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Analysis of Sting Balance Calibration Data Using Optimized Regression Models

(Extended Abstract of Proposed Conference Paper)

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Calibration data of a wind tunnel sting balance was processed using a search algorithm that identifies an optimized regression model for the data analysis. The selected sting balance had two moment gages that were mounted forward and aft of the balance moment center. The difference and the sum of the two gage outputs were fitted in the least squares sense using the normal force and the pitching moment at the balance moment center as independent variables. The regression model search algorithm predicted that the difference of the gage outputs should be modeled using the intercept and the normal force. The sum of the two gage outputs, on the other hand, should be modeled using the intercept, the pitching moment, and the square of the pitching moment. Equations of the deflection of a cantilever beam are used to show that the search algorithm's two recommended math models can also be obtained after performing a rigorous theoretical analysis of the deflection of the sting balance under load. The analysis of the sting balance calibration data set is a rare example of a situation when regression models of balance calibration data can directly be derived from first principles of physics and engineering. In addition, it is interesting to see that the search algorithm recommended the same regression models for the data analysis using only a set of statistical quality metrics.

Nomenclature

d	= distance between forward and aft moment bridge
$d_\circ$	= distance between model reference center and balance moment center
F	= normal force at balance moment center, $[lbs]$
M	= pitching moment at balance moment center, $[in - lbs]$
M_1	= moment at forward moment bridge, $[in - lbs]$
M_2	= moment at aft moment bridge, $[in - lbs]$
R_1	= electrical output at forward moment bridge, $[\mu V/V]$
R_2	= electrical output at aft moment bridge, $[\mu V/V]$
$\alpha_1, \alpha_2, \cdots, \alpha_6$	= regression coefficients
$\beta_1, \beta_2, \cdots, \beta_6$	= regression coefficients
γ_1, γ_2	= regression coefficients
δ_1, δ_2	= regression coefficients
ϵ_1, ϵ_2	= regression coefficients
η_1, η_2, η_3	= regression coefficients

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I. Summary

During the past 4 years a software package was developed at the Ames balance calibration lab that is used for the analysis of wind tunnel strain–gage balance calibration data. The software uses an innovative candidate math model search algorithm in order to find an optimized regression model for the balance calibration data analysis. This so–called recommended math model meets strict statistical quality requirements that the user specifies before the math model search is performed (see Ref. [1] and [2] for a detailed description of the algorithm). Figure 1 shows key elements of the candidate math model search algorithm.

So far, the search algorithm was applied to a wide variety of strain–gage balance calibration data sets (see, e.g., Refs. [3], [4], [5], and [6]). Recently, calibration data of a sting balance was processed using the candidate math model search algorithm. **The analysis of this data set will be discussed in great detail in the proposed paper.** The selected sting balance had two moment gages that were mounted forward and aft of the balance moment center. Figure 2 shows the location of the two moment gages relative to the balance moment center. The electrical outputs of the two balance gages are called R_1 and R_2. These gage outputs are a function of the normal force F and the pitching moment M that the balance experiences at the balance moment center. The two loads are defined by the following equations

$$F = \frac{M_2 - M_1}{d} \tag{1a}$$

$$M = \frac{M_1 + M_2}{2} \tag{1b}$$

where M_1 is the moment at the forward moment bridge, M_2 is the moment at the aft moment bridge, and d is the distance between the two balance gages (see Fig. 2).

During the analysis of the calibration data and during the regression model search (i) the difference and (ii) the sum of the two gage outputs were fitted in the least squares sense using the normal force and the pitching moment as independent variables. The regression model search algorithm needed an upper and a lower bound of the regression models in order to define the regression model search space. It was decided to use the following two regression models as upper bounds for the candidate math model search:

$$\mathcal{FIRST\ UPPER\ BOUND} \implies R_2 - R_1 = \alpha_1 + \alpha_2 \cdot F + \alpha_3 \cdot M + \alpha_4 \cdot F^2 + \alpha_5 \cdot M^2 + \alpha_6 \cdot F \cdot M \tag{2a}$$

$$\mathcal{SECOND\ UPPER\ BOUND} \implies R_1 + R_2 = \beta_1 + \beta_2 \cdot F + \beta_3 \cdot M + \beta_4 \cdot F^2 + \beta_5 \cdot M^2 + \beta_6 \cdot F \cdot M \tag{2b}$$

