JACOBS
ESTS Group

Brüel & Kjaer 4944 Microphone Grid Frequency Response Function System Identification

159th Meeting of the Acoustical Society of America
April 19-23 2010, Baltimore, Maryland

Task Order 33-040225
Fluid Dynamics Support for Flight and Integrated Test Office

Subelement – CA
Support for Ares 1 Subscale Model Acoustics Test

Reginald Bennett
Erik Lee

Introduction

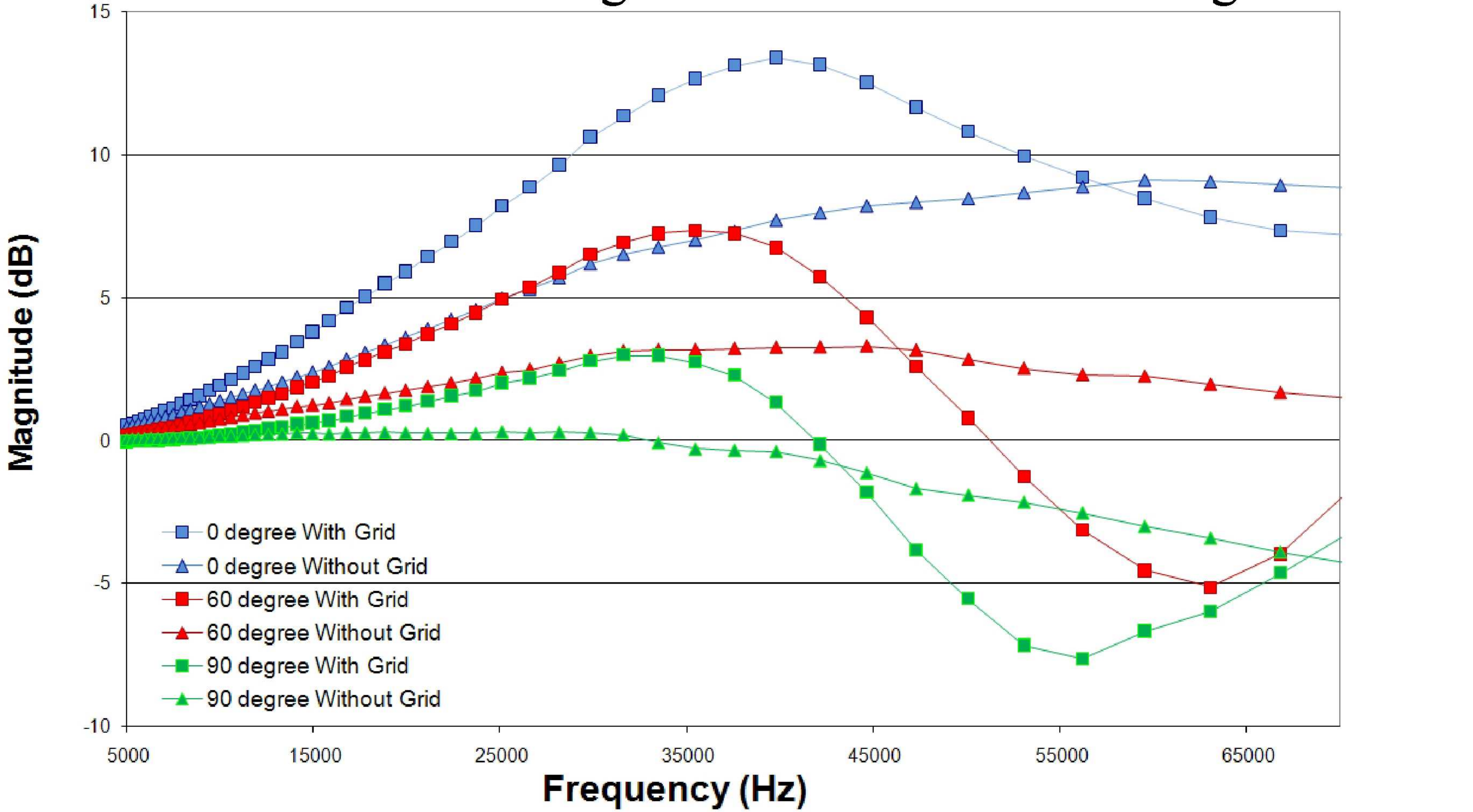
- Brüel & Kjær (B&K) 4944B pressure field microphone was judiciously selected to measure acoustic environments, 400Hz – 50kHz, in close proximity of the nozzle during multiple firings of solid propellant rocket motors
- It is well known that protective grids can affect the frequency response of microphones [1] [2]
- B&K recommends operation of the B&K 4944B without a protective grid when recording measurements above 10 to 15 kHz

Experimental Motivation

- The solid rocket plume and debris warrants diaphragm protection to preserve the integrity of the sensor and maximize its lifetime
- Quantifying the protective grid effects allow for reduced mortality rate while maintaining accurate assessment of the acoustic field
- Magnitude response corrections for measurements in a pressure field obtained with the grid in place were not provided by B&K

Free Field Magnitude Response

B&K provides the magnitude response of the 4944 in free field conditions for both configurations- with and without grid.



JACOBS
ESTS Group

Goals

- Microphone configurations with and without the grid in place were examined in an approach to establish corrections for the magnitude-frequency response of the B&K 4944 with the protective grid
  - Frequency range of interest: 10,000 hertz to 50,000 hertz
  - Acoustic field: Induced by a rocket firing
  - Proper measurement and digital signal processing (DSP) uncertainty propagation indicative to anticipated data acquisition system setup

Applying Basic Mathematics Model

- View the grid as a tube over the diaphragm and apply tube resonance equations to identify a transfer function for the magnitude response
- Tube resonance calculations suggest an approximate range between 20kHz and 25kHz for quarter wave resonance [11]
- This approach does not work well for the complex geometry of the protective grid because the openings in the grid do not create a true tube-like structure
- The transfer function cannot be modeled with simple mathematical expressions, but can more readily be derived through testing

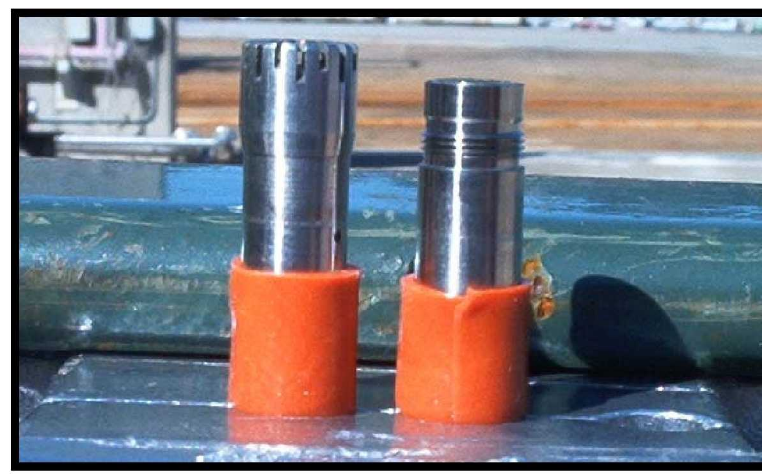
JACOBS
ESTS Group

Tests Recorded to Derive Corrections

- 1 solid rocket motor firing, Heavy Wall Motor (HWM)
- 7 liquid rocket engine firings, Gas Generator (GG)
- The microphones were placed in the near field for both test setups
- The 7 GG tests were used for frequency response function realization and the HWM data was used for verification

