

Diffraction X-ray Telescopes

Gerald K. Skinner

CRESST & NASA-GSFC, Greenbelt, MD 20771, USA;

and

Univ. Md. Department of Astronomy, College Park, MD 20742, USA

Abstract

Diffraction X-ray telescopes, using zone plates, phase Fresnel lenses, or related optical elements have the potential to provide astronomers with true imaging capability with resolution many orders of magnitude better than available in any other waveband. Lenses that would be relatively easy to fabricate could have an angular resolution of the order of micro-arc-seconds or even better, that would allow, for example, imaging of the distorted space-time in the immediate vicinity of the super-massive black holes in the center of active galaxies. What then is precluding their immediate adoption? Extremely long focal lengths, very limited bandwidth, and difficulty stabilizing the image are the main problems. The history, and status of the development of such lenses is reviewed here and the prospects for managing the challenges that they present are discussed.

Keywords: X-ray imaging, Gamma-ray imaging, X-ray interferometry, Gamma-ray interferometry

1. INTRODUCTION

Diffraction optics, in the form of zone plates and various forms of Fresnel lenses and kineforms, already play a major role in the manipulation of X-ray beams at synchrotron facilities and in X-ray microscopy. Diffraction X-ray telescopes, in contrast, exist almost entirely as concepts on paper and as proposals and suggestions, though as will be seen demonstrations of scaled systems have been made. Because of atmospheric absorption, their potential application is almost certainly limited to astronomy (and specifically

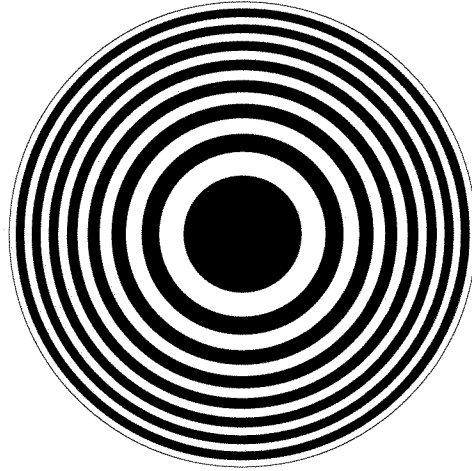
Email address: skinner@milkyway.gsfc.nasa.gov ()

to astronomy from space), perhaps including solar and planetary studies. However, for certain objectives within that field they present the prospect of enormous advances over current instrumentation, which relies largely on grazing incidence reflective optics (reviewed elsewhere in this series). The most notable prospect that diffractive telescopes offer is that of superb angular resolution, with improvements of perhaps six orders of magnitude on the current state of the art. But without taking advantage of the imaging properties, their capability of concentrating the flux received over a large effective area onto a small, and hence low background, detector may also offer unique advantages in some circumstances.

This review will consider the various concepts that have been proposed and discuss the current state of development of the technologies necessary to turn the ideas into a real system. For simplicity the term X-rays will often be used to apply to both X-rays and gamma-rays, there being no clear distinction or borderline between the two. The review will be limited to techniques exploiting the wave nature of X-ray (and gamma-ray) radiation, so excluding, for example, the use of screens with zone-plate-like patterns as coded masks [1] or in pairs to produce Moiré fringes [2, 3]. Within wave optics, systems based on multilayer optics or on crystal diffraction are not addressed (the latter are reviewed elsewhere in this series [4]).

2. History

The basic diffractive optics imaging element can be considered to be the zone plate (ZP) in which an aperture is divided into transparent and opaque regions according to whether radiation passing through them would arrive at some selected focal point with a phase such as to interfere constructively or destructively. The resulting pattern is shown in Fig. 1a. The first zone plate was made by Lord Rayleigh in 1871 though this work was never published (see [5, 6]) and it was Soret [7] who first described them in print in 1875. Already in 1888 Rayleigh[8] realised that problems of high background due to undiffracted light and low efficiency could be overcome by using phase-reversal zone plates (PZPs) in which the opaque regions are replaced by ones whose thickness is chosen to introduce a phase-shift of π (Fig. 1b). Wood demonstrated the advantages of PZPs ten years later [9]. Miyamoto [10] further extended the concept, introducing the term phase Fresnel lens (PFL below) for an optic in which the phase shift is at each radius the same as that for an ideal conventional lens (Fig. 1c).



(a)



(b)



(c)



Figure 1: Three basic forms of diffractive optics for X-ray telescopes. (a) A zone-plate (ZP), with opaque and transparent regions, (b) A phase-zone-plate (PZP), in which the shaded zones transmit radiation with a phase shift of π , (c) A phase-zone-plate (PZP), in which the thickness is everywhere such that the phase is shifted by the optimum angle.

The possibility of using ZPs for X-ray imaging seems first to have been seriously considered in the 1950s by Myers [11] and by Baez [12]. In 1974 Kirz [6] pointed out that the relative transparency of materials in the X-ray band and the fact that X-ray refractive indices differ slightly from unity would allow PZPs to be constructed even for high energy photons, so obtaining much higher efficiency.

Remarkably, as early as the 1960s simple zone plates were used for solar (soft) X-ray astronomy. Elwert [13] obtained the agreement of Friedmann to replace the pinholes in two of the pinhole cameras on a 1966 NRL solar-viewing sounding rocket flight with small zone plates designed to operate in lines 51Å (0.246 keV, Xi X) and 34Å (0.367 keV, ascribed to CVI, but actually OVI). Although the attitude control malfunctioned, blurred images were obtained. Over the next few years the technique was used, in particular by the Tübingen and Utrecht groups, for solar imagery from sounding rockets at energies up to 0.8 keV (Fe XVII) [e.g. 14, 15]. One of the disadvantages of zone plates proved to be the halo due to zero-order diffraction that surrounds the focal point of a simple zone plate. Ways were found of alleviating this problem by using only the zones within an annular region [16, 17].

At much the same time Wolter I grazing incidence telescopes were becoming available for soft X-ray solar imaging – the first was flown on a sounding rocket in 1965 [18] and two were used on the Apollo Telescope mount on Skylab [19]. With the size instrumentation feasible at that epoch, the grazing incidence technology proved superior. For cosmic observations as well as solar, it has become the imaging technique of choice except where the need for very wide fields of view or operation at high energies precludes its use, in which case non-focussing devices such as coded-mask or Compton telescopes are used.

As a result of the success of grazing angle reflective optics, diffractive X-ray optics for astronomical applications tended to be forgotten. In a 1974 PhD thesis Niemann [20] did discuss the possible use of diffractive for extra-solar astronomy and in 1996 Dewey *et al.* [21] proposed a mission concept in which patched blazed diffraction gratings based on the technology developed for the AXAF (now Chandra) mission would approximate a PZP. But it is only comparatively recently that there has been a revival of interest in the possibility of using diffractive optics for X-ray and gamma-ray astronomy in particular circumstances where it may offer unique advantages. Several authors [e.g. 22, 23, 24, 25, 26, 27] have pointed out that the angular resolu-

tion potentially available with diffractive X-ray telescopes exceeds by many orders of magnitude the practical limits of reflective optics.

Meanwhile there have been major advances in diffractive X-ray optics for non-astronomical applications, driven in particular by the availability of synchrotron sources and interest in X-ray microscopy with the best possible spatial resolution. For reasons discussed below, most of the effort has been towards lenses with structures on an extremely small scale, even if the diameter is also small. For astronomical telescopes on the other hand fineness of structure would be of secondary importance, but large apertures are essential.

3. Theory

3.1. Basic parameters

A zone plate such as illustrated in Figure 1a can be conveniently characterized by the outside diameter, d , and the pitch, p_{min} of one cycle of the opaque/transparent pattern at the periphery where it is smallest¹. The focal length for radiation of wavelength λ is then given by

$$f = \frac{p_{min}d}{2\lambda}, \quad (1)$$

a relationship that applies equally to PZPs and PFLs. In terms of photon energy E and physical units this becomes

$$f = 403.3 \left(\frac{p_{min}}{1 \mu\text{m}} \right) \left(\frac{d}{1 \text{ m}} \right) \left(\frac{E}{1 \text{ keV}} \right) \text{ m}, \quad (2)$$

making clear that focal lengths of systems of interest for X-ray astronomy are likely to be long.

Whereas diffractive X-ray optics has so far been employed almost exclusively for microscopy and for other applications for which *spatial* resolution is all-important, for telescopes, it is the *angular* resolution that counts. For ideal PFLs one can use the usual rule that the diffraction limited angular

¹The number of cycles is assumed to be large so that a characteristic period can be defined.

resolution expressed as half-power-diameter (HPD) is

$$\Delta\theta_d = 1.03\lambda/d = 263 \left(\frac{E}{1 \text{ keV}} \right)^{-1} \mu'' \quad (3)$$

$$= 0.501 \left(\frac{p_{min}}{f} \right), \quad (4)$$

(using the Rayleigh critereon, the numerical factor 1.03 would be the familiar 1.22). The same expression can be in practice be used for ZPs and PZPs with a large number of cycles (Stigliani et al[28] present an exact solution). Assuming a simple single lens system, the corresponding dimension in the image plane is then

$$\Delta x = f \Delta\theta_d = 0.501 p_{min}. \quad (5)$$

This is also approximately the spatial resolution of a microscope with a diffractive lens, explaining why the main drive in X-ray diffractive optics technology has so far been towards reducing p_{min} . ZPs with zones of less than 15 nm, corresponding to $p_{min} < 30$ nm have been reported [29]. If the angular resolution of a telescope is to be limited only by diffraction, then it is important that the distance Δx be larger than the spatial resolution of the detector. Equation 5 then implies that p_{min} should *not* be too small. Current state of the art detectors achieve spatial resolutions from 5–10 μm (CCDs at X-ray energies) to a fraction of a mm (pixelized CZT or Ge detectors for hard X-rays and gamma-rays). Because of the range of the electron which receives energy from the incoming photon, little improvement in this performance is likely, so p_{min} for diffraction-limited X-ray telescopes is likely to be at least 10s to 100s of microns.

3.2. Lens profile

A simple zone plate is inefficient because it is only 50% transmitting and because much of the radiation is not diffracted into the primary focus (order $n = 1$) but is undiffracted ($n = 0$) or diffracted into secondary focii ($n < 0, n > 1$). Table 1 shows that even if it is perfect the efficiency of a ZP is only $\sim 10\%$.

For a ZP the depth of the profile is simply dictated by the requirement that the material be adequately opaque; the only upper limit is that imposed by manufacturing considerations. High density, high atomic number, materials are generally to be preferred. For PZPs and PFLs, on the other hand,

Table 1: Comparison of the efficiency of ideal zone-plates (ZPs), phase-zone-plates (PZPs), and phase Fresnel lenses (PFLs)

	Efficiency (primary focus) $n = 1$	Lost to absorption	Undiffracted $n = 0$	Secondary focii $n = -1, \pm 3, \pm 5 \dots$
ZP	$1/\pi^2 = 10.1\%$	50%	25%	14.9%
PZP	$4/\pi^2 = 40.5\%$	0	0	59.5%
PFL	100%	0	0	0

the thickness of the profile and the transparency of the material are critical. The nominal thickness of a PZP is that which produces a phase shift of π . The complex refractive index of the material may be written

$$\mu = 1 - \frac{r_e \lambda^2}{2\pi} n_0 (f_1 + i f_2) = 1 - \delta - i\beta, \quad (6)$$

where r_e is the classical electron radius, n_0 is the number of atoms per unit volume, and $f = f_1 + i f_2$ is the atomic scattering factor. The required thickness is then $\frac{1}{2} t_{2\pi}$, where $t_{2\pi}$ is λ/δ . Well above the energy of any absorption edges f_1 approaches the atomic number, so δ is proportional to λ^{-2} and $t_{2\pi}$ to λ^{-1} , or to E .

