Modeling of the Orbital Debris Population of RORSAT Sodium-Potassium Droplets

Y.-L. Xu[1*], P. H. Krisko[1], M. Matney[2], E. G. Stansbery[2]

[1]*ESCG, Mail Code JE104, 2224 Bay Area Blvd., Houston, TX 77058, USA*
[2]*Orbital Debris Program Office, NASA Johnson Space Center, NASA, 2101 NASA Parkway, Houston, TX 77058, USA*

Abstract

A large population resident in the orbital debris environment is composed of eutectic sodium-potassium (NaK) droplets, released during the reactor core ejection of 16 nuclear-powered Radar Ocean Reconnaissance Satellites (RORSATs) launched in the 1980s by the former Soviet Union. These electrically conducting RORSAT debris objects are spherical in shape, generating highly polarized radar returns. Their diameters are mostly in the centimeter and millimeter size regimes. Since the Space Surveillance Network catalog is limited to objects greater than 5 cm in low Earth orbit, our current knowledge about this special class of orbital debris relies largely on the analysis of Haystack radar data. This paper elaborates the simulation of the RORSAT debris populations in the new NASA Orbital Debris Engineering Model ORDEM2010, which replaces ORDEM2000. The estimation of the NaK populations uses the NASA NaK-module as a benchmark. It follows the general statistical approach to developing all other ORDEM2010-required LEO populations (for various types of debris and across a wide range of object sizes). This paper describes, in detail, each major step in the NaK-population derivation, including a specific discussion on the conversion between Haystack-measured radar-cross-sections and object-size distribution for the NaK droplets. Modeling results show that the RORSAT debris population is stable for the time period under study and that Haystack data sets are fairly consistent over the observations of multiple years.

[*] Corresponding author, e-mail: yu-lin.xu-1@nasa.gov

Paper Outline

Modeling of the Orbital Debris Population of RORSAT Sodium-Potassium Droplets

Y.-L. Xu[1], P.H. Krisko[1], M. Matney[2], E.G. Stansbery[2]

[1] ESCG/Jacobs, NASA/JSC
[2] Orbital Debris Program Office, NASA/JSC

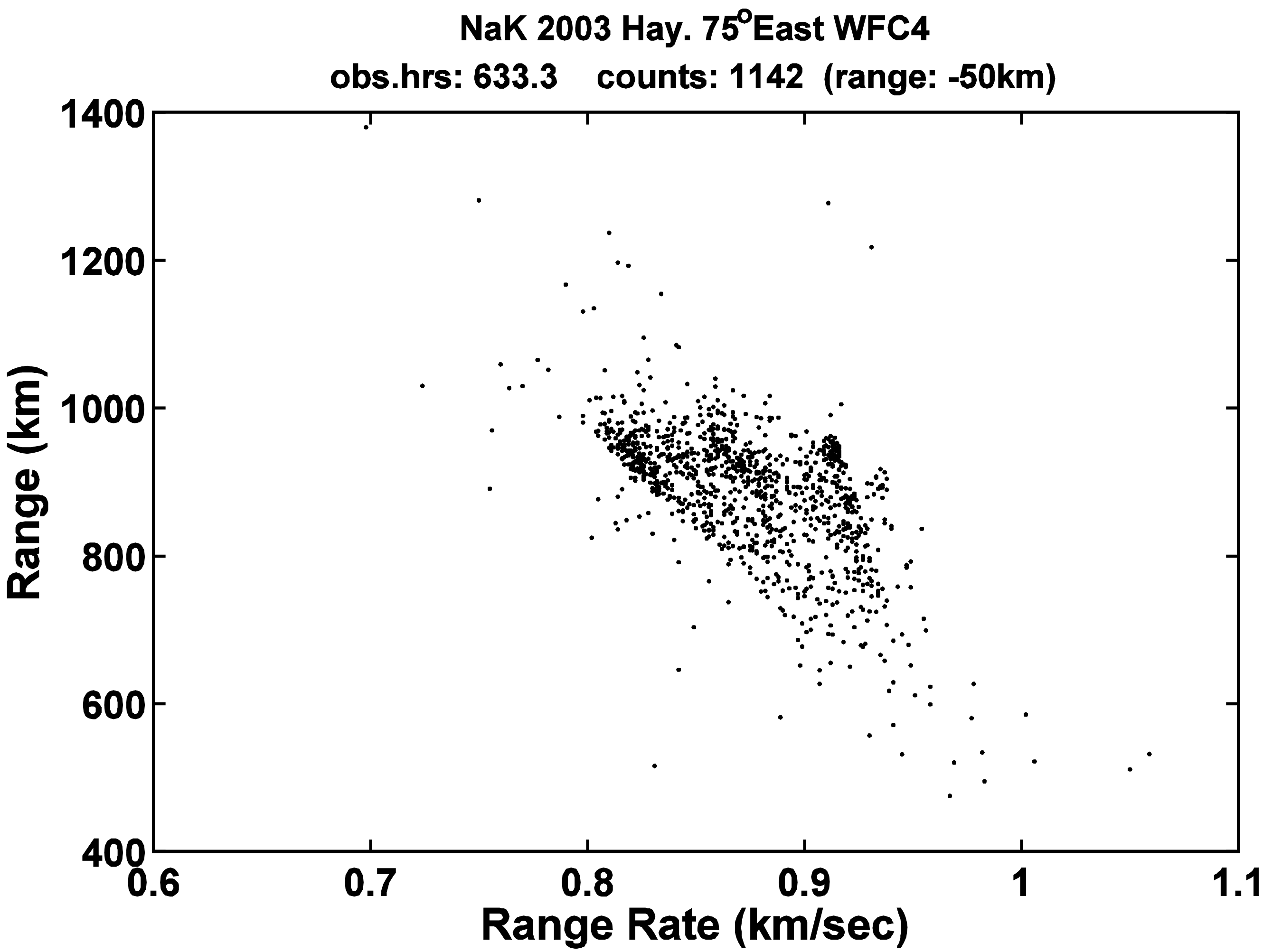
38th COSPAR Scientific Assembly
18-25 July 2010, Bremen, Germany

