HIGHLY EFFICIENT DESIGN-OF-EXPERIMENTS METHODS FOR COMBINING CFD ANALYSIS AND EXPERIMENTAL DATA

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ABSTRACT

It is the purpose of this study to examine the impact of “highly efficient” Design-of-Experiments (DOE) methods for combining sets of CFD generated analysis data with smaller sets of Experimental test data in order to accurately predict performance results where experimental test data were not obtained. The study examines the impact of micro-ramp flow control on the shock wave boundary layer (SWBL) interaction where a complete paired set of data exist from both CFD analysis and Experimental measurements By combining the complete set of CFD analysis data composed of fifteen (15) cases with a smaller subset of experimental test data containing four/five (4/5) cases, compound data sets (CFD/EXP) were generated which allows the prediction of the complete set of Experimental results No statistical difference were found to exist between the combined (CFD/EXP) generated data sets and the complete Experimental data set composed of fifteen (15) cases. The same optimal micro-ramp configuration was obtained using the (CFD/EXP) generated data as obtained with the complete set of Experimental data, and the DOE response surfaces generated by the two data sets were also not statistically different.

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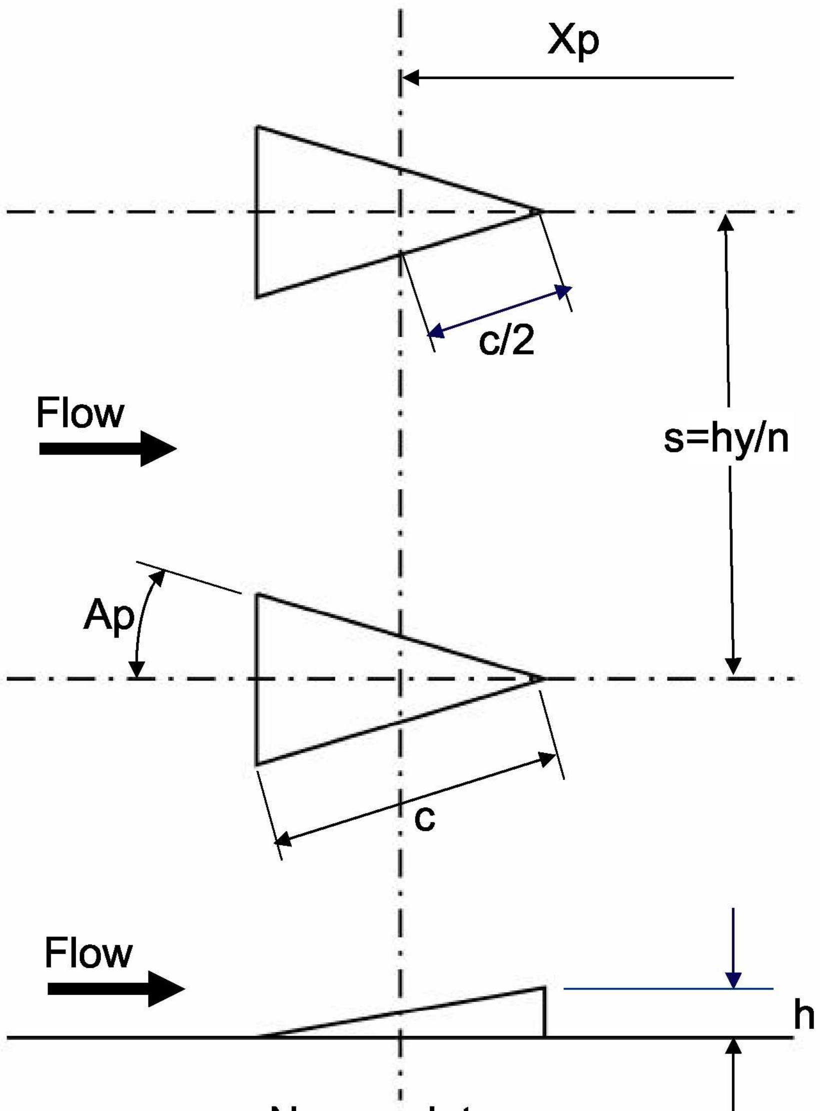
Micro-Ramp Oblique SWBL Flow Control Research Goals

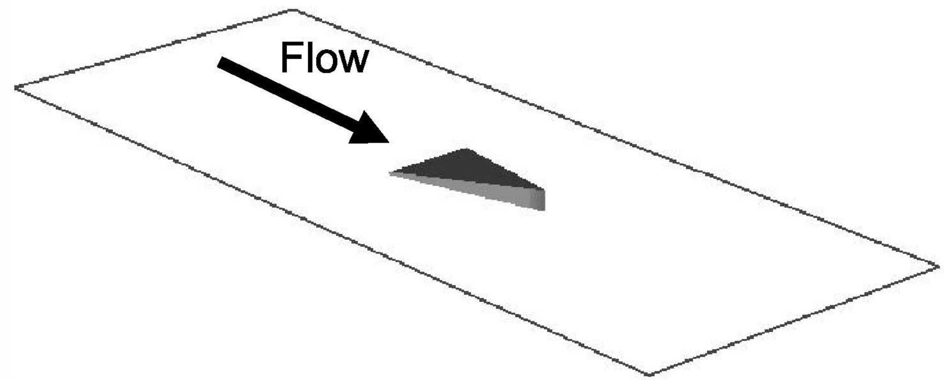
- Demonstrate the applicability of combining CFD analysis data and experimental test data to gain an efficiency and cost effectiveness advantage.

- Statistically evaluate the combined CFD/Exp. Data set in comparison to the complete Experimental Data set.

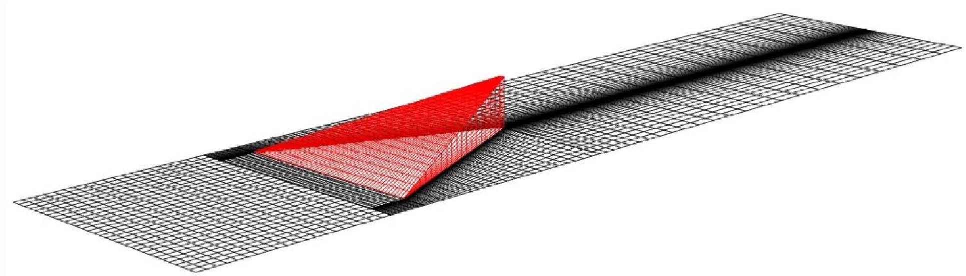
Micro-Ramp Oblique SWBL Flow Control
Nomenclature, Geometry and Grid