The practicability of X-ray PZPs relies on the fact that although the wavelengths λ are extremely short, δ is also very small. Consequently $t_{2\pi}$ is in a range, from microns to millimeters, where fabrication is practicable and where the absorption losses in traversing the required thickness of material are not too serious. Fig. 2 shows some values of $t_{2\pi}$ as a function of photon energy for some materials of interest.

In the case of PFLs, the surface of each ring is ideally part of a paraboloid, usually with a maximum height $t_{2\pi}$ (though $t_{4\pi}$ $t_{6\pi} \dots$ lenses can be made, with a correspondingly reduced number of rings). At large radii the small sections of paraboloids are almost straight and the cross-section is close to a triangular sawtooth. With some fabrication techniques it is convenient to use a stepped approximation to the ideal profile [e.g. 30]. High efficiency can be obtained with a relatively small number of levels (Table 2). Fabrication ‘errors’ in the form of rounding of corners will actually tend to *improve* efficiency.

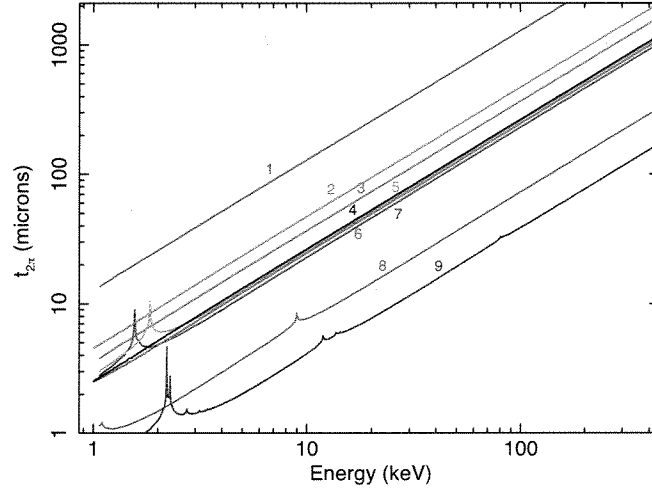


Figure 2: The thickness, $t_{2\pi}$, necessary to shift the phase of X-rays by 2π for some example materials. From top to bottom Li (1, red) Li, Polycarbonate (2, orange), Be (3, green), BC (4, light blue), Si (5, cyan), LiF (6, light blue), Al (7, dark blue), Cu (8, violet), Au (9, grey).

Table 2: The primary focus efficiency, in the absence of absorption, of a stepped approximation to a PFL [see 31].

Levels:	2 (PZP)	3	4	8	16	n
Efficiency:	40.5%	68.4%	81.1%	95.0%	98.7%	$\text{sinc}^2(\pi/n)$

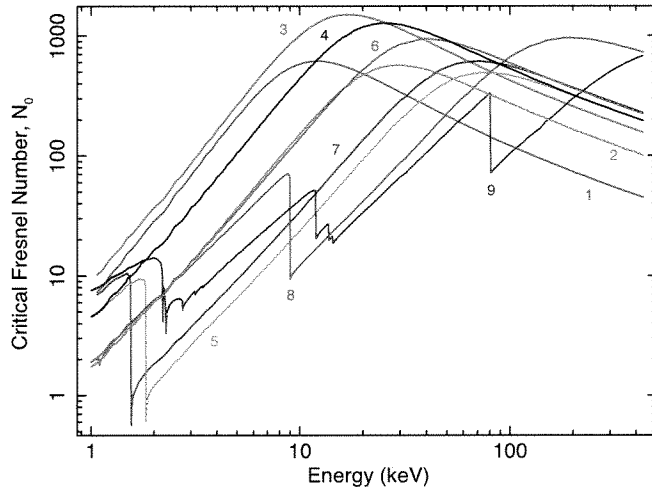


Figure 3: The critical Fresnel number, N_0 , for some example materials. Numbers, colors and materials as in Fig. 2.

3.3. Field of view

Young [32] has discussed the off-axis aberrations of ZPs and the same conclusions can be applied to PZPs and in practice to PFLs. The expressions that he derives imply that in circumstances of interest for astronomy the most important Seidel aberration is coma, which only becomes important at off-axis angles greater than $4\lambda f^2/d^3$. In other terms, this implies that the number of diffraction-limited resolution elements across the coma-free field of view is $\sim 8(f/d)^2$. With the very large focal ratios that seem to be inevitable for diffractive X-ray telescopes, aberrations are entirely negligible over a region far larger than the field of view of any practicable detector.

3.4. Limits to angular resolution

As well as the limit imposed by diffraction, given in Equation 4, two other considerations are important for the angular resolution. The most important is the limit imposed by chromatic aberration. It is apparent from Equation 2 that the focal length of a diffractive lens varies in proportional to the photon energy. Away from the nominal energy E , radiation will converge in a cone

towards a displaced focus (Figure 4). From purely geometric considerations this can be seen to lead to a focal spot whose angular size (HPD), is

$$\Delta\theta_c = \frac{1}{2\sqrt{2}} \left(\frac{\Delta E}{E} \right) \left(\frac{d}{f} \right), \quad (7)$$

where ΔE is the bandwidth.

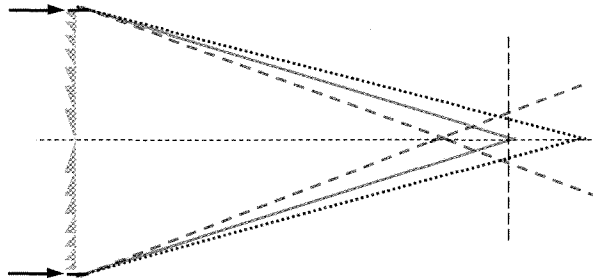


Figure 4: The effect of chromatic aberration. As the focal length of a diffractive lens is proportional to photon energy, at energies away from the nominal one radiation converges in a cone and intercepts the detector plane in a disc of size corresponding to an angular diameter $(d/f)(\Delta E/E)$.

Unless the spectrum of the radiation being imaged is a very narrow line with negligible continuum emission, blurring due to photons with energy other than the nominal energy has to be taken into account. Fortunately modern X-ray detectors of interest for the low flux levels that are encountered in astronomy are photon counting and energy resolving. Thus photons of energy outside a defined bandwidth can be disregarded when analyzing the data. Consequently in the most common case of a broad line and/or continuum spectrum it is the energy resolution of the detector that has to be used for ΔE in Equation 7 and that dictates the degree of chromatic aberration.

At X-ray energies the most widely used imaging detectors are CCDs, or arrays of CCDs, that typically have an energy resolution of ~ 150 eV at 6 keV, corresponding to $\Delta E/E = 2.5\%$. In the 100–1000 keV region Germanium detectors too can achieve $\Delta E/E$ 0.25–1%, though the (fractional) resolution is not as good at lower energies. Position sensitive Germanium detectors are now becoming available (see for example [33], where the possibility of their use in a ‘Compton’ configuration to reduce background by

selecting only those events that are consistent with photons that may have passed through a lens is also discussed). Pixelated CZT arrays are approaching the performance of Germanium detectors [34] and do not need cooling. Microcalorimeter detectors are reaching energy resolutions of 2.5–5 eV at 6 keV [35, 36], corresponding to $\Delta E/E \sim 0.05\%$, but this performance is currently only achieved in single detector elements or small arrays. Braig and Predehl [27] have even suggested that a Bragg crystal monochromator might be used in the focal plane of a diffractive telescope and that $\Delta E/E \sim 0.01\%$ might be achieved in this way.

As discussed in §3.1, the spatial resolution of the detector can be important as well. Characterizing this by a pixel size Δx , and rewriting the expression in Equation 4 in terms of energy for consistency, one can approximate the net resolution by

$$\Delta\theta^2 = \Delta\theta_d^2 + \Delta\theta_c^2 + \Delta\theta_p^2 \quad (8)$$

$$= \left(\frac{1.03 hc}{d E}\right)^2 + \frac{1}{8} \left(\frac{\Delta E}{E}\right)^2 \left(\frac{d}{f}\right)^2 + \left(\frac{\Delta x}{f}\right)^2. \quad (9)$$

Diffraction limited performance will only be obtainable if the second and third terms are negligible compared to the first.

3.5. Absorption

The efficiencies discussed so far ignore the effects of absorption. In evaluating such effects a key parameter is the critical Fresnel number defined by Yang [37]

$$N_0 = \frac{\delta}{2\pi\beta} = \frac{2}{t_{2\pi}\mu_{abs}}, \quad (10)$$

where $\mu_{abs} = 4\pi\beta/\lambda$ is the absorption coefficient. N_0 measures the relative importance of refraction and absorption. It can be thought of as the number of Fresnel zones in a refractive lens whose maximum thickness is equal to twice (??) the absorption length. Example values are shown in Fig. 2.

Kirz [6] has shown how in the presence of significant absorption the optimum thickness of a PZP is somewhat lower than $t_{2\pi}/2$. For $\beta/\delta = 1$ the (primary focus) efficiency is maximized if the thickness is reduced to 0.73 of this value. Similarly it can be shown that the efficiency of a PFL made from a material in which the absorption is important is maximized if the profile is modified as shown in Fig. 5. Efficiencies with and without this adjustment are shown as a function of N_0 in Fig. 6. The improvement is

not large, but the modified profile is actually likely to be lighter and easier to fabricate. Note that the optimum form would be different if the objective were to minimize either the undiffracted (zero-order) flux or that in higher orders, rather than maximize the flux diffracted into the prime focus.

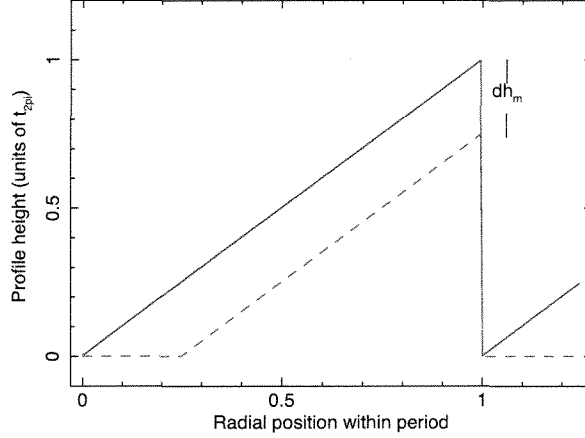


Figure 5: If significant absorption is taken into account the efficiency of a PFL can be improved by modifying the profile because improved throughput can be obtained at the expense of some phase error. The shape of the profile of a PFL that gives maximum efficiency for diffraction into the primary focus is shown.