Sodium-Potassium Droplets (NaK Populations)

- **After the re-entry of Cosmos 954 in 1980 caused concerns over the re-entry of nuclear reactors, nuclear-powered Radar Ocean Reconnaissance Satellites (RORSATs) were designed to reject their fuel cores at end-of-mission.**
- **One out of the 16 RORSATs failed to reach the normal disposal orbit region of 900-1000 km and was placed in a lower orbit (~700 km altitude).**
- **The liquid NaK metal used as coolant vented from the main coolant loop when the nuclear reactor cores were jettisoned.**
- **These electrically conducting RORSAT debris objects are believed to be spherical in shape, generating highly polarized radar returns.**
- **RORSAT orbits are nearly circular and of ~ 65 inclination. Since the NaK ejection was most likely a nonviolent release at low relative velocities, the special debris family of NaK droplets is believed to have:**
  1) Doppler inclination between 62° and 68°,
  2) Altitude below ~1000 km, and
  3) Highly-polarized radar returns (circular polarization "P" > ~0.84)

$$\boxed{P = \frac{PP\_RCS - OP\_RCS}{PP\_RCS + OP\_RCS}}$$

Haystack detections of NaK droplets: an example



2003

Radar pointing
75º elevation East

633.3 radar hours

Estimation of model NaK populations from data: two different approaches

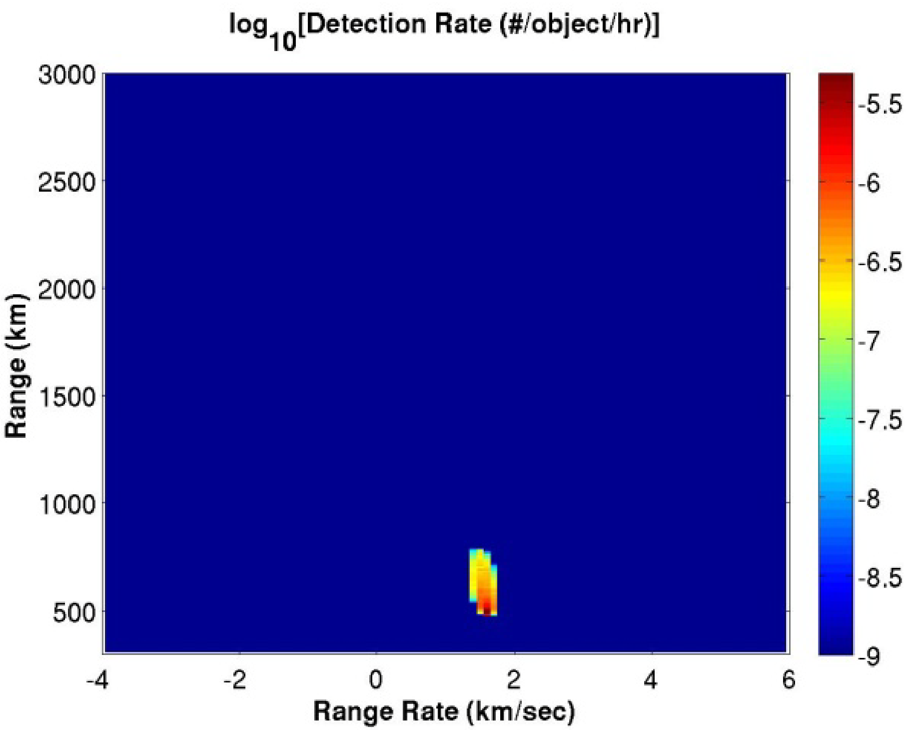
- **ORDEM2010 model NaK populations are statistically derived from Haystack data. The NaK-population derivation follows the general procedure used in the derivation of all model LEO populations in ORDEM2010, which usually requires three major elements:**
  - Data
  - An effective and reliable statistical model
  - Appropriate reference populations
    - ➢ ORDEM2010 NaK population derivations use NASA NaKModule as benchmark; **results show that the NASA NaKModule describes satisfactorily the orbital structure of NaK populations as observed by Haystack.**
- **Model NaK populations could also be derived from Haystack data via a "free-parameter searching approach," which does not require the use of any benchmark or reference population. The free-parameter searching strategy is feasible in this special case, mainly because the orbits of NaK droplets have limited spans of the orbital elements (eccentricity, inclination, and perigee altitude). However, the model populations derived in this way are easily contaminated by mysterious and stray data points. Nevertheless, results from this alternative approach can be used to validate the ORDEM2010 approach.**