Nomenclature

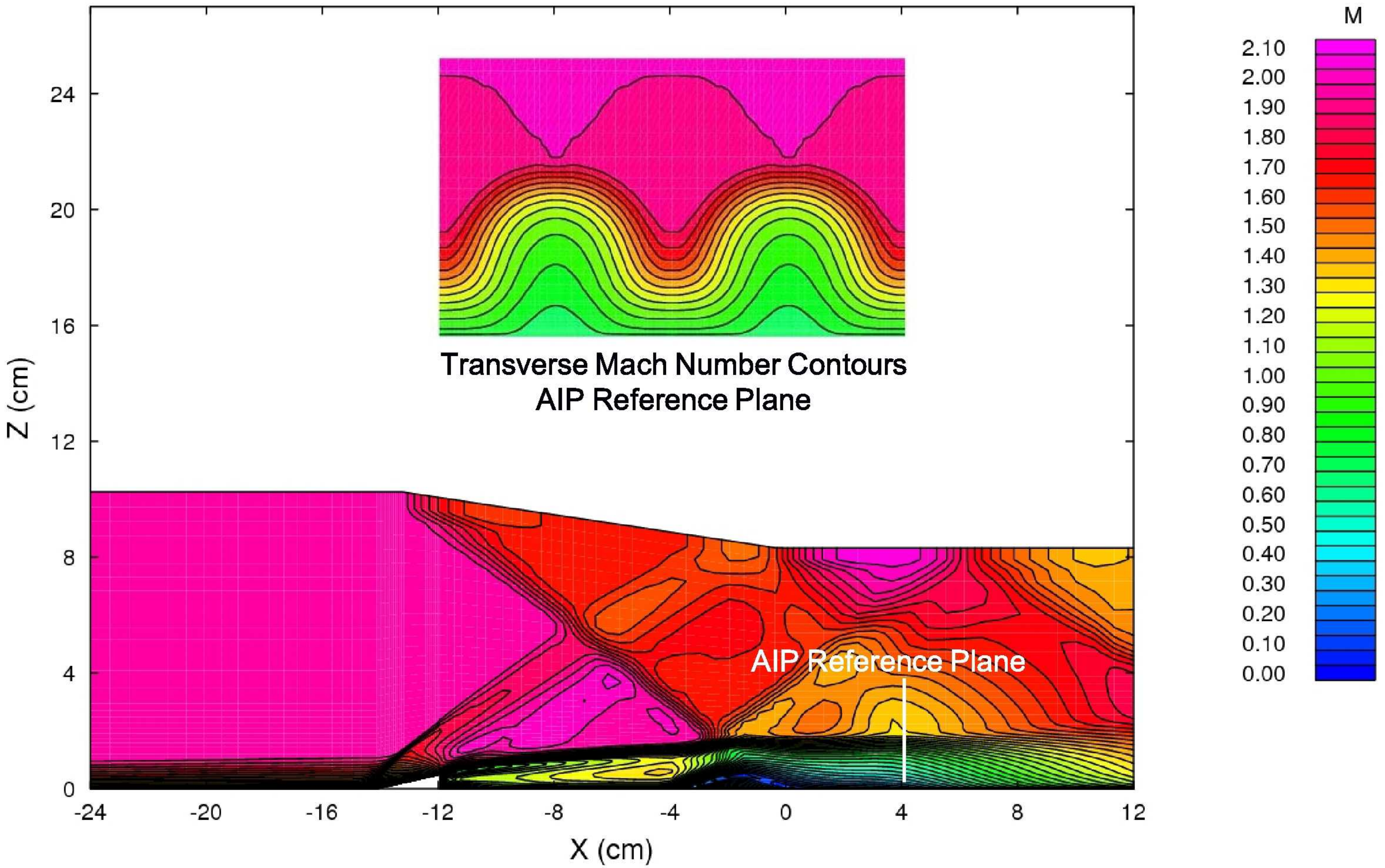


Geometry



Computational Grid

Micro-Ramp Oblique SWBL Flow Control
AIP Reference Plane Location
Mach Number Contours



Micro-Ramp Oblique SWBL Flow Control Variables Held Constant

Factor Variable	Value
Free Stream Mach Number, Mo	2.0
Free Stream Total Pressure (lbs/ft^2), Po	2112.0
Free Stream Total Temperature (^oR), To	517.0
Shock Wave Generator Angle (degs.), δ	8.461 ± 0.064
Micro-Ramp Wedge Angle (degs.), Ap	24.0

Micro-Ramp Oblique SWBL Flow Control DOE Factor Variables

Factor Variable	Range
Transverse Spacing (mm), s	25.0 – 35.0
Micro-Ramp Height (mm), h	3.0 – 5.0
Micro-Ramp Chord Length (mm), c	12.0 – 24.0

Micro-Ramp Oblique SWBL Flow Control
DOE Response Variables

Response Variable	Nomenclature
Boundary Layer Pitot Pressure Recovery	PTAVE
Boundary Layer Total Pressure Recovery	PFAVE
Compressible Displacement Thickness, (cm)	δ^*
Compressible Momentum Thickness, (cm)	θ
Transformed Form Factor	H_{tr}

Micro-Ramp Oblique SWBL Flow Control Central Composite Face Center (CCF) Design CFD Analysis Data Set

Config.	s (mm)	h (mm)	c (mm)	PTAVE	PFAVE	δ^* (cm)	θ (cm)	Htr
rvg300	____	____	____	0.55746	0.67420	0.31805	0.10940	1.30205
rvg400	____	____	____	0.63962	0.65320	0.54031	0.18391	1.89615
rvg401	25.0	3.0	12.0	0.63219	0.64284	0.54972	0.19921	1.80391
rvg402	35.0	3.0	12.0	0.62456	0.63632	0.60199	0.20737	1.83447
rvg403	25.0	5.0	12.0	0.64658	0.65530	0.61546	0.21740	1.75787
rvg404	35.0	5.0	12.0	0.64087	0.64907	0.58951	0.21108	1.75554
rvg405	25.0	3.0	24.0	0.62279	0.63195	0.64810	0.21870	1.83855
rvg406	35.0	3.0	24.0	0.62399	0.63431	0.65956	0.22384	1.83640
rvg407	25.0	5.0	24.0	0.65325	0.65770	0.65920	0.22850	1.72099
rvg408	35.0	5.0	24.0	0.65805	0.66585	0.62662	0.21618	1.74131
rvg409	25.0	4.0	18.0	0.62845	0.63684	0.64084	0.21828	1.81235
rvg410	35.0	4.0	18.0	0.63326	0.64324	0.61887	0.21210	1.80763
rvg411	30.0	3.0	18.0	0.63266	0.64054	0.67356	0.22964	1.79437
rvg412	30.0	5.0	18.0	0.64613	0.65443	0.61295	0.21409	1.73988
rvg413	30.0	4.0	12.0	0.63150	0.64190	0.60965	0.21110	1.81368
rvg414	30.0	4.0	24.0	0.63490	0.64341	0.64770	0.21940	1.80623
rvg415	30.0	4.0	18.0	0.63535	0.64491	0.60281	0.20941	1.80081

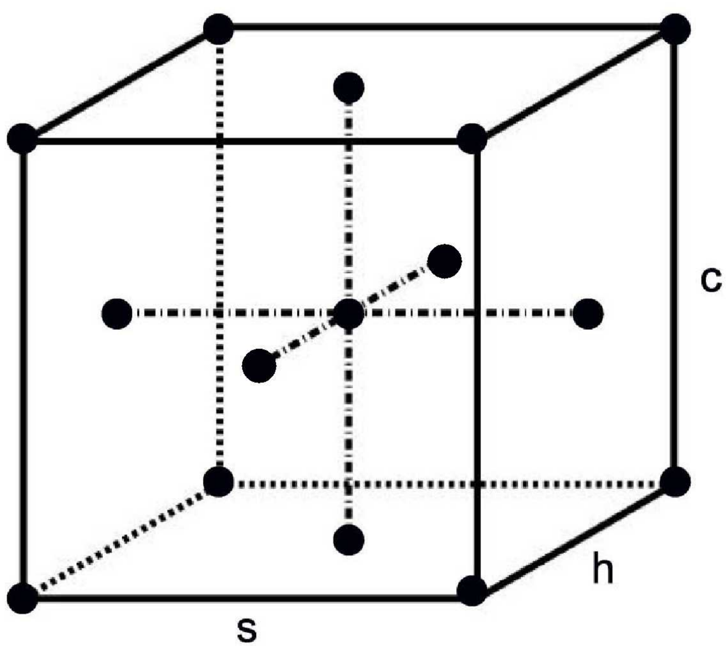
Micro-Ramp Oblique SWBL Flow Control Central Composite Face Center (CCF) Design 15 x 15 cm. SWT Experimental Data Set

