

# Design of Structurally Efficient Tapered Struts 

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## ACKNOWLEDGMENTS

The Boeing Company, its Phantom Works business unit, and the Phantom Works Structural Technology group acknowledge the following personnel for their valuable contribution to this task order:

Frank Biele (Boeing design)
Jeff Eichinger (Boeing materials and processes)
Dan Hansen (Boeing management oversight)
Karen Hirota (Boeing design)
Dawn Jegley (NASA technical monitor)
Mike Koharchik (Boeing analysis)
Ross Messinger (Boeing principal investigator)
Eric Olason (Park Aerospace Structures project manager)

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## PREFACE

This report of task order NNL08AD08T of NASA contract NAS-1-NNL04AA11B, entitled Structures and Materials and Aerodynamic, Aerothermodynamic and Acoustics Technology for Aerospace Vehicles (SMAAATAV), summarizes Boeing contractor analysis efforts performed during the task order period of performance between August 7, 2008, and November 21, 2008. This report describes in detail the analytical study of two full-scale tapered composite struts and briefly describes the fabrication of two subscale demonstration struts.


#### Abstract

This final report describes in detail the analytical study of two full-scale tapered composite struts and briefly describes the fabrication of two subscale demonstration struts. The analytical study resulted in the design of two structurally efficient carbon/epoxy struts in accordance with NASA-specified geometries and loading conditions. Detailed stress analysis was performed of the insert, end fitting, and strut body to obtain an optimized weight with positive margins. Two demonstration struts were fabricated based on a well-established design from a previous Space Shuttle strut development program.


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### 1.0 INTRODUCTION

NASA has an enduring interest in high-performance structures for a wide variety of aerospace applications. One pervasive structural element is the structurally efficient strut, which can be used in a space frame or large assembly to support subsystems such as solar panels or propellant tanks. Composite materials are particularly suited for such struts since their unique strength and stiffness properties can be leveraged to optimize the structural efficiency of the strut. However, to fully implement this performance advantage, the composite strut should be fabricated and inspected with affordable, traceable, and repeatable processes.

The objective of this task order was to perform an analytical study of two structurally efficient, full-scale, tapered composite struts, and to fabricate a subscale strut demonstration article. As summarized below, the Boeing approach to achieve this objective was to leverage and extend recent experience on a Space Shuttle composite strut development program.

The first step of the analytical study was to identify design requirements and considerations as applicable to the analytical study, a production program, and the demonstration strut. Using these requirements and considerations, detailed design and stress analysis would determine the optimum (minimum) weight of the full-scale struts. One full-scale strut was required to carry a 44,000 -pound compression load and have a 135 -inch pin-to-pin length. The second strut was required to carry a compression load of 110,000 pounds and have a pin-to-pin length of 127 inches. In this report, the $44 \mathrm{~K}-\mathrm{lb}, 135$-inch strut and the $110 \mathrm{~K}-\mathrm{lb}, 127$-inch strut will be referred to as the 44 K and 110 K struts, respectively.

The approach for the subscale strut demonstration (hereinafter, demo) article was to 1) select a strut from a set of existing designs created during the Shuttle strut program, 2) fabricate at least one strut with a focus on improving laminate quality, and 3) identify process improvements. The selected strut design would be approximately half-scale, yet represent all the design features of a full-scale strut. Using the selected design, at least one demo strut would be fabricated by Park Aerospace Structures (hereinafter, Park), which participated in the Shuttle composite strut program. Based on experience from the fabrication of the demo strut(s), various process improvements would be identified and recommended for implementation during the fabrication of a future full-scale strut test article. These improvements may increase as-built composite material properties and thus further enhance strut performance.

### 2.0 ANALYTICAL STUDY

The analytical study consisted of the detail design and analysis of two full-scale carbon/epoxy struts in accordance with NASA-specified geometries and loading conditions.

### 2.1 Requirements and Considerations

The design requirements and considerations for the two struts are tabulated in Figure 2.1-1. Design requirements as specified by NASA are provided as the first four entries in the table. In addition to the geometry and load requirements, NASA required that no joints exist in the composite tube section to preclude any joint weight penalty. The remaining entries are design considerations that may have influenced the design. Parameters considered include materials, stacking sequence, thicknesses, cross-section shape, imperfections, ability to attach joints on the ends, manufacturing methods, complexity, damage tolerance, environmental degradation, and fatigue, and minimum gage. Many of these considerations were important for the Space Shuttle composite strut program (Section 3.1). Each requirement or consideration has an assumed requirement or consideration for an Altair lunar lander. Altair is an element of the NASA Exploration program to return to the Moon for extended duration missions. Each requirement or consideration is assigned a qualitative or quantitative value as appropriate for the analytical study, for a production program, and for the strut demonstration article.

An explanation of each requirement or consideration is provided in the following.
Compression load-Limit compression load for each strut was defined by NASA for the analytical study. Production struts may be designed for a wide range of loads.

Length - The pin-to-pin length of the two full-scale analytical struts was specified by NASA, and may represent the length of typical full-scale production struts in the Altair lunar lander. The demo strut was selected from the available set of Shuttle replacement strut designs. The selected strut has a length of 65.97 inches, which is about one-half length scale compared to the analytical study struts.

End fitting boundary condition-A pinned end fitting was required by NASA, which is typically implemented to ensure transmission of axial forces and to preclude transmission of bending moments.

Number of tube joints-No joints in the tube body were specified by NASA to ensure minimum weight.

Compression stability-The two struts in the analytical study were designed not to buckle under ultimate compression load.

Tension load-A tension load condition was not specified by NASA.
Reliability-Structure reliability is typically represented by a factor of safety, which is normally 1.4 for rigorously-qualified human-rated spacecraft.

Stiffness - The Altair lunar lander may have a stiffness requirement that drives the stiffness of individual structural elements. This consideration was not identified as a requirement by NASA, so the struts of the analytical study were not designed for stiffness.

Length adjustment - Production struts should be adjustable to accommodate some variation during final assembly. The Shuttle replacement struts were adjustable with a threaded post and lock nuts. A more weight-efficient and reliable approach is to machine the end fitting to the precise length or shape during final assembly.