◀ Top View

Side View ▶



JACOBS
ESTS Group

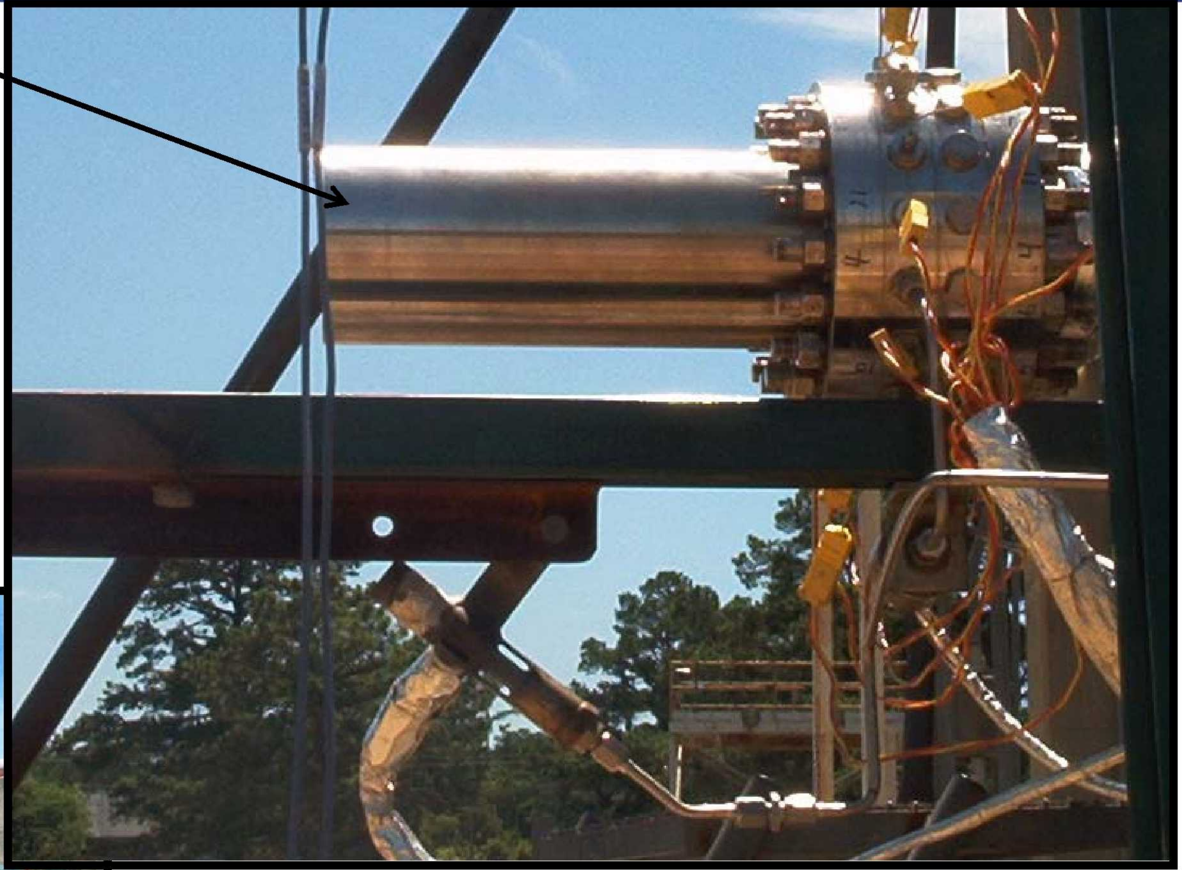
Test Details

- Microphones were mounted within 20 nozzle diameters of the center axis, to acquire measurements in the near field
- During each test, the microphones were mounted with a 90 degree incident angle to be consistent with pressure field microphone mounting configurations
- A custom portable data acquisition system was used that was designed and built by Optical Sciences Corporation
- A sample rate of 102,400 samples per second was used to acquire the data

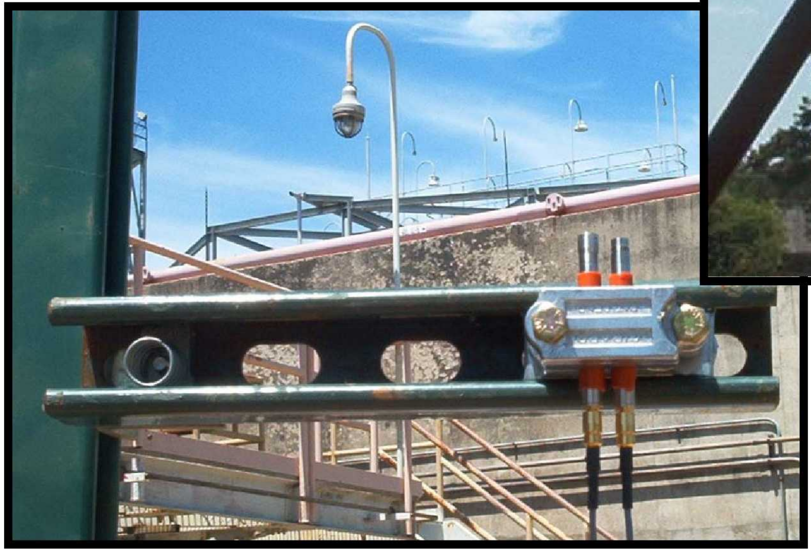
JACOBS
ESTS Group

Gas Generator Test Set up

Gas Generator
Nozzle exit plane



Sensors Mounted within 20 nozzle diameters so that the diaphragms are perpendicular to the exit plane



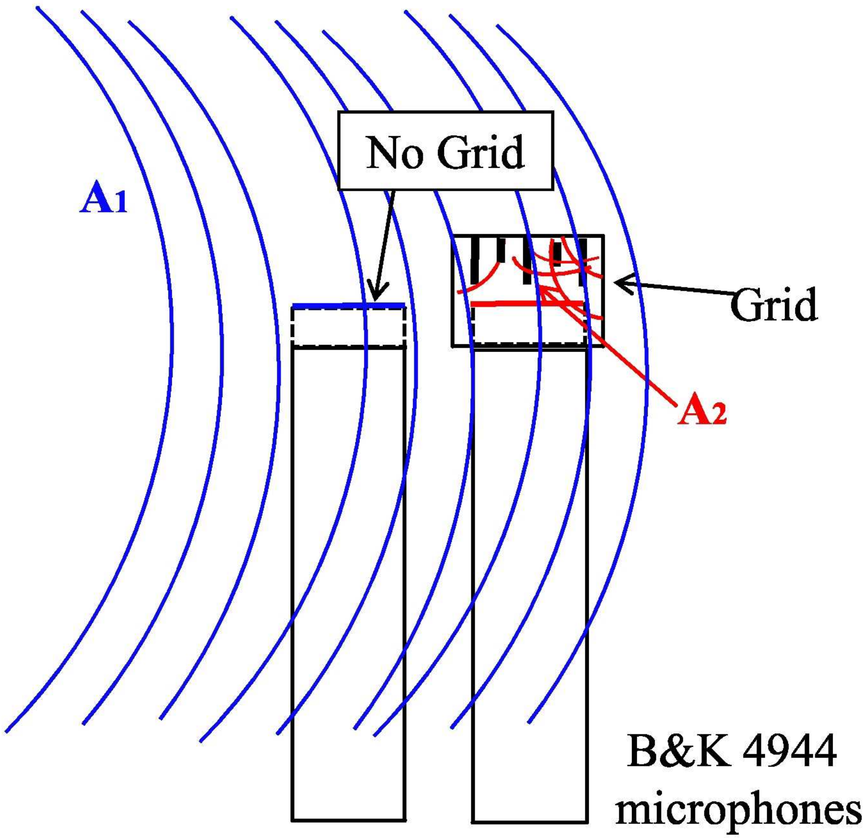
System Identification in Acoustics

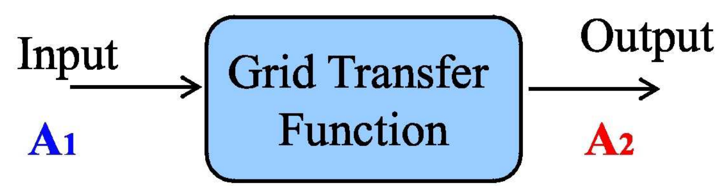
- System Identification is a procedure to realize a mathematical model from measured data
- For most engineering problems the mathematical model is termed transfer function or Frequency Response Function (FRF)
- Practical system identification methods are developed for linear systems, since this form can be used to represent many systems [3]
- For linear systems, general signal inputs include impulse signals, discrete stationary sine waves and square waves [4]
- In acoustics typical input signals are generally random
- These type of signals may exhibit
  - 1) non-stationarities
  - 2) temporal-dependent behavior
  - 3) multiple frequency constituents
- The above-mentioned signal characteristics make identification of a linear system difficult and result in less accurate realizations

Frequency Dependent Correction Curves

The mic with no grid measures acoustic field A_1. The mic with the grid measures acoustic field A_2. A_2 is created due to the grid effects.

This is a system identification technique to realize a complex transfer function for the grid from input-output data.