Config.	s (mm)	h (mm)	c (mm)	PTAVE	PFAVE	δ*(cm)	θ (cm	Htr
rvg300	____	____	____	0.5675	0.6647	0.3264	0.1180	1.3219
rvg400	____	____	____	0.6571	0.6761	0.5158	0.1903	1.7460
rvg401	25.0	3.0	12.0	0.6421	0.6605	0.5265	0.1969	1.7239
rvg402	35.0	3.0	12.0	0.6558	0.6758	0.5296	0.1963	1.7308
rvg403	25.0	5.0	12.0	0.6952	0.7180	0.5523	0.2133	1.6657
rvg404	35.0	5.0	12.0	0.6838	0.7055	0.5382	0.2062	1.6788
rvg405	25.0	3.0	24.0	0.6734	0.6939	0.5373	0.2045	1.6901
rvg406	35.0	3.0	24.0	0.6667	0.6867	0.5312	0.2008	1.7008
rvg407	25.0	5.0	24.0	0.7053	0.7226	0.5556	0.2256	1.6170
rvg408	35.0	5.0	24.0	0.6966	0.7153	0.5446	0.2165	1.6314
rvg409	25.0	4.0	18.0	0.7037	0.7267	0.5377	0.2101	1.6391
rvg410	35.0	4.0	18.0	0.6882	0.7107	0.5263	0.2019	1.6632
rvg411	30.0	3.0	18.0	0.6427	0.6659	0.5296	0.1967	1.7202
rvg412	30.0	5.0	18.0	0.6664	0.6841	0.5089	0.2044	1.6266
rvg413	30.0	4.0	12.0	0.6808	0.7035	0.5269	0.2002	1.6783
rvg414	30.0	4.0	24.0	0.7015	0.7239	0.5350	0.2090	1.6438
rvg415	30.0	4.0	18.0	0.6900	0.7131	0.5282	0.2037	1.6532

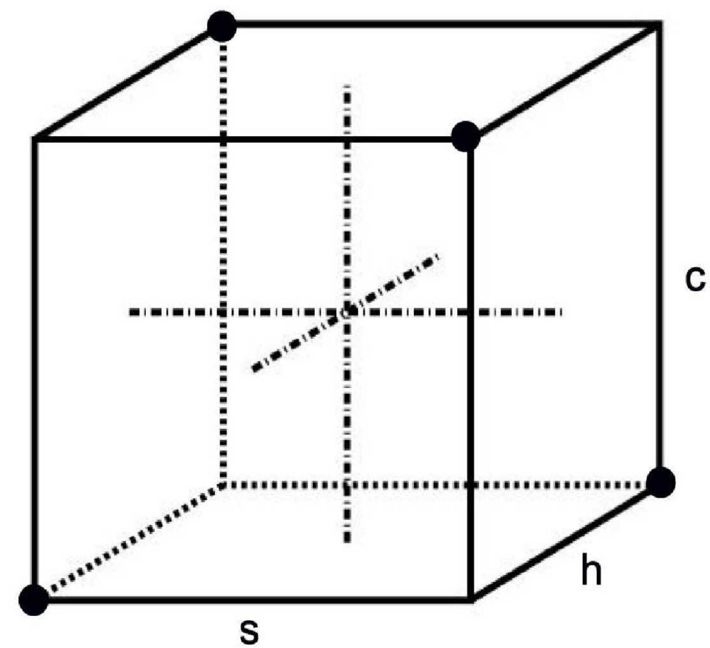
Combined Analysis/Experimental Data
D-Optimal Multi Level Design (1)

$$Y = \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij} X_i X_j + \sum_{i=1}^{N} A_{ii} X_i^2 + Rn(df, S_{y.x}, X_1, X_2, \ldots., X_N)$$

$i \neq i$



Source (1) = CFD Analysis
15 DOE Experiments



Source (2) = Experimental Data
4 DOE Experiments

Micro-Ramp Oblique SWBL Flow Control Combined Analysis/Experimental Data Significant Regression Terms (1)

Main Effect:

$$A_A X_A$$

First Order Interactions:

$$A_{sA} X_s X_A$$

$$A_{hA} X_h X_A$$

$$A_{cA} X_c X_A$$

Combined Analysis/Experimental Data
D-Optimal Multi Level Design (1)

Config.	s (mm)	h (mm)	c (mm)	X_A	PTAVE	PFAVE	δ*(cm)	θ (cm)	Htr
rvg300	____	____	____	Analysis	0.55746	0.67420	0.31805	0.10940	1.30205
rvg400	____	____	____	Analysis	0.63962	0.65320	0.54031	0.18391	1.89615
rvg401	25.0	3.0	12.0	Analysis	0.63219	0.64284	0.54972	0.19921	1.80391
rvg402	35.0	3.0	12.0	Analysis	0.62456	0.63632	0.60199	0.20737	1.83447
rvg403	25.0	5.0	12.0	Analysis	0.64658	0.65530	0.61546	0.21740	1.75787
rvg404	35.0	5.0	12.0	Analysis	0.64087	0.64907	0.58951	0.21108	1.75554
rvg405	25.0	3.0	24.0	Analysis	0.62279	0.63195	0.64810	0.21870	1.83855
rvg406	35.0	3.0	24.0	Analysis	0.62399	0.63431	0.65956	0.22384	1.83640
rvg407	25.0	5.0	24.0	Analysis	0.65325	0.65770	0.65920	0.22850	1.72099
rvg408	35.0	5.0	24.0	Analysis	0.65805	0.66585	0.62662	0.21618	1.74131
rvg409	25.0	4.0	18.0	Analysis	0.62845	0.63684	0.64084	0.21828	1.81235
rvg410	35.0	4.0	18.0	Analysis	0.63326	0.64324	0.61887	0.21210	1.80763
rvg411	30.0	3.0	18.0	Analysis	0.63266	0.64054	0.67356	0.22964	1.79437
rvg412	30.0	5.0	18.0	Analysis	0.64613	0.65443	0.61295	0.21409	1.73988
rvg413	30.0	4.0	12.0	Analysis	0.63150	0.64190	0.60965	0.21110	1.81368
rvg414	30.0	4.0	24.0	Analysis	0.63490	0.64341	0.64770	0.21940	1.80623
rvg415	30.0	4.0	18.0	Analysis	0.63535	0.64491	0.60281	0.20941	1.80081
rvg416	25.0	3.0	12.0	Exp. Data	0.6421	0.6605	0.5265	0.1969	1.7239
rvg417	25.0	5.0	24.0	Exp. Data	0.7053	0.7226	0.5556	0.2256	1.6170
rvg418	35.0	3.0	24.0	Exp. Data	0.6667	0.6867	0.5312	0.2008	1.7008
rvg419	35.0	5.0	12.0	Exp. Data	0.6838	0.7055	0.5382	0.2062	1.6788

Micro-Ramp Oblique SWBL Flow Control Comparison of CFD/Exp. and Experiment Paired t-Test Results From Set (1)

Response	MEAN	STDEV	t*	t(0.95,df)	LOW	HIGH	Comment
PTAVE	-0.0068	0.0187	0.3649	2.145	-0.0466	0.0332	Not Diff.
PFAVE	-0.0081	0.0238	0.4012	2.145	-0.0519	0.0356	Not Diff.
δ^* (cm)	-0.0002	0.0252	0.0073	2.145	0.0542	0.0538	Not Diff.
θ (cm)	-0.0016	0.0045	0.3650	2.145	-0.0080	0.0112	Not Diff.
Htr	0.0179	0.0261	0.6861	2.145	-0.0384	0.0737	Not Diff.

