Uterine Contraction Modeling and Simulation

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Abstract. Building a training system for medical personnel to properly interpret fetal heart rate tracing requires developing accurate models that can relate various signal patterns to certain pathologies. In addition to modeling the fetal heart rate signal itself, the change of uterine pressure that bears strong relation to fetal heart rate and provides indications of maternal and fetal status should also be considered. In this work, we have developed a group of parametric models to simulate uterine contractions during labor and delivery. Through analysis of real patient records, we propose to model uterine contraction signals by three major components: regular contractions, impulsive noise caused by fetal movements, and low amplitude noise invoked by maternal breathing and measuring apparatus. The regular contractions are modeled by an asymmetric generalized Gaussian function and least squares estimation is used to compute the parameter values of the asymmetric generalized Gaussian function based on uterine contractions of real patients. Regular contractions are detected based on thresholding and derivative analysis of uterine contractions. Impulsive noise caused by fetal movements and low amplitude noise by maternal breathing and measuring apparatus are modeled by rational polynomial functions and Perlin noise, respectively. Experiment results show the synthesized uterine contractions can mimic the real uterine contractions realistically, demonstrating the effectiveness of the proposed algorithm.

1. Introduction

Uterine contractions are stimulated by uterine muscle cells. Uterine contraction (UC) variations reflect the physiological changes of the uterus during both pregnancy and labor [1]. As a critical component in fetal heart rate (FHR) monitoring during labor and delivery, uterine contractions provide important information regarding maternal and fetal wellbeing. There are three main methods to record uterine contractions [2]: tocography, electrohysterography, and using intrauterine pressure catheter. Tocography measures the strain exerted by uterus on the maternal abdomen via external a tocotransducer. Electrohysterography records the electrical uterine activities from the maternal abdomen. The intrauterine pressure catheter (IUPC) measures intrauterine pressure invasively, and is mostly used during labor. Regardless of their differences, all three methods aim at providing records of the contraction patterns and their relationship to FHR. Proper interpretations of fetal heart rate and uterine contractions require special training, while monitoring of both are only available when pregnant women are hospitalized in parturiency. Building a training system that can simulate fetal heart rate and uterine contractions can help medical personnel learn critical patterns of both signals without putting patient in danger. To gain better understanding of uterine activities, it is necessary to develop mathematical models to quantitatively describe various uterine contraction patterns and this is the problem to be addressed in this paper.

Even though FHR monitoring is now the standard practice during labor and delivery, surprisingly, there were only a few studies on uterine contraction modeling and simulation. Young used polynomials to model five characteristics of uterine contractions: 1) gradual onset, 2) a linear rising segment, 3) a plateau region, 4) a symmetrical fall, and 5) gradual offset, and fitted the simulated contractions with recorded IUPC data [3]. Their results matched their physical analysis. However, their simulations were not perfect especially in the tail region of the contraction curve. Vauge et al. [4] developed a system of differential equations that describe the dynamics of uterine pressure during human parturition. This method was based on three simplified assumptions: identical contractile properties of all myometrial cells, intrauterine pressure proportional to the number of contracted myometrial cells, and that all cells have three states, namely, contraction, recovery, and resting. Their model was simple and effective especially for normal contractions which begin in the fundus, reach the apex, and then proceed symmetrically downward toward the fundus. But this method did not consider the fact that asymmetry can occur when the uterine cells function independently causing ineffective uterine contractions and minimal dilatation [2]. Recently, Kemal et al. [5] employed two methods to simulate uterine contractions. The first one is based on the same mathematical model proposed by Vauge [4]. The second approach is based on recorded patient data. They first applied Hilbert-Huang Transform (HHT) [6] to identify the contraction locations from real patient data, and then developed spatial-temporal simulations of uterine contractions. All these methods discussed above are capable of illustrating the dynamics of uterine activities. However, these methodologies are

deterministic and no comprehensive parameter estimation for these models was developed. Furthermore, the noise caused by fetal movements and maternal breathing were not considered and modeled. To address the problems in existing uterine modeling and simulation, the paper proposes a novel algorithm that integrates three major components: asymmetric generalized Gaussian function (AGGF) for modeling contractions, Perlin noise for modeling maternal breathing and instrument noise, and impulsive noise for modeling fetal movements. The parameters of the asymmetric generalized Gaussian function are estimated using the least square method based on detected uterine contractions from real patient records.