PWDM08-0021
November 21, 2008

| Strut Design Requirements and Considerations |  | Design implementation |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Requirement/ Consideration | System-level <br> (Altair) requirement | Analytical Study | Production | Demo strut |
| Compression load (requirement) | Various | $\begin{aligned} & \hline \text { A: } 44,000 \mathrm{lb} \text { (limit) } \\ & \text { B: } 110,000 \mathrm{lb} \text { (limit) } \\ & \hline \end{aligned}$ | AR | 16,464 lb (ultimate) |
| Length (requirement) | Various | A: 135 inches <br> B: 127 inches (from center of ball end fittings) | AR | 65.97-inch pin-to-pin max. length (60.88-inch tube length) |
| End fitting boundary condition (requirement) | Accommodate various attachment requirements | A: pinned on both ends $B$ : pinned on both ends | AR | NI |
| Number of tube joints (requirement) | Minimize weight | 0 | 0 | 0 |
| Compression stability | No buckling at ultimate compression load | No buckling at ultimate compression load | No buckling at ultimate compression load | NI |
| Tension load | Various | NI | AR | 18,325 lb (ultimate) |
| Reliability | Ultimate factor of safety | 1.4 | 1.4 (1.2 proof FS) | 1.4 |
| Stiffness | System-level stiffness or modal frequency: Component modal frequency | NI (strength-driven design and material) | AR | NI (IM7/8552 laminate modulus of 16.2 msi ) |
| Length adjustment | Final assembly dimensional tolerances | Machine end fitting on final assy | Final machine end fitting or node feature | NI (no adjustment for screw-on end fittings) |
| Dimensional accuracy | Final assembly dimensional tolerances | Integrated insert and end fitting, see length adjustment | Integrated insert and end fitting, see length adjustment | NI (screw-on end fittings not dimensionally accurate) |
| Straightness | Minimize end-to-end bowing | NI (assume no bowing) | AR (assume max allowable bowing) | N |
| Tube internal pressure | Accommodate 14.7psig max. during launch | NI (relieving) | AR | NI |
| Operating temperature | Acceptable properties at min/max temperatures during (Altair) mission | NI (assume ambient temp.only) | AR (dependent on system design and material) | $\begin{array}{\|l\|} \hline \mathrm{NI} \text { (IM7/8552 temperature limit ( } \\ 250 \mathrm{~F} \text { to }+250 \mathrm{~F} \text { ) } \end{array}$ |
| Operating temperature change | Minimize thermal displacement | NI (strut tube IM7/8552 material has low CTE) | AR | NI (strut tube IM7/8552 material has low CTE) |
| Fatigue | Accommodate (Altair) mission thermomechanical cycles | NI (assume not a design driver) | AR | NI |
| Low-(high) energy impact/damage tolerance | Maximize residual strength after nondetectable impact damage | Interspersed 90 degree plies (90/0(4)/90/0(4)/90) | AR (Ply layup that balances mechanical properties and DT) | $\begin{aligned} & \text { Interspersed } 90 \text { degree plies } \\ & (90 / 0(4) / 90 / 0(4) / 90) \end{aligned}$ |
| Non-destructive inspectability | Accommodate damage/flaw detection | NI | $\begin{aligned} & \text { Thermography and C-scan } \\ & \text { (with standards) } \\ & \hline \end{aligned}$ | Thermography and C-scan (no standards) |
| Ground operations damage detection | Minimize nondetectable technicianinduced impact damage during ground ops | N | Fiberglass cover ply, acoustic emission sensor, etc. | N |
| Laminate quality | Maximize laminate properties with minimum material property knockdowns | Pristine (no knockdowns) | multiple debulks, autoclave cure | Multiple autoclave debulks maximized laminate quality |
| Dissimilar material compatibility | No degradation at component interfaces | Compatible Ti insert and Gr/Ep tube | Compatible Ti insert and $\mathrm{Gr} / \mathrm{Ep}$ tube | Compatible Ti insert and Gr/Ep tube |
| Material environmental effects and compatibility (durability) | Minimize degradation due to radiation, outgassing, etc. | NI | AR (coatings, etc) | N |
| Subsystem integration | Ability to attach wiring, secondary structure, instrumentation, insulation, etc. to strut tube | NI | AR | NI |
| Insert altachment to composite tube | High reliability, and high failure and degradation tolerance | Sinusoidal insert per PAS patent (no adhesive bondline failure modes) | Sinusoidal insert per PAS patent | Sinusoidal insert per PAS patent |
| Number of struts attached to single node | 11 max. | Tapered strut tube ends, with end fitting to attach 11 struts (max) to one node | AR | Tapered strut tube ends |
| End fitting type | Accommodate variety of end fittings with | NI | AR (machine ball, clevis, etc from blank) | screw-on ball and clevis |
| Process repeatability | Minimize material property knockdowns due to process variability | No material property knockdowns | Automated tow placement | PAS manual process |
| Process traceability | Maximize part reliability and performance | NI | Complete use of documentation and specifications | Limited use of documentation and specifications |
| Tube cross section | Maximize structural efficiency while considering subsystem attachment | Circular | AR (circular, square, open section, etc) | Circular |
| Diameter | Avoid interference with adjacent subsystems and structures | Optimize for min. mass A: 6.0 -inch ID <br> B: 6.5 -inch ID | AR | 2.233-inch ID |
| Minimum gage | Tube wall thickness required for adequate damage tolerance | NI | AR | $\begin{aligned} & \hline \mathrm{NI}((90 / 0(4) / 90 / 0(4) / 90) \text { layup } \\ & \text { above min gage) } \\ & \hline \end{aligned}$ |

Figure 2.1-1. Design Requirements and Considerations

Straightness-Excessive bowing will significantly reduce compression strength and stiffness. The Space Shuttle replacement struts had a bowing requirement of less than 0.030 -inch plus 0.001 inch for each additional inch above 30 inches.

Tube internal pressure-The strut tubes will experience an internal positive pressure during launch into low-Earth orbit. This load was ignored for the analytical study struts.

Operating temperature-The struts may experience a wide range of temperatures, depending on the degree of thermal protection and location of the struts in the Altair lander or other spacecraft. Without further definition, the analytical study assumed an ambient temperature.

Operating temperature change-A change in temperature will induce thermal displacements and stresses that can affect overall stiffness, strength, and dimensional accuracy. Temperatures changes will depend on the degree of thermal protection and location of the struts in the Altair lander or other spacecraft.

Fatigue-The Altair lander may experience a series of thermal-mechanical load cycles, while in low-Earth orbit, during transfer to lunar orbit, and on the lunar surface. These load cycles were not defined at this time, so were not considered in the analytical study.

Low-(high) energy impact/damage tolerance - The primary source of damage is expected to occur during final assembly and ground operations. The strut body laminate should be designed to tolerate such low-energy ground impacts. Any low- or high-energy impact damage during flight operations is unlikely given the expected environmental protection and surrounding subsystems and in the Altair lander.