A free-parameter searching approach

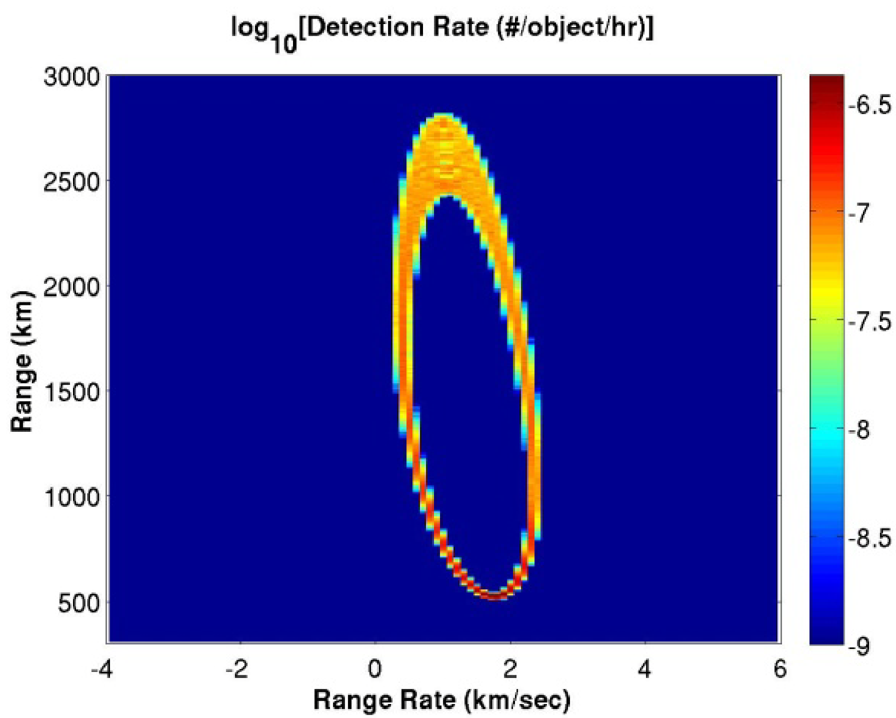
- **Definition of the population-parameter space**
  - Population parameters are the object numbers for each small bin of the three orbital elements: perigee altitude, eccentricity, and inclination
    - **Perigee altitude: 200 - 1300 km, divided in a step of 20 km, 55 bins**
    - **Eccentricity: 0 - 0.1, divided into 0-0.0001, 0.0001-0.0005, 0.0005-0.001, 0.001 bins for 0.001 < $e \leq$ 0.004, and 0.002 bins thereafter, 54 bins in total**
    - **Inclination: 62^{o} - 68^{o}, divided in a step of 0.3^{o}, 20 bins**
- **Haystack data used are the detection-number distribution in the two-dimensional space of range and range-rate**
  - Radar range is divided into 5-km bins and range-rate 0.005 km/sec bins

A free-parameter searching approach: illustrative examples for population-data connection

An orbital group
Perigee altitude: 450-470 km
Eccentricity: 0-0.02
Inclination: 48º-50º



An orbital group
Perigee altitude: 480-500 km
Eccentricity: 0.12-0.14
Inclination: 46º-48º

