

These **Point-Spread-Function Images** were obtained as the response of a test optical system (a deformable mirror) at the two noted wavelengths. The overall trefoil shape, representative of low-order aberrations, does not differ much between the two wavelengths. The “bumpy” higher-spatial-frequency image components of the two images differ noticeably, but these components represent higher-order aberrations that, typically, are smaller than the lower-order aberrations.

gorithms that recover optical phase information, and phase-retrieval algorithms constitute a subset of this class.

In phase retrieval, one utilizes the measured response of the optical system under test to produce a phase estimate. The optical response of the system is defined as the image of a point-source object, which could be a star or a laboratory point source. The phase-retrieval problem is characterized as “image-based” in the sense that a charge-coupled-device camera, preferably of scientific imaging quality, is used to collect image data where the optical system would normally form an image. In a variant of phase retrieval, denoted phase-diverse phase retrieval [which can include focus-diverse phase re-

trieval (in which various defocus planes are used)], an additional known aberration (or an equivalent diversity function) is superimposed as an aid in estimating unknown aberrations by use of an image-based wavefront-sensing algorithm.

Image-based phase-retrieval differs from such other wavefront-sensing methods, such as interferometry, shearing interferometry, curvature wavefront sensing, and Shack-Hartmann sensing, all of which entail disadvantages in comparison with image-based methods. The main disadvantages of these non-image-based methods are complexity of test equipment and the need for a wavefront reference. This concludes the background information.

The present development began with a theoretical observation that the low-order aberration content of the point-spread-function of an optical system is not strongly affected by wavelength over the visible spectrum (see figure). As a result, variations in wavelength do not significantly affect what a phase-retrieval algorithm “sees” as input. This lack of variability of effective input is what makes it possible to assume monochromaticity when processing image data acquired while using broadband light.

The validity of the assumption of monochromaticity was demonstrated by comparing wavefront-sensing performances for broadband and monochromatic light in a known aberration test case. The significance of this development is that phase-retrieval algorithms can produce accurate phase estimates when test light is passed through filters having pass bands broader than were previously thought to be useable. Because more light is transmitted by broadband than by narrow-band filters, image-detector integration times can be significantly reduced and, therefore, time needed to perform wavefront sensing can be reduced. In some applications, filters can be eliminated entirely, thereby minimizing the complexity and cost of equipment for testing optical systems.

*This work was done by Bruce H. Dean of Goddard Space Flight Center. Further information is contained in a TSP (see page 1). GSC-14899-1*

## Filter Function for Wavefront Sensing Over a Field of View

**Optical performance is more balanced when data from more field points are used.**

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A filter function has been derived as a means of optimally weighting the wavefront estimates obtained in image-based phase retrieval performed at multiple points distributed over the field of view of a telescope or other optical system. When the data obtained in wavefront sensing and, more specifically, image-based phase retrieval, are used for controlling the shape of a deformable mirror or other optic used to correct the wavefront, the control law obtained by use of the filter function gives a more balanced optical performance over the field of view than does a

wavefront-control law obtained by use of a wavefront estimate obtained from a single point in the field of view. (The terms “wavefront sensing,” “image-based,” and “phase retrieval” are defined in the immediately preceding article.)

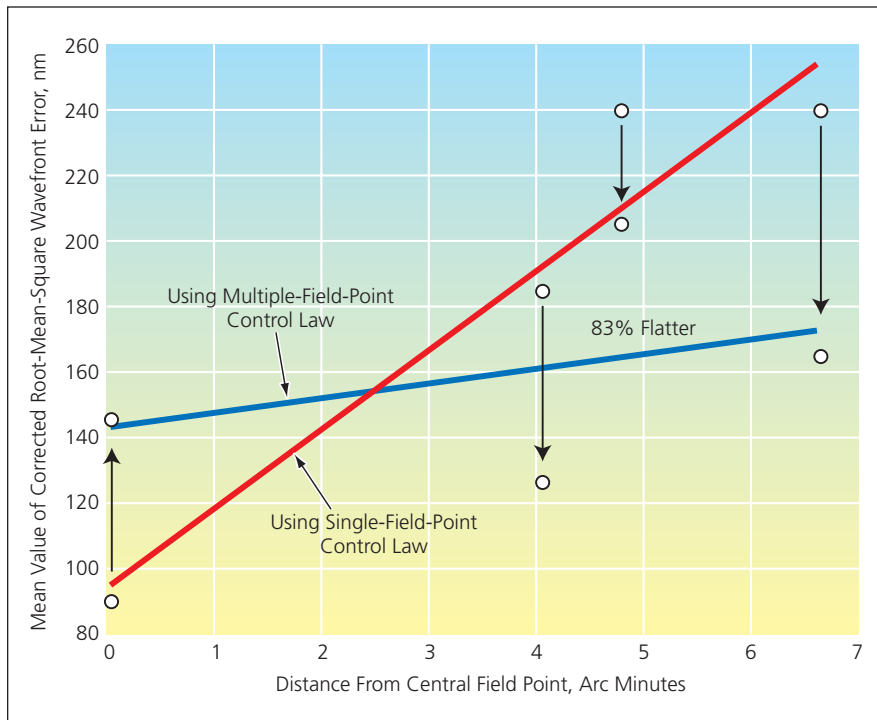
In a conventional approach to sensing and control of wavefronts, optical phase errors are estimated from the image of a single star or equivalent point source of light at a specific single location on a focal-plane image sensor. In effect, a wavefront control law is derived from a small area surrounding a