Nondestructive inspectability-After cure, the struts need to be inspectable to ensure laminate quality.

Ground operations damage detection-Impact damage should be detectable either during the impact event or during subsequent inspection. For example, the Space Shuttle replacement strut program investigated the use of a single fiberglass composite cover ply that readily and visually delaminated from the carbon/epoxy strut body when subjected to local impact.

Laminate quality-The strut body should have a minimum of wrinkles, voids, and porosity to maximize mechanical properties and minimize associate analytical knockdown factors..

Process repeatability-The production fabrication process should be highly repeatable to minimize statistical knockdowns from material properties. The analytical study assumed no knockdown from such variability, other than that inherent in the A-basis allowable values.

Dimensional accuracy-The Altair lander may require well-controlled dimensional tolerances to accurately support numerous subsystems. The production strut design incorporates a spherical node that is integrated with patented Park end fitting. The spherical node is tightly integrated with the node.

Dissimilar material compatibility-Direct contact between carbon-based composites and metals such as aluminum should be avoided to preclude galvanic corrosion. The demo struts used compatible titanium end fittings.

Material environmental effects and compatibility (durability)-The demo strut tube IM7/8552 material will be exposed to deep space vacuum and radiation during an Altair mission for period of about two weeks.

Subsystem integration-Ability to attach wiring, secondary structure, instrumentation, and insulation can affect the strut design. Since there is no current definition of this integration, the analytical struts did not consider these attachments:

Insert attachment to composite tube-Typical end fittings in a composite strut are metallic. The Park end fitting features a unique corrugated surface to create a mechanical lock between the end fitting and composite overwrap.

Number of struts attached to single node-As many as 11 struts may be attached to a single node. The tapered ends of the analytical and demo strut designs allows close nesting of struts.

End fitting type-The Altair lunar lander may have various structural requirements that will are satisfied with ball, clevis, or other end fittings.

Process repeatability-Manual fabrication processes are inherently more variable than automated processes. Higher resulting material properties knockdowns lower strut performance.

Process traceability-Documentation of materials and process ensures quality and repeatability. Altair will likely require the most rigorous traceability of the production struts.

Tube cross section-While the analytical and demo struts used a circular cross section, other sections may be more suitable for specific purposes in the Altair lander.

Diameter-The compression stability of the strut is largely determined by the body diameter, but may be constrained by adjacent subsystems in the Altair/spacecraft assembly.

Minimum gage Lightly-loaded struts may only need a few laminate plies, which will be constrained by damage tolerance or producibility.

### 2.2 Design

Based on the applicable requirements and considerations in Section 2.1, detailed design of the 44 K and 110 K struts was performed to document the results of the detailed analysis in Section 2.3.

### 2.2.1 44K Strut

The 44 K strut assembly consists of a composite body and two end fittings (Figure 2.2.1-1). The composite body is a single-piece laminate with tapered ends that capture the two end fittings. The length between the centers of the two ball end fittings is 135.0 inches. The cylindrical portion of the body has an inner diameter of 6.00 inches.

The material specification and ply layup of the 44 K strut are detailed in Figure 2.2.1-2. The carbon/epoxy materials are IM7/8552-1 prepreg tow and IM7/8552-2 prepreg tape, which are used for the circumferential ( $90-\mathrm{deg}$ ) and axial ( $0-\mathrm{deg}$ ) plies, respectively. The tube layup is $\left(90,0_{4}, 90,0_{3}, 90,0_{4}, 90\right)$ for a total of 15 plies. The ply layup in the end fitting region consists of the tube plies and additional hoop ( $90-\mathrm{deg}$ ) plies that secure the tube plies to the end fitting.

The end fitting of the 44 K strut is made of $6 \mathrm{Al}-4 \mathrm{~V}$ titanium alloy and consists of an integral insert and ball (Figure 2.2.1-3). The insert design, patented by Park (6,379,763), has three corrugations which create a high-performance and high-reliability mechanical lock between the composite and insert. The spherical ball portion of the end fitting is installed into a node to provide a pinned (zero moment) end condition. The ball is shown in an outlined block, indicating that the ball is final machined from a blank after the end fitting is cured with the composite body.


Figure 2.2.1-1. 44K Strut Assembly Design

PLY Table - 014 ASSY

| PLY No. | MATERIAL | ORIENTATION |
| :---: | :---: | :---: |
| 1 | (x) | $90^{\circ}$ |
| 2-5 | (v) | $0^{\circ}$ |
| 6 | (x) | $90^{\circ}$ |
| 7-9 | (v) | $0^{\circ}$ |
| 10 | (x) | $90^{\circ}$ |
| 11-14 | (r) | $0 \cdot$ |
| 41 | (x) | $90^{\circ}$ |

(x)C. IM7-8852-1 TOW FOR 44K STRUT: FAW $=138$ GSM (. 0055 (NCH/PLY) RESIN CONTENT $=35 \%$
TOW WIOTH $=0.125 \mathrm{iNCH}$
(7) 0 . IM7-8552-2 TAPE FOR 44K STRUT SOURCE $=$ HEXCEL
FAW RRESIN $=178$ GSNTENT $=35 \%$
REST


Figure 2.2.1-2. 44K Strut Ply Layup


Figure 2.2.1-3. 44K Strut End Fitting Detail Design

### 2.2.2 110K Strut

The 110 K strut assembly consists of a composite body and two end fittings (Figure 2.2.2-1). The composite body is a single-piece laminate with tapered ends that capture the two end fittings. The length between the centers of the two ball end fittings is 127.0 inches. The cylindrical portion of the body has an inner diameter of 6.50 inches.

The material specification and ply layup of the 110 K strut are detailed in Figure 2.2.2-2. The carbon/epoxy materials are IM7/8552-1 prepreg tow and IM7/8552-2 prepreg tape, which are used for the circumferential ( $90-\mathrm{deg}$ ) and axial ( $0-\mathrm{deg}$ ) plies, respectively. The tube layup is $\left(90,0_{3}, 90,0_{3}, 90,0_{4}, 90,0_{4}, 90,0_{3}, 90,0_{3}, 90\right)$ for a total of 27 plies. The ply layup in the end fitting region consists of the tube plies and additional hoop ( $90-\mathrm{deg}$ ) plies that secure the tube plies to the end fitting.

