Two Mathematical Models of Nonlinear Vibrations

Model parameters are fit to empirical vibration data.

NASA's Jet Propulsion Laboratory, Pasadena, California

Two innovative mathematical models of nonlinear vibrations, and methods of applying them, have been conceived as byproducts of an effort to develop a Kalman filter for highly precise estimation of bending motions of a large truss structure deployed in outer space from a space-shuttle payload bay. These models are also applicable to modeling and analysis of vibrations in other engineering disciplines, on Earth as well as in outer space.

The first model is denoted the amplitude-dependent stiffness (ADS) model to emphasize the difference between it and the classical linear harmonic-oscillator model, in which stiffness is a constant. The ADS model is embodied in the equation

$$\ddot{x} + \xi\dot{x} + K(x,\dot{x})x = 0,$$

where x is the instantaneous amplitude of the oscillating position or modal coordinate, ξ is a damping parameter, and $K(x,\dot{x})$ is the ADS.

In the initial outer-space application, the ADS was represented by the following nonlinear function:

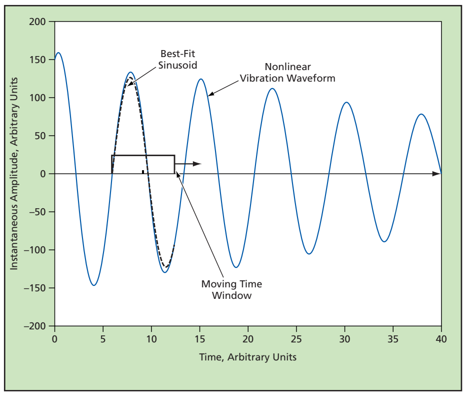
$$K(x,\dot{x}) = a + bA(x,\dot{x}) + cA(x,\dot{x})^2,$$

where a, b, and c are constant parameters to be obtained by fitting the model to empirical amplitude-versus-frequency data, and $A(x,\dot{x})$ is a modal amplitude. The amplitude-versus-frequency data are obtained by means of a moving-window estimation technique in which one analyzes the instantaneous vibration waveform during a time window of about 90 percent of the time-average vibration period. The amplitude and frequency are taken to be those of a sinusoid that makes the least-squares best fit to the instantaneous amplitude during the window (see figure). The window is then moved by about 2 percent of the average period and another best-fit sinusoid is found. This process is repeated until a suitably representative sample of the vibration waveform has been acquired.

The modal amplitude is given by

$$A(x,\dot{x}) = \sqrt{x^2 + \left(\frac{\dot{x}}{\bar{K}(x,\dot{x})}\right)^2},$$

where $\bar{K}(x,\dot{x})$ is any reasonable approximation of $K(x,\dot{x})$. One can refine the approximation iteratively, starting from $K(x,\dot{x}) = a$, then using the resulting value of $A(x,\dot{x})$ in computing a value of $\bar{K}(x,\dot{x})$ by use of the above equation for $K(x,\dot{x})$.



A **Nonlinear Decaying Waveform** is approximated with a best-fit sinusoid during a moving window. The resulting sinusoidal amplitude and frequency data are collected from fits for the entire sequence of window positions and used to characterize the frequency versus amplitude of the nonlinear waveform. These frequency versus amplitude data are then fit to an amplitude-dependent stiffness (ADS) representation.

The second model, denoted the moment-expansion (ME) model, is embodied in the equation

$$\dot{x} + M(x,\dot{x}) = 0,$$

where the function $M(x,\dot{x})$ is a moment expansion that captures damping and stiffness effects. The moment expansion is given by

$$M(x,\dot{x}) = \sum_{j=0}^{3}\sum_{i=0}^{3} p_{ij}x^i\dot{x}^j,$$

where both i and j range from 0 to 3, except that there is no $(i,j) = (0,0)$ term. In the original outer-space application, the parameters p_{ij} are obtained from (1) modal position and velocity estimates obtained from Kalman-filter states and (2) derived accelerations.