Sample Difference, $\Delta_j = (Y_{CFD/Exp} - Y_{Exp})_j$

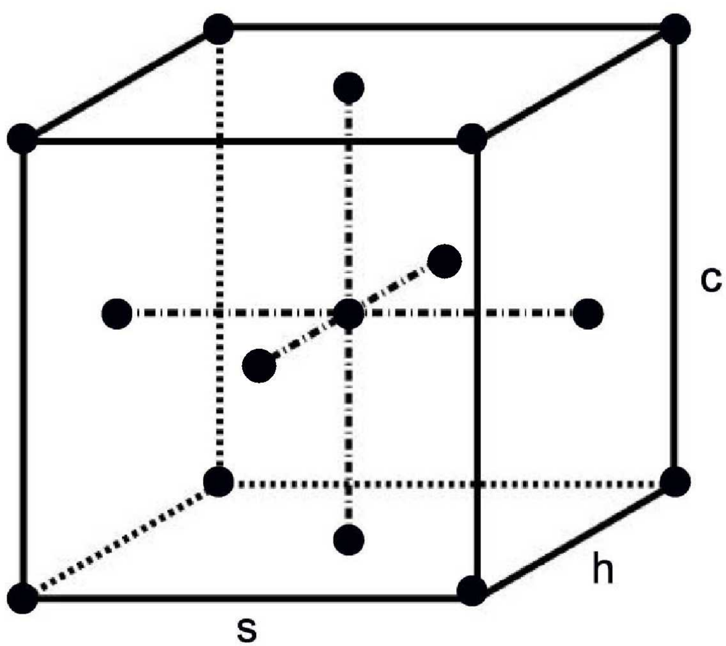
Mean of the Sample Difference, $MEAN = \frac{1}{N}\sum_{j=1}^{n}\Delta_j$

Standard Deviation of the Sample Difference, $STDEV = \sqrt{\frac{(\Delta_j - \overline{\Delta})^2}{(N-1)}}$

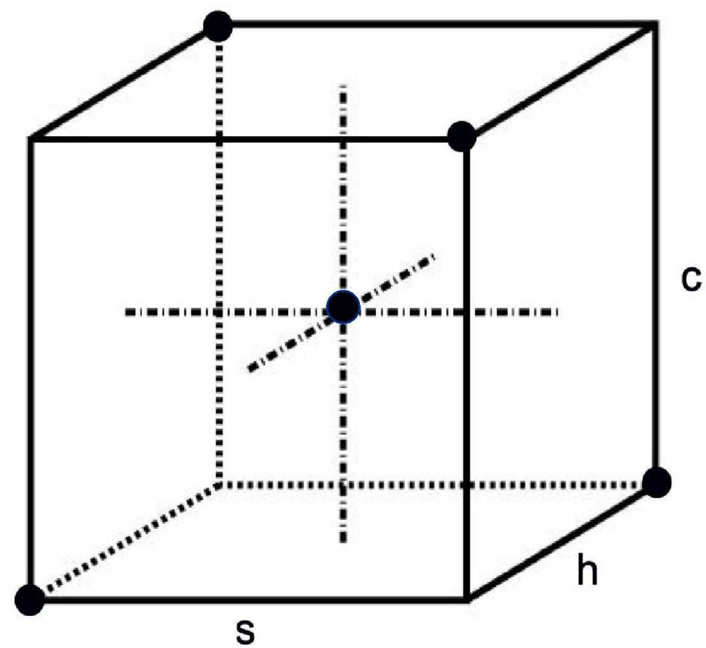
Combined Analysis/Experimental Data
D-Optimal Multi Level Design (2)

$$Y = \sum_{i=1}^{N} A_i X_i + \sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij} X_i X_j + \sum_{i=1}^{N} A_{ii} X_i^2 + Rn(df, S_{y.x}, X_1, X_2, ..., X_N)$$

$i \neq i$



Source (1) = CFD Analysis
15 DOE Experiments



Source (2) = Experimental Data
5 DOE Experiments

Micro-Ramp Oblique SWBL Flow Control Combined Analysis/Experimental Data Significant Regression Terms (2)

Main Effect:

$$A_A X_A$$

First Order Interactions:

$$A_{sA} X_s X_A$$

$$A_{hA} X_h X_A$$

$$A_{cA} X_c X_A$$

Two Possible Second Order Interactions:

$$A_{chA} X_c X_h X_A$$

$$A_{hhA} (X_h)^2 X_A$$

Combined Analysis/Experimental Data
D-Optimal Multi Level Design (2)

Config.	s (mm)	h (mm)	c (mm)	X_A	PTAVE	PFAVE	δ^* (cm)	θ (cm)	Htr
rvg300	____	____	____	Analysis	0.55746	0.67420	0.31805	0.10940	1.30205
rvg400	____	____	____	Analysis	0.63962	0.65320	0.54031	0.18391	1.89615
rvg401	25.0	3.0	12.0	Analysis	0.63219	0.64284	0.54972	0.19921	1.80391
rvg402	35.0	3.0	12.0	Analysis	0.62456	0.63632	0.60199	0.20737	1.83447
rvg403	25.0	5.0	12.0	Analysis	0.64658	0.65530	0.61546	0.21740	1.75787
rvg404	35.0	5.0	12.0	Analysis	0.64087	0.64907	0.58951	0.21108	1.75554
rvg405	25.0	3.0	24.0	Analysis	0.62279	0.63195	0.64810	0.21870	1.83855
rvg406	35.0	3.0	24.0	Analysis	0.62399	0.63431	0.65956	0.22384	1.83640
rvg407	25.0	5.0	24.0	Analysis	0.65325	0.65770	0.65920	0.22850	1.72099
rvg408	35.0	5.0	24.0	Analysis	0.65805	0.66585	0.62662	0.21618	1.74131
rvg409	25.0	4.0	18.0	Analysis	0.62845	0.63684	0.64084	0.21828	1.81235
rvg410	35.0	4.0	18.0	Analysis	0.63326	0.64324	0.61887	0.21210	1.80763
rvg411	30.0	3.0	18.0	Analysis	0.63266	0.64054	0.67356	0.22964	1.79437
rvg412	30.0	5.0	18.0	Analysis	0.64613	0.65443	0.61295	0.21409	1.73988
rvg413	30.0	4.0	12.0	Analysis	0.63150	0.64190	0.60965	0.21110	1.81368
rvg414	30.0	4.0	24.0	Analysis	0.63490	0.64341	0.64770	0.21940	1.80623
rvg415	30.0	4.0	18.0	Analysis	0.63535	0.64491	0.60281	0.20941	1.80081
rvg416	25.0	3.0	12.0	Exp. Data	0.6421	0.6605	0.5265	0.1969	1.7239
rvg417	25.0	5.0	24.0	Exp. Data	0.7053	0.7226	0.5556	0.2256	1.6170
rvg418	35.0	3.0	24.0	Exp. Data	0.6667	0.6867	0.5312	0.2008	1.7008
rvg419	35.0	5.0	12.0	Exp. Data	0.6838	0.7055	0.5382	0.2062	1.6788
rvg420	30.0	4.0	18.0	Exp. Data	0.6900	0.7131	0.5282	0.2037	1.6532

Micro-Ramp Oblique SWBL Flow Control CFD/Exp. DOE Prediction Data Set (2) Statistically Significant Terms

Term	Coeff.	p-Value	% Signif.
Intercept	0.642	0.0001	99.99
h	0.009639	0.0001	99.99
Xa	0.07104	0.0001	99.99
s*c	0.002908	0.0119	98.81
s*Xa	0.006285	0.0033	99.67
h*c	0.00401	0.0017	99.83
h*Xa	0.01349	0.0001	99.99
c*Xa	0.0183	0.0001	99.99
s^2	———	———	———
h^2	0.004711	0.0087	99.13
h^2*Xa	-0.02405	0.0001	99.99