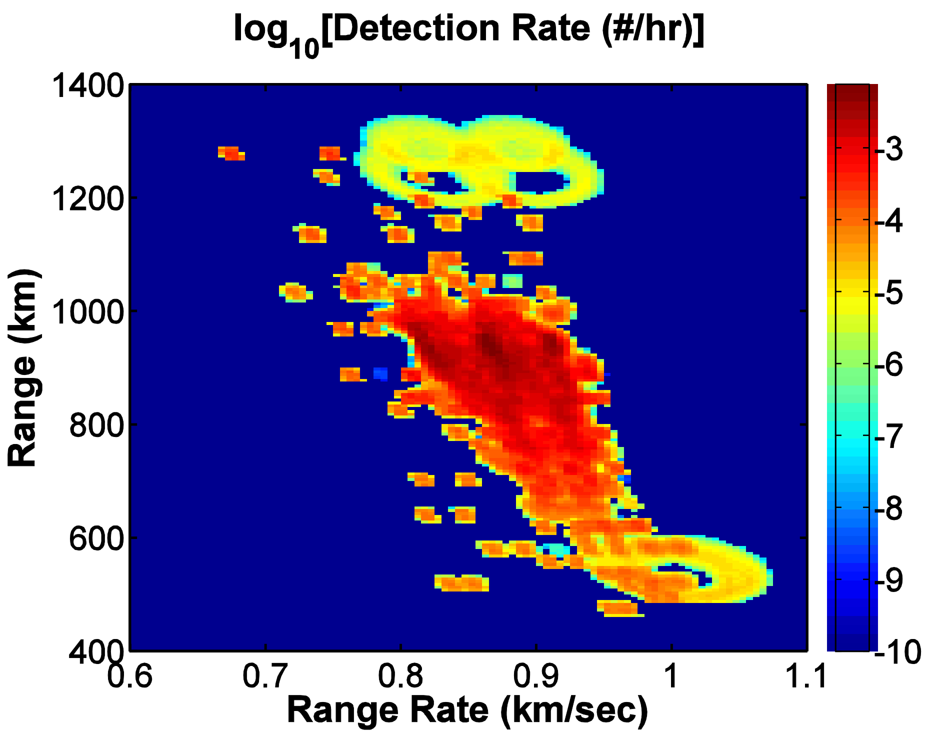


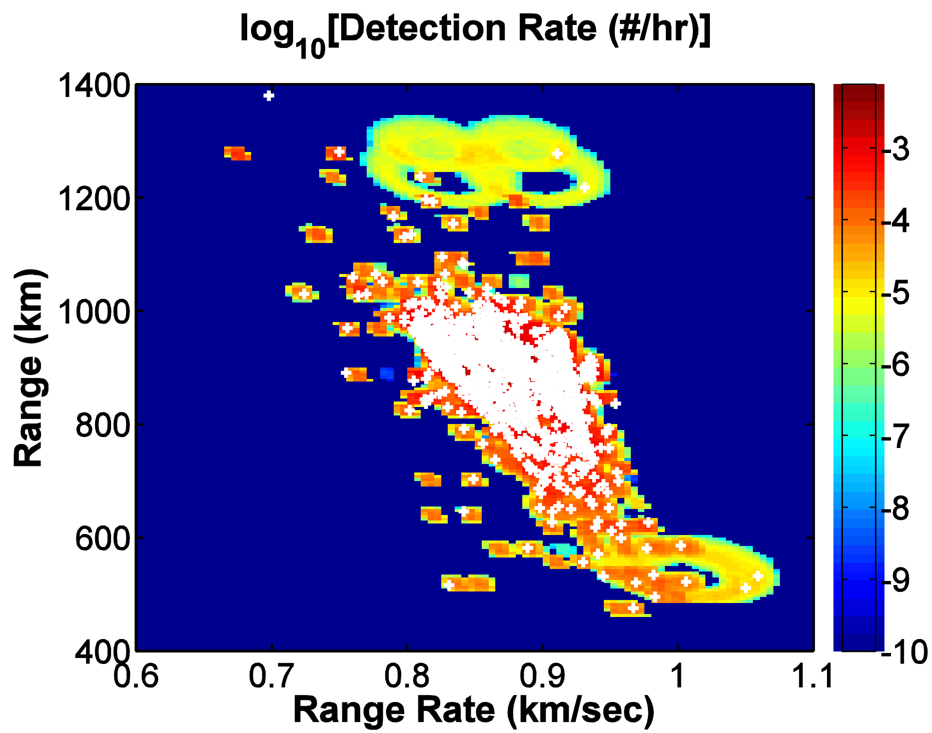
A free-parameter searching approach: A model population derived from the FY2003 data (1/2)

non-zero parameter ID#	*Perigee Altitude (km)*	*eccentricity*	*inclination (°)*	*derived object-number*	*sigma of object-number*	*sigma (%)*
…	…					
2	460-480	0-0.0001	64.1-64.4	57	54	96
10	480-500	0.0005-0.001	63.8-64.1	84	65	79
23	580-600	0.0001-0.0005	64.1-64.6	124	66	53
55	680-700	0.002-0.003	65.0-65.3	204	39	20
74	720-740	0-0.0001	65.0-65.3	395	112	28
88	740-760	0-0.0001	65.3-65.6	154	63	41
104	760-780	0.001-0.002	64.7-65.0	485	60	12
110	780-800	0-0.0001	65.0-65.3	320	87	27
155	820-840	0.0005-0.001	64.7-65.0	368	59	16
191	860-880	0-0.0001	64.7-65.0	597	106	18
213	880-900	0.0001-0.0005	64.4-64.7	761	91	12
249	920-940	0-0.0001	65.3-65.6	936	146	16
277	960-980	0.0001-0.0005	65.0-65.3	233	81	35
…	…					
			total	47800	1260	2.6

(There are in total 302 non-zero model parameters; shown are only some examples)

A free-parameter searching approach: A model population derived from the FY2003 data (2/2)





Predicted Haystack detection rate distribution of the derived model NaK population of 2003

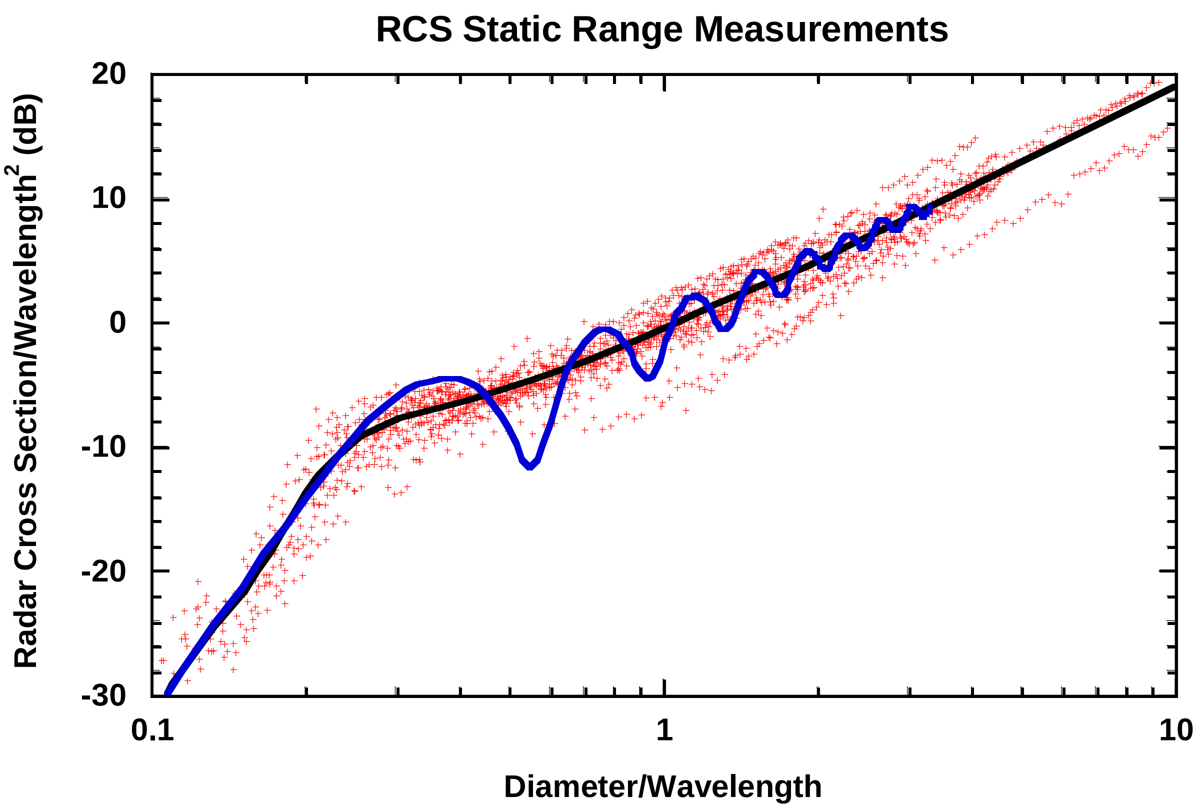
Predicted Haystack detection rate distribution of the derived model NaK population of 2003 with Haystack NaK detections shown as white plus signs

Radar cross section (RCS) and the sizes and size distribution of the NaK droplets

- **The NaK-population derivation via the free-parameter searching approach shown above uses "all" NaK detections in the data.**
  - What are the sizes and size distribution of the Haystack detected NaK droplets?
  - What is the smallest NaK sphere size that Haystack can detect?
  - What about to take look at some object-size threshold such as >7 mm or >1 cm?