In a test relevant to the original outer-space application, the ADS and ME models were compared with each other, with a linear model, and with a prior nonlinear model known as the Duffing model. The ADS model was found to yield the least error.

This work was done by Paul Brugarolas, David Bayard, John Spanos, and William Breckenridge of Caltech for NASA's Jet Propulsion Laboratory. For further information, contact iaoffice@jpl.nasa.gov. NPO-41360

Simpler Adaptive Selection of Golomb Power-of-Two Codes

The selected code-parameter value is within 1 of the optimum value.

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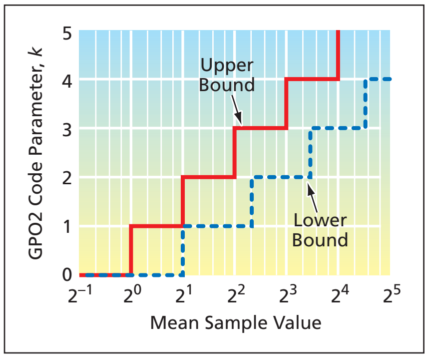
An alternative method of adaptive selection of Golomb power-of-two (GPO2) codes has been devised for use in efficient, lossless encoding of sequences of non-negative integers from discrete sources. The method is intended especially for use in compression of digital image data. This method is somewhat suboptimal, but offers the advantage in that it involves significantly less computation than does a prior method of adaptive selection of optimum codes through "brute force" application of all code options to every block of samples.

A rather lengthy discussion of background is necessary to give meaning to a brief summary of this innovation. For positive integer, m, the mth Golomb code defines a reversible, prefix-free mapping of non-negative integers to variable-length binary code words. Golomb codes are optimum for geometrically distributed sources (a model that frequently arises in image compression): In the case of a geometrically distributed random variable, δ, the appropriately selected Golomb code minimizes the expected code-word length over all possible lossless binary codes for δ.

In a GPO2 code, $m = 2^k$, where k is a non-negative integer. Such a code makes the coding process particularly simple: The code word for the integer δ consists of the unary representation of $\lfloor \delta/2^k \rfloor$ (that is, $\lfloor \delta/2^k \rfloor$ zeros followed by a one) concatenated with the k least significant bits of the binary representation of δ. More specifically, the code is called a GPO2 code of parameter k.

The problem is to calculate or estimate the value of code parameter k that minimizes the expected bit rate (the average number of encoded bits per source symbol) for an image or other source. This problem arises in Rice coding, which is a coding method well known among experts in data compression. The Rice algorithm encodes a block of samples by use of the best code option for the block from among several candidate codes that consist mostly of different GPO2 codes. A fixed number of bits are used preceding the encoded block to indicate which code was selected. The Rice method does not specify how to find the best code option, and the most common approach is to exhaustively try every code option to pick the best one for each block. Information from previously coded blocks is not utilized. This concludes the background information.



Upper and Lower Bounds on the optimum value of the code parameter make it possible to reduce the number of code options that must be considered.

In the present method, unlike in the Rice method, one utilizes the mean sample value in each block. The method is based partly on a theoretical derivation of bounds on the optimum value of k as functions of the mean sample value (see figure). These bounds are such that no more than three code choices can be optimum for a given mean sample value. For a given mean value, one of the three candidate codes is selected in a procedure that involves only integer arithmetic (without divisions) and table look-ups. It has been shown that the value of k selected in this relatively simple procedure is always within 1 of the optimum k value for the source, and that the cost added by the suboptimality of the selection is never more than 1/2 bit per sample and no more than about 13-percent inefficiency. In practical image compression experiments, the cost added by the suboptimality of the selection is negligible.

This work was done by Aaron Kiely of Caltech for NASA's Jet Propulsion Laboratory.

The software used in this innovation is available for commercial licensing. Please contact Karina Edmonds of the California Institute of Technology at (626) 395-2322. Refer to NPO-41336.