Total Pressure Recovery, PFAVE

Term	Coeff.	p-Value	% Signif.
Intercept	1.803	0.0001	99.99
h	-0.03722	0.0001	99.99
Xa	-0.1501	0.0001	99.99
s*c	———	———	———
s*Xa	———	———	———
h*c	-0.01053	0.0065	99.35
h*Xa	———	———	———
c*Xa	-0.02122	0.0024	94.76
s^2	0.01219	0.0907	90.93
h^2	-0.03069	0.0006	99.94
h^2*Xa	0.04543	0.0082	99.18

Transformed Shape Factor, Htr

Micro-Ramp Oblique SWBL Flow Control Comparison of CFD/Exp. and Experiment Paired t-Test Results From Set (2)

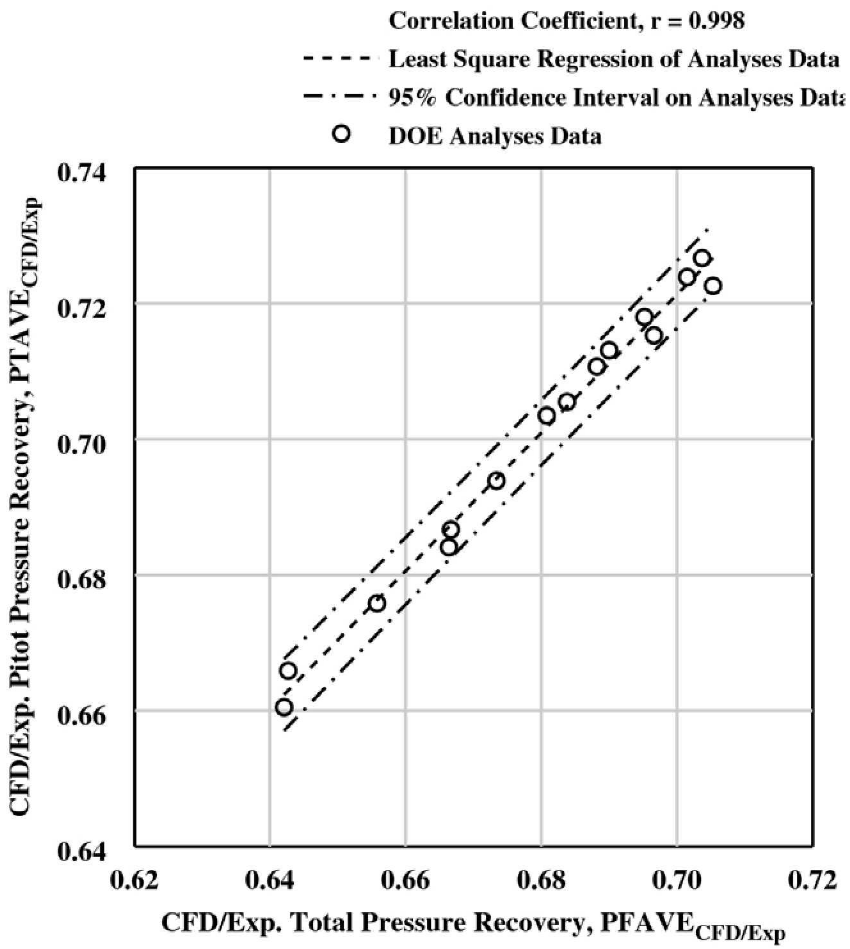
Response	MEAN	STDEV	t*	t(0.95,df)	LOW	HIGH	Comment
PTAVE	0.0002	0.0142	0.0123	2.145	-0.0301	0.0304	Not Diff.
PFAVE	0.0001	0.0155	0.0052	2.145	-0.0334	0.0332	Not Diff.
δ^* (cm)	0.0021	0.0181	0.1152	2.145	-0.0367	0.0408	Not Diff.
θ (cm)	0.0010	0.0044	0.2192	2.145	-0.0085	0.0104	Not Diff.
Htr	0.0003	0.0119	0.0241	2.145	-0.0252	0.0257	Not Diff.

Sample Difference, $\Delta_j = (Y_{CFD/Exp} - Y_{Exp})_j$

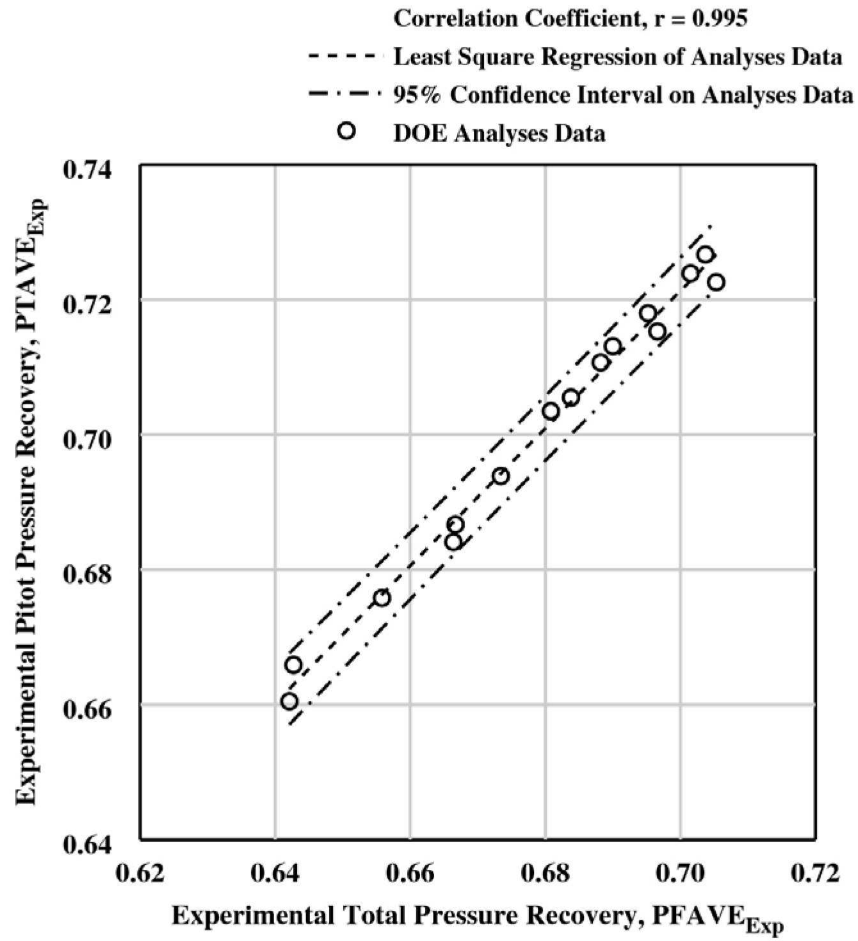
Mean of the Sample Difference, $MEAN = \frac{1}{N}\sum_{j=1}^{n}\Delta_j$

Standard Deviation of the Sample Difference, $STDEV = \sqrt{\dfrac{(\Delta_j - \overline{\Delta})^2}{(N-1)}}$

Micro-Ramp Oblique SWBL Flow Control Comparison of CFD/Exp. and Experiment Total/Pitot Pressure Correlation