From Eq. (1a) we know that the normal force is approximately proportional to the difference of the moments at the forward and aft moment gages. We also know from Eq. (1b) that the pitching moment is approximately proportional to the sum of the moments at the forward and aft moment gages. Therefore, the following two linear regression models were selected as lower bounds for the regression model search:

$$\mathcal{FIRST\ LOWER\ BOUND} \implies R_2 - R_1 = \gamma_1 + \gamma_2 \cdot F \tag{3a}$$

$$\mathcal{SECOND\ LOWER\ BOUND} \implies R_1 + R_2 = \delta_1 + \delta_2 \cdot M \tag{3b}$$

In the next step the candidate math model search algorithm was applied to the sting balance calibration data set. A total number of 20 regression models were tested during the search. Afterwards, the algorithm chose the following two recommended math models for the analysis of the data:

$$\mathcal{FIRST\ RECOMMENDED\ MATH\ MODEL} \implies R_2 - R_1 = \epsilon_1 + \epsilon_2 \cdot F \tag{4a}$$

$$\mathcal{SECOND\ RECOMMENDED\ MATH\ MODEL} \implies R_1 + R_2 = \eta_1 + \eta_2 \cdot M + \eta_3 \cdot M^2 \tag{4b}$$

It is a surprising result of the regression model search that (i) the recommended math model of the gage output difference $(R_2 - R_1)$ is only a function of the normal force and that (ii) the regression model of the sum of the gage outputs $(R_1 + R_2)$ is only a function of the pitching moment and the square of the pitching moment. An explanation of this search result had to be found.

Fortunately, the overall geometry and design of the sting balance is very simple. It may be approximated as a cantilever beam. Then, it is possible to apply the equations of the deflection of a cantilever beam in order to show that the two recommended math models, i.e., Eq. (4a) and (4b), can also be obtained after performing a rigorous analysis of the deflection of the sting balance under load (see, e.g., Ref. [7] for the description of the deflection of a cantilever beam). **This assertion will systematically be proven in the proposed conference paper.**

In conclusion, the regression analysis of the given sting balance calibration data set is a rare example of a situation when regression models of a balance calibration data set can directly be derived from first principles of physics and engineering. It is also interesting to note that the candidate math model search algorithm predicted the same regression models for the balance calibration data using only a set of statistical quality metrics.