Transfer Function in the Frequency Domain – Output/Input Ratio

Analysis Section

Analysis Outline

- Measurement uncertainty analysis
- Analysis blocking procedure
- Analysis block diagram
- Frequency response function analysis
- Uncertainty propagation through processing
- Uncertainty propagation challenges
- Uncertainty propagation through FRF smoothing

[redacted]

Measurement Uncertainty Analysis

- Included B&K 4944 Microphone and VMEbus eXtensions for Instrumentation (VXI) Technology VT1432B Digitizer with DSP
- B&K 4944
  - Sensitivity uncertainty approximately 4 percent over a range of 250 Hz to 80 kHz
  - Humidity uncertainty amounted to 1 percent worst case, assuming no condensation [5]
- VXI board
  - 1.2 percent uncertainty due to amplitude accuracy [6]

Measurement Uncertainty Analysis

- The three measurement uncertainties were assumed be of a Gaussian Distribution
- This is a common assumption and justified via the Central Limit Theorem [7]
- Each of the uncertainties were assumed to be independent of each other and therefore propagated via square root of the sum of the squares [8]
- Resultant uncertainty was 4.3 percent with a 95 percent confidence interval

Measurement Uncertainty Analysis

- B&K 4944 uncertainties not accounted for:
  1) Magnitude-frequency response of sensor
     – Both microphones were supplied with individual magnitude-frequency response curves, but differences were minimal
  2) Temperature effects on the magnitude-frequency response of the sensor
     – Significant spectral distortion can occur due to temperature effects [1]
     – Surface mount thermocouples showed that sensors remained near nominal temperature throughout experiment
  3) Temperature coefficient of sensitivity
     – Spatial positioning of sensors kept them near nominal and also resulted in minimal temperature difference between the two microphones

JACOBS
ESTS Group

Measurement Uncertainty Analysis

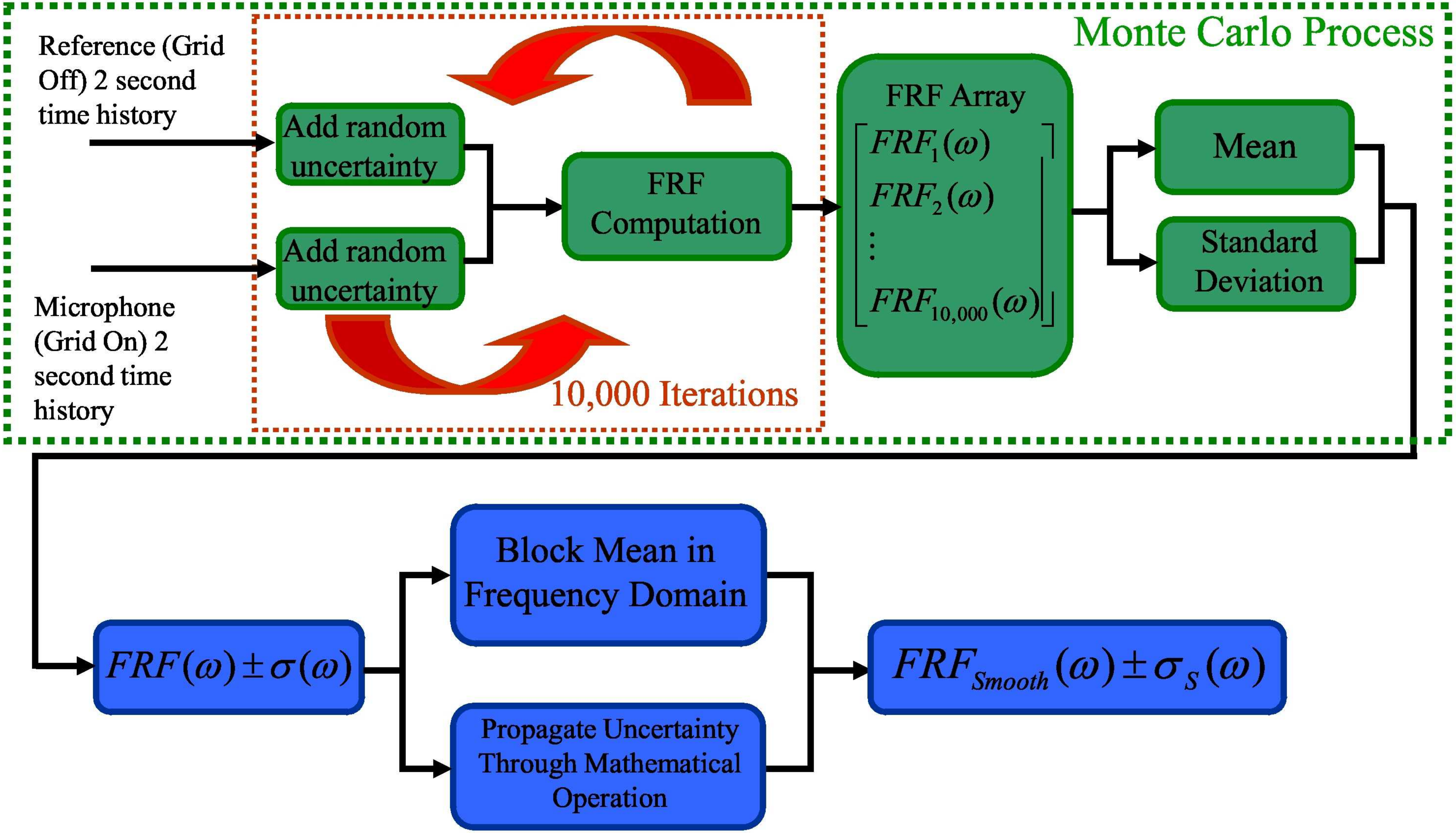
- VXI uncertainties not accounted for:
  1) Passband ripple in post-digitization filter had negligible contribution of 0.002 percent
  2) Cross channel amplitude match of 0.01 dB was not accounted for

[redacted]

Analysis Blocking Procedure

- Data were gathered for 8 individual rocket firings
- Time segment of motor/engine steady-state operation used for analysis, termed analysis region
- Analysis region divided into two second segments
- Each two second segment was used to realize a FRF
- Above procedure was performed for each test

Analysis Block Diagram



Analysis Block Diagram

- Previous analysis block diagram operations were performed for each 2 second segment of data
- Resulted in N smoothed FRFs with uncertainties
- Ensemble mean computed over all N FRF's to produce one FRF with one uncertainty as a function of frequency

Frequency Response Function Analysis

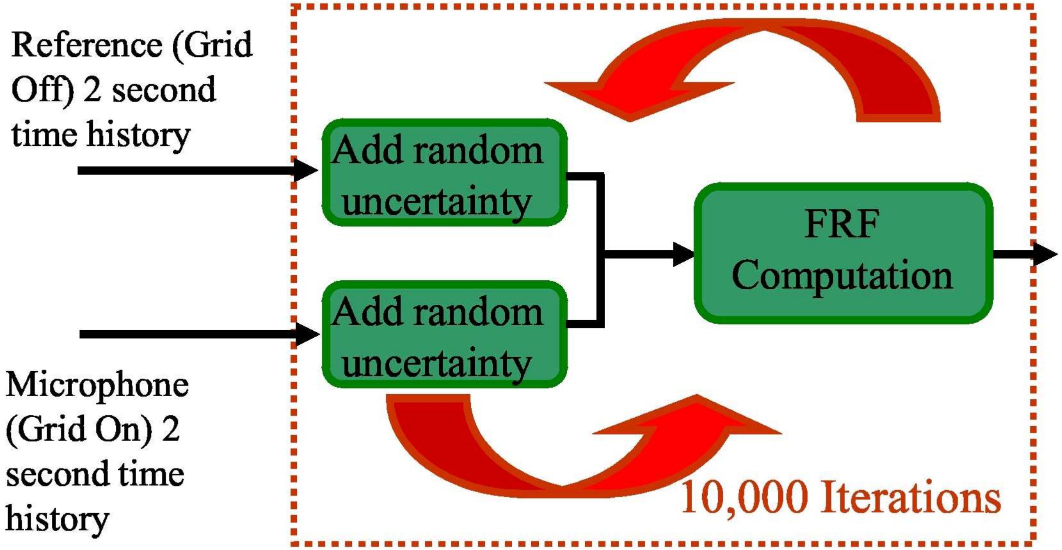
- System realization was performed using the ratio of the cross power spectral density and the power spectral density

$$frf(\omega) = \frac{P_{yx}(\omega)}{P_{xx}(\omega)}$$

- Methodology assumes the system can be represented by a linear, time-invariant transfer function [9]
- FRF $\Delta\omega$ is equal to $1/\Delta t$, same as Fast Fourier Transform (FFT)
- $\Delta\omega$ for this analysis equaled 0.5 hz
- This analysis only considered the magnitude response (ie: absolute value of complex FRF), not the phase response (ie: angle of complex FRF)

Uncertainty Propagation Through Frequency Response Function

- Monte Carlo simulations were performed to propagate measurement uncertainties on the time history through to the FRF
- 10,000 iterations were performed to ensure accurate representation of the distribution [10]



JACOBS ESTS Group

Uncertainty Propagation Through Frequency Response Function

- Computation Time Challenge
  - A total of 78 two second segments were used to realize the protective grid FRF
  - For a sample rate of 102,400 samples per second and 0.5 hz FRF resolution (ie: two second segment), each FRF was composed of 102,400 data points
  - Computation of one FRF takes approximately 0.3 seconds
  - All tests took (78)(0.3)(10,000) = 234K seconds = 65 hours

Uncertainty Propagation Through Frequency Response Function

- Computer Memory Challenge
  - Each FRF was composed of 102,400 points
  - Recall that the standard deviation is defined as

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \mu\right)^2}$$

hence knowledge of the population mean must be known

  - Each of the 10,000 FRF's from the Monte Carlo simulation must be stored because standard deviation can not be computed recursively
  - Random Access Memory (RAM) usage amounts to

(number of points)(8 bytes for double)(number of iterations)

  - RAM requirements for this analysis amounted to over 7.8 Gigabytes using this approach