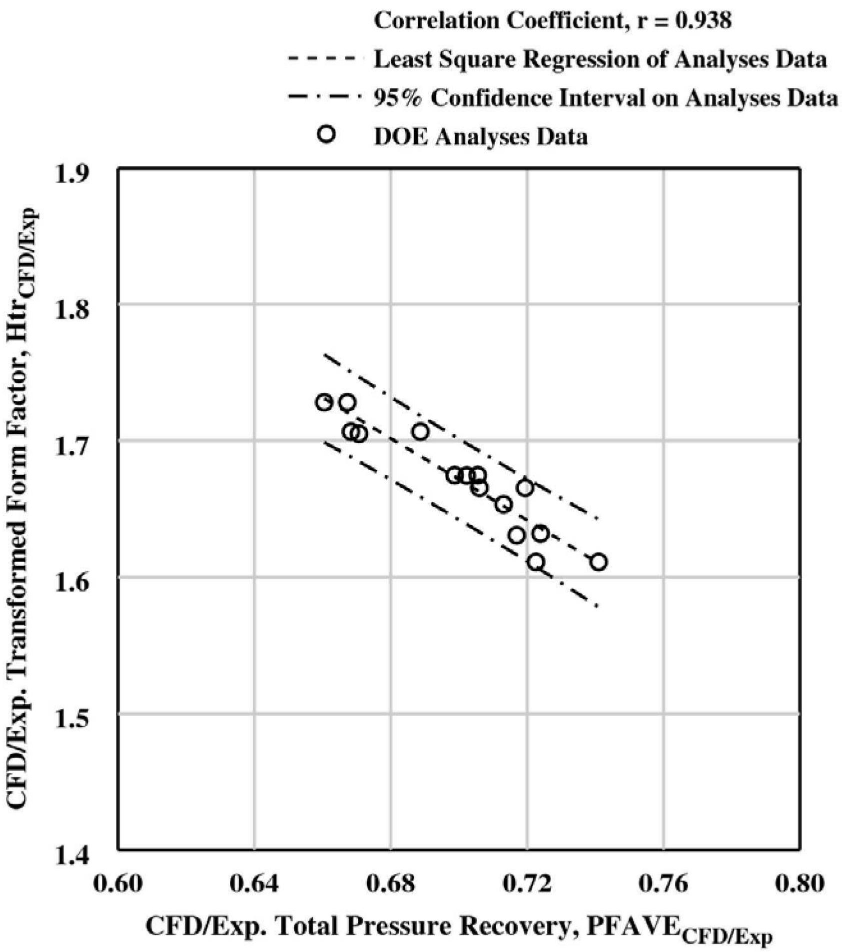


CFD/Exp Data Set (2)
Correlation Coefficient, r = 0.998

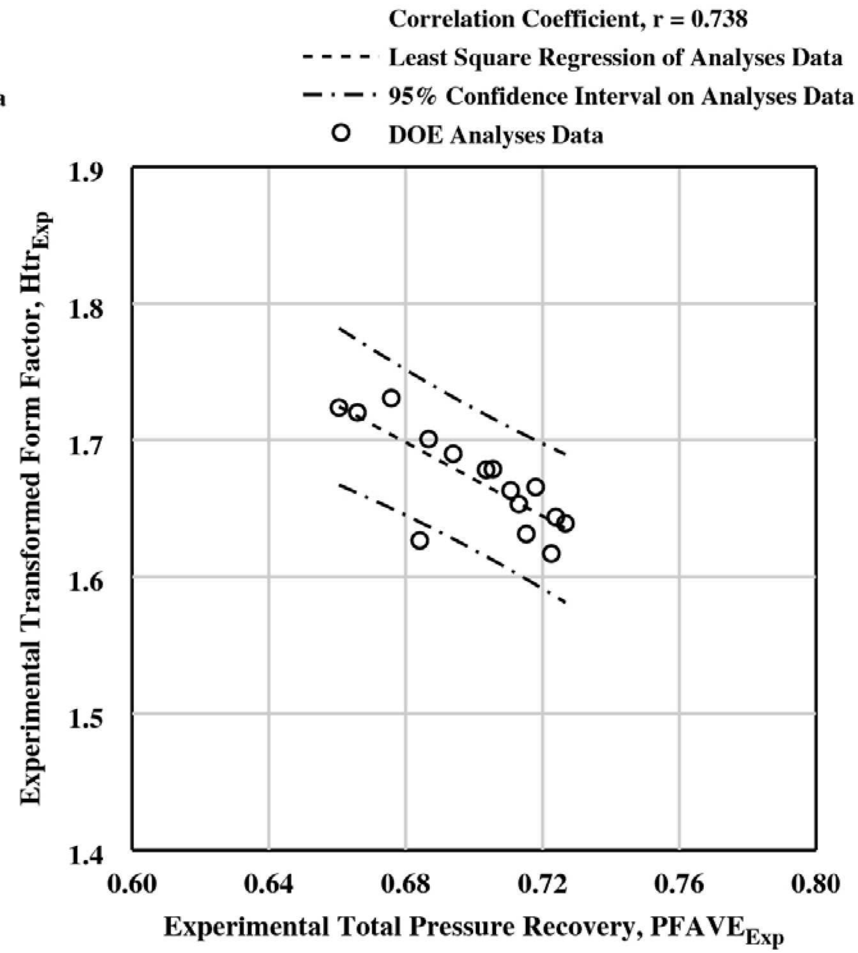


Experimental Data Set
Correlation Coefficient, r = 0.995

Micro-Ramp Oblique SWBL Flow Control Comparison of CFD/Exp. and Experiment Shape Factor/Total Pressure Correlation



CFD/Exp Data Set (2)
Correlation Coefficient, r = 0.938



Experimental Data Set
Correlation Coefficient, r = 0.738

Micro-Ramp Oblique SWBL Flow Control
s_{opt} = 25.0 mm, h_{opt} = 5.0 mm, c_{opt} = 24.0 mm
Optimal Design Point

Source	Response	Y	-95.0%Y	+95.0%Y
CFD/Exp. (2)	PTAVE	0.70530	0.69468	0.71593
	PFAVE	0.72269	0.71458	0.73104
	δ^* (cm)	0.54165	0.49242	0.59051
	θ (cm)	0.22115	0.20512	0.23718
	Htr	1.61115	1.58008	1.64223
Experiment	PTAVE	0.70110	0.68023	0.72198
	PFAVE	0.72003	0.69674	0.74327
	δ^* (cm)	0.53766	0.51496	0.56090
	θ (cm)	0.22151	0.21379	0.22923
	Htr	1.61754	1.59448	1.64069

Micro-Ramp Oblique SWBL Flow Control Comparison of Two DOE Models CFD/Exp. and Experiment

$$t^* = \frac{\left| Y_{CFD/Exp} - Y_{Exp} \right|}{\sqrt{\left(\frac{Y_A - Y_{CFD/Exp}}{t_{CFD/Exp}(0.975, df_{CFD/Exp})} \right)^2 + \left(\frac{Y_B - Y_{Exp}}{t_{Exp}(0.975, df_{Exp})} \right)^2}}$$

$Y_{CFD/Exp}$ = Response from model (CFD/Exp)

Y_{Exp} = Response from model (Exp)

Y_A = +95.0% interval from response model (CFD/Exp)

Y_B = +95.0% interval from response model (Exp)

Micro-Ramp Oblique SWBL Flow Control
Comparison of Two DOE Models
CFD/Exp. and Experiment

If $t^* > t(0.975, df)$, DOE models are different

If $t^* < t(0.975, df)$, DOE models are not different

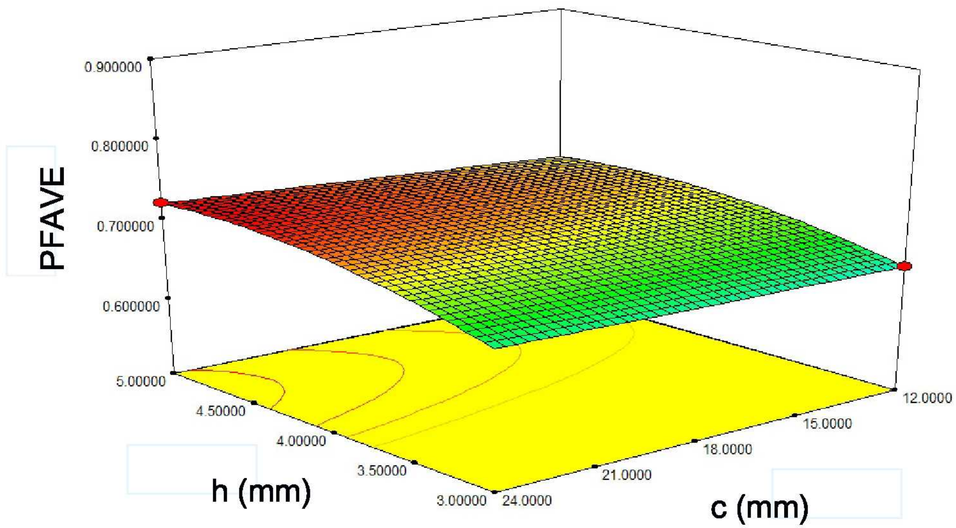
Where $t(0.975, df)$ is based on the pooled degrees of freedom:

$$df = \frac{\left(\frac{s_{CFD/Exp}{}^{2}}{df_{CFD/Exp}} + \frac{s_{Exp}{}^{2}}{df_{Exp}} \right)^{2}}{\frac{s_{CFD/Exp}{}^{4}}{df_{CFD/Exp}{}^{3}} + \frac{s_{Exp}{}^{4}}{df_{Exp}{}^{3}}}$$

