



WAVEFRONT SENSING & CONTROL GROUP



# Status on Iterative Transform Phase Retrieval applied to the GBT Data

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Richard Prestage, Todd Hunter, Frank Ghigo, Bojan Nikolic

## **Image-Based Wavefront Sensing and Control of the NRAO Green Bank Radio Telescope**

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Astronomy Observatory (NRAO) Green Bank Operations, Green  
Bank, WV, Todd Hunter, Frank Ghigo, NRAO Green Bank  
Operations, Green Bank, WV, Bojan Nikolic, Mullard Radio  
Astronomy Observatory, University of Cambridge, UK  
**Business Area:** Astrophysics, Communications / Navigation Systems, Exploration  
Systems.



# Overview

## Introduction

- Phase Retrieval / NASA Projects
- JWST TRL-6

## GBT Data / Notes:

- Data Format and Sampling
- Ray Trace Model & Wavefront
- Symmetry of GBT Data
- Pupil and Fourier Model
- Pupil Amplitude

## PR Simulations

- Wavefront derived from GBT Data symmetry
- Wavefront Sensing accuracy and Coherent / Incoherent Assumptions

## GBT Results



# Applications and Technology Development

- NASA Investments in Image-Based WFSC
  - Developments through JWST Pre Phase-A and Phase-B,
  - WFSC Demonstrated to TRL-6 using the Ball Aerospace TBT,
  - Have investigated a number of performance and implementation details, e.g., optimal diversity defocus, bandpass, phase wrapping, Branch Points,
  - Compact Supercomputing Architecture utilizing DSPs

<u>Date</u>	<u>Projects</u>
1990	Hubble Primary Mirror Aberration Determination
1994	Mars Observer Camera In-flight Diagnosis
1996	Cassini ISS Narrow Angle Camera Verification Testing
09/1998	NASA Developmental Comparative Active Telescope Testbed (DCATT)
01/1999	NASA Wavefront Control Testbed (WCT)
01/2000	NASA Wavefront Control Testbed 2 (WCT-2)
01/2002	NASA Wavefront Control Testbed 3 (WCT-3)
08/2000	IRAC Testing (Spitzer Space Telescope)
08/2001	Phase Retrieval Camera
04/2002	RIVMOS Testing
07/2002	NIRSpec Microshutter (MSA) Testbed
09/2002	HUBBLE Simulator Hardware (CASTLE)
04/2003	TPF's High Contrast Imaging Testbed (HCIT)
04/2003	Mercury Laser Altimeter (MLA)
06/2003	NASA Fixed Lens WFS Testing
08/2003	JWST AMSD Mirror Testing with a Phase Retrieval Camera
07/2003	Ball Aerospace RA-6 (Boulder, CO)
10/2003	GSFC EUNIS Testing
07/2004	IRMOS Modeling
09/2004	Keck I (Kamuela, HI)
08/2004	HST Wide-Field Camera III
07/2005	Palomar 200" Telescope Adaptive Optics System (PALAO) Calibration
10/2005	JWST Testbed Telescope (TBT; Ball Aerospace, Boulder, CO)

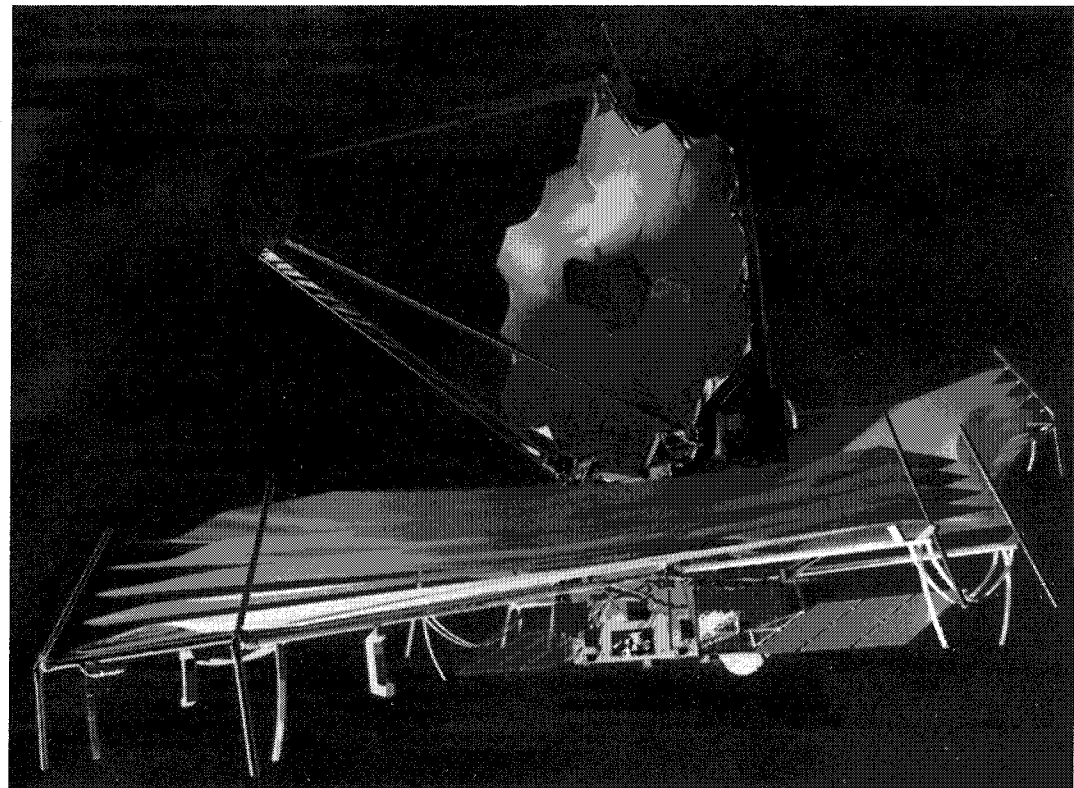


# James Webb Space Telescope (JWST)

- Successor to the Hubble Space Telescope
- Current Launch Date is 2013
  
- 18 Segment PM
- 6.5 meter aperture
- Orbit at L2

***NORTHROP GRUMMAN***  
*Space Technology*

*Ball*



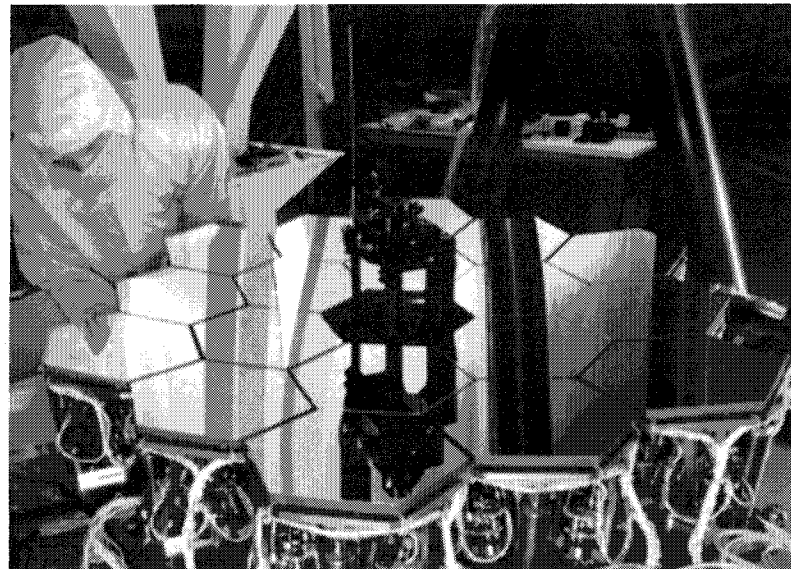
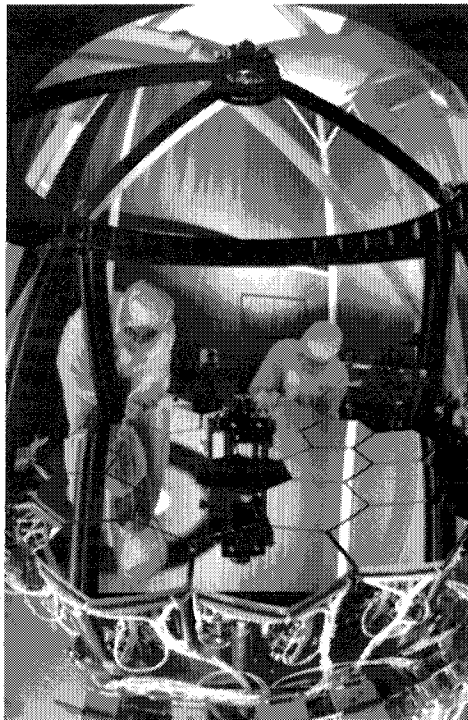