- **Radars do not directly measure the physical sizes of targets. Information on the sizes and size distributions of radar targets are contained in radar measured RCSs and RCS distributions. The RCS of a NaK (electrically conducting) sphere of arbitrary diameter can be reliably predicted using a rigorous, analytical solution to elastic radiative scattering, known as Mie theory or Lorenz-Mie solution developed a century ago. But the RCS-to-size conversion, an inverse problem, is more complicated in practice.**

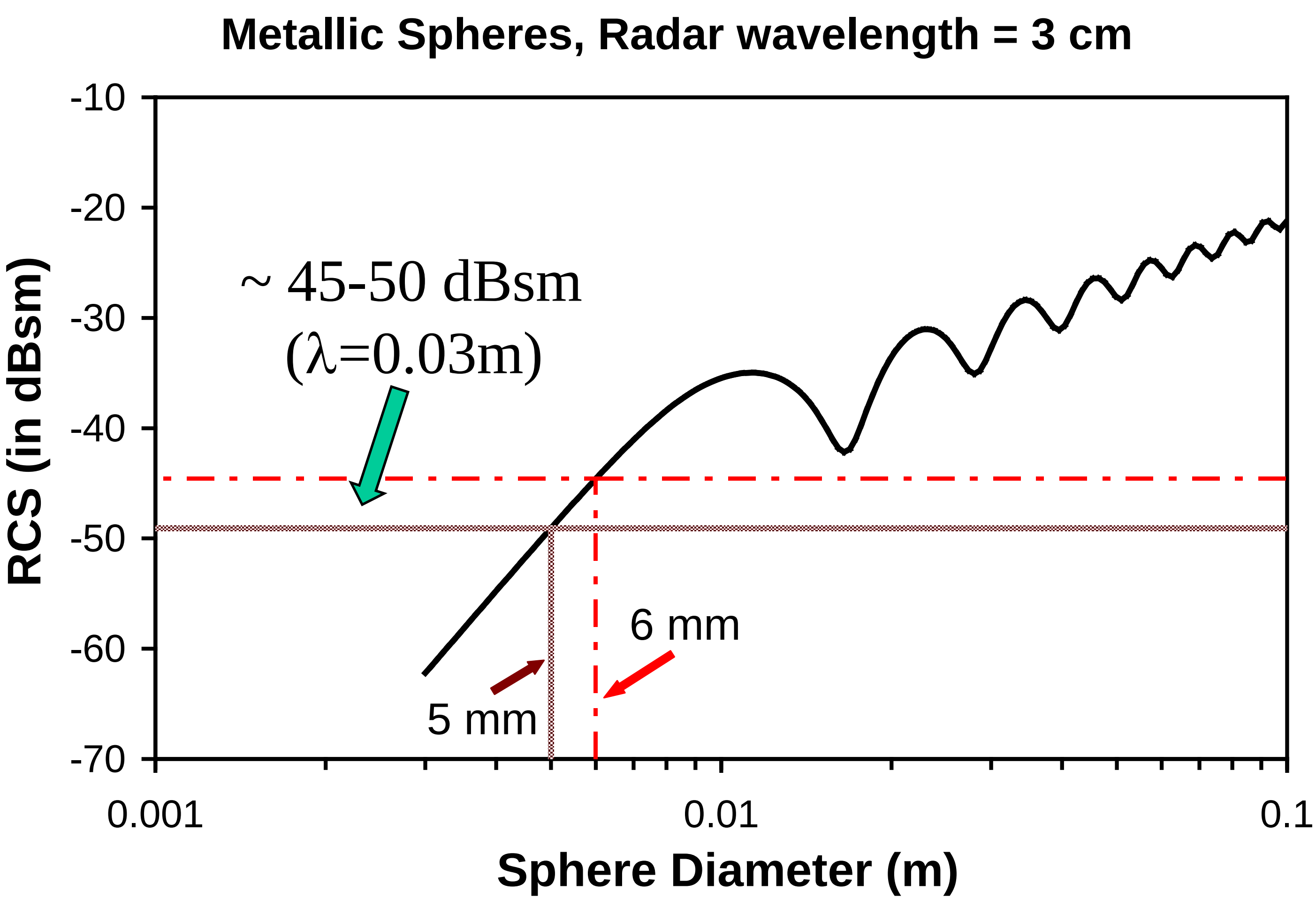
RCSs of spherical conductors and NASA SEM



Comparison of NASA Size Estimation Model (SEM), represented by smooth curve, with the theoretical prediction from Mie theory of RCS-to-size relation for metallic spheres (oscillating curve)

- **Note the overlap of SEM and Mie curves in the Rayleigh region!**
- **Haystack measured RCSs of NaK droplets suggest that the RCS floor (i.e., the lowest detectable RCS value for Haystack) is ~45 dBsm at the Haystack radar wavelength of 3 cm.**