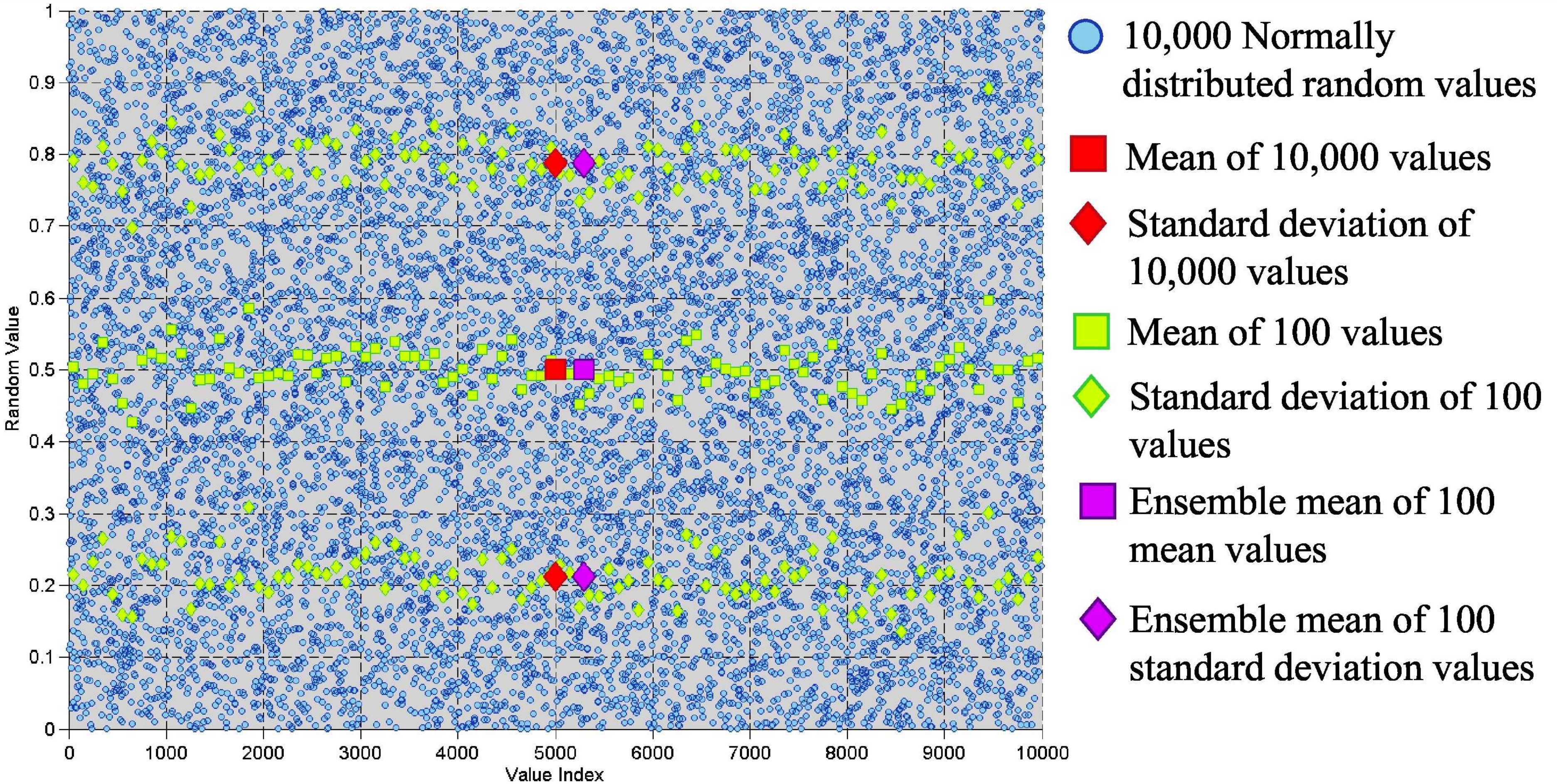
Uncertainty Propagation Through FRF

- Solution to the RAM requirements was to compute the ensemble mean every 100 iterations
- This only requires 100 FRFs to be stored, which is possible on most computer systems
- This process was repeated 100 times and an ensemble mean of the means and standard deviations was computed
- A total of 10,000 Monte Carlo iterations was still performed

Validity of Alternative Averaging Process

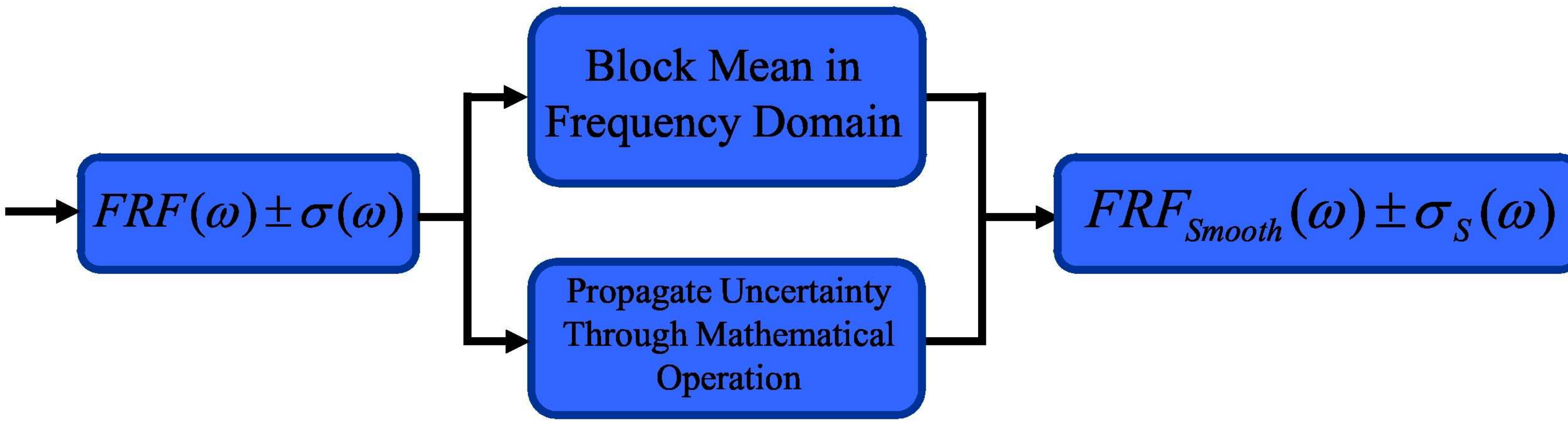
- After 100 iterations, the distribution is likely not exactly normal
- Repeating this 100 times produces 100 distributions that are not exactly normal
- Recall the Central Limit Theorem: Combining many different distributions will approach a normal distribution
- Only valid for normal distribution
- Several experimental tests were run and showed that above methodology of 100 ensemble means of 100 means and standard deviations matches mean and standard deviation of 10,000 point average and standard deviation
- The percent difference in these approaches is ~0.04%

Analysis Technique Comparison



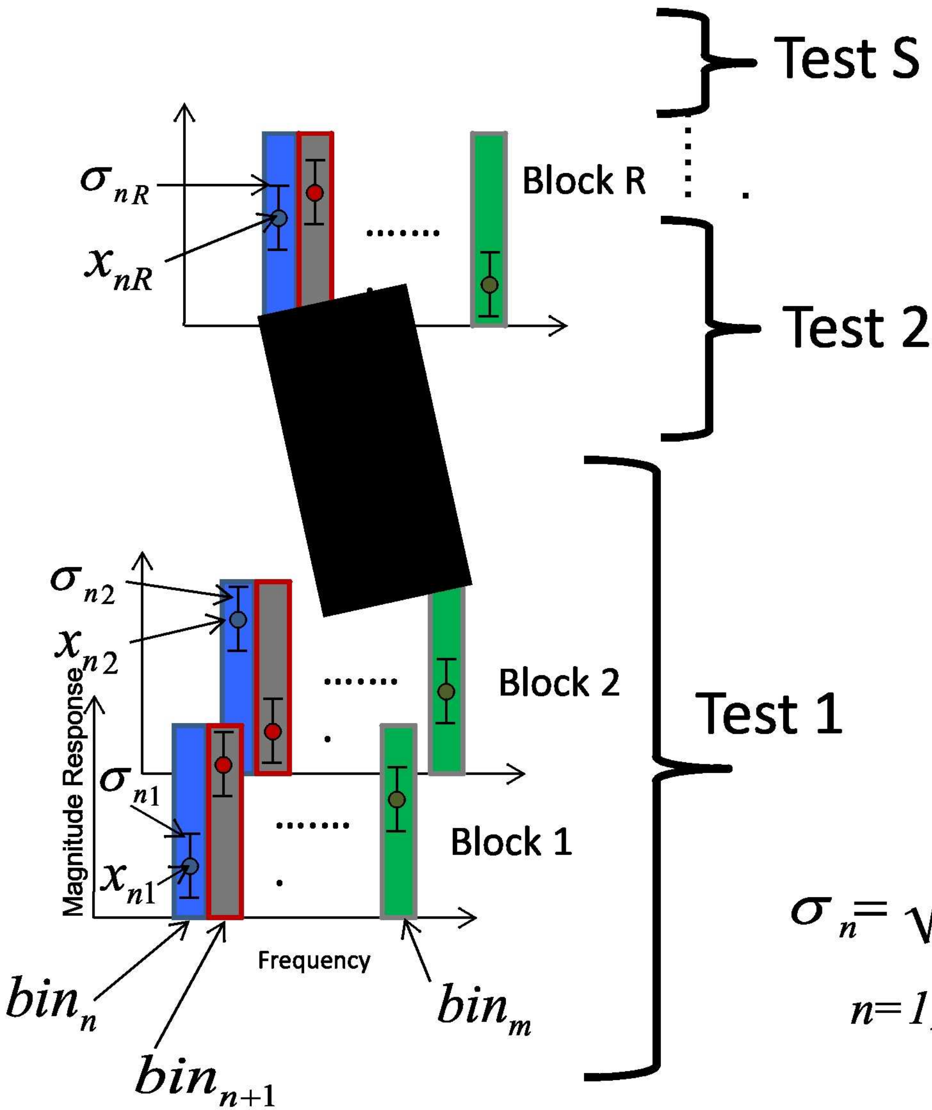
Uncertainty Propagation Through FRF Smoothing