4. The advantages and potential

4.1. Fabrication and tolerances

Compared with grazing incidence optics of a comparable aperture, diffractive optics for X-ray astronomy are expected to be relatively simple to fabricate. As noted in §3, diffraction limited diffractive telescopes are likely to have profiles with a minimum cycle length of microns to mm – well within the capabilities of current micro-electro-mechanical systems (MEMS) or single point diamond turning (SPDT) technologies. Because the refractive index of the lens is so close to unity, the tolerances on the profile are comparatively relaxed. Even for the sub-nm wavelengths involved, lens figuring to $\lambda/10$ optical precision requires only $t_{2\pi}/10$ tolerances, where $t_{2\pi}$ is $\sim 10\text{--}1000\ \mu\text{m}$ (Fig. 2). Radial tolerances for the same precision are no tighter than $p_{min}/10$, and will usually be of a similar order of magnitude to the vertical tolerance.

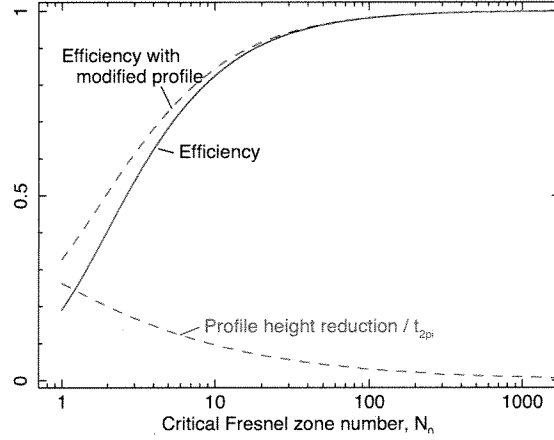


Figure 6: The efficiency of a PFL with significant absorption as a function of the critical Fresnel number, N_0 , of the material from which it is made. Also shown are the fractional amount, dh_m , by which the nominal profile height of $t_{2\pi}$ should be reduced as in Fig. 5 to optimize the performance, and the resulting efficiency.

At energies greater than a few keV Fig. 3 shows that it is possible to select materials with N_0 greater than a few, meaning that not only is absorption of X-rays in the material forming the active part of a PZP or PFL small, but the profile can be etched or machined into a somewhat thicker substrate (or deposited onto one) without serious absorption losses.

Thus the profile can be close to ideal and losses can be small so, at least at their design energy, diffractive X-ray telescope lenses can have an effective area that is very close to their geometric area, which may easily be several square meters.

A great advantage of optical elements used in transmission rather than reflection is their relative insusceptibility to tilt errors and out-of-plane distortions. Multiple reflection mirror systems in which the radiation undergoes an even number of reflections, such as Wolter grazing incidence optics, are better in this respect than single reflection systems (provided the relative alignment of the mirrors does not change). However they still act as ‘thick’ lenses and the translation of a ray leaving a section of the optic that has been tilted by angle $\delta\psi$ leads to an aberration in the image on angular scale $\sim (t/f)\delta\psi$, where t is the distance between the principal planes, a measure of the thickness of the ‘lens’. For Wolter optics t is the axial distance between the centers of the two mirrors. For the Chandra optics $(t/f) = 0.08$.

Diffractive telescope lenses are very close to ideal ‘thin’ lenses, having (t/f) ratios of $10^{-6} - 10^{-9}$ or even smaller.

4.2. Applications of high angular resolution diffractive telescopes

The most obvious attraction of diffractive optics for X-ray telescopes for astronomy is the potential they offer for superb angular resolution. Rewriting Equation 4 in terms of physical units,

$$\Delta\theta_d = 312 \left(\frac{d}{1 \text{ m}} \right)^{-1} \left(\frac{E}{1 \text{ keV}} \right)^{-1} \text{ micro arc seconds } (\mu''), \quad (11)$$

it is apparent that angular resolution better than a milli-arc-second should be readily obtainable in the X-ray band. With optics a few meters in size working with hard X-rays, sub-micro-arc-second resolution should be obtainable.

One of the original incentives for the recent reconsideration of diffractive optics for high energy astronomy was indeed the possibility that it offers of sub-micro-arcsecond resolution. As has been discussed in proposals to use X-ray interferometry for astronomy[38, 39], this is the angular resolution that would be needed to image space-time around the supermassive black holes believed to exist at the centers of many galaxies. Even our own Galaxy, the Milky Way, apparently harbors a black hole, Sgr A*, with a mass of $4.3 \times 10^6 M_\odot$ (where M_\odot is the mass of the sun)[40]. The Schwarzschild radius of a black hole of mass M is

$$R_s = \frac{2GM}{c^2} = 2.9 \frac{M}{M_\odot} km \quad (12)$$

where G is the gravitational constant and c the vacuum velocity of light. The radius of the ‘event horizon’ is R_s in the case of a non-rotating black hole or somewhat larger for one with angular momentum. For Sgr A* R_s corresponds to an angular scale of $10 \mu''$. The black holes at the centers of ‘active’ galaxies (Quasars and Seyfert galaxies, for example) can be more than 1000 times more massive, with correspondingly larger Schwarzschild radii. So despite their much greater distance, R_s in many cases still corresponds to angular scales of $0.1-4 \mu''$.

Of course one does not expect to detect radiation from the black hole itself, but the gravitational energy released by matter as it approaches the event horizon is the origin of the extremely high luminosity of some of these

objects. Simulations have been made showing how such radiation originating near to the event horizon, even from behind the black hole itself, should appear after being bent by the gravitational field (Fig. 7). The distribution expected depends not only on the mass of the black hole but on its angular momentum and inclination angle.

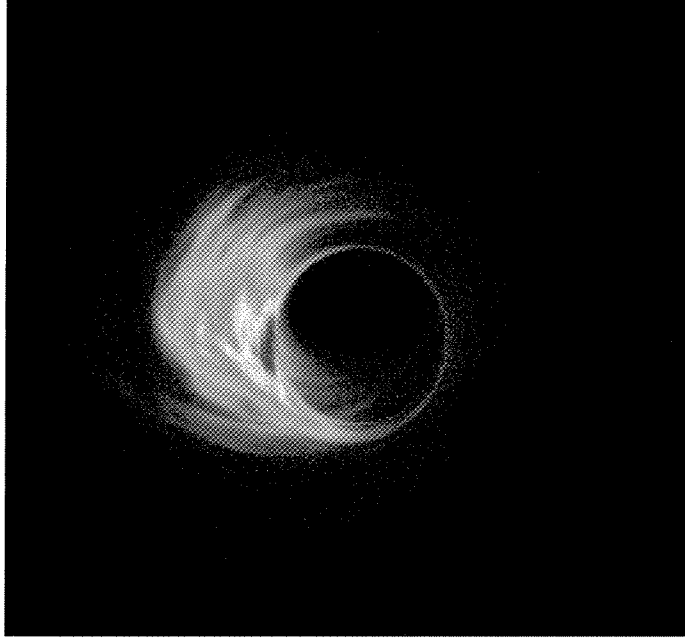


Figure 7: Simulation of radiation from the region surround a black hole. Image courtesey S. Noble. The black hole is assumed to have a spin of 0.9, inclined at 45° to the line of sight and the radiation simulated is at $\lambda = 1.3$ mm. For Sgr A* the field shown corresponds to about $100 \mu''$ by $100 \mu''$.

Fig. 8 indicates the angular resolution available with the current state-of-the-art. At the present the best angular resolution available to astronomers is obtained at mm wavelength by VLBI (very long baseline interferometry, see [41] for a recent review). The present state of the art, with transatlantic baselines and wavelengths as short as 1 mm lead to angular resolutions down to $30\text{--}40 \mu''$, though so far only with a limited number of stations [42, 43], so a characteristic size is measured rather than forming an image. The Russian RadioAstron space VLBI mission, following in the footsteps of HALCA/VSOP, is due to be launched June 2010 and will extend baselines to

350,000 km, but the shortest wavelength is 13.5 mm. Although this should allow an angular resolution of $8 \mu''$, the resolution will be limited by interstellar scattering except at high galactic latitude. Consequently Sgr A* cannot be observed with the highest resolution. The Japanese-led Astro-G/V SOP-2 mission, due for launch in 2012, will go down to 7 mm in wavelength [44] and so be less affected by interstellar scattering (which is proportional to λ^2), but with an apogee of only 25,000 km it will be limited to $38 \mu''$.

A useful basis of comparison across wavebands is the maximum baseline (or, for filled aperture instruments, the aperture diameter) in units of wavelength. Optical and infrared interferometers are pushing to higher and higher angular resolution, though as for VLBI, with a limited number baselines provide sparse $u - v$ plane coverage and allow model fitting but only an approximation to true imaging. The 640 m and 330 m baselines of the SUSI and CHARA optical interferometers correspond to about $1.5 G\lambda$ and $0.7 G\lambda$ respectively, while with radio VLBI fringes have been obtained with baseline as long as $6 G\lambda$ [45]. A modest 1 m diameter lens working with 6 keV X-rays would have a size of $50 G\lambda$, and larger lenses and those working at higher energies have corresponding greater values. In addition such lenses would provide full $u - v$ plane coverage up to this scale.

It is ironical that in the X-ray and gamma-ray bands, where the wavelengths are shortest and diffraction least limiting, the angular resolution at present obtainable is actually inferior to that possible at longer wavelengths. No currently planned X-ray mission will improve on, or even equal the $0.5''$ resolution of the Chandra grazing incidence mirror. While being subject to some constraints and limitations, discussed below, in appropriate circumstances diffractive optics offer the opportunity of improving on it by up to 6 order of magnitude. X-ray imaging may then move from the present arc-second domain to the milli-arcsecond one, where stellar surfaces can be imaged and the formation of astrophysical jets examined in detail and to micro-arc-seconds, just the resolution needed for black hole imaging.

