which implies $\vec{\theta}$ follows a *lognormal* distribution, where $\hat{\theta}$ is viewed as the mean of $\vec{\theta}$ and $\hat{\omega} = \hat{\sigma}/\hat{\theta}$ is its coefficient of variation (standard deviation, $\hat{\sigma}$, divided by mean, $\hat{\theta}$). Note that in this lognormal case, $\hat{\omega}$ is entered into the normal distribution as though it were a standard deviation parameter. Also, $\vec{\theta} = Beta\left(\hat{a},\hat{b}\right)$ means $\vec{\theta}$ follows a Beta uncertainty distribution on $\left(0,1\right)$ with parameters $\hat{a}$ and $\hat{b}$, which will be characterized later in terms of the mean and coefficient of variation of $\vec{\theta}$. In addition, $\vec{I} = Bernoulli\left(\hat{p}\right)$ means $\vec{I}$ is an indicator random variable that follows a Bernoulli distribution, which means that $\vec{I}$ has possible values 1 or 0 with probabilities p and $1-p$, respectively. Finally $\vec{\theta} = Weibull\left(\hat{\theta},\hat{\alpha}\right)$ means $\vec{\theta}$ follows a Weibull uncertainty distribution with scale parameter, $\hat{\theta}$, and shape parameter, $\hat{\alpha}$. (Note here that $\hat{\theta}$ is not the mean of the uncertainty distribution, since the mean is actually $\hat{\theta}\,\Gamma\left(1+1/\hat{\alpha}\right)$.)

With this background, the uncertainty distributions for the parameters in the model based on prior information (primarily the LLNL vessel data base) are as follows:

$$\ln\vec{\rho} = Normal\left(\ln\hat{\rho},\hat{\sigma}_{\rho}\right) \tag{2}$$

$$\ln\vec{t}_{\text{ref}} = Normal\left(\ln\hat{t}_{\text{ref}} - \hat{\rho}\ln s_{\text{piv}},\hat{\sigma}_{t_{\text{ref}}}\right) + \vec{\rho}\ln\overline{s}_{\text{piv}} \tag{3}$$

where $\overline{s}_{\text{piv}}$ is a constant called the pivot stress ratio (which is the mean of the stress ratios for all specimens for which lifetimes were obtained)

$$\ln\vec{\lambda}_{\text{LLNL}} = Normal\left(\ln\hat{\lambda}_{\text{LLNL}},\hat{\sigma}_{\lambda}\right) \tag{4}$$

Also

$$\vec{\beta} = \vec{\beta}_{\text{Orb}}\vec{I}_{\beta_{\text{Orb}}} + \vec{\beta}_{\text{LLNL}}\left(1-\vec{I}_{\beta_{\text{Orb}}}\right) \tag{5}$$

where

$$\vec{I}_{\beta_{\rm Orb}} = Bernoulli\left(\hat{p}_{\beta_{\rm Orb}}\right) \quad (6)$$

Here (5) indicates is that there will be two competing versions of the Weibull shape parameter for lifetime: one is $\beta_{\rm LLNL}$ the currently accepted value based on the LLNL data base, and one is $\beta_{\rm Orb}$ based on data from Orbiter vessels, mainly the JSC Fleet Leaders discussed elsewhere in a white paper [1]. In (6), $\hat{p}_{\beta_{\rm Orb}}$ is the pre-test or 'prior' probability that $\beta_{\rm Orb}$ is the correct value to use for reliability modeling of the large 40-inch vessels of primary interest. Also

$$\hat{p}_{\beta_{\rm LLNL}} = 1 - \hat{p}_{\beta_{\rm Orb}} \quad (7)$$

is the prior probability $\beta_{\rm LLNL}$, is correct instead. For each of the two Weibull shape parameter choices, the uncertainty distributions are respectively

$$\ln \vec{\beta}_{\rm LLNL} = Normal\left(\ln \hat{\beta}_{\rm LLNL}, \hat{\omega}_{\beta_{\rm LLNL}}\right) \quad (8)$$

and

$$\ln \vec{\beta}_{\rm Orb} = Normal\left(\ln \hat{\beta}_{\rm Orb}, \hat{\omega}_{\beta_{\rm Orb}}\right) \quad (9)$$

The stress ratio on SN007 has the uncertainty structure

$$\vec{s}_{007} = \vec{s}_{007,\rm M2}\vec{I}_{\rm M2} + \vec{s}_{007,\rm M4}\left(1 - \vec{I}_{\rm M2}\right) \quad (10)$$

where

$$\vec{I}_{\rm M2} = Bernoulli\left(\hat{p}_{\rm M2}\right) \quad (11)$$

Here (10) indicates that the stress ratio model for SN007 also involves two competing versions taken from [2]: the first is Model 2, which involves stress ratio $s_{007,\rm M2}$, and the second is Model 4, which involves stress ratio $s_{007,\rm M4}$. Model 4 is virtually the same as currently used model for these 40-inch vessels. For vessels that have high delta volumes from autofrettage, Model 4 gives significantly higher stress ratios than Model 2. In (11), $\hat{p}_{\rm M2}$ is the pretest or 'prior' probability that Model 2 is correct, in which case $s_{007,\rm M2}$ would be the correct stress ratio value to use in reliability calculations. The probability that Model 4 is the correct one is

$$\hat{p}_{\rm M4} = 1 - \hat{p}_{\rm M2} \quad (12)$$

in which case $s_{007,\rm M4}$ would be the correct stress ratio to use.

Regarding the uncertainty distribution for each stress ratio, given that its particular model is the one, we consider two versions: One version is based on the Beta distribution and the other is based on the Weibull distribution. The Beta distribution is commonly used to represent prior distributions in Bayesian analysis, but the Weibull distribution is more natural in this case since the stress ratios in the Orbiter vessels are ultimately based on the outcomes of one or two burst tests where the underlying burst strength is typically Weibull. In the Beta version we have

$$\vec{s}_{007,\rm M2} = Beta\left(\hat{a}_2, \hat{b}_2\right) \quad (13)$$

$$\vec{s}_{007,\rm M4} = Beta\left(\hat{a}_4, \hat{b}_4\right) \quad (14)$$

where $\hat{a}_i$, $\hat{b}_i$,are parameters of the Beta distribution for Model *i* as explained later. In the Weibull version

$$\vec{s}_{007,\rm M2} = Weibull\left(\hat{s}_{007,\rm M2} / \Gamma\left(1 + 1/\hat{\alpha}_2\right), \hat{\alpha}_2\right) \quad (15)$$

$$\vec{s}_{007,\rm M4} = Weibull\left(\hat{s}_{007,\rm M4} / \Gamma\left(1 + 1/\hat{\alpha}_4\right), \hat{\alpha}_4\right) \quad (16)$$

where $\hat{\alpha} = \hat{\rho}\beta$ is the Weibull shape parameter for burst strength, whose effect is also discussed in more detail later. Following a brief comparison of the two versions, which yield virtually the same mean and 95% confidence bound on predicted reliability, we use the Weibull version for all examples, since the Beta version offers no advantage.

For a standard mission duty cycle of an OMS vessel on the Orbiter, the currently used parameter values (before running the test) are

$$t_{\text{mc}} = 105 \text{ hours} \tag{17a}$$

$$t_m = 3465 \text{ hours (past survival time of SN007 at standard conditions)} \tag{17b}$$

$$\begin{aligned} \hat{\rho} &= 24 \\ \hat{\omega}_\rho &= 0.04 \end{aligned} \tag{17c}$$

$$\begin{aligned} \hat{t}_{\text{ref}} &= \hat{t}_{\text{LLNL}} = 1.43 \text{ hours} \\ \hat{\omega}_{t_{\text{ref}}} &= \hat{\omega}_{t_{\text{LLNL}}} = 0.03 \end{aligned} \tag{17d}$$

$$\overline{s}_{\text{piv}} = 0.7 \text{ (pivot stress ratio, which is the mean of LLNL vessel stress ratios)} \tag{17e}$$

$$\begin{aligned} \hat{\lambda}_{\text{LLNL}} &= 1 \\ \hat{\omega}_{\lambda_{\text{LLNL}}} &= 0.0030 \end{aligned} \tag{17f}$$

$$\begin{aligned} \hat{\beta}_{\text{LLNL}} &= 1.625 \quad \text{(lifetime shape parameter based on LLNL vessel data)} \\ \hat{\omega}_{\beta_{\text{LLNL}}} &= 0.080 \end{aligned} \tag{17g}$$