# Testbed Telescope (TBT)

Flight traceable, 1/6 scale, 18 segment design

Algorithm Performance requirements dictated by NASA's TRL-6

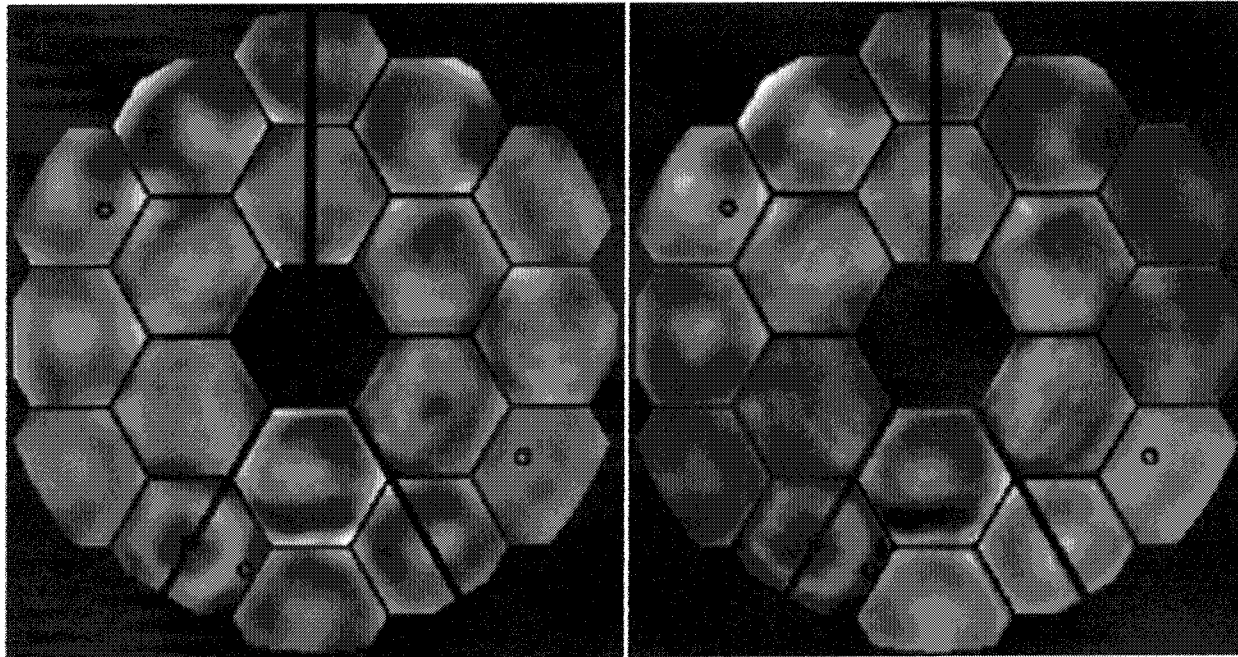
- Testbed provides functionally accurate simulation platform for developing deliverable WFSC algorithms and software,
- Used to perform TRL-6 end to end testing,
- a solution is a fine-phasing algorithm that incorporates feedback,
- an adaptive diversity function, eliminates Branch Points, and Wrapping



# TRL-6 Comparison with Interferometer

Phase Retrieval:

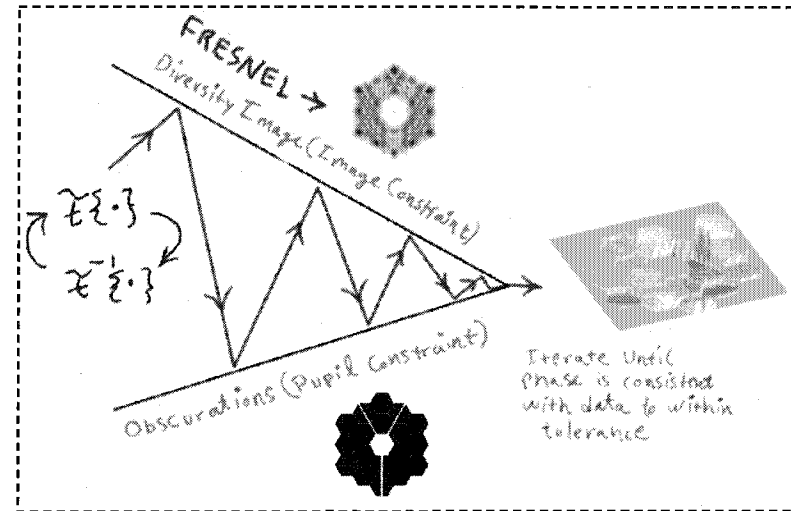
Interferometer:



# Phase Retrieval Approaches

- Two main approaches commonly used:

- Iterative Transform (ITA)
  - Gerchberg-Saxton
  - Misell-Gerchberg-Saxton
  - HDA (extends dynamic range)



- Parametric (non-linear least squares model fitting)
  - Solve for aberration coefficients
  - Solve for point-point phase in the pupil

*For JWST - adopted a hybrid approach that incorporates features of both types of algorithms.*

min [Objective Function]:

$$\frac{\partial}{\partial a_m} \sum_{j=1:N} \left\| \text{PSF}_{\text{data},j} - \text{PSF}_{\text{model},j}(\lambda, f_{\#}, \text{Pixel}, \Phi_{\text{DIV}}, \vec{a}) \right\|^2 = 0$$

† e.g., W. H. Southwell, "Wavefront Analyzer using a Maximum Likelihood Algorithm," J. Opt. Soc. Am. A3, 396-399 (1977).

# Concept:

- phase from intensity data?

$$z = x + iy = r e^{i\theta}$$

intensity
phase  
↓
↓

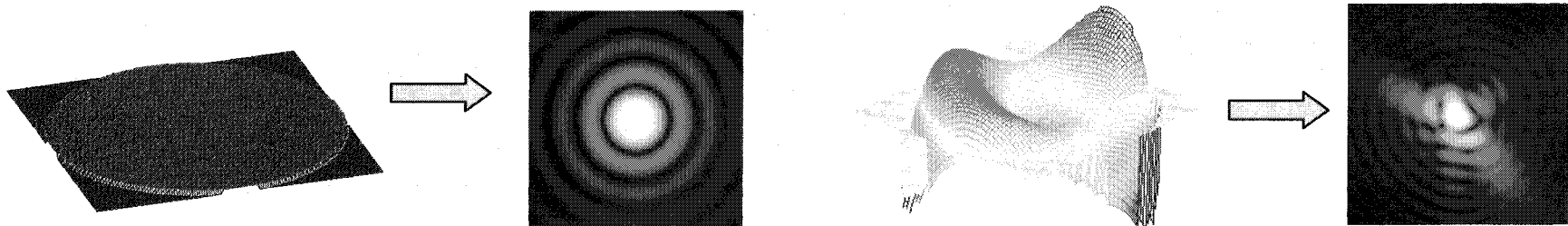
- complex numbers:

$$\Rightarrow |z|^2 = r^2 e^{i\theta} e^{-i\theta} = r^2 \neq r^2(\theta)$$

- phase part is decoupled from intensity

- phase-recovery fact - optical aperture scatters phase information into the intensity data

- star image –normally like an airy disk for a circular aperture:



$$r^2 = r^2(\theta)$$

- intensity is now a function of the phase:
- algorithm: indirectly recover phase from intensity.





## Earlier Work using ITA with Radio Antennas

- 1985, D. MORRIS 'Phase retrieval in the radio holography of reflector antennas and radio telescopes', IEEE Trans., AP-33, pp.749-755
- 1988, D. Morris, et al., "Radio holography measurement of the 30-m millimeter radio telescope ...," Astron. Astrophys., vol. 203, p. 399.
- 1991, D. MORRIS, et. al, 'Experimental assessment of phase retrieval holography of a radiotelescope', IEE Proc. H, 138, pp. 243-247
- 1994, A. Greve, D. Morris, et. al., "Astigmatism in Reflector Antennas: Measurement and Correction," IEEE Trans ANTENNAS & Prop VOL. 42, NO. 9
- 1996, D. Morris, Simulated Annealing Applied to the Misell algorithm for phase retrieval, IEE Proc - Microw Antennas Prop , Vol 143, No 4, August 1996



# Notes / Understanding of GBT Data

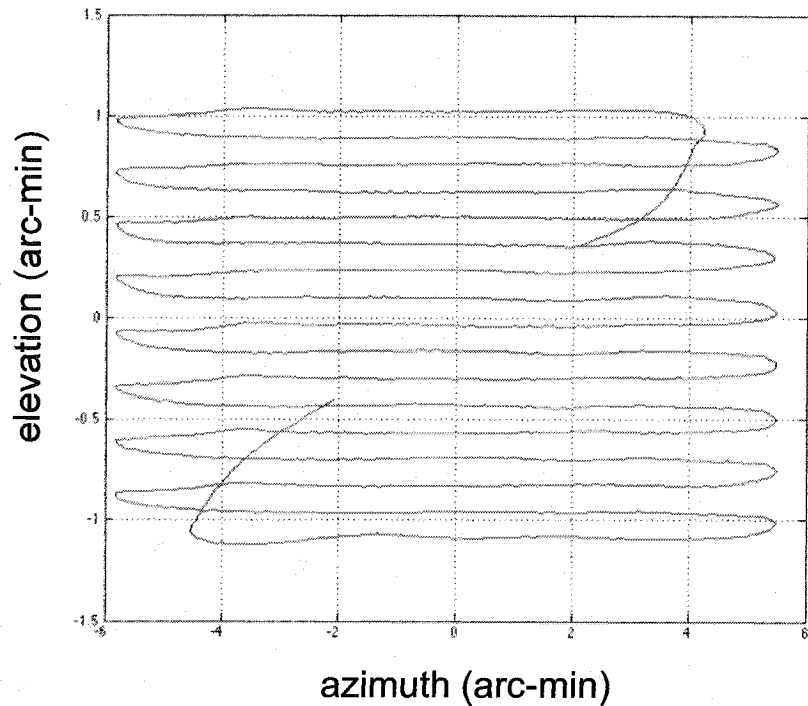
- Consists of two feeds (pixels), two polarizations,
- Separated by 58 arc-seconds,
- Output of receivers is differenced to minimize the effect of sky-brightness variations.
- Effective response of the telescope is modeled as the real beam convolved by two delta functions separated by 58" in the azimuth direction
  - aberrations due to both of the feeds being off (and on opposite sides of) the optical axis are negligible?
  - if this is not negligible, then a “single beam convolved by two delta functions” assumption may not be valid.