4.3. Applications of diffractive optics for light-buckets

Even if one does not take advantage of the imaging capabilities of diffractive X-ray optics, they may prove useful as flux concentrators. X-ray and gamma-ray astronomy is limited both by the small number of photons often detected and by background in the detector, mostly due to particles. The background is typically proportional to the detector area from which events

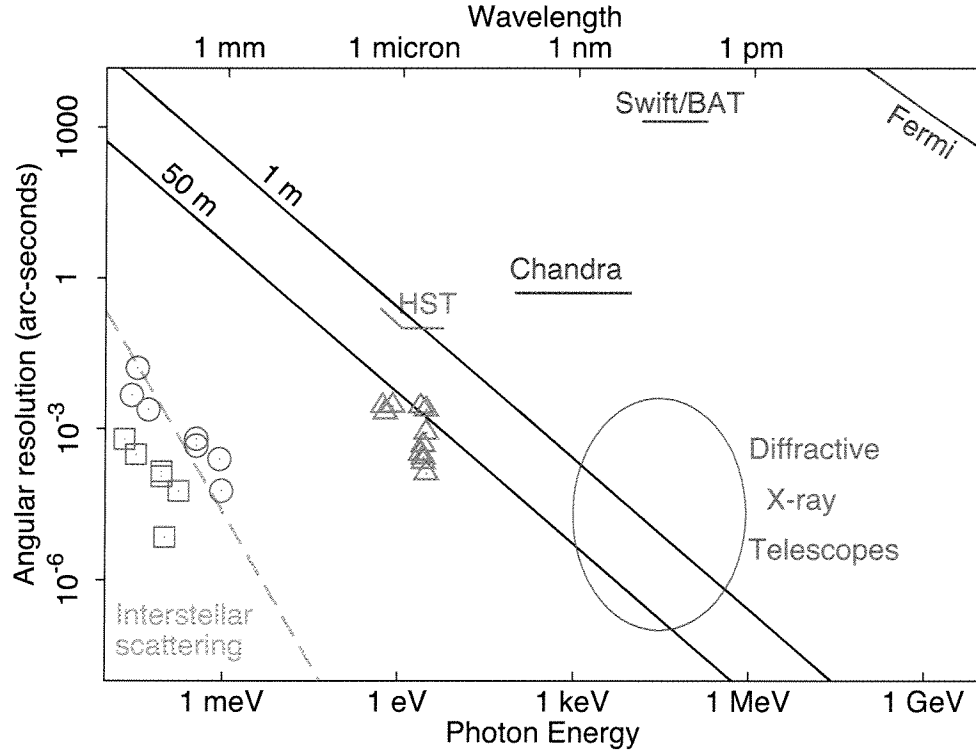


Figure 8: The angular resolution obtainable with different techniques and across the electromagnetic spectrum. The approximate domain in which diffractive X-ray optics are potentially of interest is indicated by an ellipse. Shown for comparison are (i) the Rayleigh limit for diffraction limited optics of 1 m and 50 m diameter (red lines), (ii) the angular resolution of current X-ray instruments (blue lines) and of the Hubble Space Telescope (green lines), (iii) the best resolution of some example optical interferometer systems (green triangles) and of some typical radio VLBI measurements (red circles), (iv) the diffraction-limited angular resolution possible with space VLBI (actual and near future; red squares) (v) a line roughly delineating the region in which interstellar scattering is dominant (cyan).

are accepted. Catching photons requires large collecting area whereas reducing background implies that the detector should be as small as possible. At the energies where they can be used, grazing incidence reflective optics offer a solution to the problem of how to concentrate the flux from a large collecting area onto a small detector area. Diffractive optics provide an alternative to grazing incidence mirrors – one that can be used even for high energies, and indeed whose performance is in many respects best at high energies.

If one drops the objective of being able to resolve with the detector a spot size corresponding to the diffraction-limited angular resolution of which the optic might be capable, smaller p_{min} may be used and so the focal length reduced. The bandwidth, measured as FWHM (full width at half-maximum) can be shown from simple geometric considerations to be given by

$$\frac{\Delta E}{E} = 2\sqrt{2} \left(\frac{d_{det}}{d} \right), \quad (13)$$

whereas, assuming ideal efficiency for the optic and the detector, the flux is concentrated by a factor

$$C = \frac{A_{eff}}{A_{det}} \approx \left(\frac{d}{d_{det}} \right)^2 \approx 8 \left(\frac{E}{\Delta E} \right)^2. \quad (14)$$

Thus where narrow spectral lines, or groups of lines, from a compact source are to be studied, significant advantages are available.

5. Overcoming the difficulties

5.1. Chromatic Aberration

Imaging information is lost and the peak of the PSF (Point Spread Function) is attenuated as soon as either of the other terms in Equation 9 becomes more important than the diffraction one. Rounding a numerical factor that depends on the exact definitions chosen, this sets a maximum diffraction-limited bandpass

$$\frac{\Delta E}{E} \lesssim 4 \frac{f\lambda}{d^2} \quad (15)$$

or

$$\lesssim \frac{1}{N_z}, \quad (16)$$

where the lens has N_z Fresnel zones (half periods).

This is a major constraint and minimizing the effects of chromatic aberration is one of the biggest challenges to overcome in developing diffractive X-ray telescopes. The various measures that can be taken are discussed below.

5.1.1. Refocusing

The first thing to note is that a diffractive X-ray lenses can work well over a broad range of energies provided the detector position is adjusted for each energy. Fig. 9 shows the effective area of a PFL as a function of energy *if the telescope is refocused by adjusting the detector plane to the optimum position for each energy*. Braig and Predehl have pointed out [26] that, as indicated in the figure, well below the energy, E_0 for which a lens was designed one can take advantage of the high efficiency near $E_0/2$, $E_0/3$, ... because it acts like a $t_{4\pi}$, $t_{6\pi}$... lens in the sense described in §3.2. As only one energy can be observed at a time this is not a very practical way of making broadband observations, but the approach could be useful in studying, for example, lines whose energy may be red-shifted to different extents in different sources. Note that at energies where the efficiency is less than unity, the lost flux appears in foci of other orders. The resulting halo to the PSF will have low surface brightness compared with the peak, but if it is troublesome it is possible to imagine blanking off the inner part of the lens area [26].

5.1.2. Improving chromatic aberration with long focal lengths

From Equation 7 it is clear that the effect of chromatic aberration on angular resolution is reduced if the focal ratio is large. For a given lens size, this means that the focal length should be as long as is consistent with other constraints. This is a long-known effect; before the invention of the (visible-light) achromatic doublet, astronomical telescopes were built with very long focal lengths in order to minimize chromatic aberration. Fig. 10 shows an extreme example. Long focal lengths lead to a limited field of view (§5.4) and tend to have severe implications for the logistics of maintaining the telescope in space (§5.2), but the bandwidth increases in proportion to f .

5.1.3. Segmenting lenses for multi-energy operation

Given that PFLs of large surface area are not too difficult to make but wide bandwidth is hard to achieve, it has been pointed out on a number of occasions [e.g. 23, 25, 26] that the surface of a diffractive lens can be divided into regions tuned to different energies, thus providing information

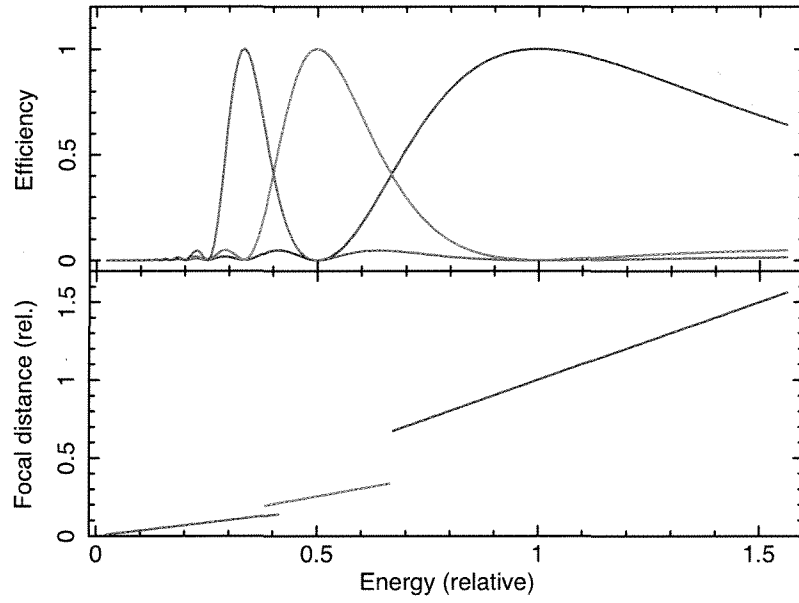


Figure 9: A PFL will work with relatively high efficiency over a broad band of energies provided that the detector plane is moved to the appropriate distance. The blue curve corresponds to the lens operating in the nominal mode. For the green and red curves the detector is assumed to be placed at the distances for the focii of the lens treated as a $t_{4\pi}$ and $t_{6\pi}$ one. Higher order responses are not shown. Absorption effects are not taken into account in the plots. Based on [27].

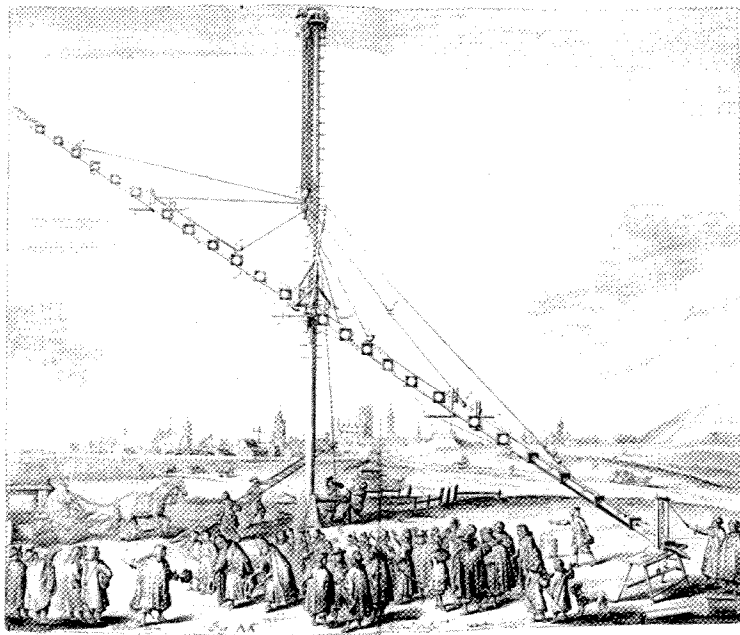


Figure 10: It has long been known that chromatic aberration is minimized by adopting a long focal length. Before the invention of the achromatic lens Hevelius built telescopes with 60 and (shown here) 140 foot focal lengths. [46]

over a wider bandpass. The division may be into a few zones or into many and an aperture may be divided radially or azimuthally, or both. The ‘sub-telescopes’ so formed may be either concentric, or parallel but offset.

An important consideration is the interference that occurs when radiation of the same energy arrives in the same part of the detector after passing through the different regions of the diffractive optic.

5.1.4. *Diffractive-diffractive correction*

In some circumstances the dispersion of a diffractive optical element can be corrected using that of a second diffractive optical element. However, as shown formally by Bennett [47] no optical system consisting of only two diffractive lenses can form a real image, free from longitudinal dispersion, from a real object. Buralli and Rogers (1989) generalized this result to any number of diffractive elements. Michette [48] has described some schemes that get around this constraint, but they do so only by using allowing diffraction into multiple orders with a consequent loss in efficiency and they only provide correction at two disparate energies. Other optical systems that do correct one dispersive element with another either involve virtual images (or objects), or depend on reflective (or refractive) relay optics. The space-based visible light diffractive imager proposed by Koechlin *et al.* [49]) is an example of the latter.

X-ray telescopes that depend on both diffractive and reflective optics risk suffering the disadvantages of both, so correction schemes that depend on reflective relay optics do not seem very attractive. On the other hand, as will be noted below (§5.2), there are perhaps other reasons for considering incorporating a reflective component, so perhaps a feasible diffractive-reflective-diffractive may eventually evolve.

5.1.5. *Diffractive-refractive correction*

Given the constraints that apply to diffractive-diffractive correction for telescopes forming real images, the possibility of correction using refractive optics has been widely considered. The constraints that preclude achromatic correction of one diffractive element with another arise because the dispersion of the two elements would be the same. Correction of a diffractive lens with a glass one is used in the visible part of the spectrum where glasses have a dispersion that differs from the E^{-1} dependence of the power of diffractive elements [e.g. 50]. The same principle applies to X-rays – purely refractive lenses can be made [51] and are widely used in X-ray microscopy and beam

manipulation, often stacked to provide adequate power. Fig. 2 shows that for most materials over most of the X-ray band $t_{2\pi}$ is proportional to E , implying that the power of a refractive lens, which depends on δ , is proportional to E^{-2} .