However based on recent analysis of Orbiter type data, primarily the JSC Fleet Leaders [1], we propose the alternative Weibull shape parameter for lifetime given by

$$\begin{aligned} \hat{\beta}_{\text{Orb}} &= 2.45 \quad \text{(lifetime shape parameter based on Orbiter type data)} \\ \hat{\omega}_{\beta_{\text{Orb}}} &= 0.30 \end{aligned} \tag{17h}$$

and we earlier introduced the idea of the prior probability, $\hat{p}_{\beta_{\text{Orb}}}$, that $\hat{\beta}_{\text{Orb}}$ is the correct choice **for** lifetime shape parameter, not $\hat{\beta}_{\text{LLNL}}$. Later we consider various cases, $\hat{p}_{\beta_{\text{Orb}}} = 0,\ 1/5,\ 1/2,\ 1$, but primarily use $\hat{p}_{\beta_{\text{Orb}}} = 1/2$ based on our prior judgment (though a strong case can be made for using $\hat{\beta}_{\text{Orb}} = 2.45$ exclusively, i.e., $\hat{p}_{\beta_{\text{Orb}}} = 1$ as is discussed in [1]). The two alternatives for the stress ratio model are

$$\begin{aligned} \hat{s}_{007,\text{M2}} &= 0.599 \quad (\text{estimate of stress ratio from Model 2}) \\ \hat{\omega}_{\text{M2}} &= 1.2/\hat{\alpha}_2 \end{aligned} \tag{17i}$$

$$\begin{aligned} \hat{s}_{007,\text{M4}} &= 0.653 \quad (\text{estimate of stress ratio from Model 4}) \\ \hat{\omega}_{\text{M4}} &= 1.2/\hat{\alpha}_4 \end{aligned} \tag{17j}$$

where

$$\hat{\alpha}_2 = \hat{\beta}_{\text{Orb}}\hat{\rho} = 59 \tag{18}$$

$$\hat{\alpha}_4 = \hat{\beta}_{\text{LLNL}}\hat{\rho} = 39 \tag{19}$$

and here too we have introduced the idea of the prior probability $\hat{p}_{\text{M2}}$ that Model 2 is correct. Among various cases we consider are $\hat{p}_{\text{M2}} = 0,\ 1/5,\ 1/2$ and 1, although most of the examples assume our first judgment value $\hat{p}_{\text{M2}} = 1/2$. Also, returning to the Beta distribution parameters $\hat{a}_2, \hat{b}_2$, and $\hat{a}_4, \hat{b}_4$, these are calculated from (19i) and (19j) as

$$\hat{a}_2 = \left[(1-\hat{s}_{007,\text{M2}})/(\hat{\omega}_{\text{M2}})^2 - \hat{s}_{007,\text{M2}}\right] \tag{20}$$

$$\hat{b}_2 = \hat{a}_2\,(1-\hat{s}_{007,\text{M2}})/\hat{s}_{007,\text{M2}} \tag{21}$$

$$\hat{a}_4 = \left[(1-\hat{s}_{007,\text{M4}})/(\hat{\omega}_{\text{M4}})^2 - \hat{s}_{007,\text{M4}}\right] \tag{22}$$

$$\hat{b}_4 = \hat{a}_4\left(1-\hat{s}_{007,\text{M4}}\right)/\hat{s}_{007,\text{M4}} \tag{23}$$

It is important to note that with the prior choices $\hat{p}_{\beta_{\text{Orb}}} = 1/2$ on the lifetime shape parameter, and $\hat{p}_{\text{M2}} = 1/2$, on stress ratio Model 2, the predicted vessel reliability is influenced most heavily by the pessimistic values $\hat{\beta}_{\text{LLNL}} = 1.625$ and $\hat{s}_{007,\text{M4}} = 0.653$ since they each also have probability $1/2$. That is, the more optimistic Orbiter based values cannot significantly improve the predicted vessel reliability unless the probabilities, $\hat{p}_{\beta_{\text{Orb}}}$ and $\hat{p}_{\text{M2}}$, are much higher than $1/2$.

A complete derivation of the two stress ratio models is given in [2]. These models capture two alternate interpretations of the various data obtained in the WSTF testing of SN011. The higher stress ratios in Model 4 are primarily due to assumed overwrap stiffness loss in a vessel that is proportional to delta volume from proof. This was about 12% in SN011 and is assumed to be about the same in SN007 when using Model 4. Model 2, however, assumes this large stiffness loss was a peculiarity only of SN011, which was sidelined as a special test vessel and was never put into service. Study of the WSTF cycling and burst test data, original manufacturer data, and outer surface profile measurements made in 2005 strongly suggest that SN011 was a singularly anomalous vessel that is uncharacteristic of other OMS vessels, especially those with serial numbers SN015 and above currently in service.

One technical note is that the model stress ratio is the applied fiber stress in the vessel (as determined from a mechanical analysis based on the applied pressure) divided by the Weibull scale parameter for effective fiber strength as determined from burst tests. In the OMS vessels, only one burst test was performed originally (SN002-Q), and as mentioned, one was performed later SN011, which itself required much interpretation. Thus the burst values obtained are not amenable to standard maximum likelihood analysis for purposes of estimating the Weibull scale parameter for effective fiber strength in the denominator. The fiber strength values obtained from one or two burst tests are more appropriately taken as estimates of the mean of the Weibull distribution. Thus, to estimate the scale parameter, the values from the burst test must be divided by $\Gamma\left(1+1/\hat{\alpha}\right)$, which increases their value by 2 or 3%. The stress ratio values given above in (17i) and (17j) for SN007 and for Models 2 and 4 already reflect this adjustment, and thus, are slightly lower than when using the burst-strength based value directly in the denominator.

It turns out, however, that when using the Weibull distribution to model the uncertainty in the stress ratio, the factor $\Gamma\left(1+1/\hat{\alpha}\right)$ enters once again since simulated stress ratio values will on average be lower in value than $\hat{s}_{007}$, when it is used directly as the scale parameter. Thus the same correction of dividing by $\Gamma\left(1+1/\hat{\alpha}\right)$ is required again, as seen in (15) and (16), but this time the effect is on the numerator of the stress ratio, and thus the two effects cancel. Thus, one might conclude that the correction can be ignored altogether, however, the first correction is needed in determining the parameters in the Beta distribution, given in (20) to (23). The need for care in these corrections is readily apparent when comparing reliability results based on the Beta uncertainty distribution on stress ratio versus the Weibull distribution.

III. Bayesian Framework for Data Analysis

We let $\underset{\sim}{\theta}$ be a vector of possible uncertainty parameter values for the basic model

$$\underset{\sim}{\theta} = \left(\rho, t_{\text{ref}}, \lambda_{\text{LLNL}}, \beta_{\text{Orb}}, \beta_{\text{LLNL}}, s_{007,\text{M2}}, s_{007,\text{M4}}\right) \tag{24}$$

which excludes for the moment the Bernoulli uncertainty regarding the correct choices of the Weibull shape parameter value and the stress ratio model. We let $\eta\left(\underset{\sim}{\theta}\right)$ be the prior likelihood function for all the parameters represented in uncertainty vector, $\underset{\sim}{\theta}$. Thus $\eta\left(\underset{\sim}{\theta}\right)$ is the product of all the Normal and Beta or Weibull uncertainty density functions above. We also let

$$R_{007}\left(n \mid \underset{\sim}{\theta}, i, j; m\right) = \exp\left\{-\left(\frac{s_{007}(j)}{\lambda_{\text{LLNL}}}\right)^{\rho\beta(i)}\left(\frac{t_{\text{mc}}}{t_{\text{ref}}}\right)^{\beta(i)}\left[\left(n+m\right)^{\beta(i)} - \left(m\right)^{\beta(i)}\right]\right\}, \quad i,j = 0,1;\ \underset{\sim}{\theta} \in \Omega_{\underset{\sim}{\theta}} \tag{25}$$

be the prior estimate of reliability after n test cycles, given possible values of these parameters in $\underset{\sim}{\theta}$ from their space $\Omega_{\underset{\sim}{\theta}}$, and all four possible pairs of choices of the shape parameter value and the stress ratio model, and where

$$\beta(i)=\begin{cases}\beta_{\text{LLNL}},\ i=0\\ \beta_{\text{Orb}},\ i=1\end{cases}\tag{26}$$

and

$$s_{007}(j)=\begin{cases}s_{007,\text{M4}},\ j=0\\ s_{007,\text{M2}},\ j=1\end{cases}\tag{27}$$