II. Acknowledgements

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III. References

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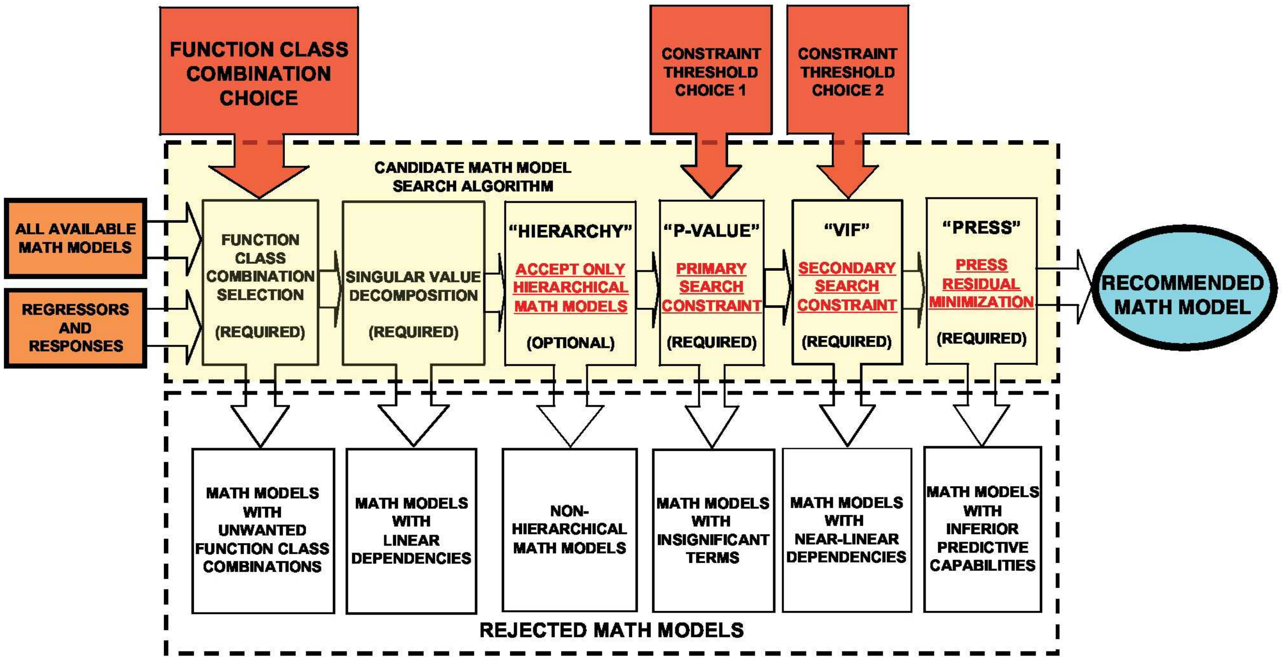

Fig. 1 Key elements of candidate math model search algorithm.

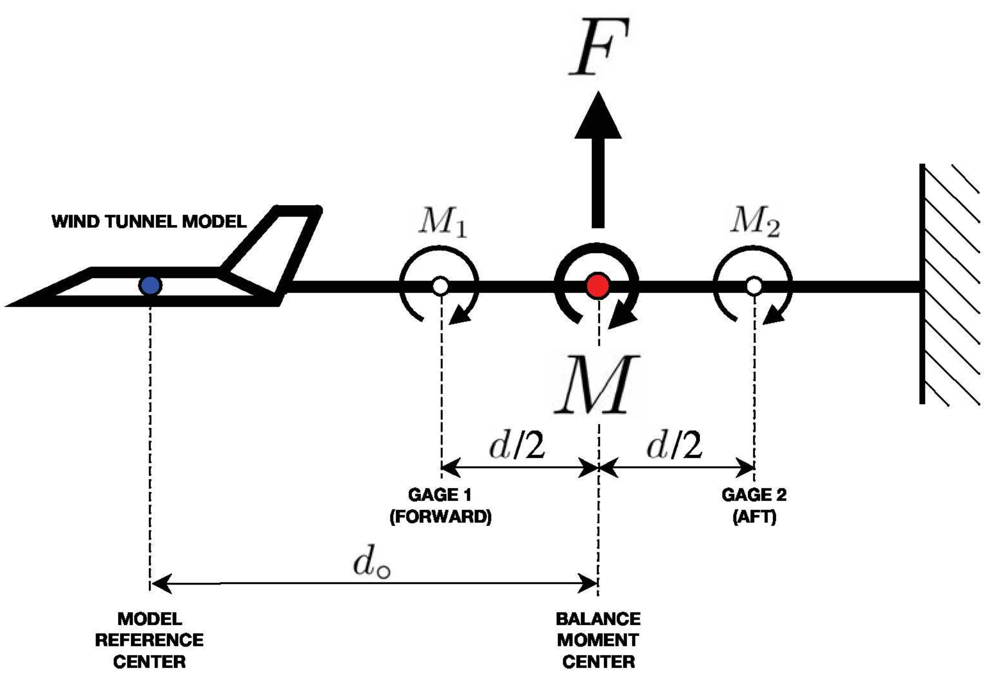

Fig. 2 Forces and moments acting on a wind tunnel sting balance.