Integrated Force Method for Indeterminate Structures

Indeterminate structural-mechanics problems can now be solved systematically.

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Two methods of solving indeterminate structural-mechanics problems have been developed as products of research on the theory of strain compatibility. In these methods, stresses are considered to be the primary unknowns (in contrast to strains and displacements being considered as the primary unknowns in some prior methods). One of these methods, denoted the integrated force method (IFM), makes it possible to compute stresses, strains, and displacements with high fidelity by use of modest finite-element models that entail relatively small amounts of computation. The other method, denoted the completed Beltrami Mitchell formulation (CBMF), enables direct determination of stresses in an elastic continuum with general boundary conditions, without the need to first calculate displacements as in traditional methods.

The equilibrium equation, the compatibility condition, and the material law are the three fundamental concepts of the theory of structures. For almost 150 years, it has been commonly supposed that the theory is complete. However, until now, the understanding of the compatibility condition remained incomplete, and the compatibility condition was confused with the continuity condition. Furthermore, the compatibility condition as applied to structures in its previous incomplete form was inconsistent with the strain formulation in elasticity.

Strength-of-materials problems have been classified into determinate and indeterminate problems. A determinate problem is analyzed primarily on the basis of the equilibrium concept and can be well understood. The solution of an indeterminate problem requires an additional compatibility condition, of which, until now, there has not been exclusive comprehension. In traditional stress-analysis methods, the compatibility condition has been improvised by manipulating the equilibrium concept, variously, by rewriting it in displacement variables or through a method known in the art as the redundant force method. Such improvisation has made traditional indeterminate analysis cumbersome.

The research that led to the development of the IMF and the CBMF included the derivation of a variational functional. The stationary condition of this functional yielded not only the traditional equations of the mechanics of solids but also new equations that were identified as constituting the boundary compatibility condition that was missing from the strain equations as originally formulated by St. Venant circa 1860. It should be noted that the IFM and the CBMF can be further specialized into four indirect methods known as the redundant force method, stiffness method, the hybrid method, and the total formulation (see table)

The use of the compatibility equations has systematized indeterminate analysis — especially for problems that involve variations in temperature and initial deformations. Solving indeterminate problems has become straightforward through use of the IFM because the IFM bestows simultaneous emphasis on force equilibrium and the deformation compatibility condition.

A report, "Integrated Force Method Solution to Indeterminate Structural Mechanics Problems" (NASA/TP-2004-207430) introduces the IFM to academia, especially to engineering students in civil, mechanical, aeronautical, and other engineering disciplines. Although the report is written for use in college education, it should be valuable to researchers who

Method		Primary Variables		Variational Functional
Elasticity	Structures	Elasticity	Structures	
Completed Beltrami-Michell Formulation (CBMF)	Integrated Force Method (IFM)	Stresses	Forces	IFM Variational Functional
Airy Formulation	Redundant Force Method	Stress Function	Redundants	Complementary Energy
Navier Formulation (NF)	Stiffness Method (DM)	Displacements	Displacements or Deflections	Potential Energy
Reissner Method (RM)	Hybrid Method (HF)	Stresses and Displacements	Forces and Deflections	Reissner Functional
Total Formulation (TF)	Washizu Method (WM)	Stresses, Strains, and Displacements	Forces, Deformations, and Deflections	Washizu Functional

Selected Characteristics of the IFM and CBMF are listed along with those of other methods of structural mechanics.

wish to work on the IFM or to complement their understanding of the compatibility condition of structural mechanics.

This work was done by Dale A. Hopkins and Gary R. Halford of Glenn Research Center and Surya N. Patnaik of Ohio Aerospace Institute. Further information is contained in a TSP (see page 1).

Inquiries concerning rights for the commercial use of this invention should be addressed to NASA Glenn Research Center, Innovative Partnerships Office, Attn: Steve Fedor, Mail Stop 4–8, 21000 Brookpark Road, Cleveland, Ohio 44135. Refer to LEW-17883-1.