Thus for any set of parameter value choices in $\underset{\sim}{\theta}$ and survival of a given number of mission cycles, n, we can easily calculate R_{007}. We also let

$$p_{0,m}(i,j)=\begin{cases}p_{\beta_{\text{LLNL}}}p_{\text{M4}},\ i=0,j=0\\ p_{\beta_{\text{Orb}}}p_{\text{M4}},\ \ i=1,j=0\\ p_{\beta_{\text{LLNL}}}p_{\text{M2}},\ i=0,j=1\\ p_{\beta_{\text{Orb}}}p_{\text{M2}},\ \ i=1,j=1\end{cases}\tag{28}$$

where we recall $p_{\beta_{\text{LLNL}}}=1-p_{\text{Orb}}$ and $p_{\text{M4}}=1-p_{\text{M2}}$. This is a bivariate Bernoulli distribution of 'prior' probabilities on the correct Weibull lifetime shape parameter and stress ratio model. Then given survival by the test vessel of n mission cycles, i.e., $N>n$, the posterior distribution for the parameter uncertainties is

$$\tilde{h}(\underset{\sim}{\theta},i,j\,|\,n;m)=\frac{R_{007}(n\,|\,\underset{\sim}{\theta},i,j;m)\eta(\underset{\sim}{\theta})p_{0,m}(i,j)}{\sum_{u,v=0,1}p_{0,m}(u,v)\int_{\Omega_{\underset{\sim}{\theta}}}R_{007}(n\,|\,\underset{\sim}{\theta},u,v;m)\eta(\underset{\sim}{\theta})d\underset{\sim}{\theta}},\quad i,j=0,1\tag{29}$$

where the integration is over the vector space $\Omega_{\underset{\sim}{\theta}}$ for all possible values of parameters in $\underset{\sim}{\theta}$. The case when failure has occurred during mission cycle, $N=n$, is handled similarly. In this case $R_{007}(n\,|\,\underset{\sim}{\theta},i,j;m)$ is replaced by

$$f_{007}(n\,|\,\underset{\sim}{\theta},i,j;m)=R_{007}(n\,|\,\underset{\sim}{\theta},i,j;m)\beta(i)\left(\frac{s_{007}(j)}{\lambda_{\text{LLNL}}}\right)^{\rho\beta(i)}\left(\frac{t_{\text{mc}}}{t_{\text{ref}}}\right)^{\beta(i)}(n+m)^{\beta(i)-1},\ \ i,j=0,1;\ \underset{\sim}{\theta}\in\Omega_{\underset{\sim}{\theta}}\tag{30}$$

based on the probability density function of the failure time multiplied by t_{mc}.

A. Independence of LLNL and Orbiter Based Uncertainty Parameters

The uncertainty parameters developed from the LLNL data base are automatically *independent* of whatever lifetime $N=n$ is observed in the test of vessel SN007. Thus the posterior marginal uncertainty distributions of these parameters will be the same as their prior marginal distributions. Secondly, the Beta or Weibull distributed uncertainty in the individual stress ratio $s_{007,\text{M2}}$ from Model 2 and the stress ratio $s_{007,\text{M4}}$ from Model 4 arose solely from (i) analysis of mechanical data (strain gages, DIC, eddy current probes) from the cycling and a burst test of SN011, (ii) the qualification reports on proof-testing, the cycling and original burst test of SN002-Q or SN003-Q, and (iii) study of the data supplied by the manufacturer for each of the 34 OMS service vessels. Thus, the posterior uncertainties reflected by Beta or Weibull distributed uncertainty parameters in $\underset{\sim}{\theta}$ for the two possible stress ratio, $s_{007,\text{M2}}$ and $s_{007,\text{M4}}$, will be the same after the test as the prior uncertainties irrespective of the observed cycles to failure. This is because the largest component of these uncertainties arises from mechanics models interpreting two burst tests, on SN011 and on SN002-Q. However, what *will* change due to the test are the uncertainty probabilities, $\hat{p}_{\beta_{\text{Orb}}}$ and $\hat{p}_{\text{M2}}$, on which stress ratio model and lifetime shape parameter are correct. The 'prior' probabilities will change to 'posterior' probabilities, denoted $\hat{p}_{\beta_{\text{Orb}},n}$ and $\hat{p}_{\text{M2},n}$, depending on the number of cycles to failure, $N=n$.

Thus, in advance of the test we can integrate out all the marginal joint distributions for all remaining uncertainty parameters reflected in $\underset{\sim}{\theta}$ since they remain unchanged. Then we can focus on the two Bernoulli distributions characterizing the correct choice of the Weibull lifetime shape parameter and the stress ratio model. Carrying out this integration over $\underset{\sim}{\theta}$ in the numerator of $\tilde{h}(\underset{\sim}{\theta},i,j\,|\,n;m)$ above, we obtain the bivariate Bernoulli posterior distribution

$$h(i,j\mid n;m) = p_{n,m}(i,j) = \begin{cases} w_{n,m}(i,j)\, p_{\beta_{\text{LLNL}}}\, p_{\text{M4}}, & i=0, j=0 \\ w_{n,m}(i,j)\, p_{\beta_{\text{Orb}}}\, p_{\text{M4}}, & i=1, j=0 \\ w_{n,m}(i,j)\, p_{\beta_{\text{LLNL}}}\, p_{\text{M2}}, & i=0, j=1 \\ w_{n,m}(i,j)\, p_{\beta_{\text{Orb}}}\, p_{\text{M2}}, & i=1, j=1 \end{cases} \tag{31}$$

where

$$w_{n,m}(i,j) = \frac{\int_{\Omega_{\theta}} R_{007}(n\mid \underset{\sim}{\theta}, i, j; m)\, \eta(\underset{\sim}{\theta})\, d\underset{\sim}{\theta}}{\sum_{u,v=0,1} p_{0,m}(u,v) \int_{\Omega_{\theta}} R_{007}(n\mid \underset{\sim}{\theta}, u, v; m)\, \eta(\underset{\sim}{\theta})\, d\underset{\sim}{\theta}}, \quad i,j = 0,1. \tag{32}$$

are special weighting factors that depend on the number of survived cycles, n. The case where failure has occurred on mission cycle $N = n$ exactly, is treated in a similar way except that $R_{007}(n\mid \underset{\sim}{\theta}, i, j; m)$ is replaced by $f_{007}(n\mid \underset{\sim}{\theta}, i, j; m)$. In this case the weights $w_{n,m}(i,j)$ will be different, especially if failure occurs very early. The above analysis can also be extended to the case of several vessels, k, put on test, with $N_1 = n_1, N_2 = n_2, \cdots, N_k = n_k$ being the failure times. In this case, $f_{007}(n\mid \underset{\sim}{\theta}, i, j; m)$ is replaced by the product

$$f_{007}(n_1\mid \underset{\sim}{\theta}, i, j; m)\, f_{007}(n_2\mid \underset{\sim}{\theta}, i, j; m) \cdots f_{007}(n_k\mid \underset{\sim}{\theta}, i, j; m) \tag{33}$$

Whatever the test circumstances, the posterior probabilities for the lifetime shape parameter β_{LLNL} and stress ratio Model 2, being the correct choices are, respectively

$$\hat{p}_{\beta_{\text{Orb}},n} = p_{n,m}(1,0) + p_{n,m}(1,1) = w_{n,m}(1,0)\, p_{\beta_{\text{Orb}}}\, p_{\text{M4}} + w_{n,m}(1,1)\, p_{\beta_{\text{Orb}}}\, p_{\text{M2}} \tag{34}$$

and

$$\hat{p}_{\text{M2},n} = p_{n,m}(0,1) + p_{n,m}(1,1) = w_{n,m}(0,1)\, p_{\text{LLNL}}\, p_{\text{M2}} + w_{n,m}(1,1)\, p_{\beta_{\text{Orb}}}\, p_{\text{M2}} \tag{35}$$