The remainder of this paper is structured as follows. Section 2 first introduces the proposed asymmetric generalized Gaussian functions for modeling uterine contractions and then estimate the parameters of generalized Gaussian functions based detected contractions. Section 3 describes Perlin noise generation for maternal breathing and low amplitude noise modeling. Section 4 presents impulsive noise generation for simulating disturbances caused by fetal movements. Section 5 summarizes the simulation procedure and compares the simulation results. Section 6 concludes this paper and discusses future research directions.

2. Uterine Contraction Modeling

2.1 Asymmetric Generalized Gaussian Function

A typical uterine contraction curve of a real patient is depicted as a continuous waveform in Figure 1. This curve is characterized by a basal tone varying from 0 to 20 units, and a deflection of the contraction curve above the baseline, whose amplitude and duration are within a certain range of values [2]. Moreover, one should note this curve is asymmetric.
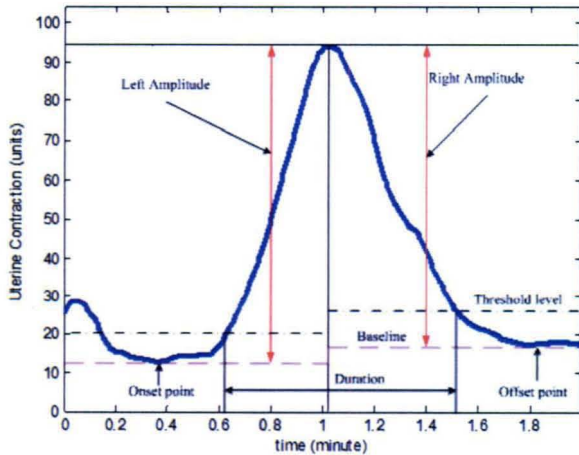
Figure 1: Typical uterine contractions. Also shown are parameters used for contraction detection.

Since the asymmetry of contraction curve matches

real cases of ineffective contractions and comprises the symmetrical case, we propose to use an asymmetric generalized Gaussian function to model uterine contractions as follows.

$$f(t) = A_l \exp\left\{-\frac{(t-t_0)^{\alpha_l}}{\beta_l}\right\}\left[u(t-t_l)-u(t-t_0)\right]$$

$$+A_r \exp\left\{-\frac{(t-t_0)^{\alpha_r}}{\beta_r}\right\}\left[u(t-t_0)-u(t-t_r)\right] \quad (1)$$

$$+b(t)\left[u(t-t_l)-u(t-t_r)\right],$$

where the parameters of (1) are shown below.

A_l, A_r Amplitudes for the left and right sides
α_l, α_r Exponents for the left and right sides
β_l, β_r Variances for the left and right sides
$u(t)$ A unit step function
t_l, t_r Left and right cut off time
t_0 The position where $f(t)$ reaches its maximum
$b(t)$ Baseline representing some basal strain exerted by the uterine muscle when contractions do not occur

To simulate uterine contractions, we need to determine the range within which the above parameters lie and how they vary with time. So in the next step we will detect uterine contractions from real patient data and estimate the parameters for the asymmetric generalized Gaussian function from the detection results.

2.2 Uterine Contraction Detection

Several methods were proposed to detect uterine contractions for different purposes. Radhakrishnan et al. [7] developed a higher-order zero crossing based method and studied the frequency of occurrence of contractions in different pregnancy stages. Novak et al. [8] described two UC detection approaches: amplitude- and derivative-based algorithm. By comparing the results from these two methods, they suggested combining both methods together to achieve better detection results. Aiming at quantitatively analyzing uterine contractions in time domain, Jezewski et al. [9] introduced a statistical method to determine the threshold and also considered duration condition for UC detection. These methods were used to calculate the regular parameters of uterine contractions such as amplitude, duration, frequency of occurrence. Our method is based on a combination of the last two methods, in which thresholding is first performed to detect the presence of uterine contractions, and derivative-based method is applied subsequently to include the samples whose amplitudes are below the threshold, but still belonging to the contractions.