Figs. 11, shows how in principle first order correction of longitudinal chromatic aberration of a PFL (or ZP, PZP) is possible with a diffractive/refractive X-ray doublet [23, 24, 52]. Because the lens remains a 'thin' one, there is almost no lateral color aberration. In the common situation where $t_{2\pi}$ is proportional to E , the corrected energy the focal length of the refractive component should be -2 times that of that of the diffractive one and the combined focal length is a factor of two larger. In this case the number of Fresnel zones in the refractive component is half the number in the PFL. In the absence of absorption the bandpass is increased from $\Delta E = E/N_z$ (Equation 16) to $\Delta E = 2EN_z^{-1/2}$ [e.g. 53].

It has already been noted (§3.5) that absorption becomes important in a refractive lens if N_z exceeds the critical Fresnel number N_0 for the material. As in practice N_0 is usually a few tens up to a few hundred (Fig.3), this sets a limit on the size of diffractive lens that can be corrected this way. Wang *et al.* [54] have suggested using the rapid energy dependence of δ just above absorption edges to reduce the thickness of the refractive component needed. This approach only works at very specific energies, but it could make possible achromatic diffractive-refractive doublets of several mm diameter for microscopy and microlithography.

A PFL can be considered as a refractive lens in which the thickness is reduced *Modulo*($t_{2\pi}$). Thus it is natural to consider reducing the absorption in a refractive correcting lens by stepping it back, not *Modulo*($t_{2\pi}$) which simply would make it another PFL, but *Modulo*($m t_{2\pi}$) for some large integer m as in Fig. 11. $t_{2\pi}$ varies with energy, so the value at some particular energy must be chosen. Coherence is then maintained across the steps at this energy and at any other for which $m t_{2\pi} = m' t_{2\pi}(E)$ for some other integer m' . This occurs at a comb of energies. In the regime where $t_{2\pi}$ is proportional to E , they occur at intervals such that $\Delta E/E \sim 1/m$. Detailed analyses of the response of PFLs with stepped achromatic correctors have been published [52, 55]. Examples are shown in Fig.13. The effect is that only a fraction ?? of the passband is effective.

If one is willing to consider configurations involving the alignment of not just two widely spaced components, but three, then the configuration

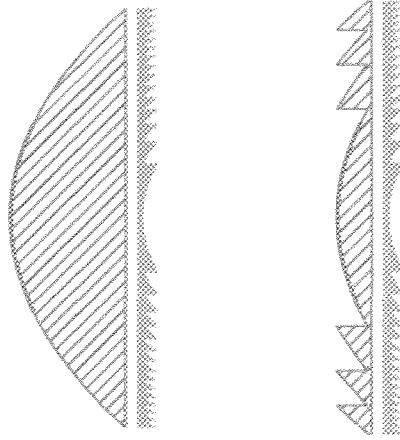


Figure 11: Left: the use of a refractive X-ray lens to chromatically correct a diffractive one. Right: a more practical configuration in which the refractive component is stepped to reduce absorption.

indicated in Fig.12 offers an even wider bandpass (see [25] and references therein). With the extra degree of freedom introduced by the separation between the two lenses it is possible to set to zero not just dz/dE (z being the axial position of the image), but also d^2z/dE^2 . The bandwidth is increased as indicated in Table 3 and the fact that the image is somewhat magnified compared with a single lens system of the same overall length could be advantageous. However as the magnification is energy dependent, care must be taken that lateral chromatic aberration does limit the useful field of view (this will not be a problem if the detector has adequate energy resolution to allow post-facto correction).

5.1.6. *Axicons, Axilenses and other wideband variants*

Variations of the ZP have been proposed in which the shape of the PSF is modified to (slightly) improve the angular resolution [e.g. 57] or in the case of ‘photon sieves’ to improve the resolution available with a given minimum feature size in the fabrication [58]. One such variant, the ‘fractal photon sieve’ has been shown to have an extended spectral response [59], achieved because some of the power from the prime focus is diverted into focii at other distances. As will be seen below, this can be accomplished in other more controlled and efficient ways.

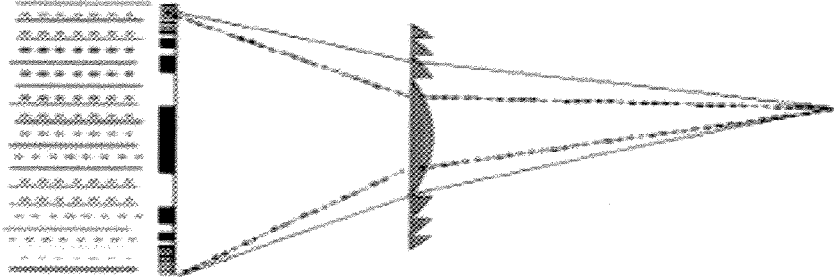


Figure 12: A variation on the refractive/diffractive achromatic doublet shown in Fig. 11 in which the components are separated, allowing second order correction of longitudinal color (figure from [56]). A stepped version of the refractive component is shown.

Table 3: Achromatic correction of PFLs. Parameters are given for systems with the same overall length, f , at the nominal design energy, E_0 , and on the assumption that $t_{2\pi}$ is proportional to energy, E . In the table E is specified relative to E_0 and d is the diameter of the refractive component. The numerical factors in the expressions for the bandwidth depend on the definition used (all in the same way, so relative values are unchanged).

	Simple PFL	Doublet	Separated pair
Diffractive focal length	f	$\frac{f}{2}$	$\frac{f}{3}$
Separation	-	0	$\frac{f}{9}$
Refractive focal length	-	$-f$	$-\frac{8}{27}f$
Image plane distance	$f(1 + \Delta E)$	$f(1 + \Delta E^2 + \dots)$	$f(1 + 1.35\Delta E^3 + \dots)$
Image scale	f	f	$\frac{4}{3}f(1 - \frac{1}{2}\Delta E + \dots)$
Diffraction-limited bandwidth (fractional)	$2.9 \left(\frac{f}{d^2} \frac{hc}{E} \right)$	$2.4 \left(\frac{f}{d^2} \frac{hc}{E} \right)^{1/2}$	$2.9 \left(\frac{f}{d^2} \frac{hc}{E} \right)^{1/2}$

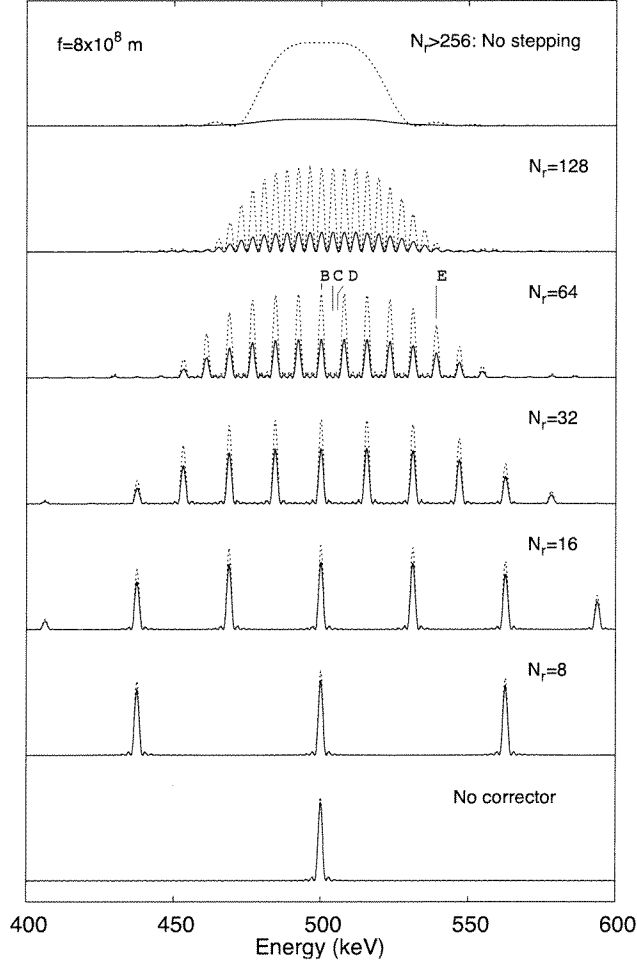


Figure 13: Simulated response of a refractive/diffractive achromatic doublet with different degrees of stepping of the refractive component. N_r corresponds to m in the text. The dotted lines indicate the response if there were no absorption. As the step size is made larger, the energy coverage becomes better but the effects of absorption become more serious. Details in [52], from which this figure is taken.

A diffractive lens can be considered as a circular diffraction grating in which the pitch varies inversely with radius so that radiation of a particular wavelength is always diffracted towards the same focal point. Regarded in this way, a PFL is a phase grating, blazed in such a way that all of the energy goes into the +1 order. Skinner [60, 61] has discussed the application to X-ray telescopes of designs in which the pitch varies radially according to laws other than r^{-1} . They can be considered as forms of radially segmented lenses in which the pitch varies smoothly rather than in steps.

In an extreme case the pitch is constant and one has the diffractive X-ray axicon, in which the pitch is constant. Interestingly, to a first approximation the point source response function of an axicon is independent of energy. According to the Rayleigh criterion, the angular resolution is as that of a PFL of the same diameter working at the lowest energy, but the secondary diffraction rings are stronger so the HPD is larger. Indeed over a wide range of radii the PSF shape is such that the enclosed power keeps on increasing approximately linearly with radius. In some respects the system should be regarded as an (achromatic) interferometer in which the information is contained as much in the fringes as in the central response.

As discussed in [60], intermediate designs are possible in which the band-pass is tailored to particular requirements based on the ideas of Sochacki [62] and of Cao & Chi [63]. Fig. 14 shows the response of some examples.

5.1.7. *Figures of merit*

Braig and Predehl [53] define an ‘Achromatic Gain’ parameter \mathcal{G} to measure the advantage that an achromatic system has over a simple lens. It is essentially the ratio of the effective areas, integrated over the energy range in the two cases.

The achromatic gain \mathcal{G} may be generalized to quantify the advantage compared to a simple PFL of other variants as well as of achromatic combinations. As the neither the resolution nor the image scale are not necessarily the same in all cases, it is best to base the effective area on the flux within a diffraction-limited focal spot rather than peak brightness. Such a figure of merit was used in [61] where it was shown that axicons and axilenses have a ‘ \mathcal{G} ’ close to unity – the extra bandwidth is obtained at the expense of effective area. The same is obviously true if a lens is segmented into areas devoted to different energies. On the other hand increasing the focal length is accompanied by an increase in bandwidth (Equation 15) and of \mathcal{G} .