NaK spheres: RCS vs. size and size floor

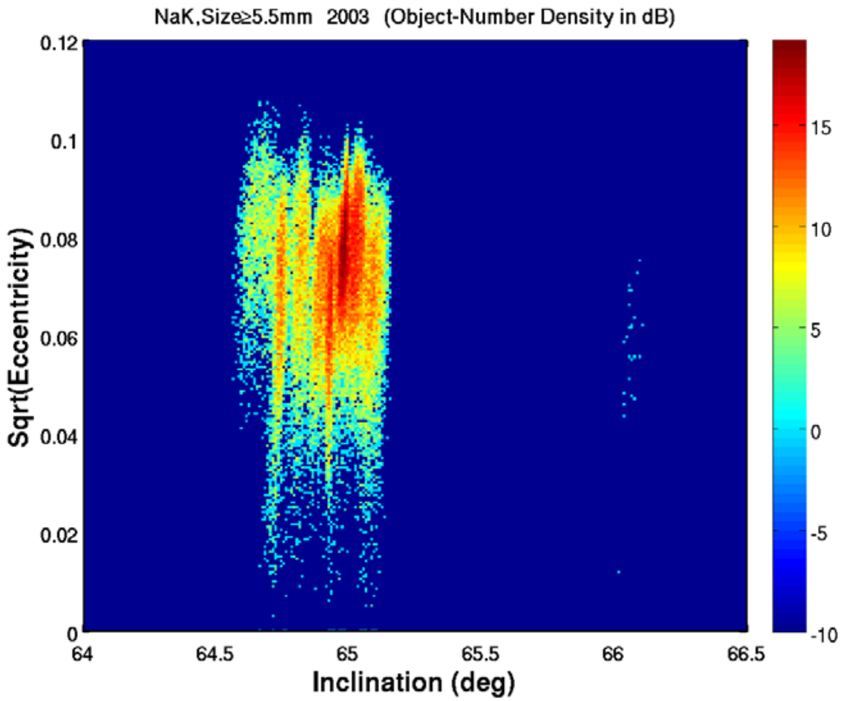


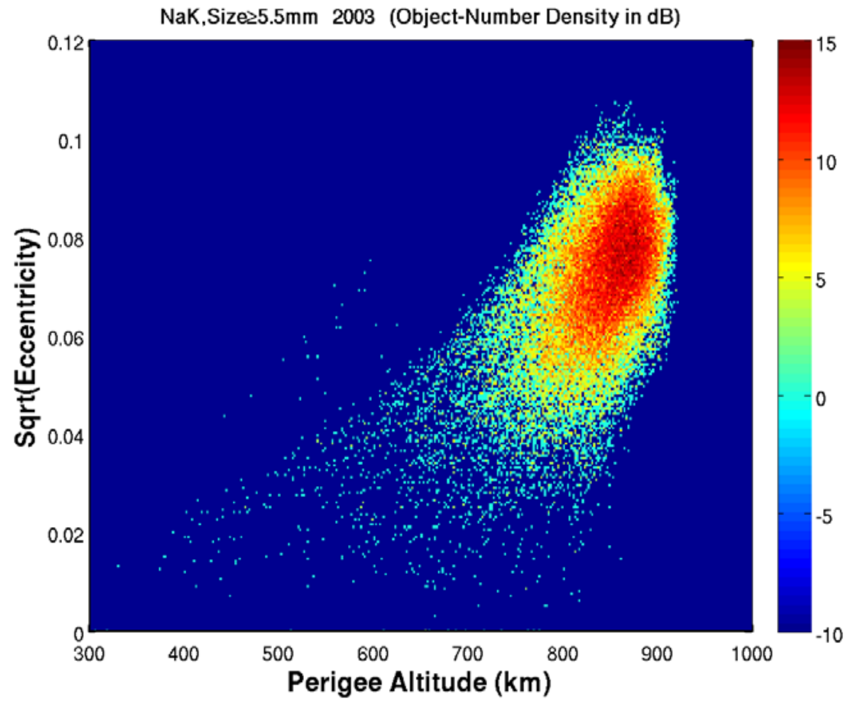
The smallest size of Nak objects that Haystack radar could detect may well be in the vicinity of 5 mm

NASA NaKModule

- **The NASA NaKModule is a source model (Foster et al., 2003; Krisko and Foster, 2004), which accounts for the 16 core-ejection/NaK deposit events of RORSATs and propagates the deposited NaK droplets in time**