The algorithm proposed by Jezewski et al. [9] starts with low pass filtering with cutoff frequency of 0.04 Hz

to suppress the artifacts caused by fetal movements and maternal breathing. Then the record is analyzed by using a moving window with a length of 4 minutes and 1-minute step. Within each window, the histogram of uterine pressure samples is constructed first and the mode of the histogram is then selected as the baseline value. Finally, the threshold level is set as 10 units above the baseline and the validity of data segment is examined. A valid contraction should remains above the threshold level for a duration longer than 30 seconds and the amplitude of contraction exceeds 20 units.
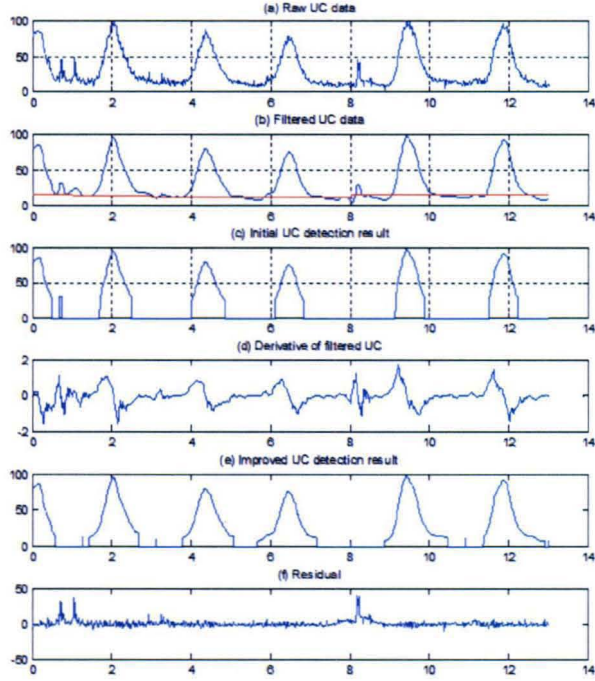
Figure 2: Intermediate results in uterine contraction detection. (a) Raw UC data. (b) Filtered UC. (c) UC detection by thresholding. (d) The derivatives of filtered UC. (e) Improved UC detection results. (f) The residual between raw UC and filtered UC which is detected.

After the uterine contractions are detected, there are still two remaining problems. The first is that the amplitude of detected samples are mostly above the threshold, thus the estimated parameters may not be accurate enough to model the tail areas. We need to include more samples from tail areas. On the other hand, the tail areas are prone to other various sources of noise. Before including them for parameter estimation, we must differentiate contaminated samples and uncontaminated or less-contaminated samples. Since the derivatives in noisy area change drastically, we propose to employ derivative-based detection method to differentiate them. In other words, for less contaminated contractions, the derivative of the left side of the contraction should be positive, while the derivative of right side should be negative. Starting from the peak of the contraction, we proceed to its left and compute the derivatives of the smoothed uterine pressure signal and then search the position where the derivative changed to negative for left side curve, and denote it as the tail point. The sample points between the peak and the tail point will be utilized for curve parameters estimation. The same principle applies to the right side of contraction with positive derives being searched.

The results of uterine contraction detection are shown Figure 2, where (a) is the original uterine pressure signal; (b) is its filtered version, the red solid line represents the base line; (c) illustrates the sample whose values are above the threshold; (d) is the derivatives of filtered UC data; (e) is the improved uterine contraction detection result; and (f) shows the residual between the detected contractions and original signal (a).

2.3 Uterine Contraction Parameter Estimation

After the uterine contractions are detected, we need to compute the parameters of the asymmetric generalized Gaussian function (1) in order to simulate the detected contractions. Here we only consider the left half of the asymmetric generalized Gaussian function (1), that is,

$$f(t) = A_l \exp\left\{-\frac{|(t-t_0)|^{\alpha_l}}{\beta_l}\right\} + b_l\left[u(t-t_l) - u(t-t_0)\right]$$

(2)

for the parameter estimation, while the right half can be handled similarly and thus is omitted in this paper.