Care must be used in interpreting this parameter. It measures the (square

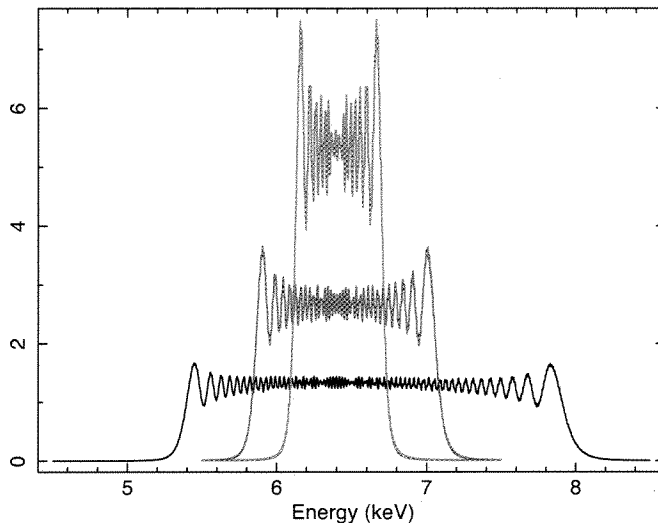


Figure 14: The on-axis response as a function of photon energy at a fixed distance of 100 m for Axilenses with different parameters [60].

of) the improvement in signal-to-noise ratio on the assumption that the noise is dominated by photon statistics from the source. If the observation is dominated by detector background that is not source-related, such as that due to particles, then a dependence on the square-root of the bandwidth would be more appropriate². In the detector background limited case, the background will also depend on the detector area over which the signal is spread³.

5.2. Focal length; Formation flying

Equation 2 makes it clear that focal length, f , of any practical system is likely to be long, particularly at high energies. In addition in various aerhaps spectrs discussed above *very* large f will give the best performance.

First, in section §3.1 it was seen that a minimum focal length is required if spatial resolution of a detector at the prime focus is not to be a limiting factor. For example even if the detector pixels are as small as $10\ \mu\text{m}$ a resolution of

²Provided of course that the passband of the optic is wider than the energy resolution of the detector

³Provided in this case that the focal spot is larger than the detector spatial resolution

$1 \mu''$ implies $f > 2000$ km. Gorenstein [64] has suggested that this problem might be alleviated if a grazing incidence reflecting telescope were used to re-image the prime focus, perhaps with a magnification by a factor of 20. Radiation from a particular sky pixel would in practice illuminate a very small part of the telescope surface because the latter would be very close to the focal plane. The reflecting surface would in effect remap positions over its aperture to pixels in a detector plane.

A second reason that long f may give the best performance is that as noted in §5.1 chromatic aberration is minimized if f is long. Finally, based on Equation 2, unless f is large diffractive lenses of a reasonable size will have a very small period p_{min} and fabrication will be more difficult and perhaps less precise.

For focal lengths of up to ~ 100 m, it is perhaps possible to consider telescopes with a boom connecting the lens and the detector assembly. Deployable booms of up to 60 m have been flown in space [65]; the record for the largest rigid structure in space is now held by ISS at over 100 m.

On the other hand even for a focal length of 50 m, ESA studies for the proposed XEUS grazing incidence mirror X-ray mission concluded that a boom was unnecessary and that formation flying of separate spacecraft carrying the optic and the detector assemble offered a more viable solution [66].

Formation flying for a long focal length telescope mission implies maintaining two spacecraft such that the line joining them has a fixed direction in inertial space. Because of gravity gradients within the solar system, as well as disturbances such as those due to differences in solar radiation pressure, a continuous thrust will be needed on at least one spacecraft. Another consideration is that a major repointing of the telescope will require moving maneuvering one of the spacecraft by a distance $\sim f$.

Some of the issues associated with formation flying of two spacecraft for a diffractive X-ray telescope mission have been briefly discussed in the literature [22, 26]. Internal studies at NASA-GSFC's Integrated Mission Design Center of possible missions based on this technique have considered the issues more deeply. Krizmanic *et al.* [67] has reported on one of these and updated some of the conclusions. The missions studied were considered ambitious, but possible. A single launcher would launch both both lens and detector spacecraft either to the vicinity of the L2 Lagrangian point or into a 'drift-away' orbit around the sun. Existing ion thrusters can provide the necessary forces both to maintain the pointing direction against disturbances and for

re-orienting the formation. The fuel and power needed for the thrusters are acceptable. In short, no ‘show-stoppers’ were identified.

5.3. Pointing knowledge

Although precise control of the *distance* between lens and detector is not needed, the knowledge of the *direction* in celestial space of their vector separation is crucial. As information about every photon detected can be time-tagged, *control* is needed ‘only’ to the extent necessary to ensure that the image of the region to be studied falls on the detector area (§5.4). *Knowledge* of the orientation is needed, however, to a level corresponding to the angular resolution required. This amounts to a need to establish the transverse position of the detector with respect to a line passing from the source through the center of the lens, and to do so with a precision that can be from a millimeter down to a few microns.

The problem of precise attitude determination in celestial coordinates is one common to all missions attempting to work at the μ'' level. It has already been considered in the context of the studies of MAXIM, a proposed mission to address the problem of imaging space-time around black-holes using X-ray interferometry. Gendreau *et al.* reviewed a number of approaches to the problem [69]. If a laser beacon can be on the lens spacecraft and viewed against background stars, a ‘super-startracker’ on the detector spacecraft can in principle provide the necessary information. Obtaining measurements with the necessary precision is not out of the question – astrometry is already in an era where milli-arc-second accuracy is the norm and micro-arc-second the target. The problem is obtaining that precision with faint stars on short enough time-scales. Fortunately various technologies discussed by Gendreau *et al.* offer the prospect of gyroscopic systems that would have sufficient precision to allow interpolation between absolute fixes from the stellar observations. To further alleviate the difficulties, Luquette and Sanner [70] have discussed how knowledge of the dynamics dictating the spacecraft displacements can help with determining short term changes to the alignment.

5.4. Field of View

The fields of view of ultra-high angular resolution telescopes such as those possible using diffractive optics are necessarily limited. With a single lens configuration, or with a contact achromatic doublet, the field of view will simply be the detector size divided by the focal length. With long focal lengths, reasonable detector sizes imply small fields of view. Although optical

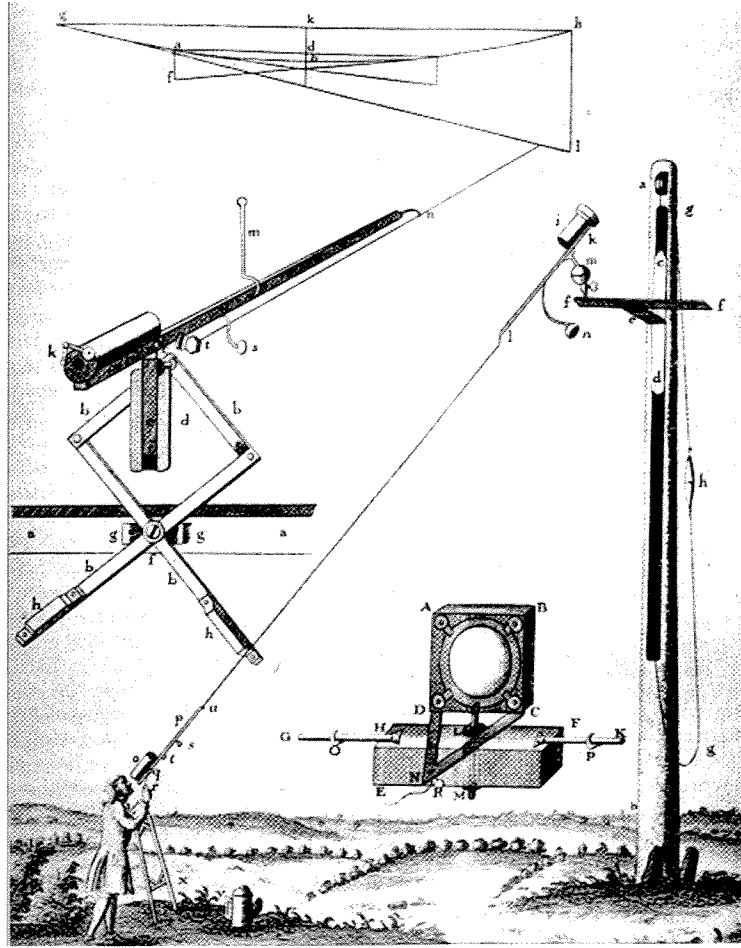


Figure 15: The concept that the objective of a telescope and the detector need not be rigidly connected to each other is not new. Christian Huygens' (1629-1687) used this 210 foot focal length telescope [68]. Note that the design takes advantage of the relative immunity of a thin lens to tilt errors.

systems with separated lenses (§5.1.5) or with relay mirrors (§5.2) can change the image scale, this would probably be in the sense of increasing it, so reducing the field of view.

There is, anyway, a basic limitation. If for example micro-arc-second resolution were wanted over a $1''$ field of view, a detector with $> 10^{12}$ pixels would be needed. The present state of the art for X-ray detectors in space is indicated by a few examples. The EPIC camera on XMM-Newton has an array of 7 CCDs with a total of 2.5×10^6 pixels. The imaging array of the ACIS focal plane instrument on Chandra has 4×10^6 pixels in 4 CCDs. Although the e-Rosita telescopes will have a total area of CCDs about an order of magnitude greater than present generation X-ray instruments, the pixel size will be larger to match the telescopes and the number of pixels will be only just over 10^6 [71]. Instruments planned for the IXO X-ray observatory are also in the few megapixel range.

Larger arrays of larger format CCDs are possible. Although the requirements are a little different for visible radiation, ESA's Gaia astrometry mission will have an array of 106 CCDs covering about half a square meter, with $\sim 9 \times 10^8$ pixels [72]. The Gaia pixels are rectangular $20 \times 30 \mu\text{m}$. Taking $20 \mu\text{m}$ as a typical dimension, a similar array would provide a field of view 30 milli-arc-seconds across with $1 \mu''$ pixels at a focal distance of just over 4000 km.

Of course, even were a 10^{12} pixel detector available, it would require an extremely strong source to produce a significant signal in even a small fraction of the pixels. Sources of interest for study at the resolution possible with diffractive X-ray optics are necessarily rather compact.

6. Alternatives to simple lenses

Equation 4 implies that sub-micro-arcsecond angular resolution should be possible with modest sized lenses working at a few hundred keV. There is a special interest, though, in such measurements in the X-ray band. Not only are the photons more numerous, but the emission spectra of AGN typically show strong emission lines in the region of 6.7 keV associated with highly ionized Fe. These lines carry important diagnostic information because they are shifted in energy both by gravitationally redshifts and by the Doppler effect. Ultra-high angular resolution mapping at energies in this band would be particularly valuable but would require lenses 50 m or more in diameter.

Although membrane/unfoldable Fresnel lenses of 25-100 m diameter for visible light have been proposed [73] and even been demonstrated on the ground on scales of up to 5 m [74], X-ray lenses of this size would not be easy to engineer and would actually have an effective area larger than needed from the point of view of photon flux.

A PFL with a partially filled aperture can be envisioned. It could comprise subsections of a PFL mounted on multiple spacecraft distributed over a plane [75, 60]. The system then becomes somewhat similar to that proposed within the studies of the proposed MAXIM (Micro-Arcsecond-X-ray IMager) mission in which mirror assemblies on spacecraft distributed over a plane would divert radiation to form fringes on a detector situated at a large distance [76]. In both cases tight control of the distances of the spacecraft from the center of the array would be needed (to a fraction of p_{min} for the partially filled PFL; similar for a MAXIM formation of the same size). The differences are (1) that the subsections of a partially filled aperture PFL would concentrate flux whereas the plane mirrors proposed for MAXIM would not and (2) mirror reflection is achromatic whereas the PFL subsections would divert radiation by a wavelength-dependent angle and fringes would only be formed where the concentrated, deflected, beams cross. The bandwidth over which fringes can be improved by altering the way the pattern pitch in the PFL in ways analogous to X-ray axicons and axilenses. In the constant pitch (axicon) case one has a MAXIM-like interferometer in which the beam diverters are blazed diffraction gratings and the borderline between imaging and interferometry becomes blurred.