The end fitting of the 110 K strut is made of $6 \mathrm{Al}-4 \mathrm{~V}$ titanium alloy and consists of an integral insert and ball (Figure 2.2.2-3). The insert design, patented by Park $(6,379,763)$, has three corrugations, which create a high-performance and high-reliability mechanical lock between the composite and insert. The spherical ball portion of the end fitting is installed into a node to provide a pinned (zero moment) end condition. The ball is shown in an outlined block, indicating that the ball is final machined from a blank after the end fitting is cured with the composite body.


Figure 2.2.2-1. 110K Strut Assembly Design

| PLY Table - 023 ASSY |  |  |
| :---: | :---: | :---: |
| PLYNO. | MATERIAL | ORIENTATION |
| 1 | ( $\times$ | $90^{\circ}$ |
| 2-4 | (Y) | $0^{\circ}$ |
| 5 | (x) | $90^{\circ}$ |
| 6-9 | (Y) | $0{ }^{\circ}$ |
| 10 | (x) | $90^{\circ}$ |
| 11-14 | (Y) | $0^{\circ}$ |
| 15 | X ${ }^{\text {x }}$ | $90^{\circ}$ |
| 16-19 | (Y) | $0^{\circ}$ |
| 20 | X ${ }^{\text {a }}$ | $90^{\circ}$ |
| 21-23 | (r) | $0^{\circ}$ |
| 80 | (x) | $90^{\circ}$ |

(X) F. IM7-8852-1 IOW FOR 110 K STRUT: FAW $=138$ GSM (. 0055 INCH/PLY)
RESIN CONTENT
I REW WIDTH $=0.125 \mathrm{iNCH}$
(Y) G. IM7-8552-2 TAPE FOR 110 K STRUT: SOURCE $=$ HEXCEL
FAW 178 GSM $(.0071$ INCH/PLY $)$
RESIN CONTENT $=35 \%$


149764-005.1
Figure 2.2.2-2. 110K Strut Ply Layup


149764-006
Figure 2.2.2-3. 110K Strut End Fitting Detailed Design

### 2.2.3 Node Attachment

The ball end fitting described in Sections 2.2.1 and 2.2.2 is but one of a wide variety of end fittings that can be incorporated into the strut assembly. The ball end fitting would be installed into a spherical recess in the receiving node. Other examples of node attachment options include a post, receiving ball, single clevis, double clevis, and receiving cruciform (Figure 2.2.3-1). The figure also illustrates how these end fittings can be combined to create complex assemblies. Furthermore, each end of the strut can have a unique end fitting. To illustrate this feature, the demo strut may include the ball end fitting described in Sections 2.2.1 and 2.2.2 on one end, and/or the clevis end fitting on the other (Section 3.0).


Figure 2.2.3-1. Node Attachment Concepts

### 2.3 Analysis

Detailed stress analysis of the 44 K and 110 K struts was performed to obtain optimized (minimum) weights. The sizing satisfies the requirements and considerations applicable to the analytical study as described in Section 2.1. The analysis began with the optimization (minimum-weight) of the strut tube diameter. Given the optimum diameter, detailed stress analysis was performed to confirm buckling strength and other positive margins. The optimized weight of the two struts is summarized in Figure 2.3-1. The following analysis shows that both struts have positive margins of safety and satisfy other important parameters (Figure 2.3-2).

|  | Weight (lb) |  |
| :--- | :---: | :---: |
|  | 44 K Strut | 110 K Strut |
| Body | 13.31 | 23.28 |
| Insert | 1.20 | 3.56 |
| Ball | 1.50 | 5.40 |
| Total | 16.01 | 32.24 |

Figure 2.3-1. 44K and 110K Strut Weight Summary

|  | MSNalue |  |
| :--- | :---: | :---: |
|  | 44 K Strut | 110 K <br> Strut |
| End fitting <br> bearing | 5.58 | 5.14 |
| End fitting hoop <br> stress | 0.52 | 0.33 |
| Insert tension <br> stress | 0.30 | 0.30 |
| Insert hoop <br> stress | 0.52 | 0.33 |
| Insert joint shear <br> stress (<5000 <br> psi) | 3856 psi | 4997 psi |
| Strut buckling, <br> Pult > Pcr (lb) | $64,451>$ | $160,623>$ |
| First natural <br> frequency, f1 <br> (Hz) | 61,600 | 154,000 |
| Compression | 3.63 | 79 |
| Crippling/Local <br> Instability | 0.26 | 0.11 |
| Figure 2.3-2. Sun |  | 0.31 |

Figure 2.3-2. Summary of Margins of Safety and Critical Parameters

### 2.3.1 Strut Diameter Optimization

### 2.3.1.1 44K Strut

The 44 K strut diameter optimization was performed by Park using an established Excelbased analysis method. The diameter of the constant-section of the strut body and the number of tube plies were varied to achieve the required compression strength (Figure 2.3.1.1-1). Based on those results, the minimum weight is shown in Figure 2.3.1.1-2 when the tube inner diameter is 6.0 inches and the number of plies is 10 .

| ID <br> (in) | No. of <br> O-deg <br> Plies | OD (in) | Failure <br> Mode | Weight <br> (Ib) |
| ---: | :---: | ---: | :--- | :--- |
| 4.0 | 27 | 4.47 | Buckling | 24.41 |
| 4.5 | 20 | 4.85 | Buckling | 20.32 |
| 5.0 | 15 | 5.27 | Buckling | 17.37 |
| 5.5 | 12 | 5.72 | Buckling | 15.39 |
| 6.0 | 10 | 6.19 | Crippling | 14.61 |
| 6.5 | 10 | 6.69 | Cripling | 15.66 |
| 7.0 | 9 | 7.17 | Crippling | 15.56 |
| 7.5 | 9 | 7.67 | Crippling | 16.53 |
| 8.0 | 9 | 8.17 | Crippling | 17.51 |
| 8.5 | 9 | 8.67 | Crippling | 18.48 |
| 9.0 | 9 | 9.17 | Crippling | 19.45 |
| 9.5 | 9 | 9.67 | Crippling | 20.42 |
| 10.0 | 9 | 10.17 | Crippling | 21.40 |