First, the baseline b_l is estimated as the contraction value at the onset point t_l which is the lowest point between two filtered contractions. Thus

$$A_l = f(t_0) - b_l.$$

After simple manipulation, the remaining known variables and unknown parameters are separated by taking the logarithm of both sides of (2),

$$\ln\frac{f(t)-b_l}{A_l} = -\frac{|(t-t_0)|^{\alpha_l}}{\beta_l}.$$ (3)

Since $f(t) - b_l < A_l$, adding minus sign to both sides of (3) and taking logarithm again gives

$$\ln\left(-\ln\frac{f(t)-b_l}{A_l}\right) = \alpha_l \ln|t-t_0| - \ln\beta_l.$$ (4)

Now we denote the detected uterine contraction data set as $\{f(t_i), t_i\}$, $1 \le i \le N$, where N is the number of samples. Substituting the data set into equation (4) and writing each term in matrix form, we

have

$$\mathbf{Y} = \begin{bmatrix} \ln\left(-\ln\dfrac{f(t_1)-b_l}{A_l}\right) \\ \ln\left(-\ln\dfrac{f(t_2)-b_l}{A_l}\right) \\ \vdots \\ \ln\left(-\ln\dfrac{f(t_N)-b_l}{A_l}\right) \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \ln|t_1-t_0|, & -1 \\ \ln|t_2-t_0|, & -1 \\ \vdots & \vdots \\ \ln|t_N-t_0|, & -1 \end{bmatrix},$$

$$\mathbf{p} = \begin{bmatrix} \alpha_l \\ \ln\beta_l \end{bmatrix}.$$

(5)

Then equation (4) can be transformed into a concise linear equation represented by a matrix product,

$$\mathbf{Y} = \mathbf{Xp}, \tag{6}$$

where $\mathbf{X}$ and $\mathbf{Y}$ are matrices containing detected sample information, $\mathbf{p}$ is the parameter vector to be estimated. The parameter estimation problem can be expressed as the following minimization problem,

$$\mathbf{p}_o = \arg\min_{\mathbf{p}\in R^2} \|\mathbf{Y} - \mathbf{Xp}\|_2^2, \tag{7}$$

where $\mathbf{p}_o$ is the solution. This is a least square estimation problem whose solution is

$$\mathbf{p}_o = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}. \tag{8}$$

Thus we have obtained the parameter estimator for the proposed asymmetric generalized Gaussian function for modeling uterine contractions.

3. Perlin Noise

The low amplitude noise caused by maternal breathing and measuring apparatus are random yet exhibiting both low and high frequency characteristics as shown in Figures 2(a) and (f). Common random number generators cannot be used directly to generate such noise, since they are too random to exhibit the natural outlook of continuity and self-similarity of the noise. To address this problem, we propose to use Perlin noise generator [10] to simulate the low amplitude noise.

A Perlin noise generator is composed of two components: a noise function and an interpolation function. The basic procedure of Perlin noise generation is
(1) Generate a series of random numbers n_l of length N from uniform distribution $U[-0.1, 0.1]$;
(2) Decimate the series to size $N/2$;
(3) Upsample the decimated series to size N by B-spline interpolation and increase the amplitude of the new series by a factor of p;
(4) Repeat (2) and (3) until reaching the specified level s, and we obtain a set of series $\{n_1, n_2, ..., n_s\}$.

By adding the above generated series together, we have the Perlin noise

$$f_p(t) = \sum_{i=0}^{s} p^i n_i(t), \tag{9}$$

where $t \in [1, N]$, and p is called persistence factor which controls the amplitude.

(a)

(b)

(c)

(d)

(e)

(f)