B. Calculation of Posterior Uncertainty Distribution on Reliability

To calculate the posterior uncertainty distribution on the reliability of SN007, we must carry out the integration in all four $w_{n,m}(i,j)$ components, and this is accomplished using Monte Carlo simulation. Then the posterior probability components $p_{n,m}(i,j)$ can be calculated as well as the posterior probabilities, $\hat{p}_{\beta_{\text{Orb}},n}$ and $\hat{p}_{\beta_{\text{LLNL}},n} = 1 - \hat{p}_{\beta_{\text{Orb}},n}$ and $\hat{p}_{\text{M2},n}$ and $\hat{p}_{\text{M4},n} = 1 - \hat{p}_{\text{M2},n}$ regarding which of the two Weibull lifetime shape parameters and stress ratio models are correct. The calculation of the posterior uncertainty distribution on the reliability must be performed over the full space of possible parameter values, $\Omega_{\theta} \otimes [(0,0),(1,0),(0,1),(1,1)]$, where the quantity in square parentheses represents the possible bivariate Bernoulli values which determine the particular stress ratio model being used (Model 2 or Model 4) and the Weibull shape parameter being used (β_{LLNL} or β_{Orb}) as defined in (26) and (27). One technical note is that numerical study of the various components shows that the posterior Bernoulli random variables, $\vec{I}_{\beta_{\text{Orb}},n}$ and $\vec{I}_{\text{M2},n}$ are virtually independent in the posterior, as was assumed in the prior. Thus, in keeping with (30), the posterior bivariate Bernoulli probabilities can be taken as

$$p_{n,m}(i,j) \approx \begin{cases} \hat{p}_{\beta_{\text{LLNL}},n}\hat{p}_{\text{M4},n}, & i=0, j=0 \\ \hat{p}_{\beta_{\text{Orb}},n}\hat{p}_{\text{M4},n}, & i=1, j=0 \\ \hat{p}_{\beta_{\text{LLNL}},n}\hat{p}_{\text{M2},n}, & i=0, j=1 \\ \hat{p}_{\beta_{\text{Orb}},n}\hat{p}_{\text{M2},n}, & i=1, j=1 \end{cases} \tag{36}$$

As the final step, to calculate the posterior uncertainty distribution on the reliability we have used Monte Carlo simulation to determine > 50,000 replicated outcomes of all model parameters in the extended vector space, $\hat{\Omega} = \Omega_{\theta} \otimes [(0,0),(1,0),(0,1),(1,1)]$, including Bernoulli outcomes generated using (36). From each replication we calculate a reliability value using (1), and the uncertainty distribution is the empirical distribution function generated from the set of calculated reliabilities. Conceptually, we are calculating the uncertainty distribution function

$$\begin{aligned} P\left(\vec{R}_{007}\left(n \mid m\right) < R\right) &= \frac{\left[\sum_{\hat{\Omega}(R_{007}<R)} p_{n,m}(i,j)\int R_{007}\left(n \mid \underset{\sim}{\theta}, i, j; m\right)\eta(\underset{\sim}{\theta})d\underset{\sim}{\theta}\right]}{\sum_{u,v=0,1} p_{n,m}(u,v)\int_{\Omega_{\theta}} R_{007}\left(n \mid \underset{\sim}{\theta}, u, v; m\right)\eta(\underset{\sim}{\theta})d\underset{\sim}{\theta}} \\ &= \sum_{i,j=0,1} p_{n,m}(i,j) P\left(\vec{R}_{007}\left(n \mid i,j;m\right) < R\right), \quad 0 < R < 1 \end{aligned} \tag{37}$$

where the numerator calculates reliabilities only for combinations of all parameter values in $\hat{\Omega}$ which yield $R_{007} < R$, given reliability level, R. Below, R is expressed in numbers of 'nines'.

C. Effects of Prior Probability Assumptions and Beta vs. Weibull Uncertainty on Stress Ratio

Next we consider a comparison of uncertainty distributions on the predicted reliability for one mission cycle of SN007 based on the Beta versus Weibull distribution for modeling stress ratio uncertainty. The Beta parameter values have been chosen as given above, and the Beta and Weibull coefficients of variation have been chosen to match. Results are calculated for various prior or posterior probabilities $\hat{p}_{\beta_{\text{Orb}}} = 1 - \hat{p}_{\beta_{\text{LLNL}}}$ and $\hat{p}_{\text{M2}} = 1 - \hat{p}_{\text{M4}}$.

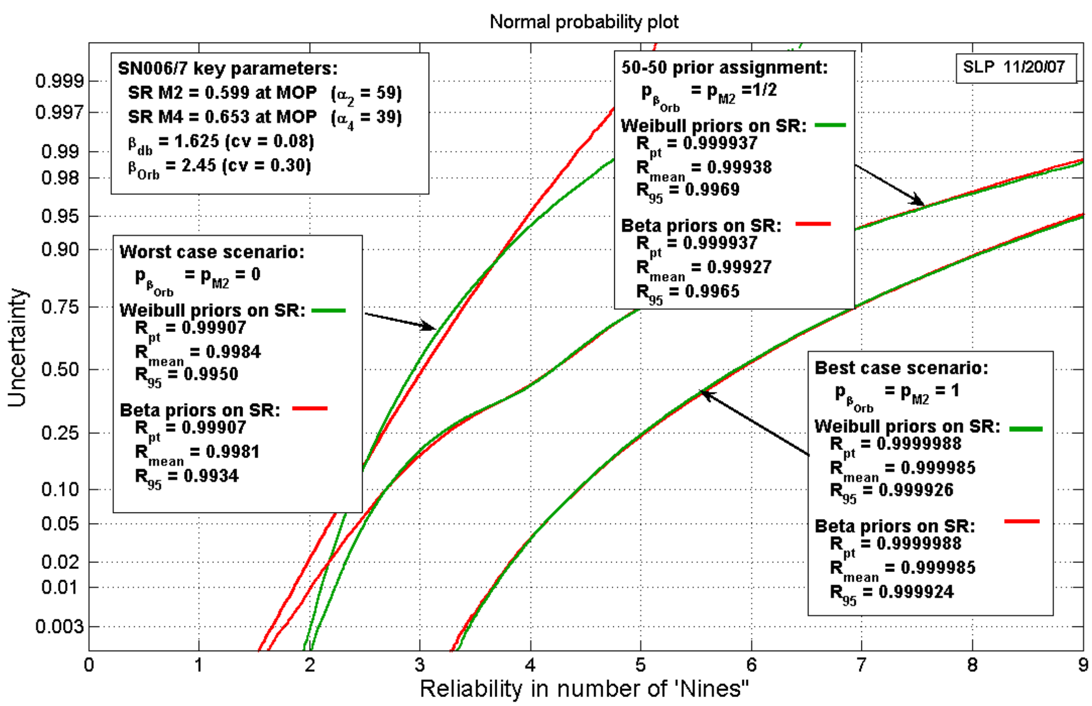

Figure 1. Comparison of reliability predictions on SN007 for a mission cycle $t_{\text{mc}} = 105$ hrs., given pessimistic $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 0$, mixed $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 1/2$ and optimistic $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 1$ cases.

Predictions applicable to one mission of a vessel similar to SN007 are shown in Figure 1 for $t_{\text{mc}} = 105$ hours, and past survival $t_m = 3465$ hours. The most pessimistic case is $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 0$, so $\hat{\beta}_{\text{LLNL}} = 1.625$ and $\hat{s}_{007,\text{M4}} = 0.653$. The mixed case assumes $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 1/2$, and the most optimistic case $\hat{p}_{\beta_{\text{Orb}}} = \hat{p}_{\text{M2}} = 1$, where $\hat{\beta}_{\text{Orb}} = 2.45$ and $\hat{s}_{007,\text{M2}} = 0.599$ applies. Monte Carlo simulation was used on all model parameters including the Bernoulli random variable reflecting the choices of the lifetime shape parameter and stress ratio Model. We find that the point estimates of the predicted reliabilities are the same for both the Beta and Weibull versions and the mean reliabilities and point reliabilities are close, especially for the most optimistic case. For the more pessimistic cases, however, the Beta distribution gives slightly lower predictions.

The cause of this more pessimistic behavior in the Beta distribution case rests in the behavior of the deep tail, which reflects the premise that the stress ratios calculated from one or two burst tests may, in rare cases, be much worse than the true value. This would require that the original burst test (one or two) reflected unrealistically strong vessels from the population. However, this is more a characteristic of the behavior of the upper tails of the Beta distribution itself, which assumes a power form for the probability of strong vessels occurring in the original burst tests to set stress ratio. The Weibull distribution, however has an exponentially decaying upper tail, and consistent with experimental observation on various data sets of strands and pressure vessels, indicates that vessels significantly weaker than the mean strength of the population are in fact much more likely to be selected than vessels much stronger than the average. Thus the Weibull uncertainty approach is judged more realistic and we shall henceforth use it.