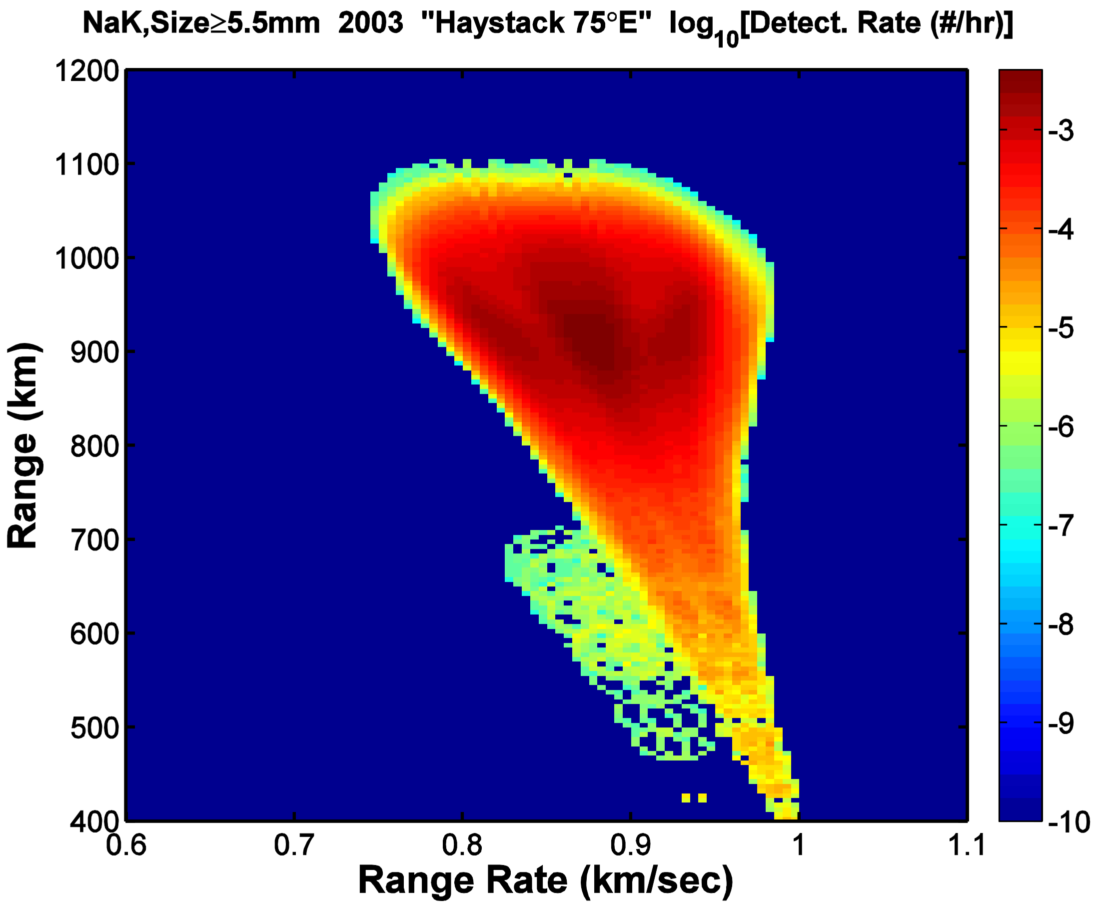
NASA NaKModule: Example orbital distributions





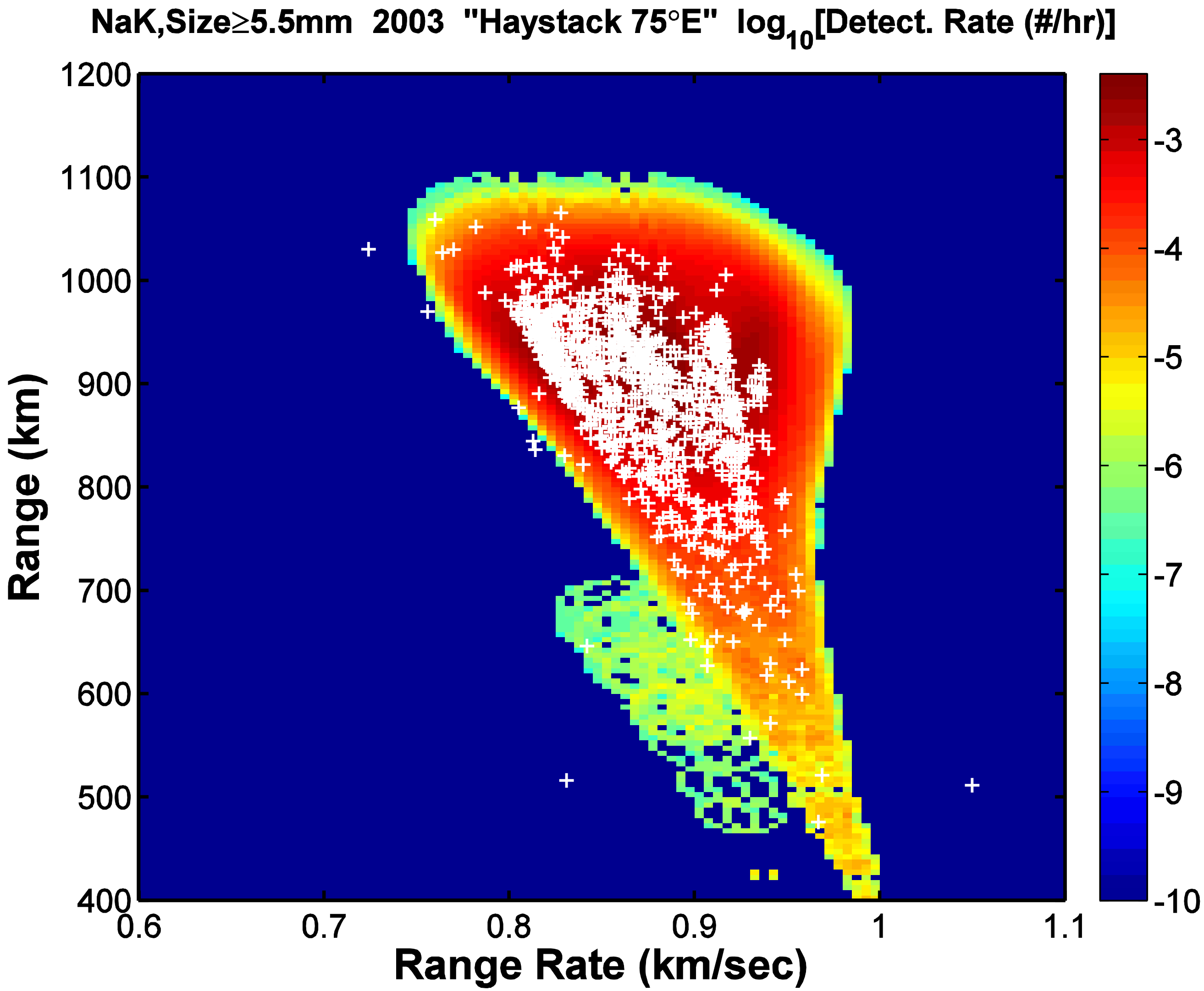
Orbital distributions of NaK droplets (with size ≥ 5.5 mm) in the NASA NaKModule for FY2003