(g)
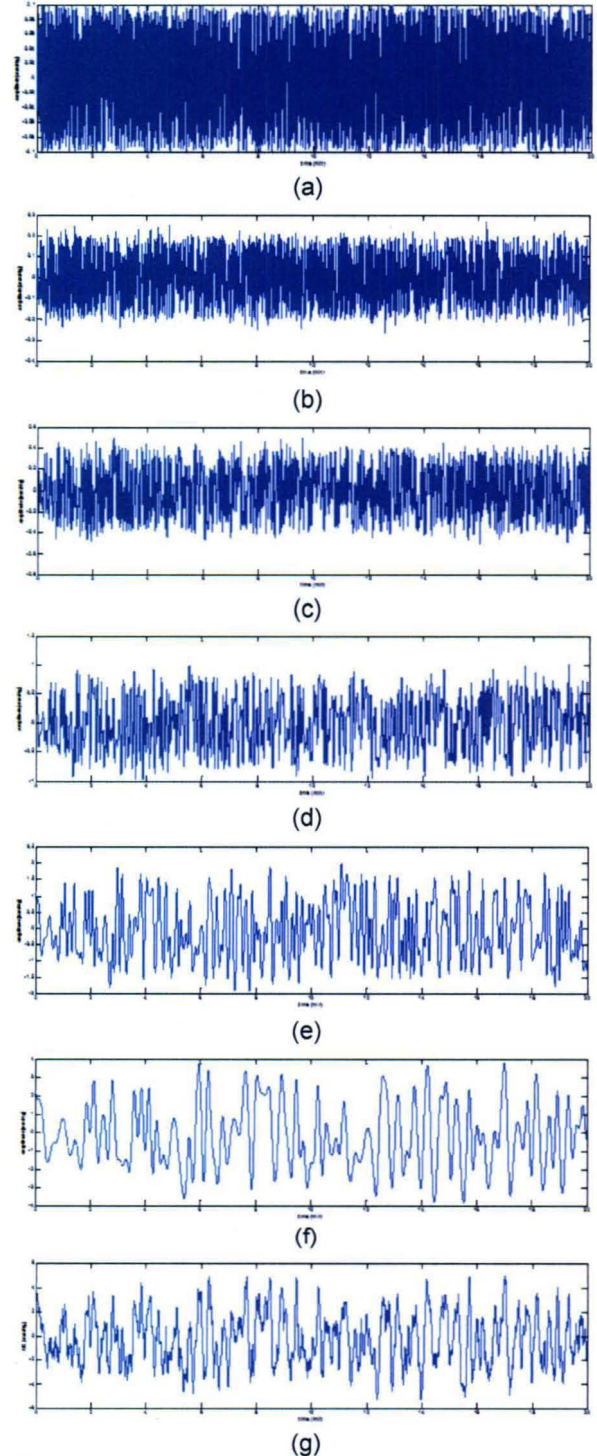
Figure 3: (a) - (f) are waveforms corresponding to 6 levels noise series $\{n_1, n_2, ..., n_6\}$. (g) is the Perlin noise.

One fact worth of explaining is the mechanism for how the continuous noise is produced. Since n_1 is generated from i.i.d. (independent and identically-distributed) uniform distribution, it is actually white noise. Through repeated downsampling and upsampling, the new noise series become the low-pass filtered version of the previous noise series. In other words, decimation by a factor of 2 reduces one half the Nyquist frequency of previous noise, then B-spline interpolation restores the sample number of noise without incurring new frequency contents. Thus the noise n_s generated in the last step, occupies the lowest frequency band. So the waveforms of the noise series from n_1 to n_s, become increasingly smoother. Figure 3 illustrates the components of the Perlin noise generated in this work, in which (a) is the noise generated from uniform distribution $U[-0.1, 0.1]$, (b) – (f) are low-pass filtered version of noise generated from one level before with $p = 2$. Note that the sub-figures in Figure 3 have different vertical scales and low-frequency components have much larger amplitudes than high-frequency components. The final synthesized Perlin noise is shown in Figure 3(g).

4. Impulsive Noise Modeling

The last component to simulate is the spikes or large magnitude impulsive noise in uterine contractions, which suggest possible fetal movements [2]. After examining the shape of those spikes, we propose to use the following rational polynomial function to model them

$$f(t) = \frac{a}{1 + \left(\dfrac{t - t_0}{b}\right)^2}, \qquad (8)$$

where a is the amplitude which follows uniform distribution with value between 0 and 50 units, b is the scale parameter following uniform distribution with value between 0 and 10, and t_0 is the spike position. A typical simulation result of the impulsive noise is shown in Figure 4.
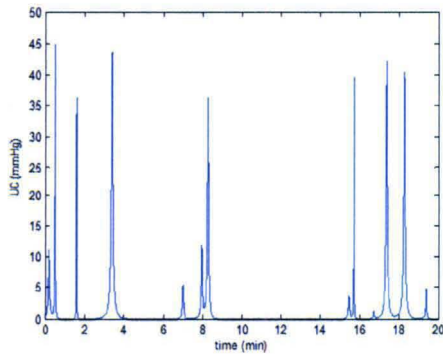