- Linear Smoothing with different block sizes, as well as 1/12 Octave and 1/3 Octave based smoothing was performed

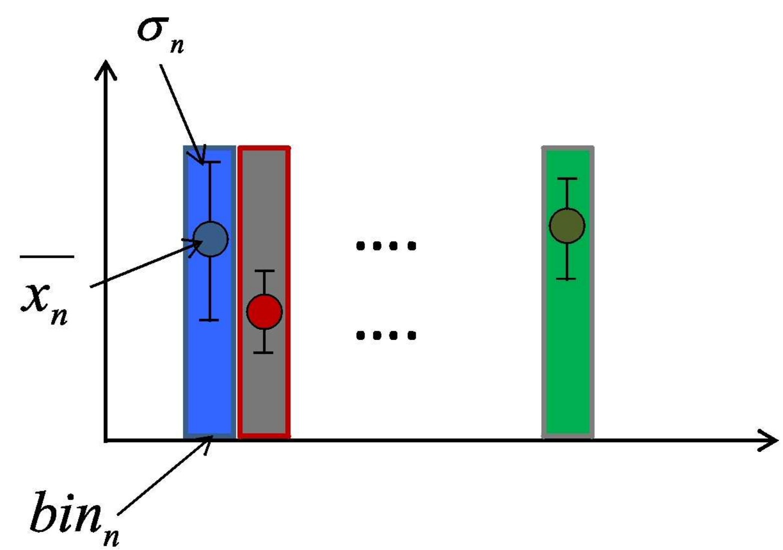


JACOBS
ESTS Group

Uncertainty Propagation Accounting For All Tests



Representative FRF

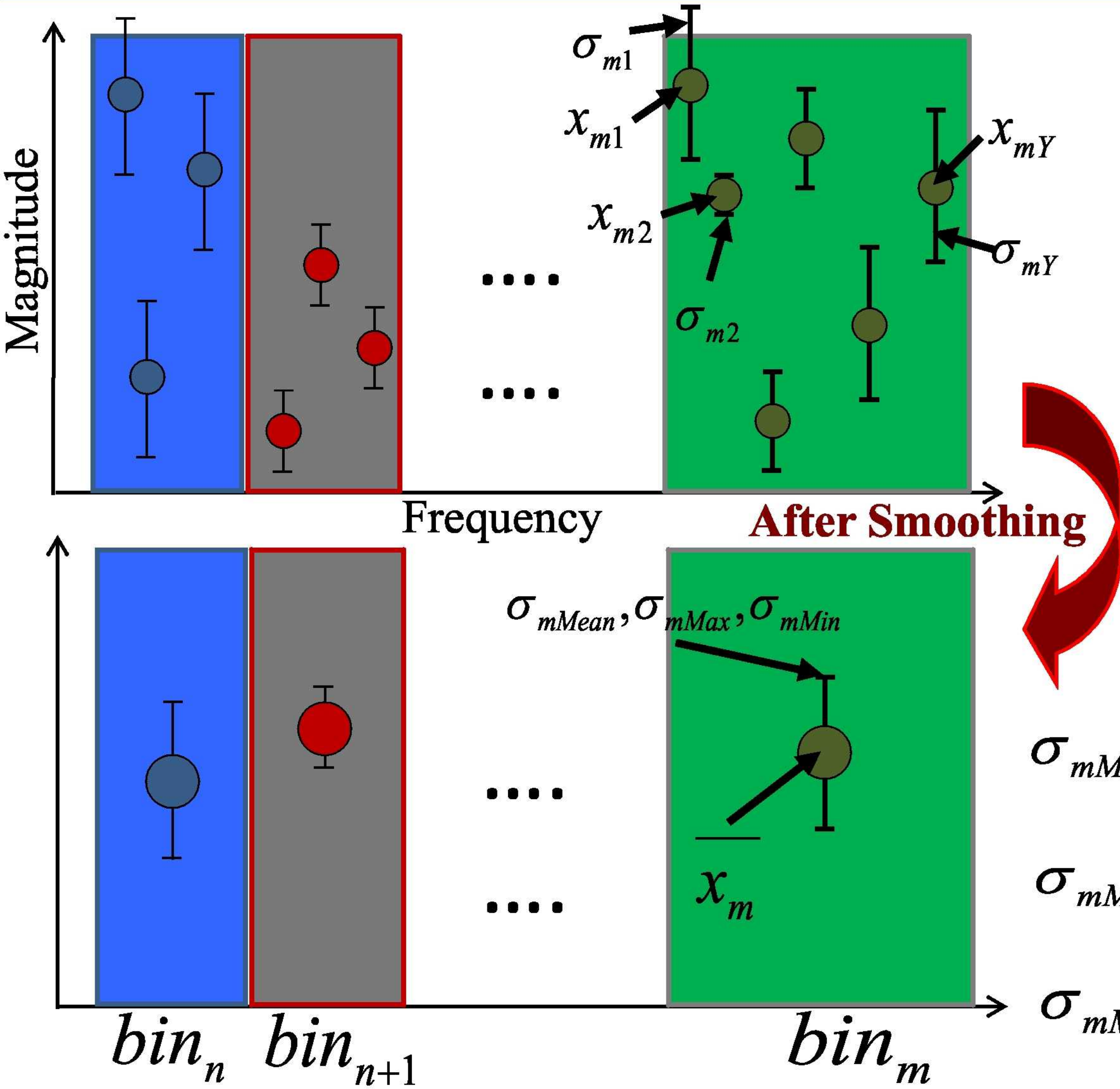


$$\overline{x_n} = mean(x_{ni}), i = 1,2,\ldots R$$

$$\sigma_n = \sqrt{\left(std(x_{ni})\right)^2 + \left(mean(\sigma_{ni})\right)^2}, i = 1,2,\ldots R$$

$n = 1,2,\ldots m$

Uncertainty Propagation Through FRF Smoothing



$n = 1, 2, \ldots m$

$$\overline{x_m} = \sum_{i=1}^{Y} \frac{x_{mi}}{Y}$$

$$\sigma_{mMean} = \sqrt{\left(std\left(x_{mi}\right)\right)^2 + \left(mean\left(\sigma_{mi}\right)\right)^2}$$

$$\sigma_{mMax} = \sqrt{\left(std\left(x_{mi}\right)\right)^2 + \left(\max\left(\sigma_{mi}\right)\right)^2}$$

$$\sigma_{mMin} = \sqrt{\left(std\left(x_{mi}\right)\right)^2 + \left(\min\left(\sigma_{mi}\right)\right)^2}$$