7. Conclusions; Status and prospects

As mentioned in the introduction, diffractive X-ray telescopes presently exist almost entirely as concepts and proposals on paper. Some work has been conducted on verifying that no problems exist and on demonstrating feasibility using scaled systems [77] fabricated by gray scale lithography [78]. By scaling down the form of a large PFL that might be used for astronomy in radius (but not in thickness) the focal length is reduced. Lenses a few mm in diameter and with a focal length of ~ 100 m can act as models of ones, say, a few meters in diameter for which $f \sim 100$ km. With this reduced focal length testing is possible using existing ground-based facilities such as the 600 m long the 600 m X-ray interferometry testbed at NASA-GSFC [79, 80]

Ironically, the smaller lenses are *more* difficult to make than a flight lens would be, because the period of the profile is correspondingly reduced.

Tests and developments of small-scale PFLs, using various fabrication techniques and including achromatic combinations are continuing [81]. Progress to an micro-arc-second mission would probably be through an intermediate-level ‘pathfinder mission’. A specific proposal has been made for such a mission in the form MASSIM, a milli-arc-second imaging mission with working in 5-11 keV the [82]. Another possible route forward is through even lower resolution solar imagery, where small diffractive lenses, little larger than

In conclusion, diffractive optics offers a new family of possibilities for telescopes. Despite drawbacks in terms of inconvenient focal lengths, and limited field of view bandwidth, the potential that they offer flux concentration even at high energies and, in particular, for superb angular resolution suggests that in due course they will find their place in the range of techniques available to X-ray and gamma-ray astronomy.

References

- [1] L. Mertz, Transformations in optics, 1965.
- [2] D. T. Wilson, G. D. Demeester, H. H. Barrett, E. Barsack, A new configuration for coded aperture imaging, Optics Communications 8 (1973) 384–386. doi:10.1016/0030-4018(73)90222-8.
- [3] U. D. Desai, J. P. Norris, R. J. Nemiroff, Soft gamma-ray telescope for space flight use, in: D. B. Cline (Ed.), Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 1948 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 1993, pp. 75–81.
- [4] F. Frontera, P. von Ballmoos, Diffractive optics: Laue/Bragg ?TBC?, This series.
- [5] R. W. Wood, Physical Optics, 3rd Edition, Macmillan, New York, 1934, p. 37.
- [6] J. Kirz, Phase zone plates for X rays and the extreme UV., Journal of the Optical Society of America (1917-1983) 64 (1974) 301–309.

- [7] J. L. Soret, Ueber die durch Kreisgitter erzeugten Diffractionsphänomene, *Annalen der Physik* 232 (1875) 99–113. doi:10.1002/andp.18752320906.
- [8] L. Rayleigh, *Encyclopedia Britannica*, 9th Edition, Vol. 24, University of Chicago Press, 1888, p. 429.
- [9] R. W. Wood, ?tbd?, *Phil. Mag.* 45 (1898) 511.
- [10] K. Miyamoto, The Phase Fresnel Lens, *Journal of the Optical Society of America* (1917-1983) 51 (1961) 17–+.
- [11] O. E. Myers, Studies of Transmission Zone Plates, *American Journal of Physics* 19 (1951) 359–365. doi:10.1119/1.1932827.
- [12] A. V. Baez, A study in diffraction microscopy with special reference to x-rays, *Journal of the Optical Society of America* (1917-1983) 42 (1952) 756–+.
- [13] G. Elwert, X-Ray Picture of the Sun Taken with Fresnel Zone Plates, in: K. O. Kiepenheuer (Ed.), *Structure and Development of Solar Active Regions*, Vol. 35 of IAU Symposium, 1968, pp. 439–+.
- [14] H. Bräuninger, H. J. Einighammer, J. V. Feitzinger, H. H. Fink, D. H. Höhn, H. Koops, G. Krämer, U. Mayer, G. Möllenstedt, M. Mozer, EUV and Soft X-Ray Images of the Sun on March 11th, 1971, *Sol. Phys.* 20 (1971) 81–84. doi:10.1007/BF00146098.
- [15] M. Burger, J. H. Dijkstra, Photographs of the Sun in the XUV-Region, *Sol. Phys.* 24 (1972) 395–404. doi:10.1007/BF00153381.
- [16] J. H. Dijkstra, W. de Graaff, L. J. Lantwaard, Construction of Apodised Zone Plates for Solar X-Ray Image Formation, in: F. Labuhn & R. Lust (Ed.), *New techniques in Space Astronomy*, Vol. 41 of IAU Symposium, 1971, pp. 207–+.
- [17] G. Elwert, J. V. Feitzinger, The image improvement of field-sources with zone plates, especially of the sun in the XUV and soft X-region., *Optik* 31 (1970) 600–612.

- [18] R. Giacconi, W. P. Reidy, T. Zehnpfennig, J. C. Lindsay, W. S. Muney, Solar X-Ray Image Obtained Using Grazing-Incidence Optics., *Astrophys. J.* 142 (1965) 1274–1278. doi:10.1086/148404.
- [19] G. S. Vaiana, L. van Speybroeck, M. V. Zombeck, A. S. Krieger, J. K. Silk, A. Timothy, The S-054 X-ray telescope experiment on SKYLAB, *Space Science Instrumentation* 3 (1977) 19–76.
- [20] B. Niemann, Investigations of zone plate telescopes for the use of non-solar sources in the range of soft X-radiation., Ph.D. thesis, Diss. Göttingen Univ., F.R. Germany, Math.-Naturwiss. Fak., 96 p. (1974).
- [21] D. Dewey, T. H. Markert, M. L. Schattenburg, Diffractive-optic telescope for x-ray astronomy, in: R. B. Hoover & A. B. Walker (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 2805 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 1996, pp. 224–235.
- [22] G. K. Skinner, Diffractive/refractive optics for high energy astronomy. I. Gamma-ray phase Fresnel lenses, *Astron. & Astrophys.* 375 (2001) 691–700. doi:10.1051/0004-6361:20010745.
- [23] G. K. Skinner, Diffractive-refractive optics for high energy astronomy. II. Variations on the theme, *Astron. & Astrophys.* 383 (2002) 352–359. doi:10.1051/0004-6361:20011700.
- [24] P. Gorenstein, Concepts: x-ray telescopes with high-angular resolution and high throughput, in: J. E. Truemper & H. D. Tananbaum (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 4851 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2003, pp. 599–606. doi:10.1117/12.461605.
- [25] P. Gorenstein, Role of diffractive and refractive optics in x-ray astronomy, in: O. Citterio & S. L. O’Dell (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 5168 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2004, pp. 411–419. doi:10.1117/12.506443.

- [26] C. Braig, P. Predehl, X-ray astronomy with ultra-high-angular resolution, in: G. Hasinger & M. J. L. Turner (Ed.), Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 5488 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2004, pp. 601–612. doi:10.1117/12.551416.
- [27] C. Braig, P. Predehl, Large-scale diffractive X-ray telescopes, *Experimental Astronomy* 21 (2006) 101–123. doi:10.1007/s10686-006-9077-7.
- [28] D. J. Stigliani, Jr., R. Mittra, R. G. Semonin, Resolving power of a zone plate, *Journal of the Optical Society of America* (1917-1983) 57 (1967) 610–+.
- [29] W. Chao, E. H. Anderson, P. Fischer, D. Kim, Toward sub-10-nm resolution zone plates using the overlay nanofabrication processes, in: Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 6883 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2008. doi:10.1117/12.768878.
- [30] E. di Fabrizio, F. Romanato, M. Gentili, S. Cabrini, B. Kaulich, J. Susini, R. Barrett, High-efficiency multilevel zone plates for keV X-rays, *Nature* 401 (1999) 895–898. doi:10.1038/44791.
- [31] H. Dammann, Blazed synthetic phase-only holograms, *Optik* 31 (1) (1970) 95–104.
- [32] M. Young, Zone plates and their aberrations., *Journal of the Optical Society of America* (1917-1983) 62 (1972) 972–976.
- [33] C. B. Wunderer, G. Weidenspointner, A. Zoglauer, P. v. Ballmoos, N. Barrière, S. E. Boggs, H. Halloin, J. Knödlseider, N. Lund, Germanium (Compton) focal plane detectors for gamma-ray lenses, in: Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 6266 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2006. doi:10.1117/12.672128.
- [34] Q. Li, A. Garson, P. Dowkontt, J. Martin, M. Beilicke, I. Jung, M. Groza, A. Burger, G. De Geronimo, H. Krawczynski, ., ., ., ., Fabrication and Test of Pixelated CZT Detectors with Different Pixel Pitches and Thicknesses, *ArXiv e-prints* arXiv:0811.3201.

- [35] C. K. Stahle, D. McCammon, K. D. Irwin, *Quantum Calorimetry*, *Physics Today* 52 (1999) 32–+. doi:10.1063/1.882776.
- [36] R. L. Kelley, S. R. Bandler, W. B. Doriese, Y. Ezoe, R. Fujimoto, L. Gottardi, R. den Hartog, J. den Herder, H. Hoevers, K. Irwin, Y. Ishisaki, C. A. Kilbourne, P. de Korte, J. van der Kuur, K. Mitsuda, T. Ohashi, L. Piro, F. S. Porter, K. Sato, K. Shinozaki, P. Shirron, S. J. Smith, Y. Takei, P. Whitehouse, N. Y. Yamasaki, *The X-Ray Microcalorimeter Spectrometer for the International X-Ray Observatory*, in: B. Young, B. Cabrera, & A. Miller (Ed.), *American Institute of Physics Conference Series*, Vol. 1185 of *American Institute of Physics Conference Series*, 2009, pp. 757–760. doi:10.1063/1.3292449.
- [37] B. X. Yang, *Fresnel and refractive lenses for X-rays*, *Nuclear Instruments and Methods in Physics Research A* 328 (1993) 578–587. doi:10.1016/0168-9002(93)90678-B.
- [38] N. White, *Imaging Black Holes*, *Nature* 407 (2000) 146–+.
- [39] W. Cash, *Imaging a black hole: MAXIM*, *Advances in Space Research* 35 (2005) 122–129. doi:10.1016/j.asr.2003.09.057.
- [40] S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins, T. Ott, *Monitoring Stellar Orbits Around the Massive Black Hole in the Galactic Center*, *Astrophys. J.* 692 (2009) 1075–1109. arXiv:0810.4674, doi:10.1088/0004-637X/692/2/1075.
- [41] E. Middelberg, U. Bach, *High resolution radio astronomy using very long baseline interferometry*, *Reports on Progress in Physics* 71 (6) (2008) 066901–+. arXiv:0803.2983, doi:10.1088/0034-4885/71/6/066901.
- [42] T. P. Krichbaum, D. A. Graham, M. Bremer, W. Alef, A. Witzel, J. A. Zensus, A. Eckart, *Sub-Milliarcsecond Imaging of Sgr A* and M 87*, *Journal of Physics Conference Series* 54 (2006) 328–334. arXiv:arXiv:astro-ph/0607072, doi:10.1088/1742-6596/54/1/052.
- [43] S. S. Doeleman, J. Weintroub, A. E. E. Rogers, R. Plambeck, R. Freund, R. P. J. Tilanus, P. Friberg, L. M. Ziurys, J. M. Moran, B. Corey, K. H. Young, D. L. Smythe, M. Titus, D. P. Marrone, R. J. Cappallo, D. Bock, G. C. Bower, R. Chamberlin, G. R. Davis, T. P. Krichbaum, J. Lamb,