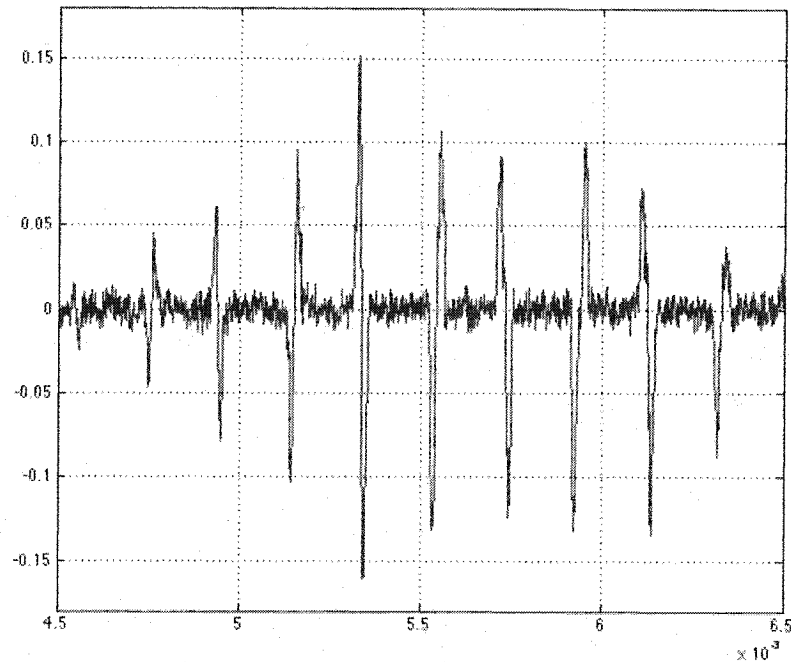
# Raw Data Contributed by the NRAO

- Data Filename: s114-l-db.fits, April 2005
- Read: dx, dy, fnu, ufnu, ttime ([5806×1 double])

Scan Pattern: (plot dx, dy) :



Signal vs time (plot fnu, ttime):



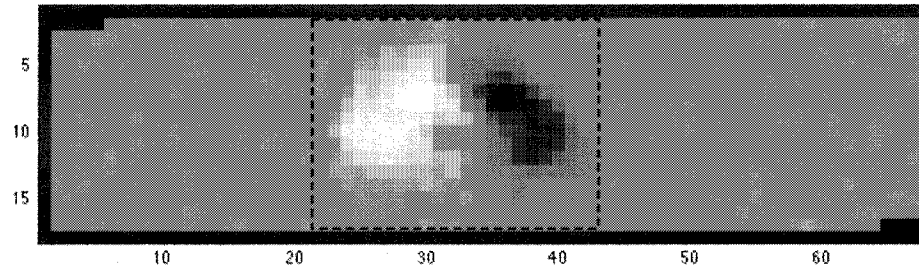


# NRAO Data: Non-uniform data samples are interpolated:

- Data values: dx (azm) , dy (elv) are used to form a rectangular coordinate array.

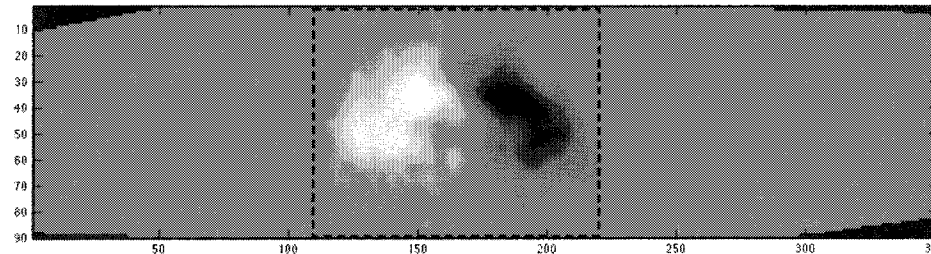
## Two Options:

down-sample  
in x:



17 × 68

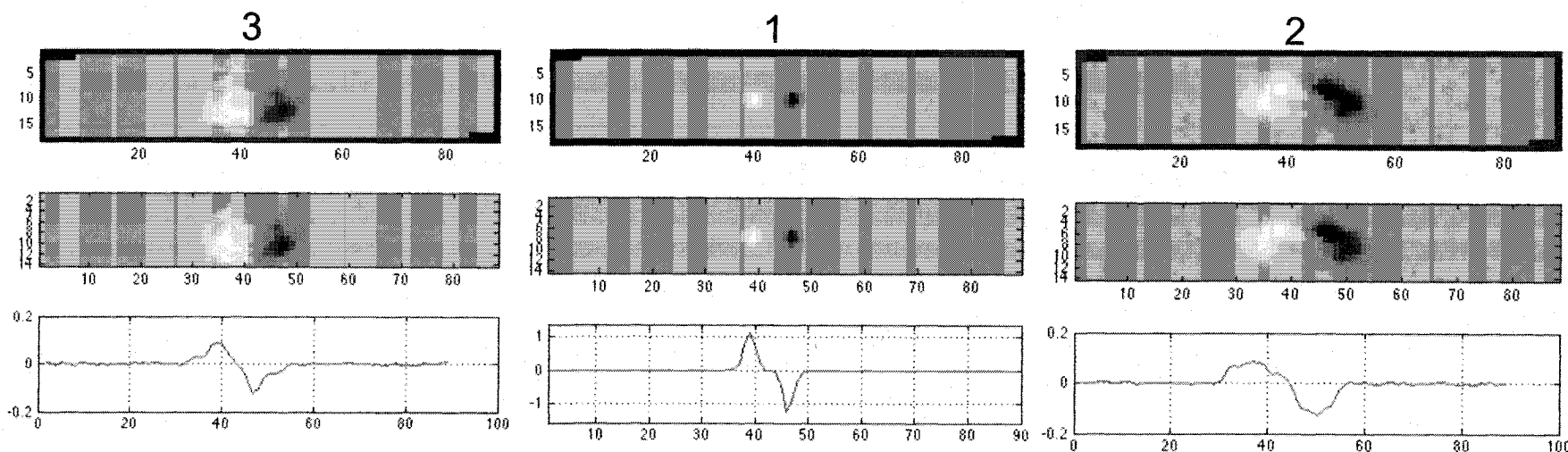
up-sample  
in y:



90 × 351

- First interpolated to a uniform rectangular grid (azm-elev),
- A rectangular coordinate grid of 17 by 68 is formed and then the 5806 fnu data values are interpolated to this grid using cubic interpolation.

# NRAO Data & Sampling



$\nu = 43.1$  GHz;  $\lambda = 6.96$  mm

Azimuth direction (x), approximately 350 samples/per scan line.

Sampling in Azimuth =  $3600 \cdot (180/\pi) \cdot (dx(251) - dx(250)) = 2.42$  Arcsec / pixel =  $p_x = 1.1732e-05$  radians

$Q_x = \lambda / (D \cdot p_x) = 6.96e-3 / (100 \cdot 1.1732e-05) = 5.9325$

Elevation direction (y), 17 scan lines

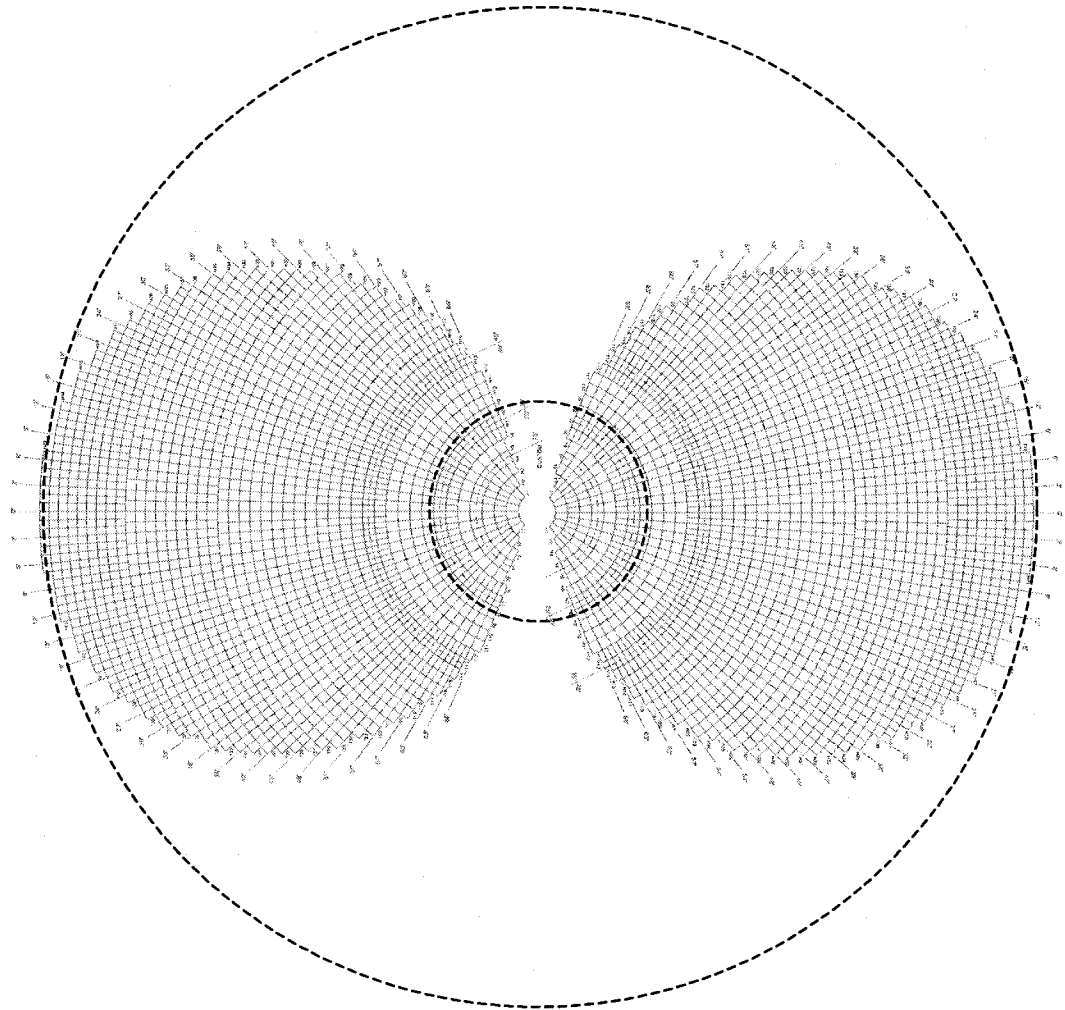
Sampling in Elevation =  $3600 \cdot (180/\pi) \cdot (dy(5600) - dy(250)) / 17 = 7.5$  Arcsec / pixel =  $p_y = 3.6361e-05$  radians