single field point and is subsequently used to correct the performance of the optical system over the entire field of view. The disadvantage of this approach is that the performance of the system at other field points can suffer additional degradation because the wavefront information obtainable at those field points can differ from that obtained at the chosen field point.

A mathematically complete description of the filter function and its derivation would exceed the space available for this article; it must suffice to summarize. The derivation of the filter function

begins with the concept of an anisoplanatic function, defined as a phase function representative of the degree to which imaging performance varies over

the field of view. The wavefront phase at a given field point is assumed to be given by the sum of the isoplanatic and anisoplanatic contributions. It is further as-



Mean Values of Corrected Root-Mean-Square Wavefront Error were computed for several field points and fitted with straight lines to show that errors can be reduced and/or distributed more evenly when multiple field points and the filter function are used.

sumed that an estimate of an isoplanatic phase function at a given field point can be modeled as a sum, over all other field points, of the convolutions of the filter function with the wavefront phase. Then the filter-function problem is formulated as one of choosing the filter coefficients to minimize the sum, over all field points, of the squares of the differences between the estimated and exact phase values of the isoplanatic phase function. To minimize this sum, one sets the partial derivatives of this sum with respect to the filter coefficients equal to zero. After some further algebraic manipulations, one obtains equations for the filter coefficients and an equation for the corrected wavefront generated by use of the filter function.

The filter function was tested in a computational simulation based on the optical design of the James Webb Space Telescope. Among the results, the variation of phase error over the field of view was 83 percent less in the case of a multiple-field-point/filter-function control law than in the case of a single-field-point control law (see figure).

*This work was done by Bruce H. Dean of Goddard Space Flight Center. Further information is contained in a TSP (see page 1). GSC-14900-1*

## Iterative-Transform Phase Retrieval Using Adaptive Diversity

High- and low-spatial-frequency contents are recovered with high dynamic range.

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A phase-diverse iterative-transform phase-retrieval algorithm enables high-spatial-frequency, high-dynamic-range, image-based wavefront sensing. [The terms “phase-diverse,” “phase retrieval,” “image-based,” and “wavefront sensing” are defined in the first of the two immediately preceding articles, “Broadband Phase Retrieval for Image-Based Wavefront Sensing” (GSC-14899-1).] As described below, no prior phase-retrieval algorithm has offered both high dynamic range and the capability to recover high-spatial-frequency components.

Each of the previously developed image-based phase-retrieval techniques can be classified into one of two categories: iterative transform or parametric. Among the modifications of the original iterative-transform approach has been the introduction of a defocus diversity function (also defined in the cited com-

panion article). Modifications of the original parametric approach have included minimizing alternative objective functions as well as implementing a variety of nonlinear optimization methods. The iterative-transform approach offers the advantage of ability to recover low, middle, and high spatial frequencies, but has disadvantage of having a limited dynamic range to one wavelength or less. In contrast, parametric phase retrieval offers the advantage of high dynamic range, but is poorly suited for recovering higher spatial frequency aberrations.

The present phase-diverse iterative-transform phase-retrieval algorithm offers both the high-spatial-frequency capability of the iterative-transform approach and the high dynamic range of parametric phase-recovery techniques. In implementation, this is a focus-diverse iterative-transform phase-

retrieval algorithm that incorporates an adaptive diversity function, which makes it possible to avoid phase unwrapping while preserving high-spatial-frequency recovery.

The algorithm includes an inner and an outer loop (see figure). An initial estimate of phase is used to start the algorithm on the inner loop, wherein multiple intensity images are processed, each using a different defocus value. The processing is done by an iterative-transform method, yielding individual phase estimates corresponding to each image of the defocus-diversity data set. These individual phase estimates are combined in a weighted average to form a new phase estimate, which serves as the initial phase estimate for either the next iteration of the iterative-transform method or, if the maximum number of iterations has been reached, for the next