Figure 4: Impulsive noise simulation

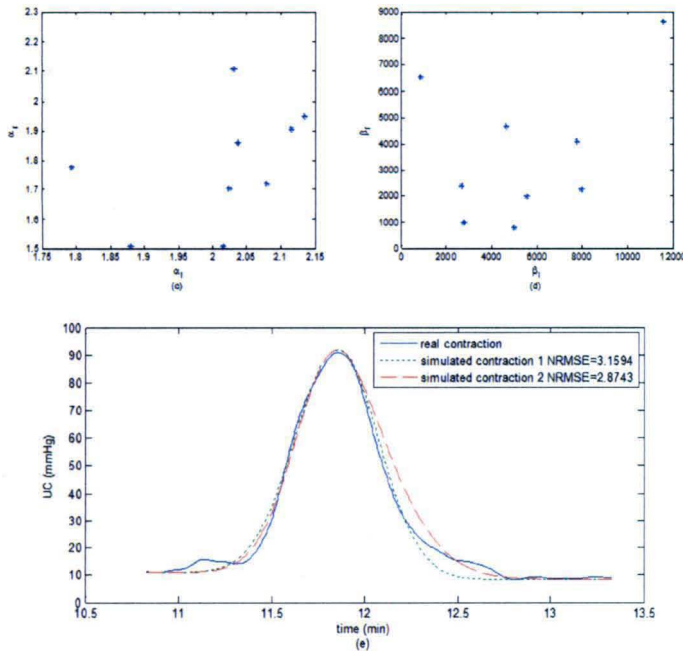## 5. Uterine Contraction Synthesis

Uterine contraction simulation is finalized by superimposing the components generated by the asymmetric generalized Gaussian function, Perlin noise, and impulsive noise. First, a typical segment of 20 minutes was extracted from real patient record and the parameters for the asymmetric generalized Gaussian function were estimated. Figure 5 compares the parameters estimated from two detected data sets. The first data set contains only samples whose amplitude is above the threshold. The second data set is the expanded version of the first set by incorporating samples whose amplitude are below the threshold but are belong to the contraction. Figures 5 (a) and (b) plot parameters estimated from the first set. Figures 5(c) and (d) are estimated from the second set. It can be seen that the parameters of the left side and right side are different, validating the asymmetry of uterine contractions.

To compare the impact of different parameters on uterine contraction simulation, we select one set of parameters from Figures 5(a) and (b), and another set from Figures 5(c) and (d) to simulate two contractions, namely, contraction 1 (green dotted line) and contraction 2 (red dashed line), as shown in Figure 5(e). It can be seen that the red dash line achieves a better fit to the real contraction at both the peak and tail area of the contraction and has smaller normalized root mean square error (NRMSE), which is defined as

$$\text{NRMSE} = \frac{\sqrt{\dfrac{\sum_{i=1}^{N} \left(f_1(i) - f_0(i)\right)^2}{N}}}{\max_{1 \le j \le N}\{f_0(j)\} - \min_{1 \le k \le N}\{f_0(k)\}}. \qquad (10)$$

This result indicates that the derivative-based analysis could help to recruit more effective data for parameter estimation, and thus increase the estimation accuracy. Finally, by adding together the asymmetric Gaussian function, the Perlin noise, and the impulsive noise, we obtain the final simulation results in Figure 6(b). Comparing the original uterine contraction record in Figure 6(a) and the simulated results in Figure 6(b), it can be easily seen that the proposed algorithm is very effective and produces superb results.