$Q_y = \lambda / (D \cdot p_y) = 6.96e-3 / (100 \cdot 3.6361e-05) = 1.9141$

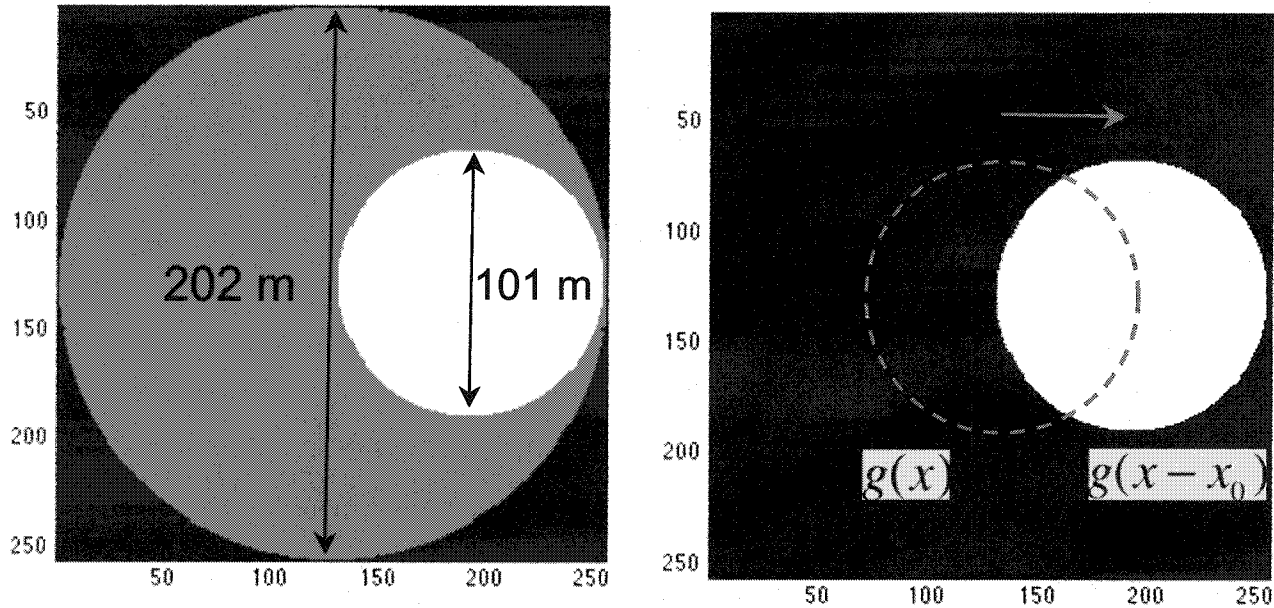
Nyquist sampling is 7.2 arcsec / sample,  $Q = 2$ ; Under-sampled by 0.96 in Elevation; over-sampled by 2.97 in Azimuth

# NRAO GBT Aperture

- Panels are arranged in such a way that rings are concentric with a parent parabola.
- Zemax design: GBT is setup as a single off-axis section of the parent parabola.



# GBT Fourier Model



Note that Translation Shift of Fourier Transform produces a phase factor:

$$\mathcal{F}[g(x-x_0)] = G(\omega)e^{-i2\pi\omega x_0}$$



# Pupil Illumination - I

Cool Link: edge taper in radio astronomy (Cheng / Mangum):

<http://www.alma.nrao.edu/memos/html-memos/alma197/memo197.html>

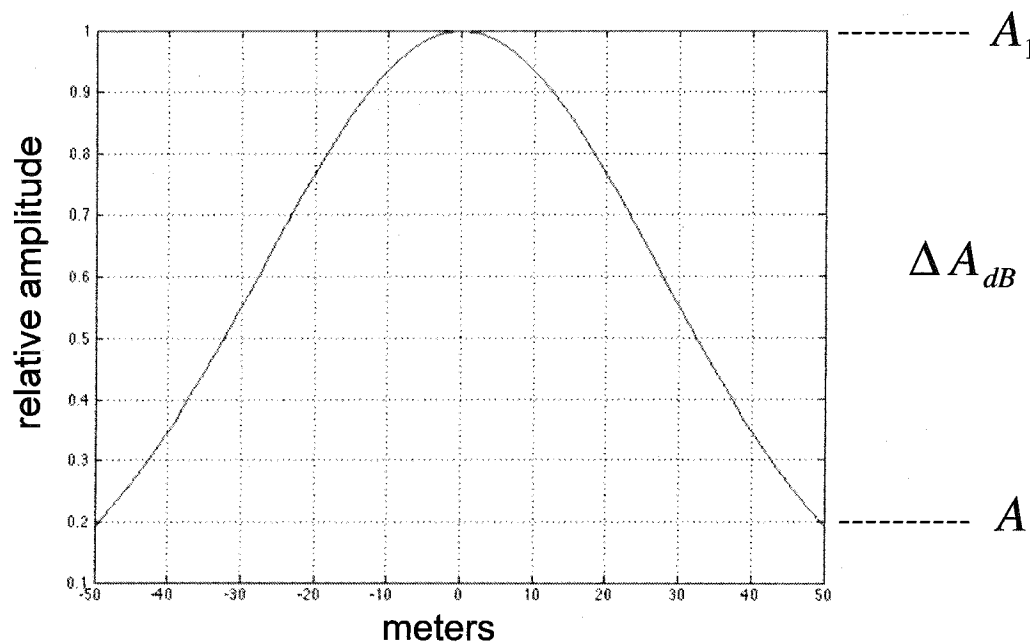
$$A(r) = 1, \quad (\text{uniform})$$

$$= \exp[-\alpha(r/r_0)^2], \quad (\text{tapered Gaussian})$$

$$\alpha = (T_e / 20) \ln 10, \quad (\text{edge taper factor})$$

$T_e \equiv$  edge taper in dB

Using the formula for edge taper in dB:



amplitude variation:

$$\Delta A_{dB} = 10 \log[(A_1 / A_2)^2] = 20 \log(1 / 0.18)$$

$$\approx 15 \text{ dB}$$





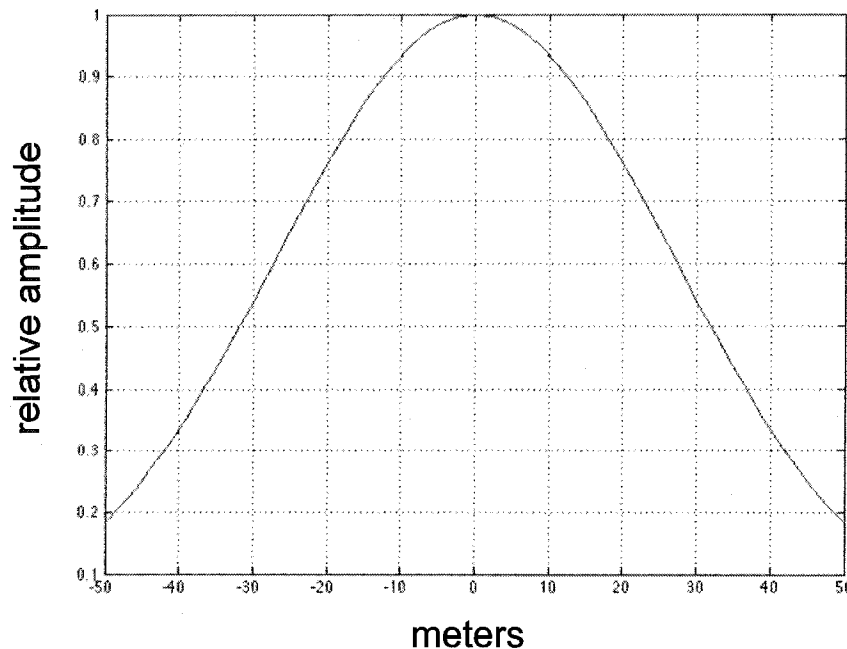
# Pupil Illumination - II

$$A(r) = 1, \quad (\text{uniform})$$

$$= \exp[-\hat{r}^2 / 2\sigma^2], \quad (\text{tapered Gaussian})$$

with  $\sigma = 0.3$ , from PTCSPN47.pdf

... the aperture plane amplitude distribution, that is, the illumination of the primary surface. This was approximated as a well-centered and circular Gaussian with a width (in radius-normalized units) defined by  $\sigma = 0.3$ , which corresponds to 14.5 dB of illumination taper at the edge of the dish ...



$$\sigma \approx 0.55$$

*amplitude variation:*

$$\Delta A_{dB} = 10 \log[(A_1 / A_2)^2] = 20 \log(1 / 0.18) \\ \approx 15 \text{ dB}$$



# Challenge for ITA Phase Retrieval

- Two incoherently subtracted irradiance values appear in the GBT data.
- Data collection process,  $I = I_1 - I_2$
- For the ITA approach to work, these irradiance values should be the result of one FFT.
- So make the approximation that:

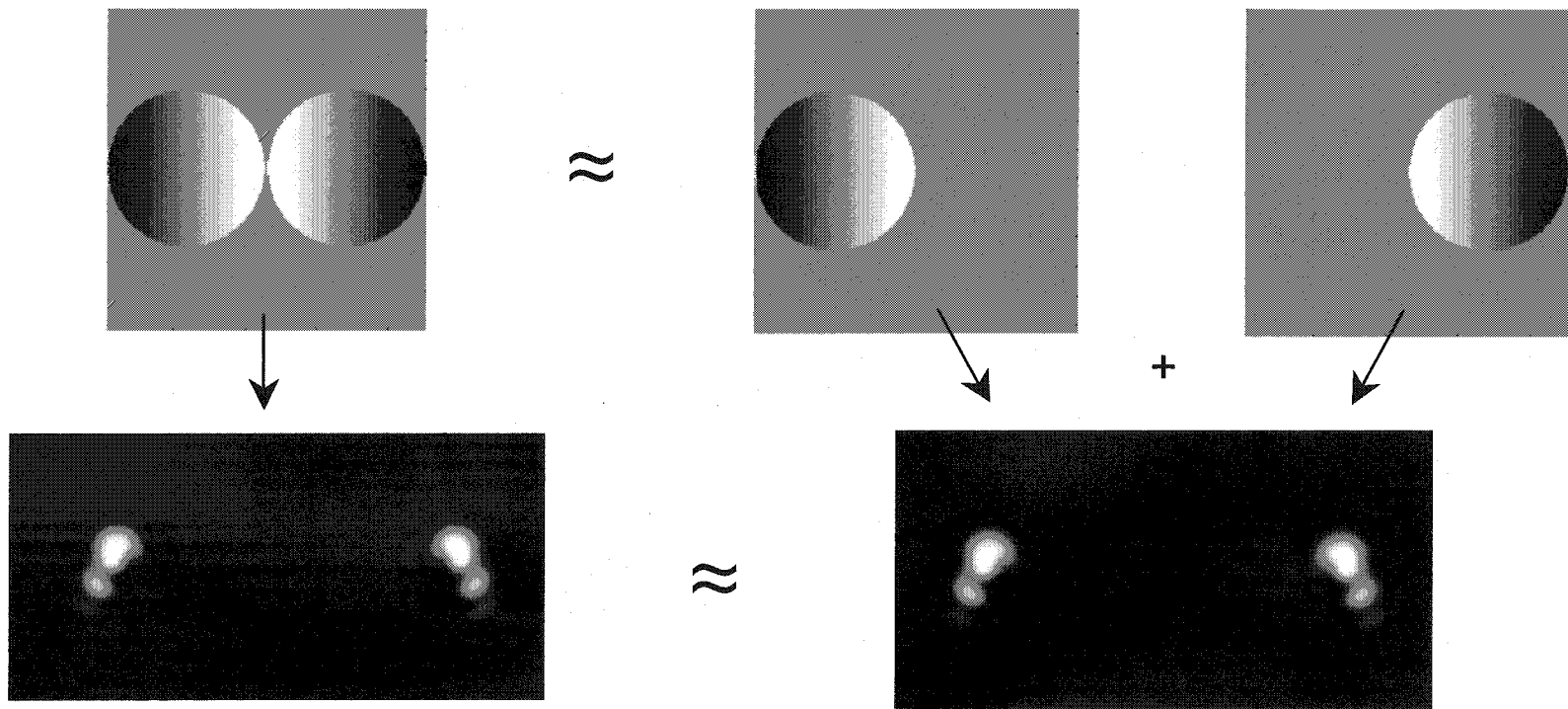
Coherent Approximation for Incoherent Data:

$$|\mathfrak{I}\{A_L(-\theta_t) + A_R(+\theta_t)\}|^2 \approx |\mathfrak{I}\{A_L(-\theta_t)\}|^2 + |\mathfrak{I}\{A_R(+\theta_t)\}|^2$$

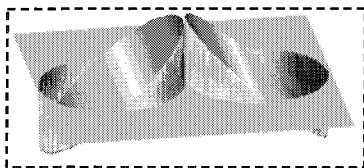
or simply  $I \approx I_L + I_R$

# Coherent Approximation for Incoherent Data

$$|\mathfrak{F}\{A_L(-\theta_t) + A_R(+\theta_t)\}|^2 \approx |\mathfrak{F}\{A_L(-\theta_t)\}|^2 + |\mathfrak{F}\{A_R(+\theta_t)\}|^2$$



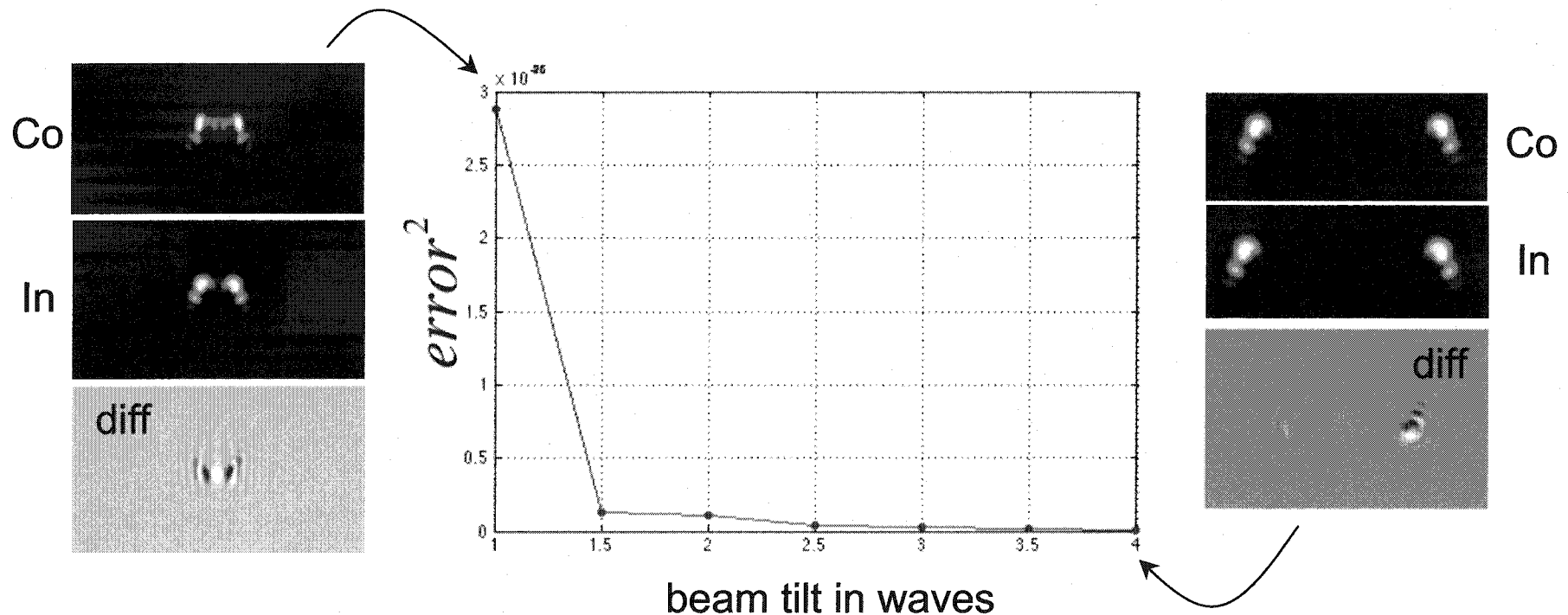
beam tilt:



# Validity of Approximation?

- Good approximation for large tilt (i.e., there is little interference)
- Plot of squared error as a function of tilt:

$$error^2 = \sum_k \sum_j |I_{coherent}(j, k) - I_{incoherent}(j, k)|^2$$

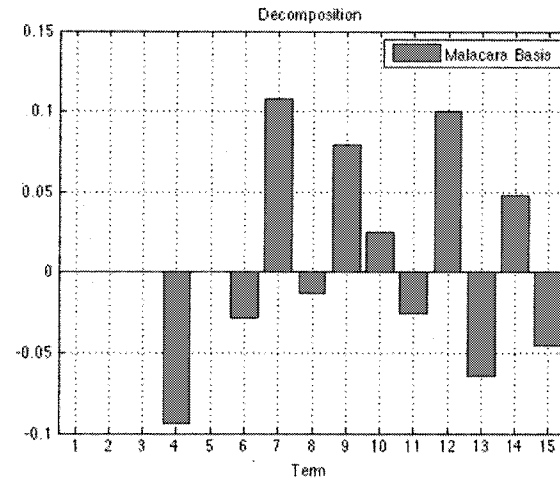
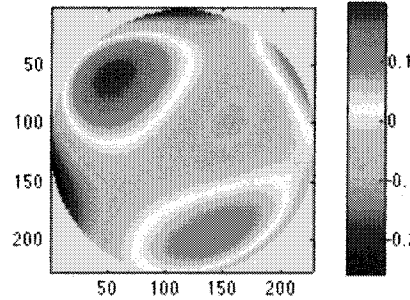
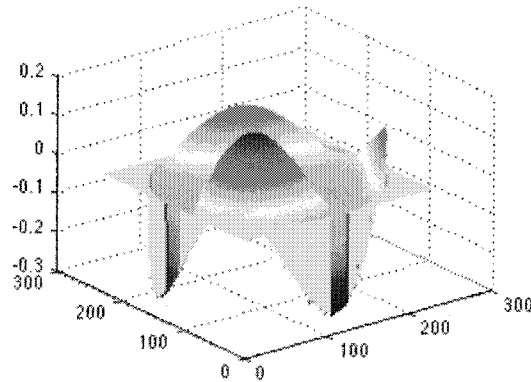


*How does error propagate to Phase Retrieval?*



# Proof of Concept: PR Simulation

Input Wavefront:



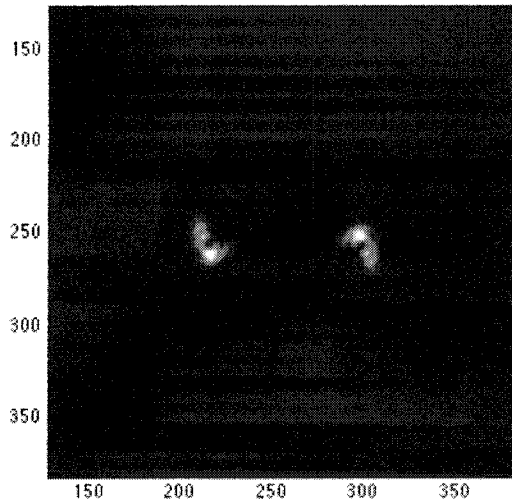
RMS = 557  $\mu$ ;  $\lambda/13$ ; PV = 0.4561  $\lambda$

Malacara Basis Set:

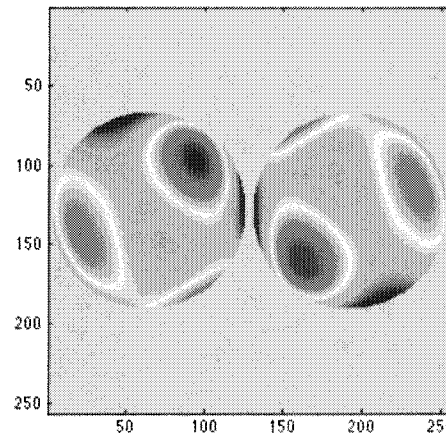
#	radial	azimuth	term	aberration
1	0	0	1	piston
2	1	0	$r \sin \alpha$	y-tilt
3	1	1	$r \cos \alpha$	x-tilt
4	2	0	$r^2 \sin 2\alpha$	45° astig (1 <sup>st</sup> order)
5	2	1	$2r^2 - 1$	defocus
6	2	2	$r^2 \sin 2\alpha$	0° astig (1 <sup>st</sup> order)
7	3	0	$r^3 \sin 3\alpha$	30° trefoil
8	3	1	$r(3r^2 - 2) \sin \alpha$	y-coma
9	3	2	$r(3r^2 - 2) \cos \alpha$	x-coma
10	3	3	$r^3 \cos 3\alpha$	0° trefoil
11	3	3	$r^4 \sin 4\alpha$	22.5° tetrafoil
12	3	3	$(4r^4 - 3r^2) \sin 2\alpha$	45° astig (2 <sup>nd</sup> order)
13	3	3	$6r^4 - 2r^2 - 1$	spherical
14	3	3	$(4r^4 - 3r^2) \cos 2\alpha$	0° astig (2 <sup>nd</sup> order)
15	3	3	$r^4 \cos 4\alpha$	0° tetrafoil

# Check: Coherent PR Simulation

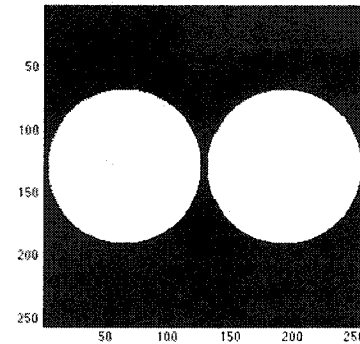
Image on 1-side of focus:



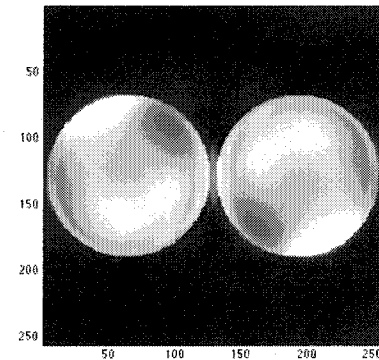
Dual aperture model:



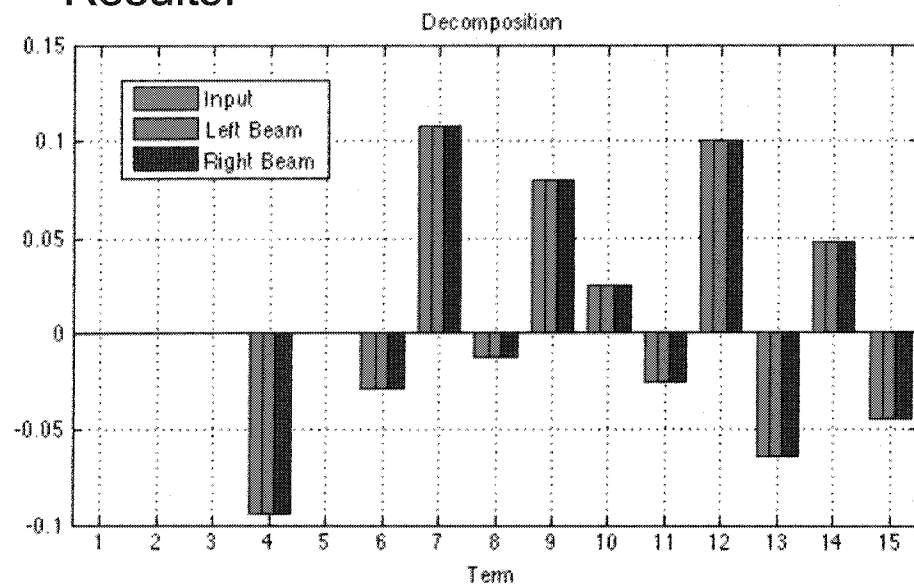
Pupil Amplitude:



*Recovered:*



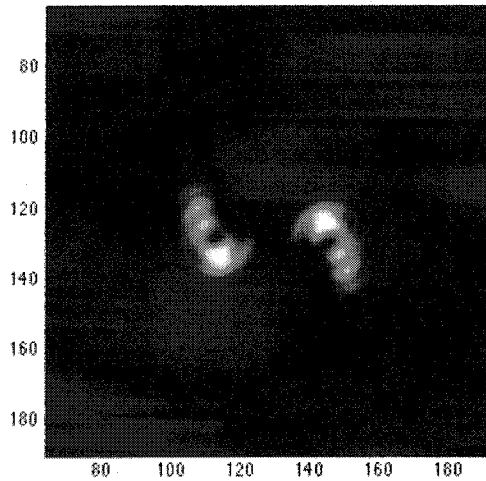
Results:



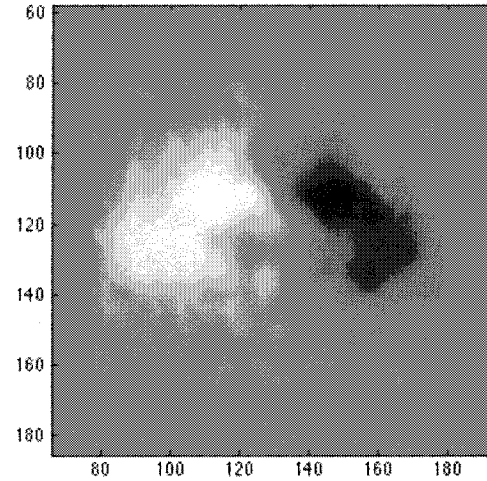


# Comment on GBT Beam Symmetry

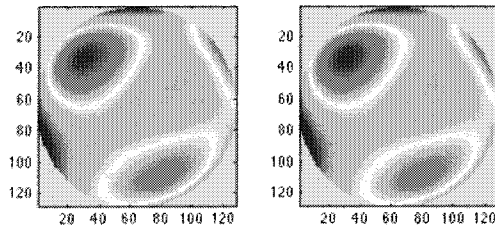
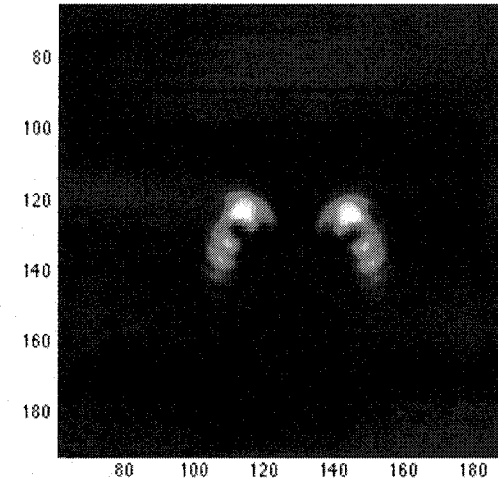
Model



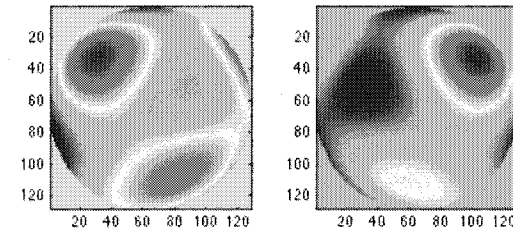
Data



Incoherent Model

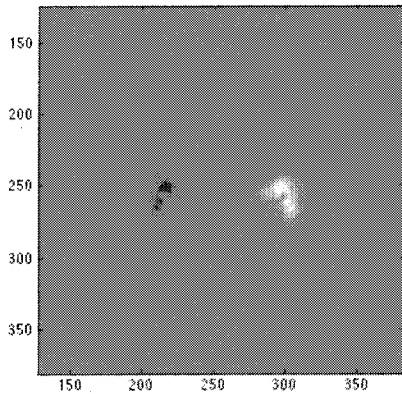


Let PR solve for  
the wavefront  
that is consistent  
with the data:

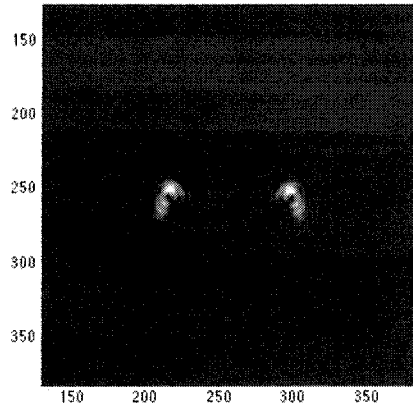


# Incoherent PR Results (simulation)

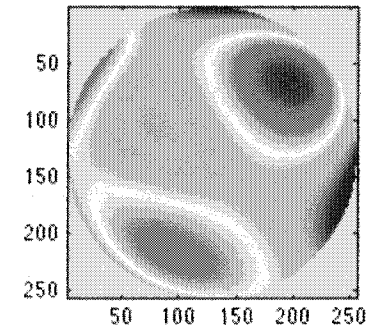
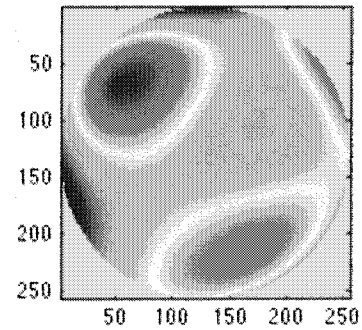
Incoherent Data:



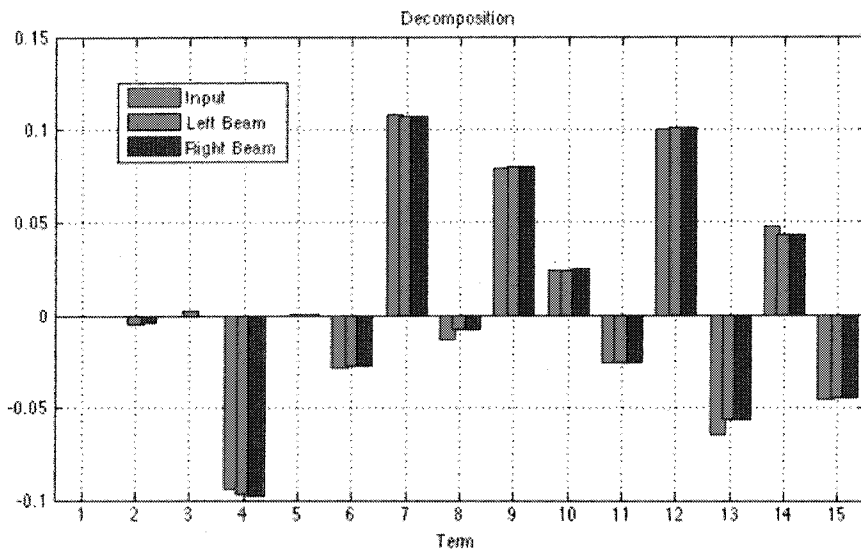
abs()



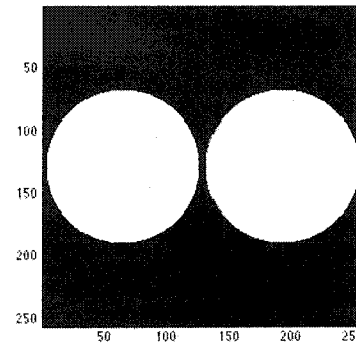
dual wavefront:



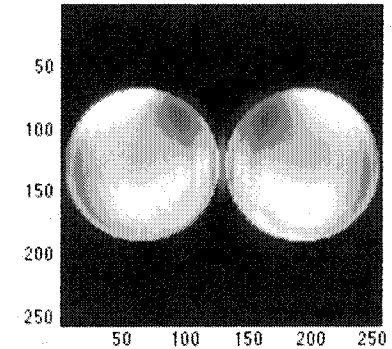
Results:



Pupil Amplitude:



Recovered:

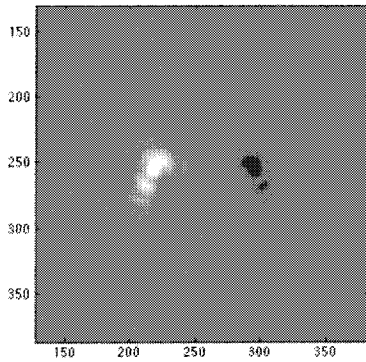




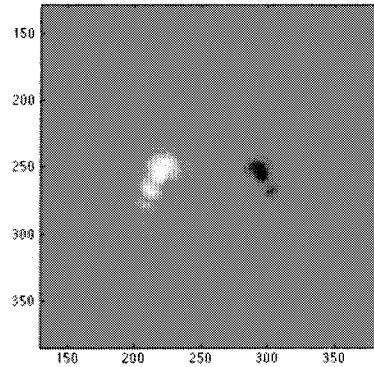
# Incoherent PR Results - worst case

-- Simulation: 2 waves beam tilt

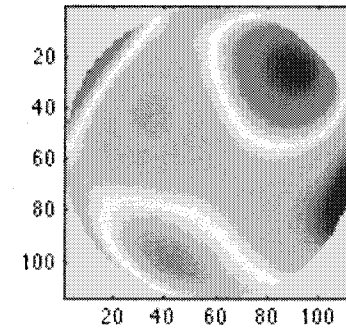
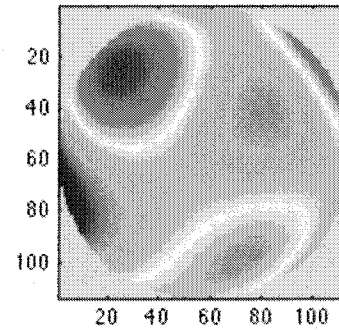
Incoherent Data:



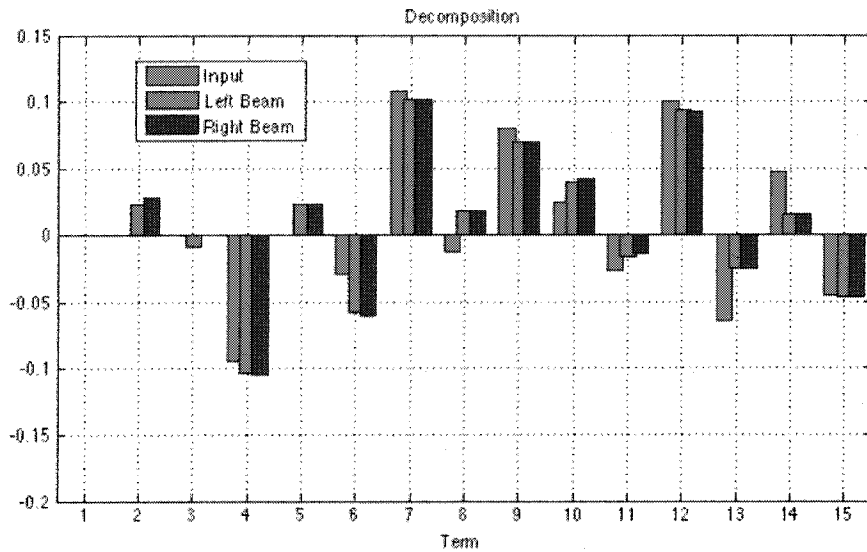
Model:



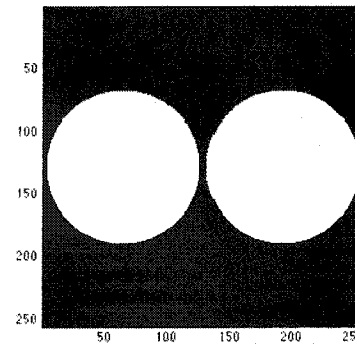
dual wavefront:



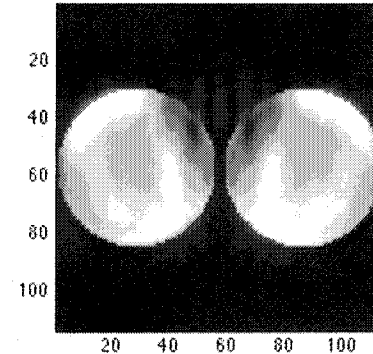
Results:



Pupil Amplitude:

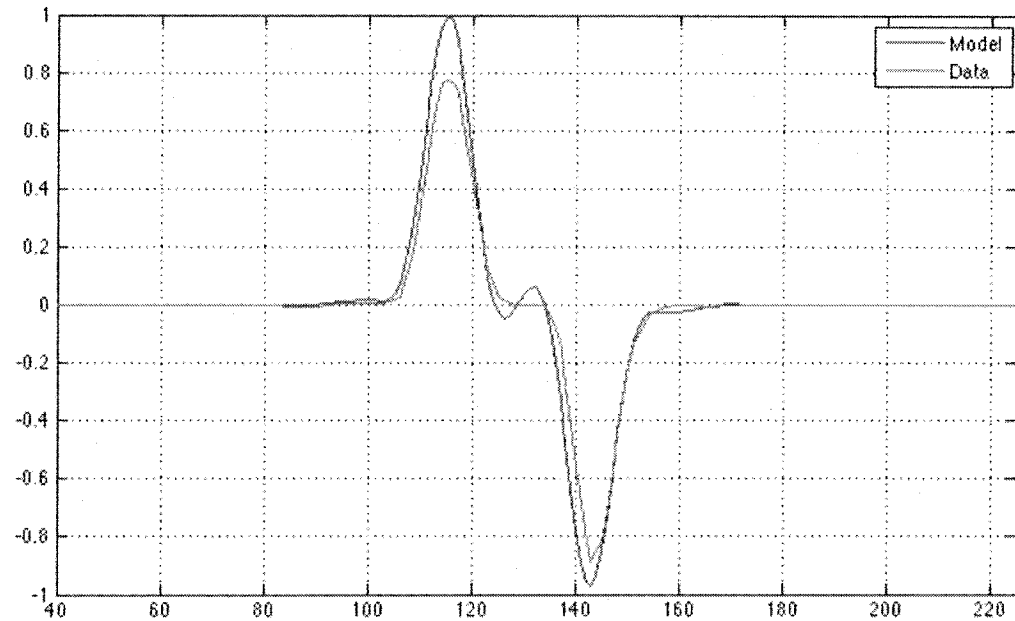
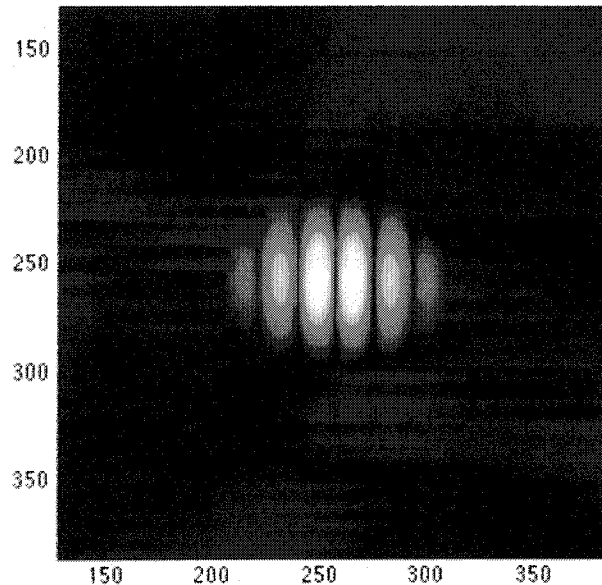