IV. Case Studies and Main Results

A. Testing and Results under Standard Operating Conditions

Next we describe what would happen in a stress-rupture test on SN007, using standard operating conditions: pressure $p_{\mathrm{MOP}} = 4875$ psi and temperature $T_{\mathrm{ref}} = 81°\mathrm{F}$. In the analytical framework above, to obtain significant gains in predicted reliability of OMS-type COPVs, it is necessary to have test conditions under which significantly increased posterior probabilities are possible compared to prior probabilities, that is $\hat{p}_{\beta_{\mathrm{Orb}},n} > \hat{p}_{\beta_{\mathrm{Orb}}}$ and $\hat{p}_{\mathrm{M2},n} > \hat{p}_{\mathrm{M2}}$. This means that the test must be run long enough that under the most optimistic scenario $\hat{\beta}_{\mathrm{Orb}} = 2.45$ and $\hat{s}_{6/7,\mathrm{M2}} = 0.599$ the vessel has at least a 50% chance of failure, and preferably as high as 80%; that is, if it is tested to shorter than this time and the test is stopped, say, for budgetary reasons, there can be no significant improvement in the predicted reliability and the 'status quo' will remain. In order to limit the required test time to the maximum of 100 test cycles, each test cycle must be nominally 4000 hours in duration. (Each 'cycle' can be viewed merely as a convenient time block for analysis, and does not imply that the vessel must be depressurized and repressurized every 4000 hours.)

Thus standard test conditions are set to be, $t_{\mathrm{mc,t}} = 4000$ hours, and $t_{\mathrm{m,past}} = t_m = 3465$ hours, which is the past survival time under standard conditions. Thus the number, m_{t}, of past mission cycles survived is $m_{\mathrm{t}} = t_{\mathrm{m,past}} / t_{\mathrm{mc,t}} = 3465/4000 \approx 1$. Hence, under standard test conditions, the number of 'test cycles', n_{t}, at any point of the test will be the integer value of the total time on test divided by n_{t}. What is seen immediately is that under the most optimistic test scenario, the past mission cycles survived amounts to only one cycle. Consequently in subsequent figures and discussion we abbreviate $p_{n,m}(i,j)$ to $p_n(i,j)$.

We now consider various results regarding posterior estimates of the probability components $p_n(i,j)$ and the posterior probabilities $\hat{p}_{\beta_{\mathrm{Orb}},n}$ and $\hat{p}_{\mathrm{M2},n}$ as a result of running the stress rupture test and surviving varying numbers of mission cycles, n. Figure 2 presents posterior results for the case where the vessel is known to have survived n mission cycles, i.e., $N > n$, at standard conditions $p_{\mathrm{MOP}} = 4875$ psi and $T_{\mathrm{ref}} = 81°\mathrm{F}$. Before the test, the 'prior' probabilities for the shape parameter and stress ratio model, $\hat{p}_{\beta_{\mathrm{Orb}}}$ and $\hat{p}_{\mathrm{M2}}$, were taken as ½. Also shown is the probability that the vessel will fail by mission cycle n both for the most optimistic starting case $\hat{p}_{\beta_{\mathrm{Orb}}} = \hat{p}_{\mathrm{M2}} = 1$ and the most pessimistic case $\hat{p}_{\beta_{\mathrm{LLNL}}} = \hat{p}_{\mathrm{M4}} = 1$.

Note in Figure 2 that as more and more mission cycles are survived, the posterior probabilities $\hat{p}_{\beta_{\mathrm{LLNL}},n}$ and $\hat{p}_{\mathrm{M4},n}$, for the pessimistic parameter values, shift to lower values especially the latter. This means that the corresponding posterior probabilities, $\hat{p}_{\beta_{\mathrm{Orb}},n}$ and $\hat{p}_{\mathrm{M2},n}$, increase for the optimistic vessel parameter values, and the increase is dramatic in favor of the optimistic stress ratio Model 2. Should the vessel survive to the median time 248,000 hours or 62 missions of the most optimistic scenario of parameter choices, we obtain $\hat{p}_{\beta_{\mathrm{Orb}},62} = 0.55$ and $\hat{p}_{\mathrm{M2},62} = 0.945$. The inset table in Figure 2 shows the resulting increases in predicted single mission reliability of an OMS vessel in current service similar to SN007 and for $t_{\mathrm{mc}} = 105$ hours and $t_m = 3465$ hours. Unfortunately, this length of test time, 248,000 hours is not feasible, so shortly we consider accelerating temperature conditions to reduce the 248,000 hours to more manageable time.

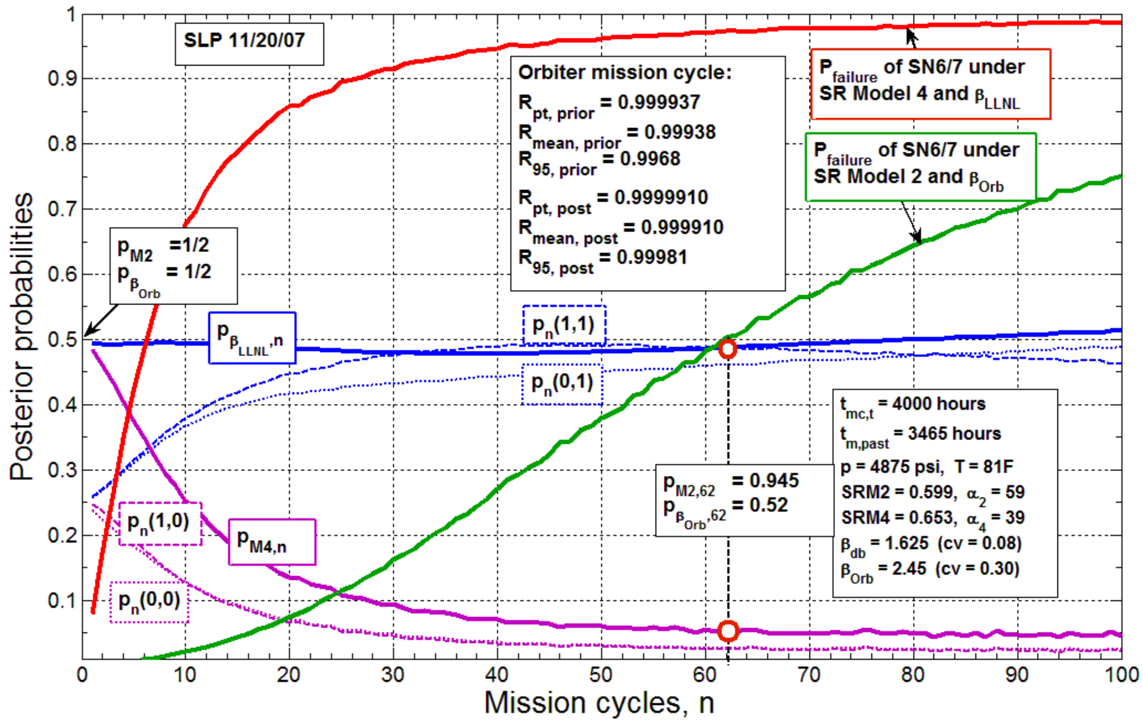

Figure 2. Posterior probability components vs. mission cycles, *n*, survived assuming prior probabilities $p_{\beta_{\rm Orb}} = p_{\rm M2} = 1/2$, $p_{\rm MOP} = 4875$ psi and $T_{\rm ref} = 81°$F. Also shown are probabilities of vessel survival to various cycles, n, as well as prior and posterior reliability at $n = 62$.

Nonetheless, to obtain the corresponding OMS COPV service mission reliability predictions ($t_{\rm mc} = 105$ hours), assuming test survival as described above, we used Monte Carlo simulation on all model parameters including the Bernoulli random variable reflecting the choices of the lifetime shape parameter and stress ratio Model. For these Bernoulli random variables we used the posterior values $\hat{p}_{\beta_{\rm Orb},62} = 0.52$ and $\hat{p}_{\rm M2,62} = 0.945$ obtained in Figure 2. The most important observation is that thee reliabilities relevant to one mission cycle in service increase on the order of one 'nine' (slightly less for the mean and point estimate and slightly more for the 95% bound).

In Figure 3 we present posterior results for the case where the vessel is known to have failed exactly on mission cycle n, i.e., $N = n$, under the conditions of Figure 2. As failure occurs at later and later cycle numbers the posterior probabilities, $\hat{p}_{\beta_{\rm Orb},n}$, for the more optimistic Weibull lifetime shape parameter, eventually become slightly higher than in Figure 2 (about 0.55 compared to the value 0.5 at test start), whereas the posterior probabilities, $\hat{p}_{{\rm M2},n}$, for optimistic stress ratio Model 2 are not quite as high as in Figure 2, though still very high. However, if failure occurs very early (even early for the pessimistic model scenario), $\hat{p}_{{\rm M4},n}$ jumps to almost 1 and $\hat{p}_{\beta_{\rm LLNL},n}$ jumps to 0.6, as ought to be the case. This does not occur in Figure 2 since knowing $N > n$ early on provides little true information. Generally in Figure 3 we note that the predicted Orbiter OMS mission reliabilities for the case $N = 62$ are only slightly different from those for $N > 62$ in Figure 2, and again reflect a reliability gain of about one 'nine'.