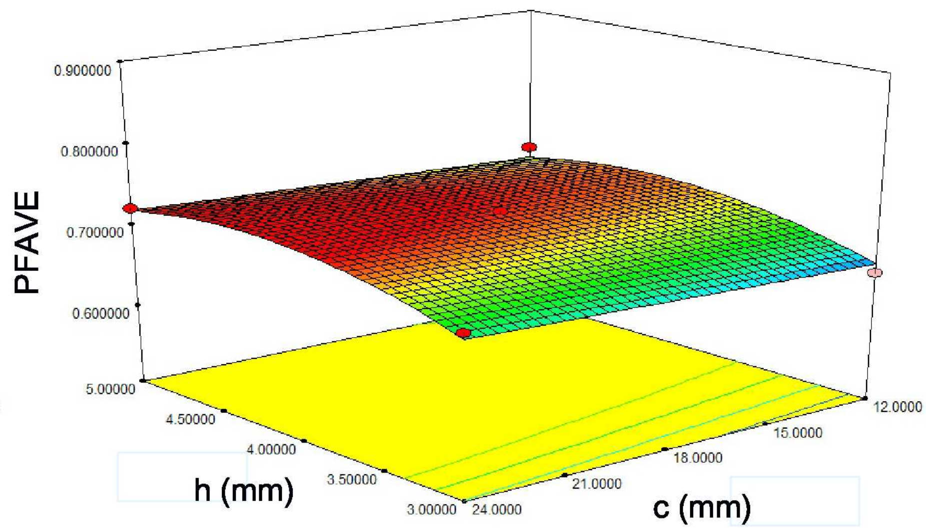
Micro-Ramp Oblique SWBL Flow Control Comparison of Two DOE Models Optimal Design Point

Response	$Y_{CFD/EXP}$	Y_{Exp}	$S_{y.x}$	t*	df	t(0.975,df)	Comment
PTAVE	0.7053	0.7011	0.0109	0.3854	23	2.069	Not Diff.
PFAVE	0.7227	0.7200	0.0112	0.2307	20	2.088	Not Diff.
δ^* (cm)	0.5416	0.5377	0.0249	0.1600	22	2.074	Not Diff.
θ (cm)	0.2212	0.2215	0.0083	0.0432	23	2.069	Not Diff.
Htr	1.6112	1.6175	0.0201	0.3185	22	2.074	Not Diff.

Micro-Ramp Oblique SWBL Flow Control Total Pressure Response Surface hxc Statistical Interaction

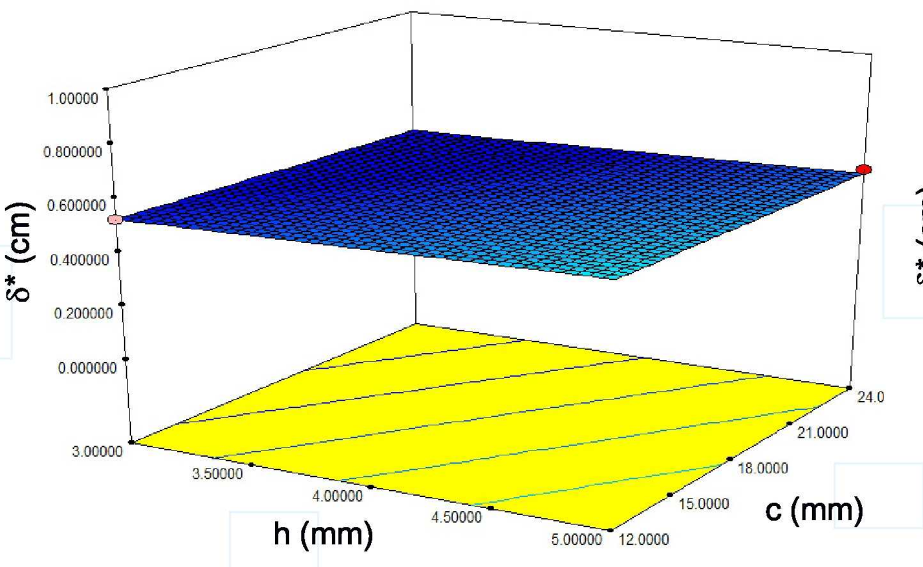


CFD/Exp. DOE Data Set (2)

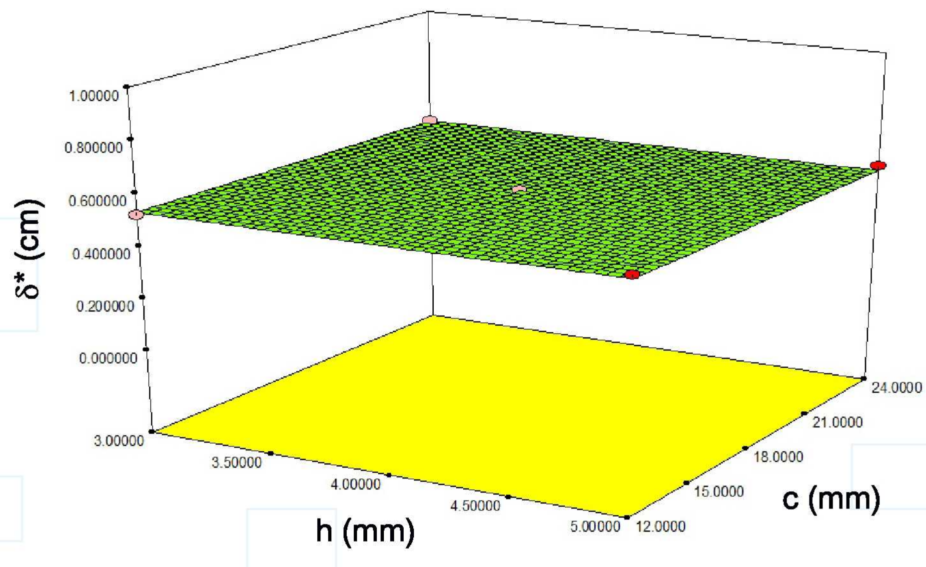


Experimental DOE Data Set

Micro-Ramp Oblique SWBL Flow Control Displacement Thickness Response Surface hxc Statistical Interaction