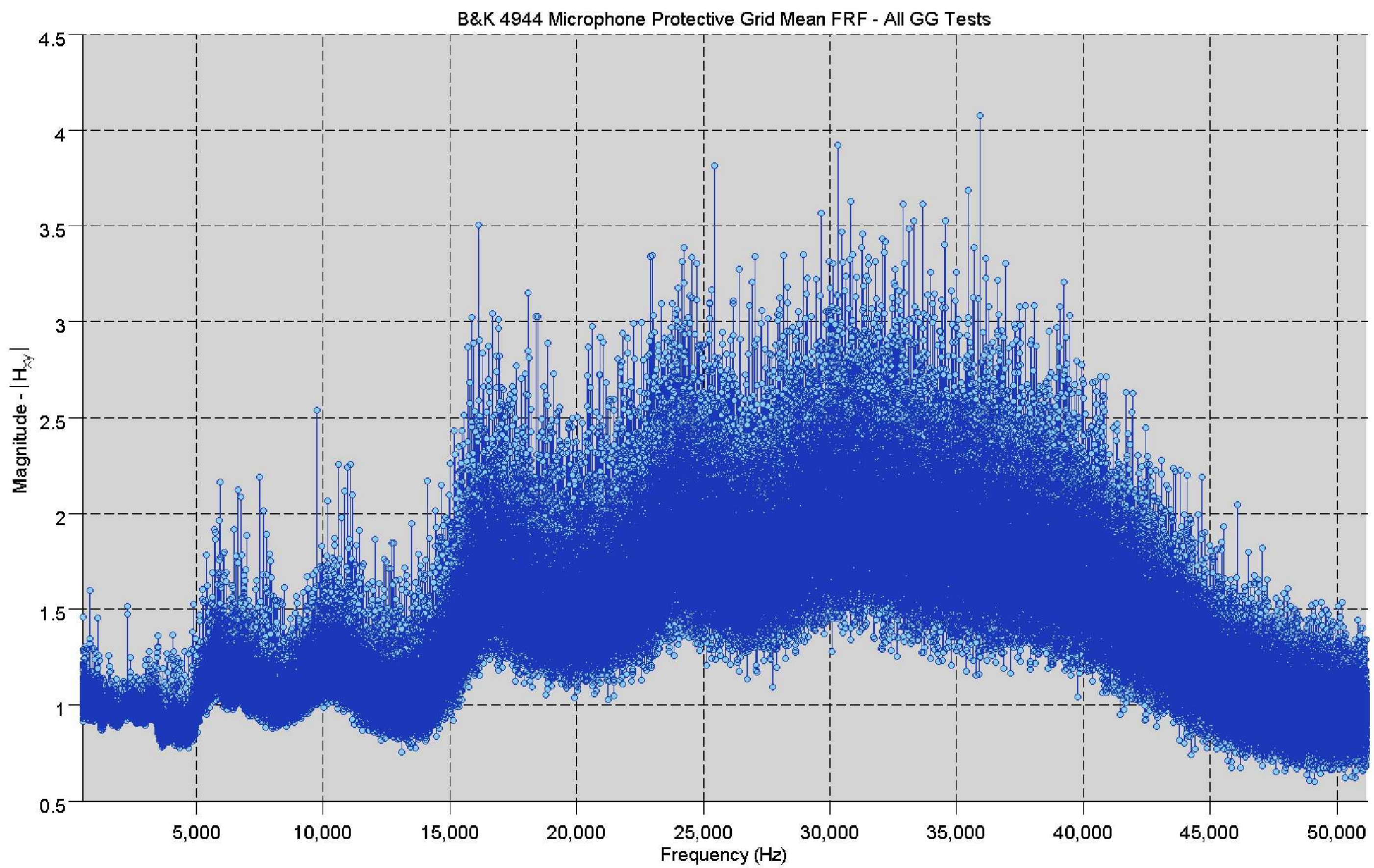
JACOBS
ESTS Group

Results/Conclusions Section

Why Average and Smooth The FRF

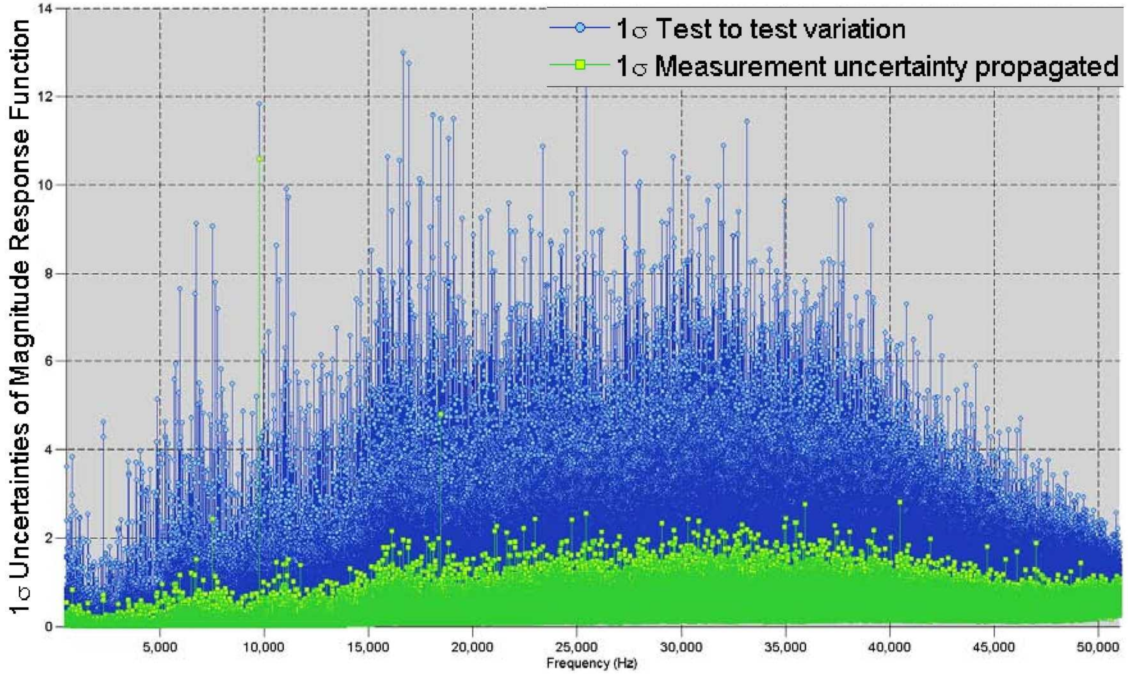
- Difficult to implement as a correction
- Practical systems do not have rapidly changing FRFs

[redacted]



Uncertainty Analysis Results

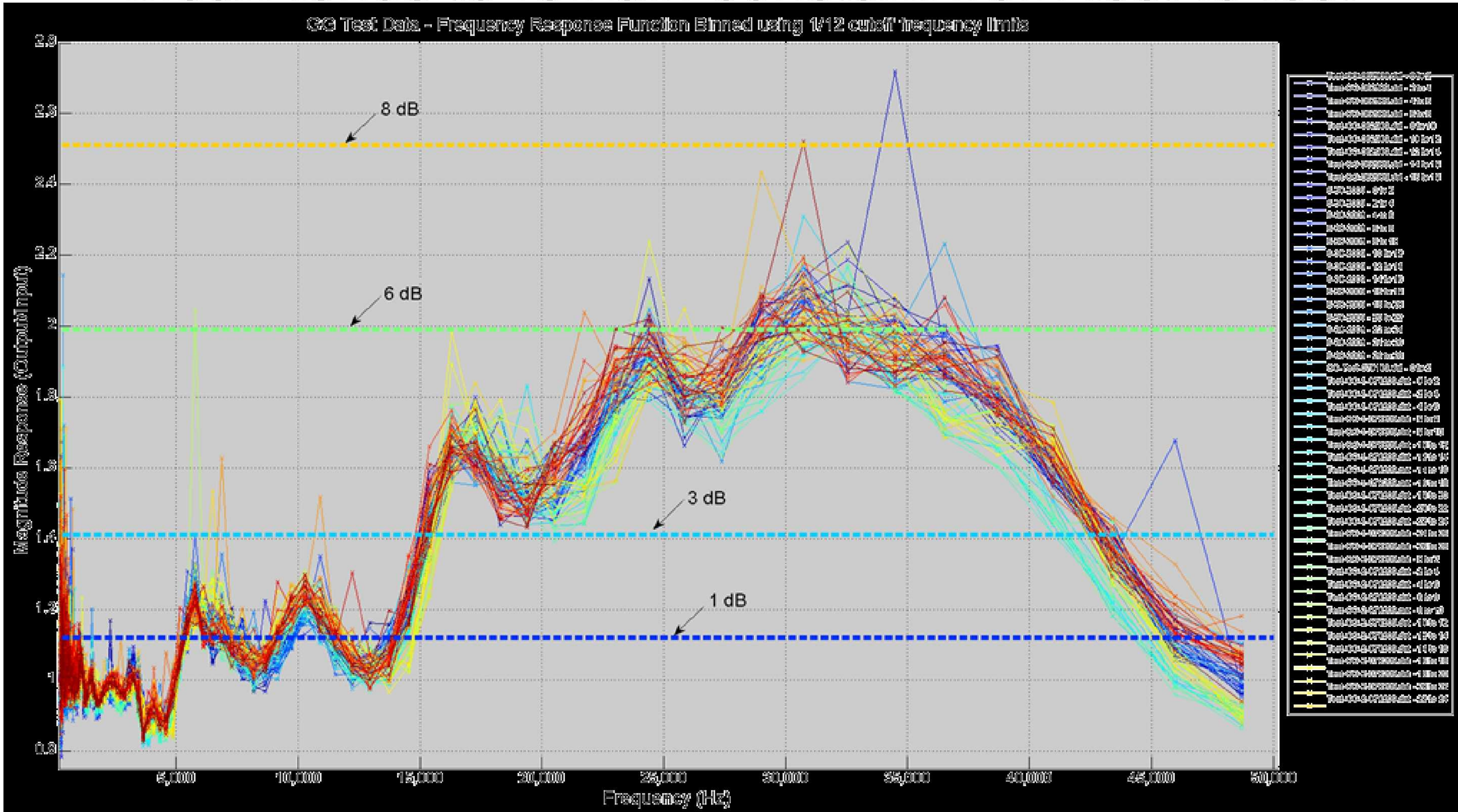
- Test to test spread includes all variation, <u>including measurement uncertainty</u>
- Uncertainty other than measurement could be due to model form
- System likely linear time varying or nonlinear