Figure 2.3.1.1-2. 44K Strut Weight as a Function of Diameter

Figure 2.3.1.1-1. 44K Strut Optimization
Results

### 2.3.1.2 110K Strut

The 110 K strut diameter optimization was performed using an established Excel-based analysis method at Park. The diameter of the constant-section of the strut body and the number

| ID (In) | No. of <br> O-deg <br> Plies | OD (in) | Failure <br> Mode | Weight <br> (Ib) |
| :---: | :---: | :---: | :--- | :--- |
| 4.0 | 52 | 4.89 | Buckling | 47.30 |
| 4.5 | 40 | 5.19 | Buckling | 40.67 |
| 5.0 | 31 | 5.54 | Buckling | 35.44 |
| 5.5 | 24 | 5.92 | Buckling | 30.64 |
| 6.0 | 19 | 6.34 | Buckling | 27.33 |
| 6.5 | 17 | 6.80 | Crippling | 26.27 |
| 7.0 | 17 | 7.30 | Crippling | 27.83 |
| 7.5 | 17 | 7.80 | Crippling | 28.39 |
| 8.0 | 16 | 8.28 | Crippling | 29.70 |
| 8.5 | 16 | 8.78 | Crippling | 31.18 |
| 9.0 | 16 | 9.28 | Crippling | 32.67 |
| 9.5 | 15 | 9.77 | Crippling | 32.69 |
| 10.0 | 15 | 10.27 | Crippling | 34.09 |
| 10.5 | 15 | 10.77 | Crippling | 35.50 |
| 11.0 | 15 | 11.27 | Crippling | 36.91 |
| 11.5 | 14 | 11.75 | Crippling | 36.56 |
| 12.0 | 14 | 12.25 | Crippling | 37.89 |

Figure 2.3.1.2-1. 110K Strut Optimization Results
of tube plies were varied to achieve the required compression strength (Figure 2.3.1.2-1). Based on those results, the minimum weight is shown in Figure 2.3.1.2-2 when the tube inner diameter is 6.5 inches and the number of plies is 17 .


Figure 2.3.1.2-2. 110K Strut Weight as a Function of Diameter

### 2.3.2 End Fitting Analysis

The end fitting (Figure 2.3.2-1) is a titanium part consisting of a shaft with a ball end. An axial hole through the center is used during fabrication for access to and removal of the drill rod and plaster mandrel.

## Material Properties

$6 \mathrm{Al}-4 \mathrm{~V}$ annealed titanium
$\mathrm{E}=16,000,000 \mathrm{psi}$
$\mathrm{F}_{\mathrm{TU}}=130,000 \mathrm{psi}$
$\mathrm{F}_{\mathrm{CY}}=126,000 \mathrm{psi}$
$\mathrm{F}_{\mathrm{brU}}=191,000 \mathrm{psi}$


## End Fitting Dimensions

|  | 44 k | 110 k |
| :--- | :---: | :---: |
| $\mathrm{D}_{\text {bal }}[\mathrm{in}]$ | 2.7 | 4.2 |
| $\left.\mathrm{D}_{\text {shaft }} \mathrm{in}\right]$ | 1.336 | 2.145 |
| $\mathrm{D}_{\text {hole }}[\mathrm{in}]$ | 1.0 | 1.625 |
| Length $[\mathrm{in}]$ | 3.0 | 4.0 |

Figure 2.3.2-1. End fitting dimensions

## Compression

Compression loads are transmitted to the end of the ball through surface bearing, and distributed over the effective surface area.

$$
\begin{aligned}
& \theta_{\text {min }}=\sin ^{-1}\left(\frac{D_{\text {hole }}}{D_{\text {ball }}}\right) \\
& \theta_{\max }=45^{\circ} \\
& \theta_{\text {ave }}=\left(\theta_{\text {max }}+\theta_{\text {max }}\right) / 2
\end{aligned}
$$

|  | 44 k | 110 k |
| :---: | :---: | :---: |
| $\theta_{\min }$ | $21.74^{\circ}$ | $22.76^{\circ}$ |
| $\theta_{\max }$ | $45^{\circ}$ | $45^{\circ}$ |
| $\theta_{\text {ave }}$ | $33.37^{\circ}$ | $33.88^{\circ}$ |


$149764-011$

## Buckling

Buckling of the end fitting depends on the stiffness of the rest of the strut and therefore cannot be performed independently of the strut. The end fitting is included in the overall strut buckling analysis.

## Surface Bearing

Compression loads are distributed over the effective surface contact area at the end of the ball. The surface area of a spherical cap of radius $r$ and height $h$ is given by
$A=2 \pi r h$
In the case of the ball end, this will be the area of the cap above $\theta_{\text {max }}$ minus the area above $\theta_{\text {min }}$
$A=2 \pi\left(D_{\text {ball }} / 2\right)\left[\left(D_{\text {bal }} / 2\right)\left(1-\cos \theta_{m a x}\right)\right]$
$-2 \pi\left(D_{\text {ball }} / 2\right)\left[\left(D_{\text {ball }} 2\right)\left(1-\cos \theta_{\min }^{m a x}\right)\right]$

|  | 44 k | 110 k |
| :---: | :---: | :---: |
| A | $2.54 \mathrm{in}^{2}$ | $5.96 \mathrm{in}^{2}$ |



Assuming a frictionless interface, the bearing forces act normal to the surface of the ball

RbULT $=1.4 \mathrm{P}_{\mathrm{app}} / \cos \theta_{\mathrm{ave}}$


149764013
The bearing stress is then
$\sigma_{b r U}=R_{b U L T} / A$
For the 44 k strut,
$\sigma_{\text {bru }}=73,760 / 2.54=29,040 \mathrm{psi}$
$\mathrm{MS}=191,000 / 29,040-1=5.58$
For the 110 k strut,
$\sigma_{\text {bru }}=185,498 / 5.96=31,124 \mathrm{psi}$
$\mathrm{MS}=191,000 / 31,124-1=5.14$

## Hoop Stress

The radial component of the contact force will result in a hoop stress in the end of the ball.
The minimum radius of the annular section is the radius of the hole.
The maximum radius is given by
$\mathrm{r}_{\text {max }}=\left(\mathrm{D}_{\text {ball }} / 2\right) \sin \theta_{\text {max }}$
and the average radius is the radius at the centroid of the (approximate) triangle
$r_{\text {ave }}=r_{\text {min }}+\left(r_{\text {max }}-r_{\text {min }}\right) / 3$
the running force is then the radial force divided by the average circumference
$\mathrm{R}_{\mathrm{h}}=\mathrm{R}_{\mathrm{bULT}} \sin \theta_{\text {ave }} / 2 \pi \mathrm{r}_{\mathrm{ave}}$

|  | 44 k | 110 k |
| :---: | :---: | :---: |
| $\mathrm{r}_{\min }$ | 0.5 in | 0.8125 in |
| $\mathrm{r}_{\max }$ | 0.9546 in | 1.4849 in |
| $\mathrm{r}_{\text {ave }}$ | 0.6515 in | 1.0366 in |
| $\mathrm{R}_{\mathrm{h}}$ | $9910 \mathrm{lb} / \mathrm{in}$ | $15876 \mathrm{lb} / \mathrm{in}$ |