- H. Maness, A. E. Niell, A. Roy, P. Strittmatter, D. Werthimer, A. R. Whitney, D. Woody, Event-horizon-scale structure in the supermassive black hole candidate at the Galactic Centre, *Nature* 455 (2008) 78–80. arXiv:0809.2442, doi:10.1038/nature07245.
- [44] Y. Murata, N. Mochizuki, H. Saito, H. Hirabayashi, M. Inoue, H. Kobayashi, P. G. Edwards, The next generation space VLBI project, VSOP-2, in: J. M. Chapman & W. A. Baan (Ed.), *IAU Symposium*, Vol. 242 of *IAU Symposium*, 2007, pp. 517–521. doi:10.1017/S1743921307013695.
- [45] T. P. Krichbaum, I. Agudo, U. Bach, A. Witzel, J. A. Zensus, VLBI at the highest frequencies - AGN studies with micro-arcsecond resolution, in: *Proceedings of the 8th European VLBI Network Symposium*, 2006.
- [46] J. Hevelius, *Machinae coelestis*, Vol. 2, 1738, p. 353.
- [47] S. J. Bennett, Achromatic combinations of hologram optical elements, *Appl. Opt.* 15 (1976) 542–545.
- [48] A. G. Michette, C. J. Buckley, F. Gallo, K. Powell, S. J. Pfauntsch, Zone plate achromatic doublets, in: A. K. Freund, T. Ishikawa, A. M. Khounsary, D. C. Mancini, A. G. Michette, & S. Oestreich (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 4145 of *Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, 2001, pp. 303–310.
- [49] L. Koechlin, D. Serre, P. Deba, The Fresnel interferometric imager, *Ap. Space Sci.* 320 (2009) 225–230. doi:10.1007/s10509-008-9793-8.
- [50] T. Stone, N. George, Hybrid diffractive-refractive lenses and achromats, *Appl. Opt.* 27 (1988) 2960–2971.
- [51] B. Lengeler, C. G. Schroer, M. Kuhlmann, B. Benner, T. Florian Günzler, O. Kurapova, F. Zontone, A. Snigirev, I. Snigireva, Refractive x-ray lenses, *Journal of Physics D Applied Physics* 38 (26) (2005) A260000–A222. doi:10.1088/0022-3727/38/10A/042.
- [52] G. K. Skinner, Design and Imaging Performance of Achromatic Diffractive-Refractive X-Ray and Gamma-Ray Fresnel Lenses,

- Appl. Opt. 43 (2004) 4845–4853. arXiv:arXiv:astro-ph/0407457, doi:10.1364/AO.43.004845.
- [53] C. Braig, P. Predehl, Advanced Fresnel X-ray telescopes for spectroscopic imaging, *Experimental Astronomy* (2009) 43–+doi:10.1007/s10686-009-9180-7.
 - [54] Y. Wang, W. Yun, C. Jacobsen, Achromatic Fresnel optics for wideband extreme-ultraviolet and X-ray imaging, *Nature* 424 (2003) 50–53.
 - [55] C. Braig, P. Predehl, A diffraction limited dual-band x-ray telescope, in: *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 6688 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2007. doi:10.1117/12.734350.
 - [56] P. Gorenstein, W. Cash, N. Gehrels, K. Gendreau, J. Krizmanic, M. C. Miller, C. S. Reynolds, R. M. Sambruna, G. K. Skinner, R. E. Streitmatter, D. L. Windt, The future of high angular resolution x-ray astronomy, in: *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 7011 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2008. doi:10.1117/12.789978.
 - [57] M. J. Simpson, A. G. Michette, Imaging Properties of Modified Fresnel Zone Plates, *Optica Acta* 31 (1984) 403–413. doi:10.1080/713821522.
 - [58] L. Kipp, M. Skibowski, R. L. Johnson, R. Berndt, R. Adelung, S. Harm, R. Seemann, Sharper images by focusing soft X-rays with photon sieves, *Nature* 414 (2001) 184–188.
 - [59] F. Giménez, J. A. Monsoriu, W. D. Furlan, A. Pons, Fractal photon sieve, *Optics Express* 14 (2006) 11958–11963. arXiv:arXiv:physics/0610212, doi:10.1364/OE.14.011958.
 - [60] G. K. Skinner, X-ray and gamma-ray focusing and interferometry, in: *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 7437 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2009. doi:10.1117/12.826853.

- [61] G. Skinner, Development of optics for sub-micro-arcsecond angular resolution in the X-ray and gamma-ray domains, in: *astro2010: The Astronomy and Astrophysics Decadal Survey*, Vol. 2010 of *Astronomy*, 2009, pp. 20–+.
- [62] J. Sochacki, A. Klodziejczyk, Z. Jaroszewicz, S. Bara, Nonparaxial design of generalized axicons, *Appl. Opt.* 31 (1992) 5326–5330.
- [63] Q. Cao, S. Chi, Axially symmetric on-axis flat-top beam, *Journal of the Optical Society of America A* 17 (2000) 447–455. doi:10.1364/JOSAA.17.000447.
- [64] P. Gorenstein, J. D. Phillips, R. D. Reasenberg, Refractive/diffractive telescope with very high angular resolution for X-ray astronomy, in: O. Citterio & S. L. O'Dell (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 5900 of *Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, 2005, pp. 369–376. doi:10.1117/12.620144.
- [65] T. G. Farr, P. A. Rosen, E. Caro, R. Crippen, R. Duren, S. Hensley, M. Kobrick, M. Paller, E. Rodriguez, L. Roth, D. Seal, S. Shaffer, J. Shimada, J. Umland, M. Werner, M. Oskin, D. Burbank, D. Alsdorf, The Shuttle Radar Topography Mission, *Reviews of Geophysics* 45 (2007) 2004–+. doi:10.1029/2005RG000183.
- [66] A. N. Parmar, M. Arnaud, X. Barcons, J. Bleeker, G. Hasinger, H. Kunieda, G. Palumbo, T. Takahashi, M. Turner, P. de Korte, R. Willingale, N. Rando, A. Lyngvi, P. Gondoin, D. Lumb, M. Bavdaz, P. Verhoeve, XEUS: The x-ray evolving universe spectroscopy mission, in: *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 6266 of *Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, 2006. doi:10.1117/12.674336.
- [67] J. Krizmanic, G. Skinner, N. Gehrels, Formation Flying for a Fresnel Lens Observatory Mission, *Experimental Astronomy* 20 (2005) 497–503. arXiv:arXiv:astro-ph/0601047, doi:10.1007/s10686-006-9038-1.
- [68] R. Smith, *A Compleat System of Optiks*, Gedani, 1673, p. 37.

- [69] K. C. Gendreau, J. Leitner, L. Markley, W. C. Cash, A. F. Shipley, Requirements and options for a stable inertial reference frame for a 100-micro-arcsecond imaging telescope, in: M. Shao (Ed.), Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 4852 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2003, pp. 685–694. doi:10.1117/12.460746.
- [70] R. J. Luquette, R. M. Robert M. Sanner, Spacecraft Formation Control: Managing Line-of-Sight Drift Based on the Dynamics of Relative Motion, in: Proceedings of the 3rd International Symposium on Formation Flying Missions and Technologies, April 2008, ESA/ESTEC, Noordwijk, The Netherlands; NASA document ID 20080040729, 2008.
- [71] N. Meidinger, R. Andritschke, S. Ebermayer, J. Elbs, O. Hälker, R. Hartmann, S. Herrmann, N. Kimmel, P. Predehl, G. Schächner, H. Soltau, L. Strüder, L. Tiedemann, CCD detectors for spectroscopy and imaging of x-rays with the eROSITA space telescope, in: Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 7435 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2009. doi:10.1117/12.825234.
- [72] A. Laborie, R. Davancens, P. Pouny, C. Vétel, F. Chassat, P. Charvet, P. Garé, G. Sarri, The Gaia focal plane, in: Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 6690 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2007. doi:10.1117/12.741062.
- [73] R. A. Hyde, S. N. Dixit, A. H. Weisberg, M. C. Rushford, Eyeglass: A Very Large Aperture Diffractive Space Telescope, in: H. A. MacEwen (Ed.), Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 4849 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2002, pp. 28–39. doi:10.1117/12.460420.
- [74] R. Hyde, S. Dixit, A giant leap for space telescopes, LNL Science and Technology Review <https://www.llnl.gov/str/March03/Hyde.html>.
- [75] G. K. Skinner, J. F. Krizmanic, X-ray interferometry with transmissive beam combiners for ultra-high angular resolution astron-

- omy, *Experimental Astronomy* 27 (2009) 61–76. arXiv:0907.5365, doi:10.1007/s10686-009-9175-4.
- [76] W. C. Cash, Maxim: micro-arcsecond x-ray imaging mission, in: M. Shao (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 4852 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2003, pp. 196–209. doi:10.1117/12.460915.
- [77] J. Krizmanic, B. Morgan, R. Streitmatter, N. Gehrels, K. Gendreau, Z. Arzoumanian, R. Ghodssi, G. Skinner, Development of ground-testable phase fresnel lenses in silicon, *Experimental Astronomy* 20 (2005) 299–306. arXiv:arXiv:physics/0601012, doi:10.1007/s10686-006-9030-9.
- [78] B. Morgan, C. M. Waits, J. Krizmanic, R. Ghodssi, Development of a Deep Silicon Phase Fresnel Lens Using Gray-Scale Lithography and Deep Reactive Ion Etching, *J. Microelectromechanical Systems* 13 (2004) 113–120.
- [79] Z. Arzoumanian, K. C. Gendreau, W. C. Cash, A. F. Shipley, S. Z. Queen, Laboratory testbeds for broadband x-ray interferometry, in: G. Hasinger & M. J. L. Turner (Ed.), *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 5488 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2004, pp. 623–629. doi:10.1117/12.551731.
- [80] G. Gendreau, Gsfc x-ray interferometry testbed, <http://lhea-www.gsfc.nasa.gov/kgc/beamline/home.html>.
- [81] J. Krizmanic, In preparation.
- [82] G. K. Skinner, Z. Arzoumanian, W. C. Cash, N. Gehrels, K. C. Gendreau, P. Gorenstein, J. F. Krizmanic, M. C. Miller, J. D. Phillips, R. D. Reasenberg, C. S. Reynolds, R. M. Sambruna, R. E. Streitmatter, D. L. Windt, The milli-arc-second structure imager (MASSIM): a new concept for a high angular resolution x-ray telescope, in: *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 7011 of Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference, 2008. doi:10.1117/12.789568.