Figure 4 presents corresponding results to Figure 3 for the cases where the prior probabilities are reduced to 0.2, respectively, for choosing the optimistic scenario $\beta_{\rm Orb} = 2.45$ and stress ratio Model 2. That is, the prior probabilities are slanted heavily to the most pessimistic assumption. Otherwise the parameter choices are the same as in Figures 2 and 3 namely. Despite beginning with the more pessimistic prior probabilities, as more and more missions are survived there is still a major shift in the posterior probabilities, $\hat{p}_{\beta_{\rm Orb},n}$ and $\hat{p}_{{\rm M2},n}$, but there is also further room to grow as the posterior values are not as high as starting with 1/2. This also means that the prior reliabilities are also slightly worse but in the end, the relative gain in reliability from prior to posterior is approximately the same. The general features of the behavior for the case $N = n$ in Figure 4 as compared to Figure 3 remain qualitatively the same.

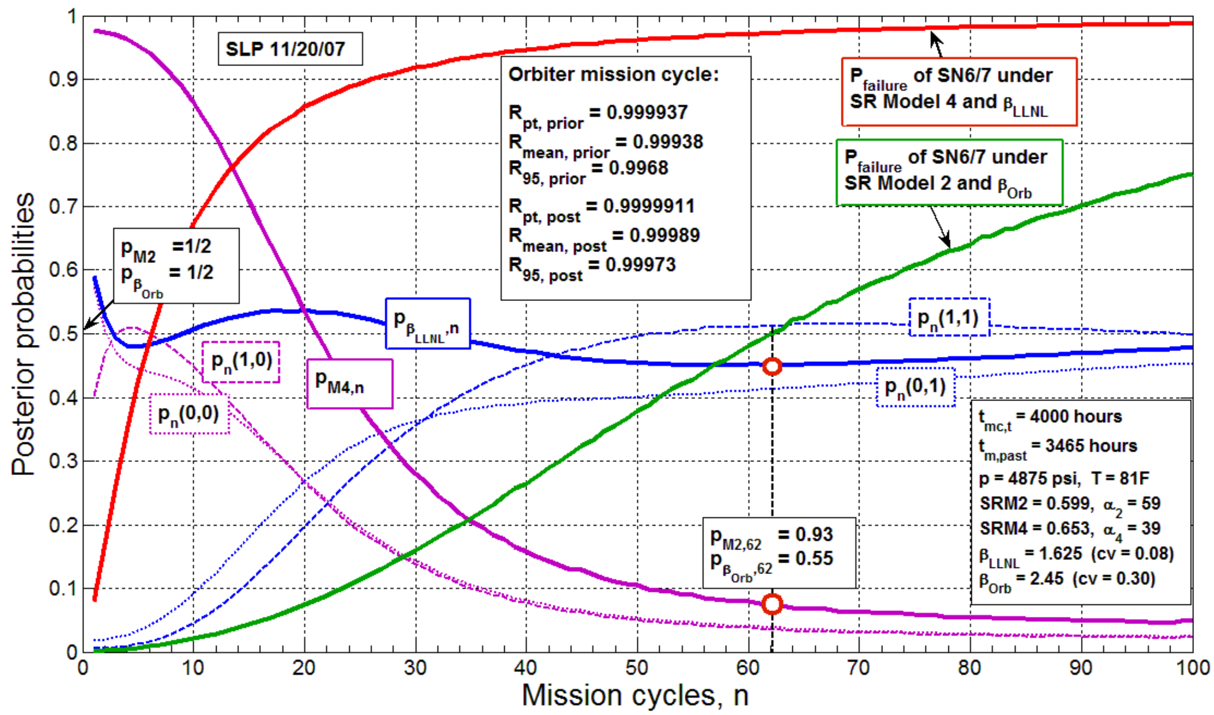

Figure 3. Posterior probabilities vs. mission cycle, *n*, when failure occurs for the prior values $p_{\beta_{\text{Orb}}} = p_{\text{M2}} = 1/2$, $p_{\text{MOP}} = 4875$ psi and $T_{\text{ref}} = 81°\text{F}$. Also shown are probabilities of vessel failure at various test cycles, n, as well as prior and posterior reliability at $n = 62$.

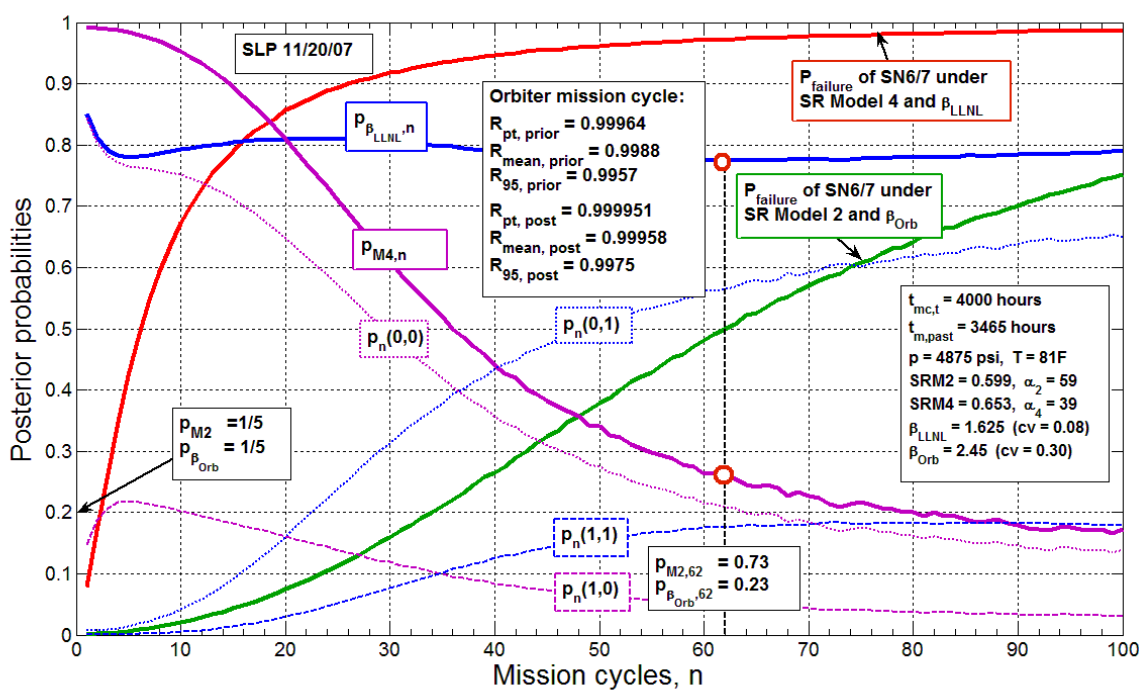

Figure 4. Posterior probabilities vs. mission cycle, *n*, when failure occurs for the case $p_{\beta_{\text{Orb}}} = p_{\text{M2}} = 1/5 = 0.2$, $p_{\text{MOP}} = 4875$ psi and $T_{\text{ref}} = 81°\text{F}$. Also shown are probabilities of vessel survival to various cycles, n, as well as prior reliability and posterior reliability at $n = 62$.

B. Test Acceleration using Increased Temperature and Pressure

It is clear from the above example that unless some form of acceleration is used, the test time is impractically long (248,000 hours or 28 years) to achieve the outcome that stress ratio Model 2 is correct. Thus it is necessary to accelerate the test using an increase in pressure or temperature or both. We let T_{el} and p_{el} be elevated temperatures and pressures relative to the standard conditions T_{ref} and p_{ref}, the latter being maximum operating pressure, 4875 psi, for an OMS vessel. It is shown in [2] that for any elevated pressure, p_{el}, the stress ratios for Model 2 and Model 4 are, respectively,

$$S_{\text{M2}}\left(p_{\text{el}},T_{\text{ref}}\right)=\frac{W_{2,\Delta V_{007}}}{rc_{\text{Orb,LLNL}}}\frac{\left(\Delta V_{007}+\dfrac{p_{\text{el}}}{8.674}\right)}{1430} \tag{38}$$

and

$$S_{\text{M4}}\left(p_{\text{el}},T_{\text{ref}}\right)=\frac{W_{4,\Delta V_{007}}}{rc_{\text{Orb,LLNL}}}\frac{\left(\Delta V_{007}+\dfrac{p_{\text{el}}}{\left(8.674-5.415\left(0.1421\right)\right)}\right)}{1430}, \tag{39}$$

where $W_{2,\Delta V_{007}}=0.978$ and $W_{4,\Delta V_{007}}=1.011$ are Weibull based correction factors for through-thickness gradients in the tow (wrap layer) tensions, and for SN007 the delta volume (permanent volume growth from autofrettage) is approximately $\Delta V_{007}=340\ \text{in}^3$. Also $rc_{\text{Orb,LLNL}}=1.02$ is a pressure rate correction factor in interpreting the burst tests since the LLNL database COPVs were pressurized at a slower rate than the OMS vessels.