The cross-sectional area of the ring is determined from the areas of the various segments

$$
\begin{aligned}
& A_{1}=1 / 2\left(D_{\text {ball }} / 2\right)^{2} *\left[2 \theta_{\max }-\sin \left(2 \theta_{\text {max }}\right)\right] \\
& A_{2}=1 / 2\left(D_{\text {ball }} / 2\right)^{2 *}\left[2 \theta_{\min }-\sin \left(2 \theta_{\min }\right)\right] \\
& A_{3}=D_{\text {hole }}\left[\left(D_{\text {ball }} / 2\right) \cos \theta_{\min }-\left(D_{\text {ball }} / 2\right) \cos \theta_{\text {max }}\right] \\
& A_{4}=\left(A_{1}-A_{2}-A_{3}\right) / 2
\end{aligned}
$$



|  | 44 k | 110 k |
| :---: | :---: | :---: |
| $A_{1}$ | $0.5201 \mathrm{in}^{2}$ | $1.2586 \mathrm{in}^{2}$ |
| $A_{2}$ | $0.0645 \mathrm{in}^{2}$ | $0.1786 \mathrm{in}^{2}$ |
| $A_{3}$ | $0.2994 \mathrm{in}^{2}$ | $0.7337 \mathrm{in}^{2}$ |
| $A_{4}$ | $0.0781 \mathrm{in}^{2}$ | $0.1731 \mathrm{in}^{2}$ |


$149764-015$
The hoop stress is then given by
$\sigma_{\text {hoop }}=\mathbf{R}_{\mathrm{b}} \mathrm{r}_{\mathrm{ave}} / \mathrm{A}_{4}$
For the 44 k strut,
$\sigma_{\text {hoop }}=9910 * 0.6515 / 0.0781=82,641 \mathrm{psi}$
$\mathrm{MS}=126,000 / 82,641-1=0.52$
For the 110 k strut,
$\sigma_{\text {hoop }}=15,876 * 1.0366 / 0.1731=95,060 \mathrm{psi}$
$\mathrm{MS}=126,000 / 95,060-1=0.33$

### 2.3.3 Insert Analysis

The strut insert is a titanium part with a basically cylindrical shape (Figure 2.3.3-1). The outer surface consists of a series of peaks and valleys. The strut body laminate is mechanically locked into these features by the overwrap plies, eliminating the need for adhesives or fasteners to transmit the load from the metal end fitting to the composite strut body. The hole through the center is used during fabrication for access to and removal of the drill rod and plaster mandrel.

## Material Properties

$6 \mathrm{Al}-4 \mathrm{~V}$ annealed titanium
FTU $=130 \mathrm{ksi}$


| Dimensions |  |  |
| :---: | :---: | :---: |
|  | 44k | 110k |
| $\mathrm{D}_{\text {waist }}$ [in] | 1.336 | 2.145 |
| $\mathrm{D}_{\text {bul }}$ [in] | 1.772 | 2.686 |
| $\mathrm{D}_{\text {hole }}$ [in] | 1.000 | 1.625 |
| Length [in] | 3.272 | 4.061 |

Figure 2.3.3-1. Insert dimensions
Assuming the insert reacts the entire tension load (neglecting any load transmitted to the composite by adhesion or friction), stress at the waist is given by

$$
\sigma=P / A=\frac{P}{\frac{\pi}{4}\left(\mathrm{D}_{\text {waist }}{ }^{2}-\mathrm{D}_{\text {hole }}{ }^{2}\right)}
$$

For the 44 k strut,
$\sigma=\frac{1.4 * 44,000}{\frac{\pi}{4}\left(1.336^{2}-1.000^{2}\right)}=99,926 \mathrm{psi}$
$\mathrm{MS}=130,000 / 99,926-1=0.30$
For the 110 k strut,
$\sigma=\frac{1.4 * 110,000}{\frac{\pi}{4}\left(2.145^{2}-1.625^{2}\right)}=100,020 \mathrm{psi}$
MS $=130,000 / 100,020-1=0.30$
In addition to the tensile stress calculation, a "joint shear" calculation is performed by the vendor, with a maximum equivalent shear stress design limit of $5,000 \mathrm{psi}$.

The shear area is determined at the average diameter of the insert:
$A=\pi D_{\text {ave }} L=\pi\left[\left(D_{\text {waist }}+D_{\text {bulb }}\right) / 2\right] L$
For the 44 k strut,
$\mathrm{A}=\pi[(1.336+1.772) / 2](3.272)=15.974 \mathrm{in}^{2}$
$\tau=\mathrm{P} / \mathrm{A}=1.4 * 44,000 / 15.974=3856 \mathrm{psi}$
For the 110 k strut,
$\mathrm{A}=\pi[(2.145+2.686) / 2](4.061)=30.817 \mathrm{in}^{2}$
$\tau=\mathrm{P} / \mathrm{A}=1.4 * 110,000 / 30.817=4997 \mathrm{psi}$
In both cases, the equivalent shear stress is below the design limit of 5000 psi . A margin of Safety is not calculated since this is a design limit and not a true material allowable.

### 2.3.4 Buckling Analysis

A Newmark buckling analysis method was used to account for the variable cross-section of the strut. This iterative approach begins with an assumed buckling mode shape; each successive iteration generates a mode shape and corresponding buckling load. The iteration has converged when the difference in successive buckling load predictions is within the desired accuracy.

The strut is divided axially into elements and nodes, such that a node lies at each critical change in geometry (such as the transition between the end fitting and the composite strut body). Smooth transitions, such as those between tapered composite sections and straight composite sections, do not require a node to be located at the transition.

## Material Properties (A-basis)

IM7-8552 Gr/E
$\mathrm{F}_{\mathrm{CU}}{ }^{\mathrm{L}}=191.0 \mathrm{ksi}$
$\mathrm{E}_{\text {longitudinal }}=21,000,000 \mathrm{psi}$
$\mathrm{E}_{\text {transverse }}=1,100,000 \mathrm{psi}$
$\rho=0.0555 \mathrm{lb} / \mathrm{in}^{3}$

### 2.3.4.1 44K Strut

The 44 K strut is divided into 90 axial segments, located in the end fitting, insert, taper, and straight sections of the strut. Given the strut length of 135 inches, this corresponds to a segment length of 1.5 inches. Along with dividing evenly into the overall strut length, this segment length also divides evenly into the 3 inch length of the end fitting, conveniently placing a node at the transition between the end fitting and strut body.