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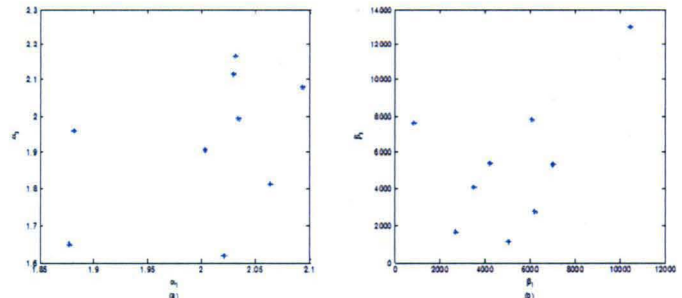
Figure 5: Parameters estimation and uterine contraction simulation. (a) is the scatter plot of α, and (b) is the scatter plot of β for left and right curves. (c) and (d) are α and β plots of the second data set. (e) shows the simulations of one uterine contraction from two data sets.
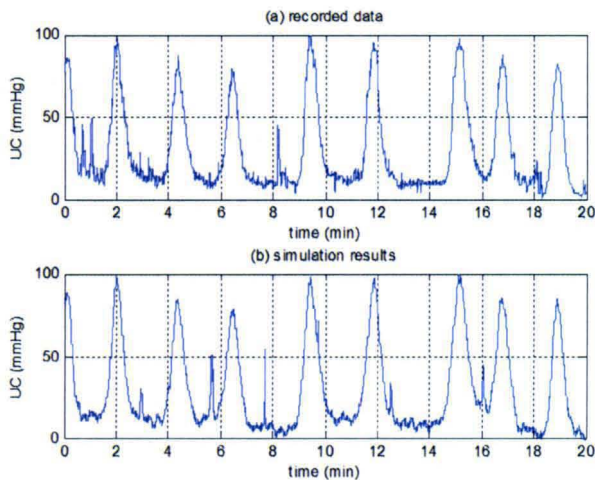
Figure 6: Simulation results. (a) The original uterine contraction recording. (b) Simulated uterine contraction based on (a). It can be seen that the proposed algorithm is very effective, producing realistic simulations.

6. Conclusion

This paper proposed a set of parametric models to simulate uterine contractions. The proposed algorithm contains three major components: AGGF model for contractions, Perlin noise for maternal breathing and instrument noise, and rational polynomial functions for fetal movements.

Parameter estimation for the asymmetric generalized Gaussian function is derived based on real uterine contractions. The proposed algorithm is effective and produces realistic simulation results. Future work includes parameter estimation for the Perlin noise and impulsive noise in order to further improve the quality of uterine contraction simulations.

References

1. M Holger, M William, S George, G Robert (2003): The physiology of uterine contractions. *Clinics in Perinatology 30*:665.
2. M Michelle, H Gayle, R Patricia (2007): Essentials of Fetal Monitoring, Springer, New York.
3. RC Young (1997): A computer model of uterine contractions based on action potential propagation and intercellular calcium waves. *Obstetrics and Gynecology 89*:604.
4. C Vauge, B Carhonne, E Papiernik, F Ferre (2000): A Mathematical Model of Uterine Dynamics and Its Application to Human Partruition. *Acta Biotheoretica 48*:95.
5. A Kemal, H Tansel, B Coskun, L Kamran (2006): Computer Simulation of Uterine Contraction Dynamics. In: Proceedings of the 5th WSEAS International Conference on Circuits, Systems, Electronics, Control & Signal Processing, WSEAS, Dallas, Texas, 1790.
6. NE Huang, Z Shen, SR Long, MC Wu, HH Shin, Q Zheng, N-C Yen, CC Tung, HH Liu (1998): The Empirical Mode Decomposition and Hilbert Spectrum for Nonlinear and Non-stationary Time Series Analysis. In: The Royal Society, 903.
7. N Radhakrishnan, JD Wilson, C Lowery, H Eswaran, P Murphy (2000): A Noninvasive Method Utilizing Higher-Order Zero Crossing for Signal Analysis. *IEEE Eng. Med. Biol 19*:89.
8. D Novak, A Macek-Lebar, D Rudel, T Jarm (2007): Detection of contractions during labour using the uterine electromyogram. In: IFMBE, 148.
9. J Jezewski, K Horoba, A Matonia, J Wrobel (2005): Quantitative analysis of contraction patterns in electrical acitivity signal of pregnant uterus as alternative to mechanical approach. *Phyiological Measurement 26*:753.
10. Perlin Noise, http://freespace.virgin.net/hugo.elias/models/m_perlin.htm