We also have temperature acceleration adjustments we can make to produce higher effective stress ratios to substitute into the model as discussed in [1]. The effective stress ratios from temperature acceleration are

$$S_{\text{M2}}\left(p_{\text{el}},T_{\text{el}}\right)=\Phi^{\left(T_{\text{el},K}-T_{\text{ref},K}\right)/T_{\text{el},K}}\,S_{\text{M2}}\left(p_{\text{el}},T_{\text{ref}}\right)^{T_{\text{ref},K}/T_{\text{el},K}} \tag{40}$$

$$S_{\text{M4}}\left(p_{\text{el}},T_{\text{el}}\right)=\Phi^{\left(T_{\text{el},K}-T_{\text{ref},K}\right)/T_{\text{el},K}}\,S_{\text{M4}}\left(p_{\text{el}},T_{\text{ref}}\right)^{T_{\text{ref},K}/T_{\text{el},K}} \tag{41}$$

Where $\Phi=2.86$ is the $0°\text{K}$ stress ratio convergence point determined from experiments

$$T_{\text{ref,K}}=300°\text{K}\quad\left(80.6°\text{F}\right),\qquad T_{\text{ref,F}}=32+\left(9/5\right)\left(300-273\right)=80.6°\text{F} \tag{42}$$

and

$$T_{\text{el,K}}=300+\left(5/9\right)\left(T_{\text{el,F}}-T_{\text{ref,F}}\right)°\text{K} \tag{43}$$

Finally we adjust the past survival times to correspond to the new stress ratios according to

$$t_{\text{m,past,el}}=t_m\left(p_{\text{el}},T_{\text{el}}\right)=t_m\left(p_{\text{ref}},T_{\text{ref}}\right)\left[S_{\text{M4}}\left(p_{\text{ref}},T_{\text{ref}}\right)/S_{\text{M4}}\left(p_{\text{el}},T_{\text{el}}\right)\right]^{\hat{\rho}} \tag{44}$$

as well as the Weibull shape parameters at elevated temperatures

$$\hat{\beta}_{\text{Orb}}\left(T_{\text{el}}\right)=\hat{\beta}_{\text{Orb}}\,T_{\text{el,K}}/T_{\text{ref,K}} \tag{45}$$

and

$$\hat{\beta}_{\text{LLNL}}\left(T_{\text{el}}\right)=\hat{\beta}_{\text{LLNL}}\,T_{\text{el,K}}/T_{\text{ref,K}} \tag{46}$$

B. Main Results under Accelerated Test Conditions

We first considered the case $p_{\text{el}}=5750$ psi and $T_{\text{el}}=T_{\text{ref}}=80.6°\text{F}$; that is, a greatly increased pressure is used to accelerate the test but the temperature remains ambient at $T_{\text{ref}}=80.6°\text{F}$. This reduced the test mission cycle time to 306 hours but for 61 mission cycles to reach the median time to failure under stress ratio Model 2, the test would still take 18,666 hours or 26 months, again longer than desirable. Such a large increase in test pressure also poses risks to the titanium liner and ultimately is not viable as a test option.

The next case considered was the standard service pressure, $p_{\text{mop}}=4875$ psi, but temperature accelerated to $T_{\text{el}}=145°\text{F}$. Figure 5 shows results where to reach the median time to failure under Model 2 the test now takes 4560 hours or about 6 months. Apart from shorter tests times in this accelerated case, the basic pattern of vessel probabilities of failure under SR Model 2 and Model 4 as well as prior and posterior probabilities are about the same, though accelerating the temperature does show a slight advantage in posterior probabilities and reliabilities.

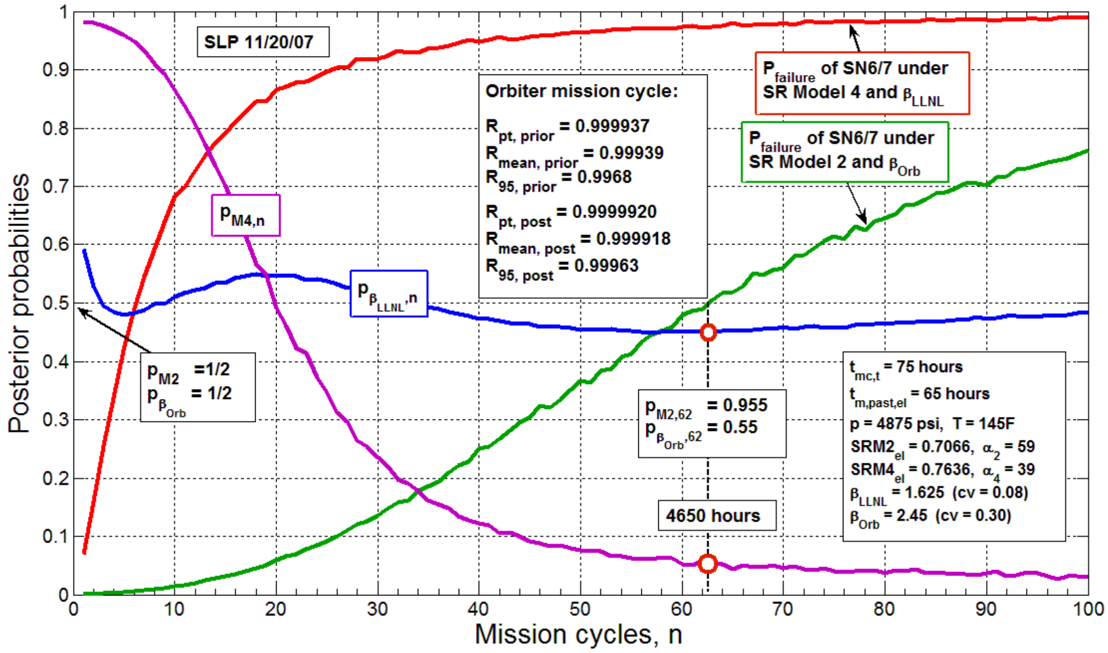

Figure 5. Posterior reliabilities versus number of mission cycles survived assuming prior probabilities $p_{\beta_{\rm Orb}} = p_{\rm M2} = 1/2$ a test pressure of $p_{\rm test} = p_{\rm mop} = 4875$ psi, and $T_{\rm test} = T_{\rm el} = 145°$F.

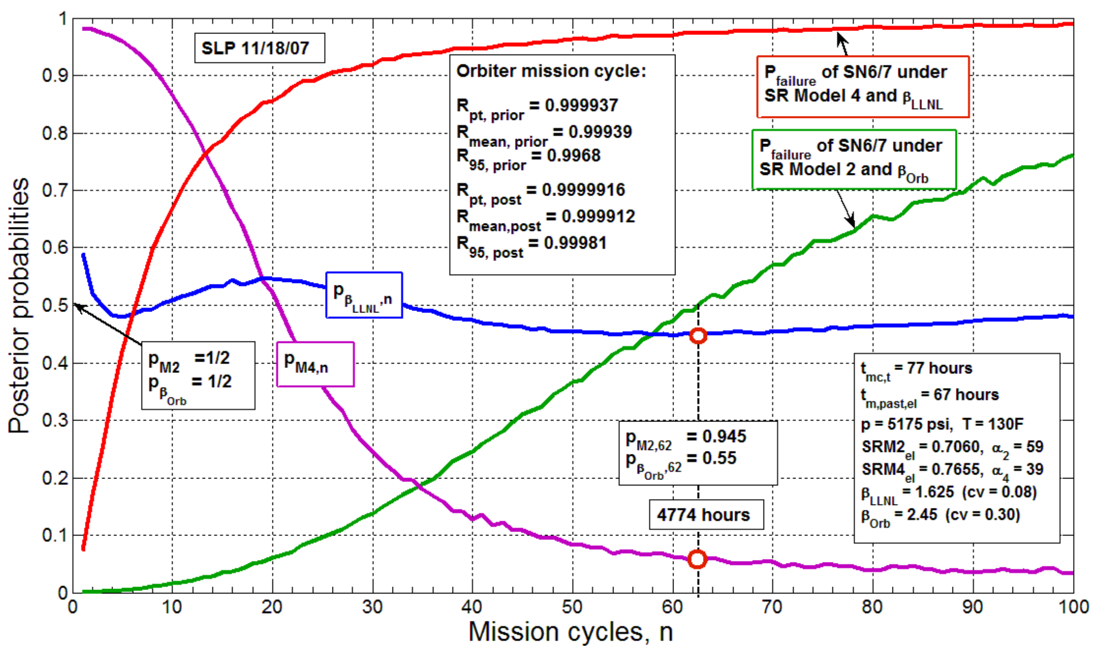

Figure 6. Posterior reliabilities versus number of mission cycles survived assuming prior probabilities $p_{\beta_{\rm Orb}} = p_{\rm M2} = 1/2$, a test pressure of $p_{\rm test} = 5175$ psi, and $T_{\rm test} = T_{\rm el} = 130°$F.