## End Fitting Segments

The titanium end fitting portion is analyzed as a straight cylindrical shaft - the added material at the ball is neglected. This assumption is only slightly conservative, since the properties near the end have little effect on the buckling capability.

$$
\begin{aligned}
& \mathrm{E}=160,000,000 \mathrm{psi} \\
& \mathrm{I}=\pi / 4\left(\mathrm{r}_{\mathrm{o}}^{4}-\mathrm{r}_{\mathrm{i}}^{4}\right)=\pi / 4\left(\left(\mathrm{D}_{\text {shaft }} / 2\right)^{4}-\left(\mathrm{D}_{\text {hole }} / 2\right)^{4}\right) \\
& \mathrm{I}=\pi / 4\left[(1.336 / 2)^{4}-(1.0 / 2)^{4}\right]=0.1073 \mathrm{in}^{4} \\
& \mathrm{EI}=1,716,766 \mathrm{lb}-\mathrm{in}^{2}
\end{aligned}
$$



Dimensions

|  | 44 k | 110 k |
| :--- | :---: | :---: |
| Length [in] | 135 | 127 |
| $\mathrm{~L}_{\text {end fiting }}$ [in] | 3 | 4 |
| $\mathrm{~L}_{\text {insert }}$ [in] | 3.272 | 4.061 |
| $\mathrm{D}_{\mathrm{i} / \text { midspan }}$ [in] | 6 | 6.5 |
| $\# 0^{\circ}$ plies | 11 | 18 |
| $\# 90^{\circ}$ plies | 4 | 6 |
| $\mathrm{t} 0^{\circ}$ plies [in] | 0.00711 |  |
| $\mathrm{t} \mathrm{90}^{\circ}$ plies [in] | 0.0055 |  |
| $\theta_{\text {taper }}$ | $4^{\circ}$ |  |

Figure 2.3.4-1. Strut Parameters

## Straight Segments (Midspan)

The straight section in the middle of the strut is a simple layup of $0-\mathrm{deg}$ and $90-\mathrm{deg}$ plies. The diameter of the mandrel is 6.0 inches at midspan, and it is wrapped with $110-\mathrm{deg}$ plies and $490-$ deg plies. Each 0 -deg ply is 0.00711 inch thick; each 90 -deg ply is 0.0055 inch thick

$$
\begin{aligned}
& \mathrm{r}_{\mathrm{i} / \text { midspan }}=6 / 2=3 \mathrm{in} \\
& \mathrm{t}_{0 / \text { midspan }}=11 * 0.00711=0.07821 \mathrm{in} \\
& \mathrm{t}_{90}=4 * 0.0055=0.022 \mathrm{in} \\
& \mathrm{t}_{\mathrm{lam} / \text { midspan }}=\mathrm{t}_{0 / \text { midspan }}+\mathrm{t}_{90}=0.07821+0.022=0.10021 \mathrm{in} \\
& \mathrm{r}_{0} / \text { midspan }
\end{aligned}=\mathrm{r}_{\mathrm{i} / \mathrm{midspan}}+\mathrm{t}_{\text {lam } / \text { midspan }}=3+0.10021=3.10021 \mathrm{in} \text {. }
$$

The modulus of elasticity of the laminate was approximated using the moduli and relative areas of $0-\mathrm{deg}$ and $90-\mathrm{deg}$ plies.
$\mathrm{E}_{\text {longitudinal }}=21,000,000 \mathrm{psi}$
$\mathrm{E}_{\text {transverse }}=1,100,000 \mathrm{psi}$
$\mathrm{E}_{\text {lam } / \text { midspan }}=\left(\mathrm{t}_{0} /\right.$ midspan $\left./ \mathrm{t}_{\text {lam } / \text { midspan }}\right) * \mathrm{E}_{\text {longitudinal }}+\left(\mathrm{t}_{90} / \mathrm{t}_{\text {lam } / \text { midspan }}\right) * \mathrm{E}_{\text {transverse }}$
$\mathrm{E}_{\text {lam } / \text { midspan }}=(0.07821 / 0.10021) * 21,000,000+(0.022 / 0.10021) * 1,100,000=16,631,175 \mathrm{psi}$
The moment of inertia of the midspan section is

$$
\begin{aligned}
& I=\pi / 4\left(r_{0} / \text { midspan }{ }^{4}-r_{i} / \text { midspan }_{4}^{4}\right) \\
& I=\pi / 4\left[(3.10021)^{4}-(3.0)^{4}\right]=8.9356 \mathrm{in}^{4} \\
& E I=148,609,101 \mathrm{lb}^{4}-\mathrm{in}^{2}
\end{aligned}
$$

## Insert Segments

The insert section of the strut was analyzed considering only the composite strut body plies the titanium insert and the overwrap plies are ignored.

The diameter of the bulb ( $\mathrm{D}_{\text {bulb }}$ ) on the insert was taken as the inner diameter of the composite
$\mathrm{r}_{\mathrm{i} / \text { insert }}=1.772 / 2=0.886$ in
The thickness of the $0-\mathrm{deg}$ plies on the insert can be determined from the fact that the cross sectional area of the $0-\mathrm{deg}$ plies is constant over the length of the strut. The diameter and thickness of the $0-\mathrm{deg}$ plies is known at the midspan.
$\left.\mathrm{A}_{0}=\pi\left(\mathrm{r}_{\mathrm{i} / \mathrm{midspan}}+\mathrm{t}_{0 / \mathrm{midspan}}\right)^{2}-\mathrm{r}_{\mathrm{i} / \mathrm{midspan}}{ }^{2}\right]=\pi *\left[(3+0.07821)^{2}-3^{2}\right]=1.493$ in $^{2}$
$\mathrm{t}_{0} /$ insert $=\left(\left[\left(\mathrm{A}_{0} / \pi\right)+\mathrm{r}_{\mathrm{i} / \text { insert }}\right]^{2 / 2}-r_{i / \text { insert }}\right.$
$\mathrm{t}_{0} /$ insert $=\left(\left[(1.493 / \pi)+0.886^{2}\right]^{1 / 2}-0.886=0.237\right.$ in
Since the thickness of the $90-\mathrm{deg}$ plies is constant over the length of the strut, the thickness of the laminate at the insert can be determined.

$$
\mathrm{t}_{\text {lam } / \text { insert }}=\mathrm{t}_{0 / \text { insert }}+\mathrm{t}_{90}=0.237+0.022=0.259 \mathrm{in}
$$