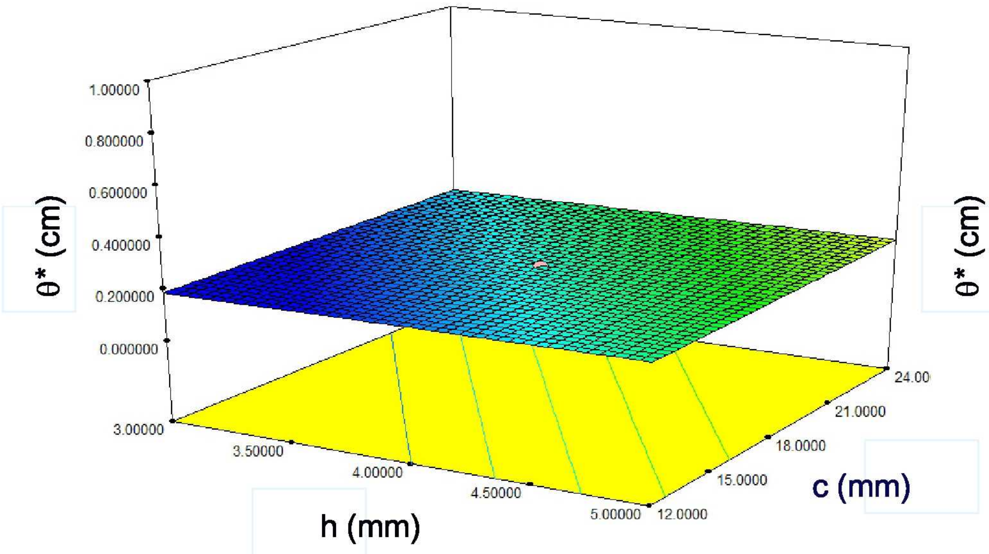


CFD/Exp. DOE Data Set (2)

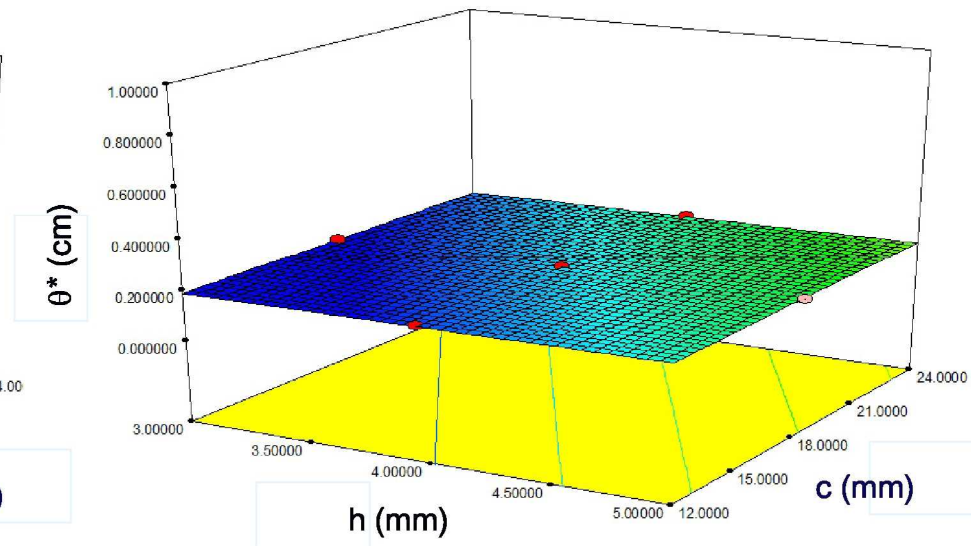


Experimental DOE Data Set

Micro-Ramp Oblique SWBL Flow Control Momentum Thickness Response Surface hxc Statistical Interaction

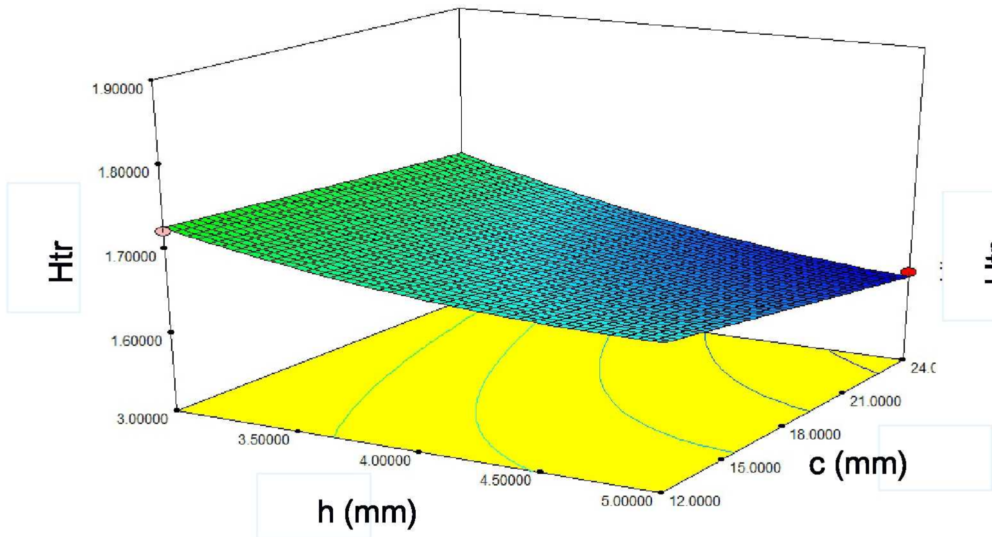


CFD/Exp. DOE Data Set (2)

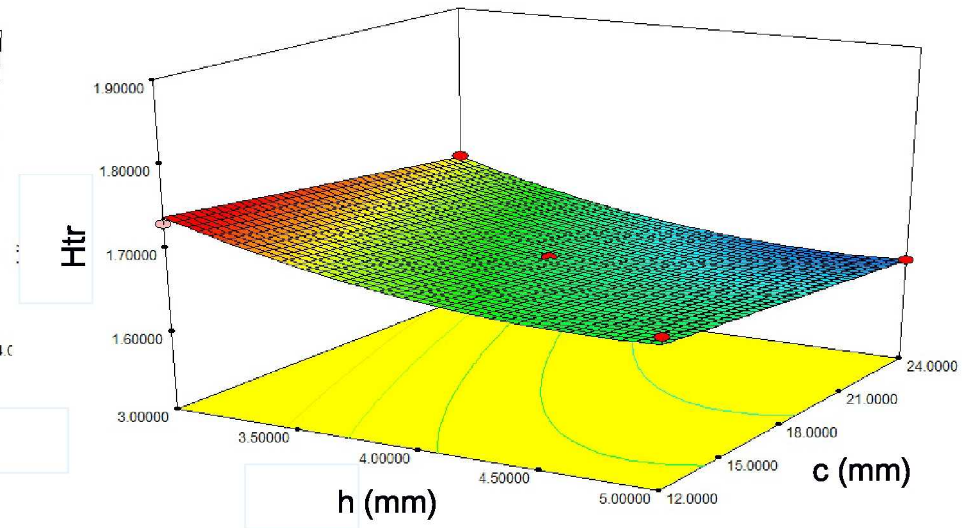


Experimental DOE Data Set

Micro-Ramp Oblique SWBL Flow Control
Shape Factor Response Surface
hxc Statistical Interaction



CFD/Exp. DOE Data Set (2)



Experimental DOE Data Set

Advanced Flow Control for Supersonic Inlets
Conclusions

- A CFD analysis data set and a smaller set of experimental data was combined into a single data set and analyzed to gain an efficiency and cost effectiveness advantage.

- There was no statistical difference between the CFD/Exp. data set and the complete experimental data set.

- There was no statistical difference between the DOE model generated by the CFD/Exp. data set and the DOE model generated by the complete experimental data set.

Advanced Flow Control for Supersonic Inlets Concluding Advice on Data Scaling

- Care must be exercised in choosing both the CFD data set and the smaller experimental data set in order to maintain an average error of prediction ratio (EOPR) close to 1.0 for the combined data set.

- Choose only the statistically significant terms in the regression model during the DOE analysis.

- Choose a linear approach to data scaling by starting from the main and first order interaction effects and adding higher order interaction terms as statistically necessary.