JACOBS
ESTS Group

FRF Overlay of Tests

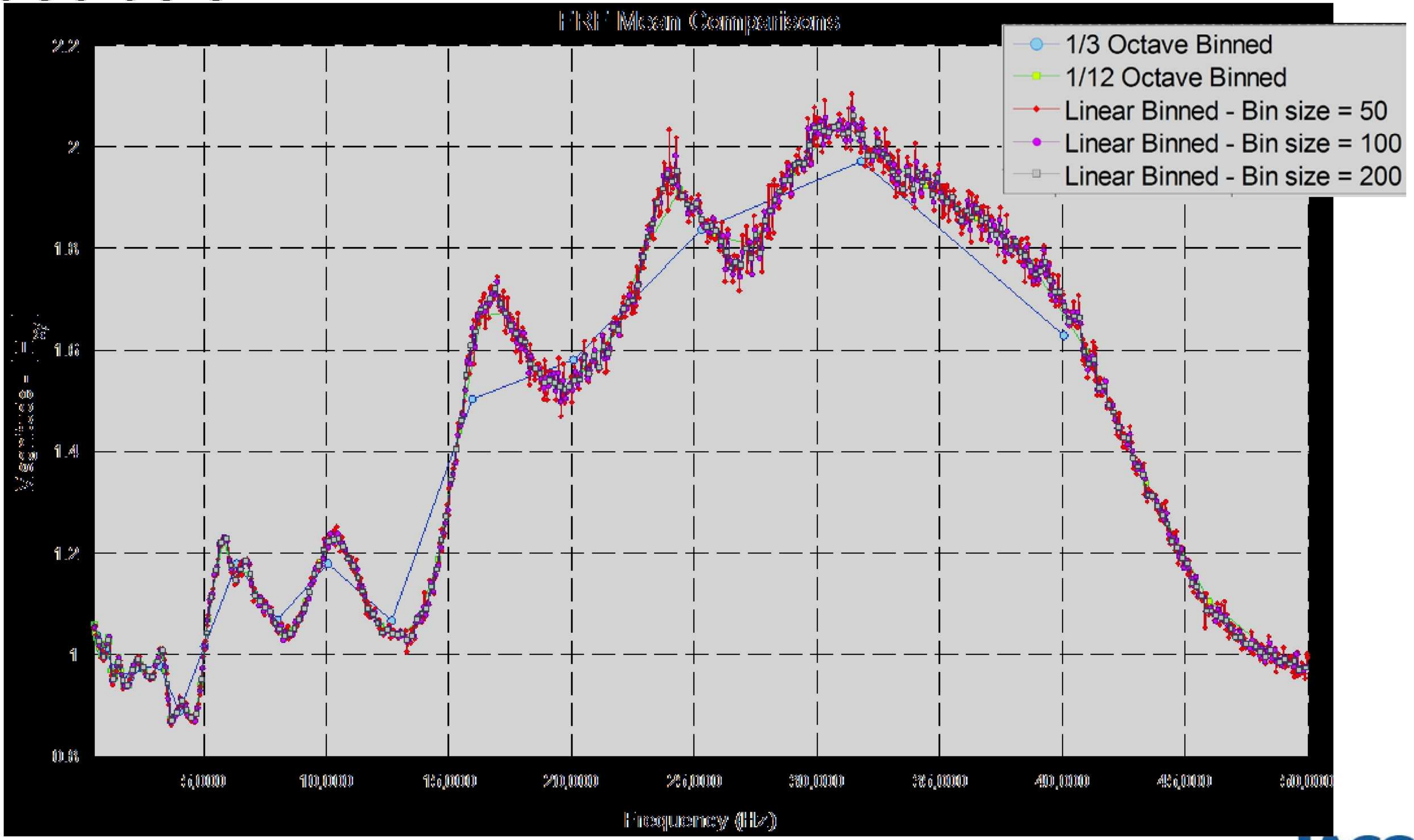
- Mean tends to remain constant from test to test



JACOBS
ESTS Group

Mean Binned FRF

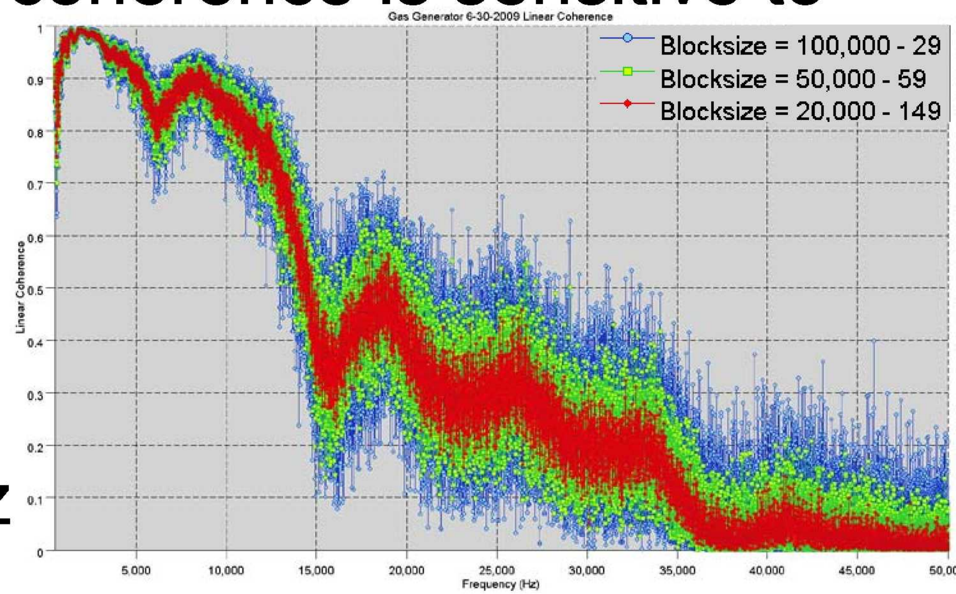
- Smoothed Mean FRF shows amplification over 6 decibels