NASA NaKModule predicted Haystack NaK detections



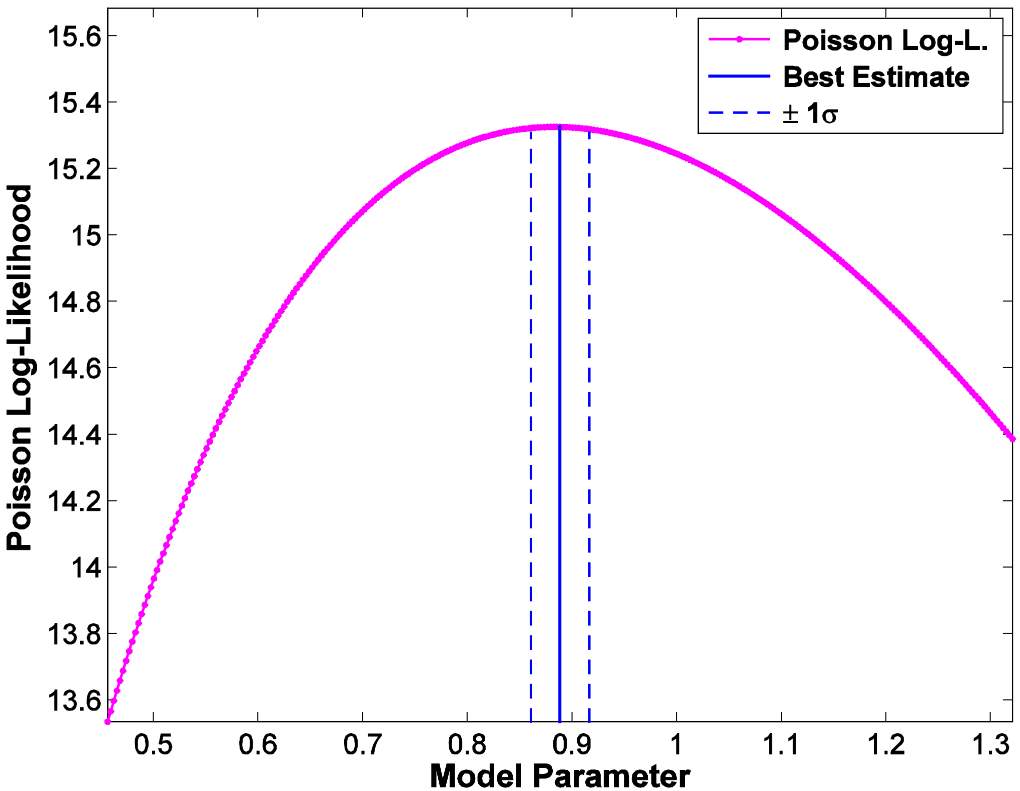
Predicted Haystack detection rate map in the two-dimensional space of range and range rate for the model NaK population (size ≥5.5 mm, 2003) from the NASA NaKModule. The predictions refer to the case with Haystack radar pointing to 75° East.

Connection between data and model NaK populations provided in NASA NaKModule



White cross signs are Haystack detections

Derivation of ORDEM2010 NaK populations



Convergence of the best estimate of the model parameter at maximum Poisson log-likelihood in a single-parameter model for the statistical derivation of a model NaK FY2003 ≥5.5 mm population from Haystack FY2003 data (using NASA NaKModule as benchmark)

Best estimated model parameter:

41800±1300

(For FY2003 >5mm population, the result is 46200±1380, which is consistent with what derived from the free-parameter searching approach shown in slide 7)

Examples for the Results from Haystack data of different years

year	*# of radar observing hours*	*smallest NaK obj. diameter (mm)*	*total # of NaK detections (75 °E)*	*total # of objects in the NaKModule*	*Model Estimates*		
					Adjustment factor	total # of objects	sigma (%)
1999	210.2	5.0	433	53932	0.97	52300	4.8
		5.5	384	48156	0.96	46200	5.1
		5.7	363	46201	0.94	43600	5.3
		6.0	336	43246	0.93	40300	5.5
2000	264.9	5.0	585	53547	1.05	56400	4.1
		5.5	490	48010	0.98	47100	4.5
		5.7	476	46113	0.99	45600	4.6
		6.0	440	43199	0.97	42100	4.8
2001	250.6	5.0	554	53018	1.07	56900	4.3
		5.5	476	47715	1.02	48600	4.6
		5.7	456	45875	1.01	46500	4.7
		6.0	424	43072	1.00	43100	4.9
2002	171.3	5.0	409	52165	1.18	61800	5.0
		5.5	332	47211	1.06	49900	5.5
		5.7	316	45470	1.04	47400	5.6
		6.0	302	42795	1.05	45200	5.8
2003	633.3	5.0	1118	51651	0.89	46200	3.0
		5.5	1026	46967	0.89	41800	3.1
		5.7	990	45264	0.90	40600	3.2
		6.0	960	42628	0.92	39200	3.2
				average:	0.99		

Summary

- **ORDEM2010 model NaK populations are derived from Haystack data, using the NASA NaKModule as a benchmark. Modeling results show that the Bayesian statistical approach works properly in the model NaK population derivations for both model-parameter estimation and uncertainty analysis.**
- **Mie theory, a precise theoretical interpretation of the interaction between electromagnetic radiation and homogeneous spheres, helps to obtain reliable results for the Haystack data analysis specifically for NaK spheres.**
- **The NASA NaKModule provides a reasonable description of the special RORSAT debris population for the size and orbital distributions of the NaK droplets.**
- **Haystack data of different years is fairly consistent, at least over the time period under study.**