Figure 6 shows results for the compromise choices, $p_{\rm el} = 5175$ psi and $T_{\rm el} = 130°$F. In this case the test takes 4774 hours again about 6 months for the vessel to reach the median time to failure under stress ratio Model 2.

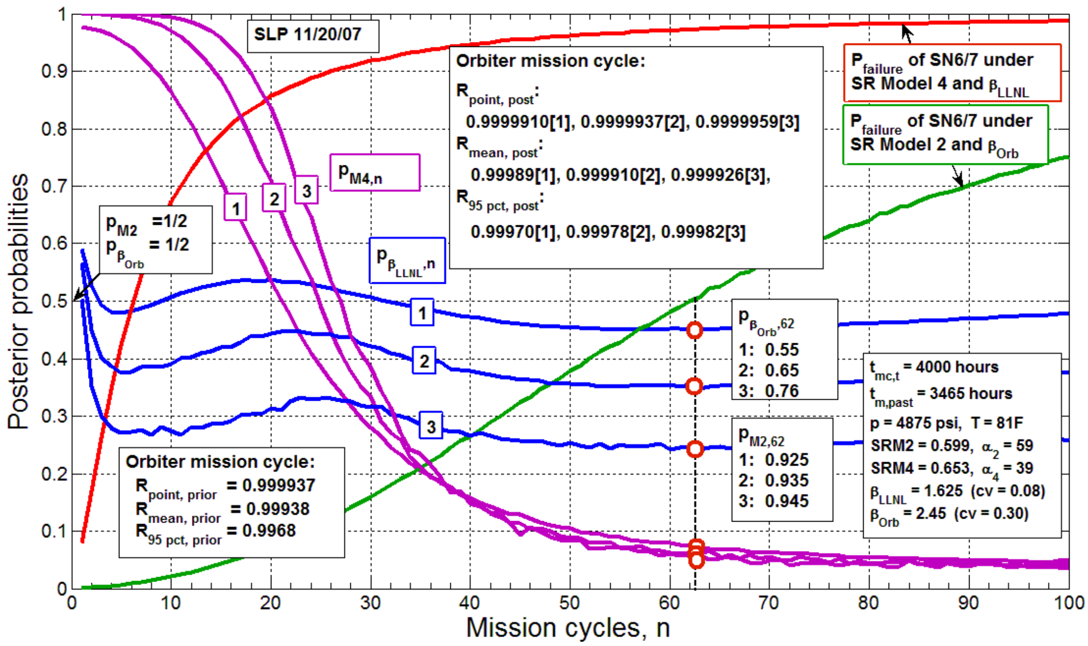

Figure 7. Posterior probabilities vs. mission cycle, *n*, for three vessels on test and all failing close together in time (so very little variability). Other test parameters are $p_{\beta_{\rm Orb}} = p_{\rm M2} = 1/2$, $p_{\rm MOP} = 4875$ psi and $T_{\rm ref} = 81°{\rm F}$.

Since the posterior probabilities on the choice of the Weibull lifetime shape parameter are little changed from the prior values, we consider the case of testing two and three vessels, respectively, under standard test conditions and where they all fail within a few cycles of each other. Figure 7 shows posterior probabilities vs. mission cycle, *n*, for three vessels put on test and all failing close together in time so there is very little variability. Other parameters are $p_{\beta_{\rm Orb}} = p_{\rm M2} = 1/2$, and standard test conditions are assumed, $p_{\rm MOP} = 4875$ psi and $T_{\rm ref} = 81°{\rm F}$.

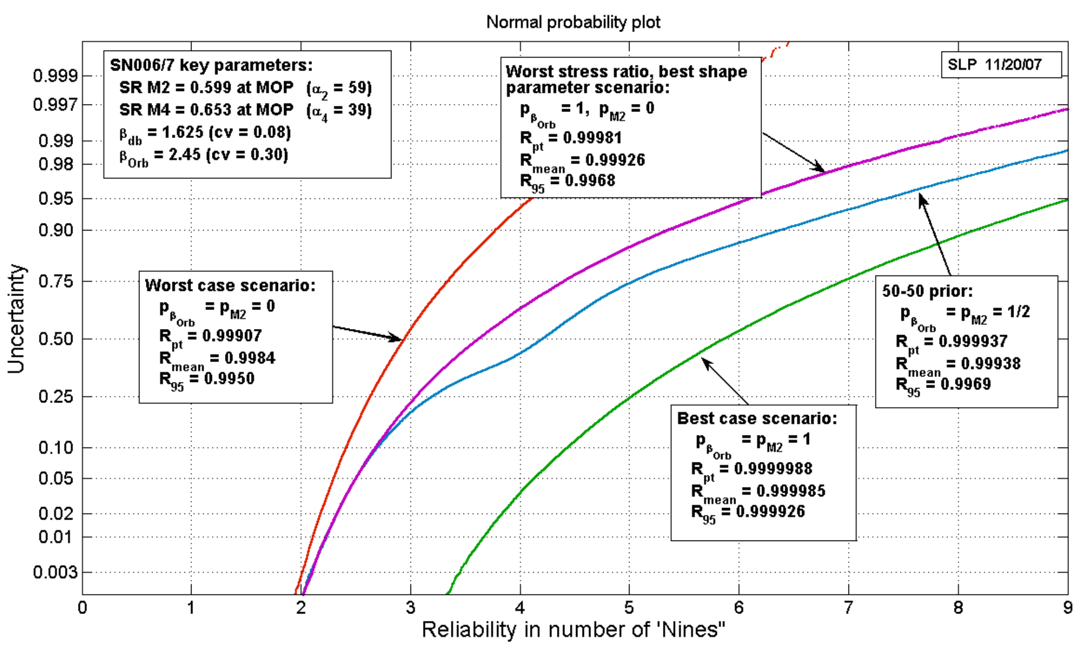

Figure 8. Uncertainty distributions and predicted reliabilities for one Orbiter mission of an OMS vessel like SN007 under various stress ratio models and lifetime shape parameter assumptions.

Figure 7 shows that if failure occurs for all three vessels near $n = 62$ mission cycles, then the posterior probabilities all improve and are $\hat{p}_{\beta_{\rm Orb},62} = 0.76$ and $\hat{p}_{\rm M2,62} = 0.945$. While the posterior reliabilities also increase for an OMS mission cycle on a vessel like SN007, dramatic gains stemming from a high posterior probability of the optimistic shape parameter would require testing many more vessels. Thus, without testing many vessels, the test does little to resolve whether the Weibull shape parameter $\beta_{\text{Orb}} = 2.45$ is correct or $\beta_{\text{LLNL}} = 1.625$ is correct as the posterior probability is increased only slightly over the prior value $p_{\beta_{\text{Orb}}}$. One must rely on studying Fleet Leader and Orbiter data itself to make that choice [1].

Finally Figure 8 shows a comparison of the cases in Figure 1 together with the case of stress ratio Model 4 being correct but also $\beta_{\text{Orb}} = 2.45$ being correct. The predicted reliabilities for one service mission cycle of a vessel like SN007 are about the same for $p_{\beta_{\text{Orb}}} = p_{\text{M2}} = 1/2$. However for the most optimistic case where stress ratio Model 2 and $\beta_{\text{Orb}} = 2.45$ are correct, the predicted mean reliability is about five 'nines' and 95% bound exceeds four 'nines'.

V. Concluding Comment

It remains a misconception that testing one vessel can validate the reliability model; such a test can do no such thing. Validating the model would require several vessels tested at each of several stress ratios. What the test ***can do*** is sort out questions about the correct stress ratio model to use, which in turn would point to SN011 being a singularly anomalous vessel for which there is much evidence. Clearly a properly run test can yield great benefits in improving the reliability, of the order of one order of magnitude or one 'nine'.

Acknowledgments

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[2] S.L. Phoenix, *New Stress Ratio Formulas Based on Study of the Mechanical Response of OMS SN011 in WSTF Tests*, **White Paper**, November 2, 2007.