JACOBS
ESTS Group

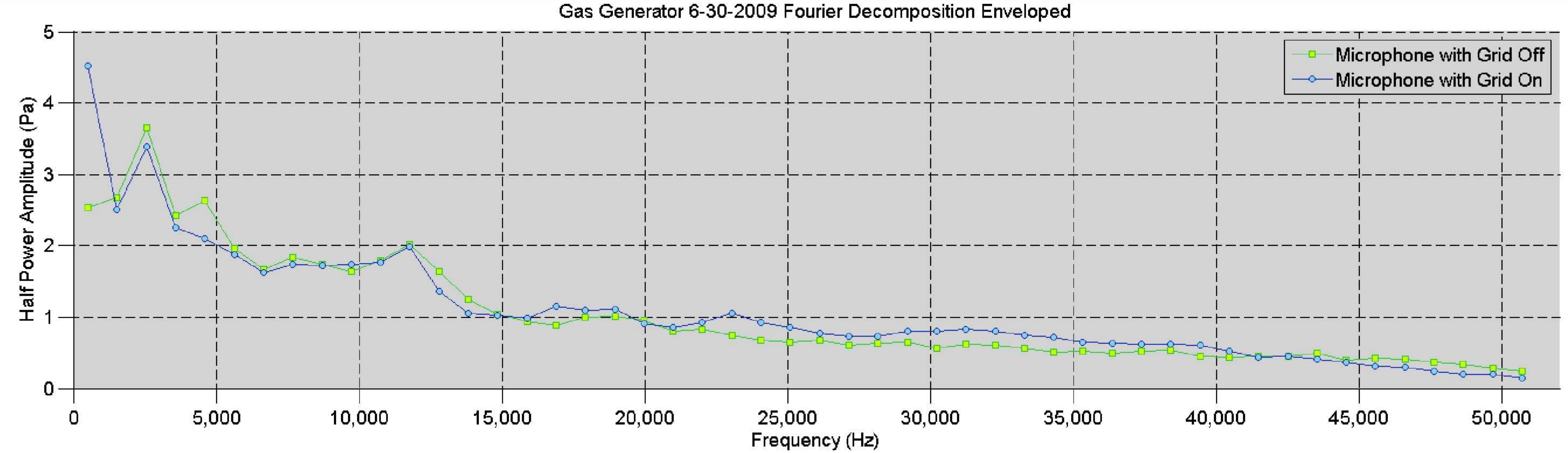
Linear Coherence of Data

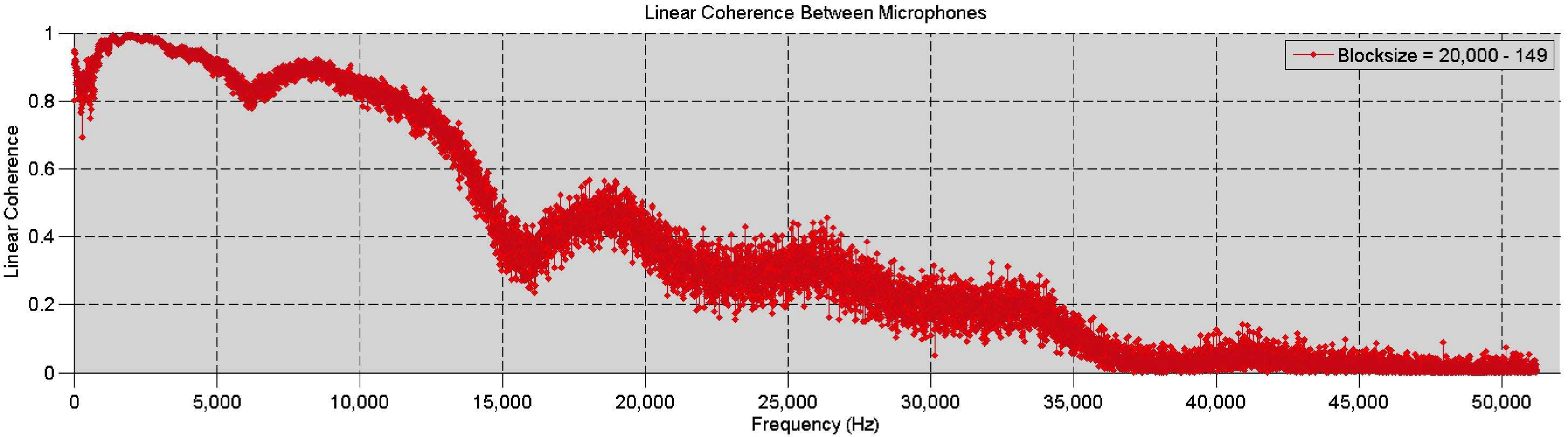
- Linear coherence decreases at higher frequencies
- Nonlinear system, insufficient amplitude, phasing?
- Recall test to test repeatability of magnitude response
- Tests show that linear coherence is sensitive to phasing
- Higher frequency increases phase shift
- FFT does not flatten up to 50,000 Hz



JACOBS
ESTS Group

Fast Fourier Transformation of Data

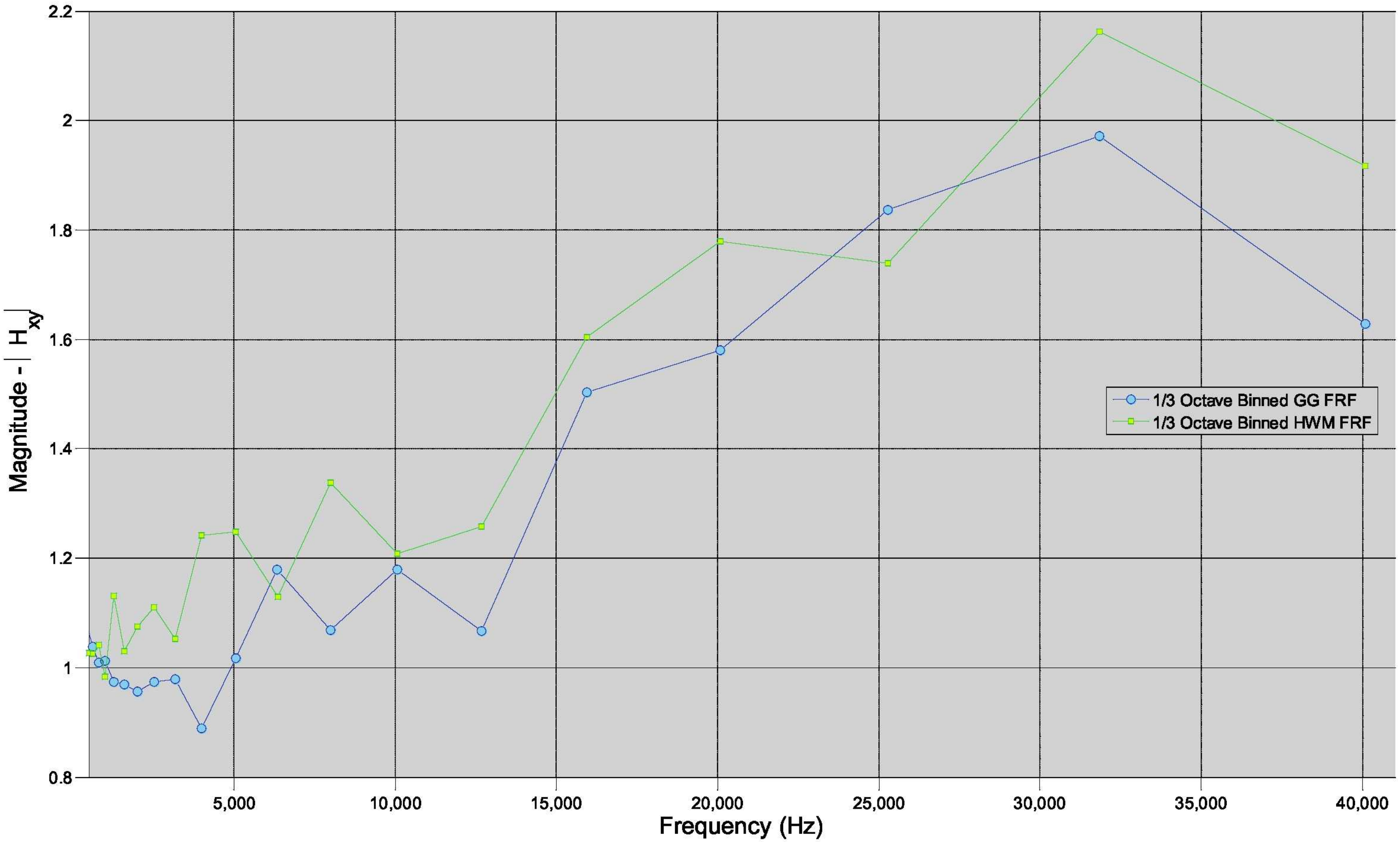




JACOBS
ESTS Group

Response comparisons

- Overlay free field grid response at 0 degs, FRF from HWM and FRF from GG



JACOBS
ESTS Group

Conclusions

- Mean FRF is best estimate to correct for grid frequency response distortions, even though the uncertainties are large
- While low coherence is concerning, repeatability of mean FRF supports the application of the mean FRF as a correction
- Distortions due to grid are sufficiently large in amplitude to warrant a correction

Conclusions Continued

- Mean FRF is NOT recommended for stability analysis or other system parameter identification
- For model realization purposes regarding stability and parameter identification, a time-varying or nonlinear model is recommended
- Phase response should only be considered in frequency regions where linear coherence is high

[redacted]

JACOBS
ESTS Group

References

[1] "Brüel & Kjær Microphone Handbook". Denmark, July 1996.

[2] Allen, C.S. and Vandra, K. "Microphone Corrections for Accurate In-Flow Acoustic Measurements at High Frequency. First Joint CEAS/AIAA Aeroacoustics Conference, CEAS/AIAA-95-150, June 1995.

[3] Heij, Christian et. Al. Introduction to Mathematical Systems Theory. Boston: Springer Science, 2007.

[4] Ljung, Lennart. System Identification: Theory for the User. New Jersey: Prentice Hall, 1987.

[5] "Product Data Sheet: DeltaTron Pressure-field ¼-inch Microphone – Type 4944A." Bruell & Kjaer. Naerum, Denmark.

[6] "Product Data Sheet: VT1432B Digitizer plus DSP." VXI Technology. Irvine, California. Vxitech.com.

[7] Lapin, Lawrence. Probability and Statistics for Modern Engineering. Waveland Press Inc: Illinois, 1998.

[8] Beckwith, Thomas et. al. Mechanical Measurements. Massachusetts: Addison-Wesley Publishing Company, 1995.

[9] Bendat, J.S. and Piersol, A.G., *Engineering Applications of Correlation and Spectral Analysis*, John Wiley & Sons, 1980.

[10] Coleman, Hugh W. and Steele, W. Glenn, Experimentation and Uncertainty Analysis for Engineers. 2nd ed. John Wiley & Sons, 1999.

[11] Liljencrants, Johan. "End Correction at a Flue Pipe Mouth."2006. http://www.fonema.se/mouthcorr/mouthcorr.htm

JACOBS
ESTS Group