Recovered:



# Estimate Initial Sampling Parameters from focused GBT Data

- $Q \approx 4.5$
- Beam tilt  $\approx 1.5 \lambda$

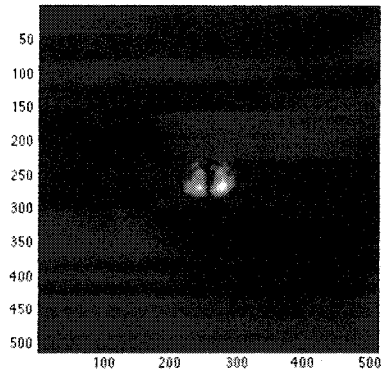


Can also tune parameters by  
matching the FFT of the data

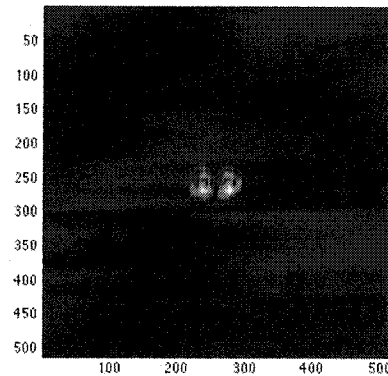


# Wavefront Sensing Results applied to GBT Data - 3

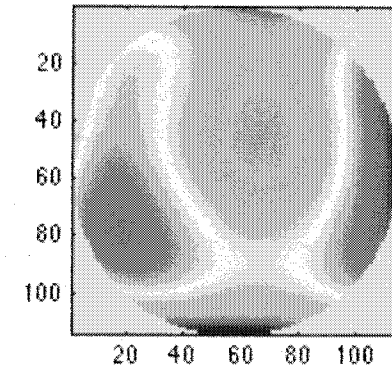
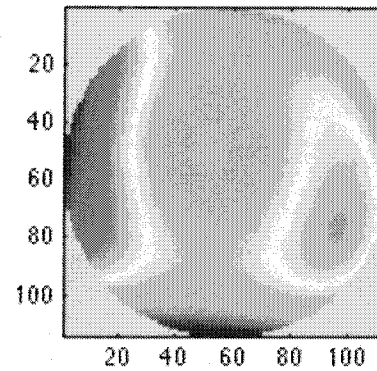
GBT Data:



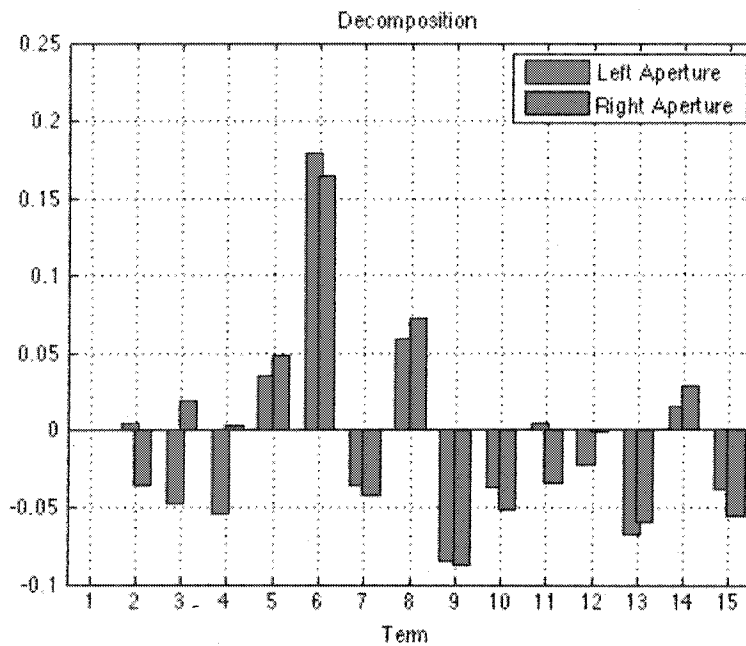
Model:



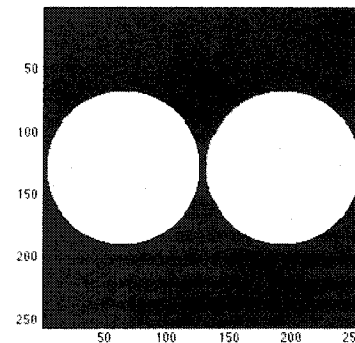
wavefront:



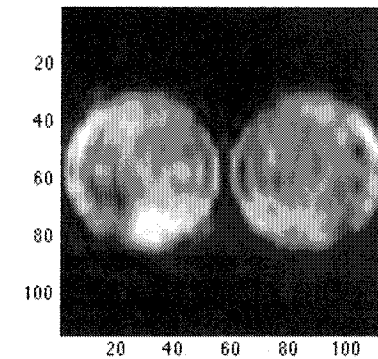
Results:



Pupil Amplitude:



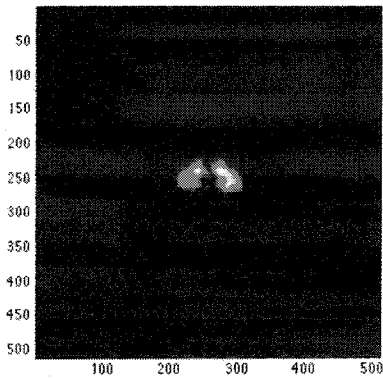
Recovered:



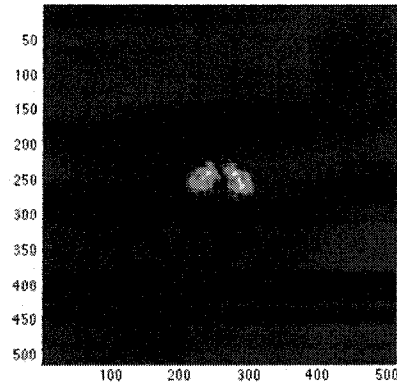


# Wavefront Sensing Results applied to GBT Data - 2

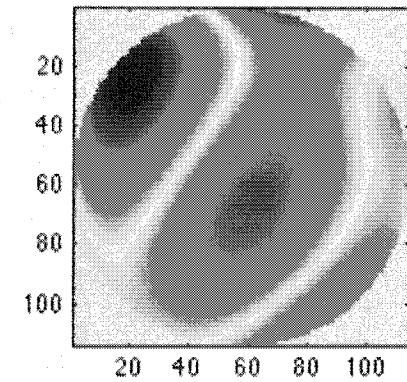
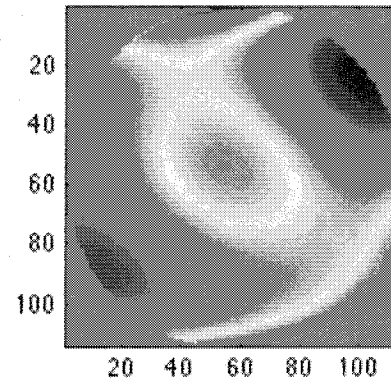
GBT Data:



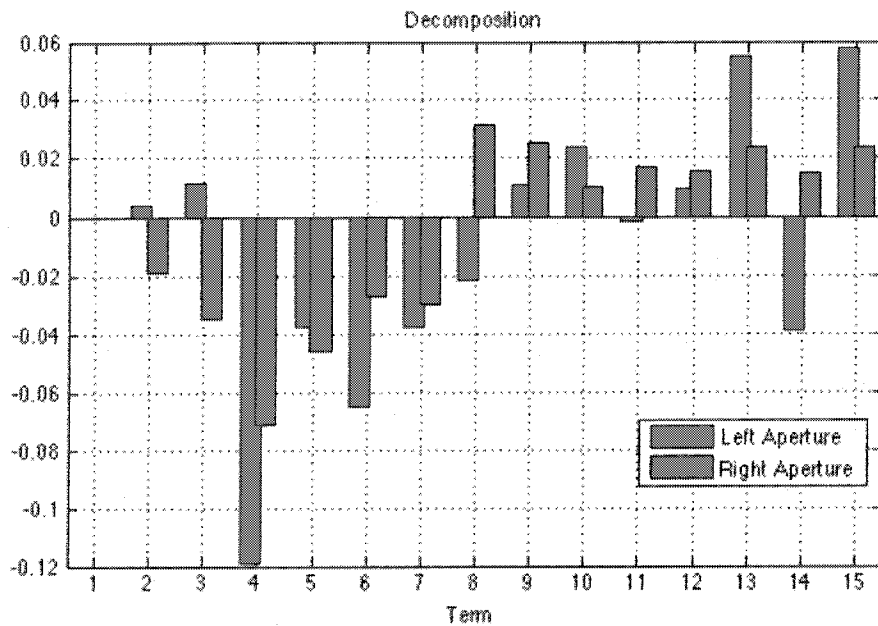
Model:



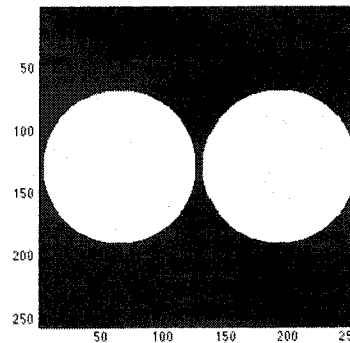
wavefront:



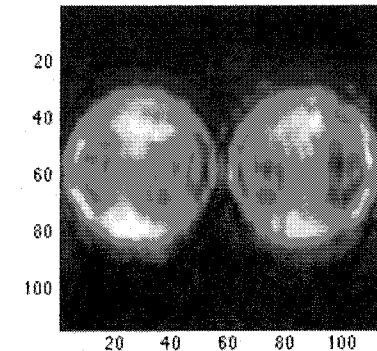
Results:



Pupil Amplitude:



Recovered:





# Summary

- In principle, coherent ITA PR may work on incoherent GBT data,
- Errors increases as beam tilt decreases