The outer radius of the laminate at the insert is
$r_{0} /$ insert $=r_{i / \text { insert }}+t_{\text {lam } / \text { insert }}$
$\mathrm{r}_{0} /$ insert $=0.886+0.259=1.145$ in
The modulus of elasticity of the laminate is approximated using the moduli and relative areas of $0-\mathrm{deg}$ and $90-\mathrm{deg}$ plies.
$\mathrm{E}_{\text {lam } / \text { insert }}=\left(\mathrm{t}_{0} /\right.$ insert $\left./ \mathrm{t}_{\text {lam } / \text { insert }}\right) * \mathrm{E}_{\text {longitudinal }}+\left(\mathrm{t}_{90} / \mathrm{t}_{\text {lam } / \text { insert }}\right) * \mathrm{E}_{\text {transverse }}$
$\mathrm{E}_{\text {lam } / \text { insert }}=(0.237 / 0.259) * 21,000,000+(0.022 / 0.259) * 1,100,000=19,307,452 \mathrm{psi}$
The moment of inertia of the composite at the insert is

$$
\begin{aligned}
& \mathrm{I}=\pi / 4\left(\mathrm{r}_{\mathrm{o}} / \text { insert }^{4}-\mathrm{r}_{\mathrm{i} / \text { insert }}{ }^{4}\right) \\
& \mathrm{I}=\pi / 4\left[(1.145)^{4}-(0.886)^{4}\right]=0.8644 \mathrm{in}^{4} \\
& \mathrm{EI}=16,688,733{\mathrm{lb}-\mathrm{in}^{2}}^{2}
\end{aligned}
$$

## Taper Segments

The inner radius of the taper section of the strut and the modulus of elasticity were interpolated between the values at the insert and at the straight section. The ply thicknesses and outer radius were then calculated using the same groundrules mentioned previously - the crosssectional area of the 0 -deg plies is constant over the length of the strut, and the thickness of the $90-\mathrm{deg}$ plies is constant over the length of the strut. Moments of inertia were then calculated from these thicknesses.

The taper section begins after the end fitting and insert
$\mathrm{x}_{0 / \text { taper }}=\mathrm{L}_{\text {end fitting }}+\mathrm{L}_{\text {insert }}=3.000+3.272=6.272$ in
The taper angle is $4^{\circ}$; the length of the taper is given by
$\mathrm{L}_{\text {taper }}=\left(\mathbf{r}_{\mathbf{i} / \text { midspan }}-\mathbf{r}_{\mathbf{i} / \text { insert }}\right) / \tan \theta_{\text {taper }}$
$\mathrm{L}_{\text {taper }}=(3-0.886) / \tan 4^{\circ}=30.232$ in
The inner radius of the taper at station $x$ is

$$
\begin{aligned}
r_{i} / \text { taper } & =r_{i / \text { insert }}+\tan \left(\theta_{\text {taper }}\right) *\left(x-x_{0 / \text { taper }}\right) \\
r_{i / \text { taper }} & =0.886+\tan \left(4^{\circ}\right) *(x-6.272)
\end{aligned}
$$

The corresponding outer radius is

$$
\begin{aligned}
& \mathrm{r}_{0 / \text { taper }}=\mathrm{t}_{90}+\left[\left(\mathrm{A}_{0} / \pi\right)+\mathrm{r}_{\mathrm{i} / \text { taper }}^{2}\right]^{1 / 2} \\
& \mathrm{r}_{0} / \text { taper }
\end{aligned}=0.022+\left[(1.493 / \pi)+\mathrm{r}_{\mathrm{i} / \text { taper }}^{2}\right]^{1 / 2} .
$$

The modulus of elasticity at station $x$ is

$$
\begin{aligned}
& \mathrm{E}_{\text {lam } / \text { taper }}=\mathrm{E}_{\text {lam } / \text { midspan }}-\left[\left(\mathrm{x}-\mathrm{x}_{0 / \text { taper }}\right) / \mathrm{L}_{\text {taper }}\right] *\left(\mathrm{E}_{\text {lam } / \text { insert }}-\mathrm{E}_{\text {lam } / \text { midspan }}\right) \\
& \mathrm{E}_{\text {lam } / \text { taper }}=16,631,175-[(\mathrm{x}-6.272) / 30.232] *(19,307,452-16,631,175) \\
& \mathrm{E}_{\text {lam } / \text { taper }}=16,631,175-88524.64(\mathrm{x}-6.272)
\end{aligned}
$$

And the moment of inertia is

$$
\mathrm{I}=\pi / 4\left(\mathrm{r}_{0} / \text { taper }^{4}-\mathrm{r}_{\mathrm{i}} / \mathrm{taper}^{4}\right)
$$

The values of E, I, and (EI) are tabulated for each segment of the strut in Figure 2.3.4.1-1.

| Node | Description | Material | Sta | E [psi] | 1 [in'] | El [ [1b-in ${ }^{2}$ ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | end fitting | metal | 0 | 16000000 | 0.10730 | 1,716,766 |
| 2 | end fitting | metal | 1.5 | 16000000 | 0.10730 | 1,716,766 |
| 3 | ftg/insert | metal | 3 | 16000000 | 0.10730 | 1,716,766 |
| 3 | ftg/insert | composite | 3 | 19307452 | 0.86437 | 16,688,733 |
| 4 | insert | composite | 4.5 | 19307452 | 0.86437 | 16,688,733 |
| 5 | insert | composite | 6 | 19307452 | 0.86437 | 16,688,733 |
| 6 | taper | composite | 7.5 | 19,198,742 | 1.00301 | 19,256,453 |
| 7 | taper | composite | 9 | 19,065,954 | 1.19148 | 22,716,724 |
| 8 | taper | composite | 10.5 | 18,933,165 | 1.40142 | 26,533,372 |
| 9 | taper | composite | 12 | 18,800,376 | 1.63330 | 30,706,566 |
| 10 | taper | composite | 13.5 | 18,667,587 | 1.88757 | 35,236,292 |
| 11 | taper | composite | 15 | 18,534,799 | 2.16470 | 40,122,339 |
| 12 | taper | composite | 16.5 | 18,402,010 | 2.46518 | 45,364,279 |
| 13 | taper | composite | 18 | 18,269,221 | 2.78947 | 50,961,462 |
| 14 | taper | composite | 19.5 | 18,136,433 | 3.13805 | 56,913,005 |
| 15 | taper | composite | 21 | 18,003,644 | 3.51139 | 63,217,787 |
| 16 | taper | composite | 22.5 | 17,870,855 | 3.90997 | 69,874,449 |

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| Node | Description | Material | Sta | E [psi] | [ [in ${ }^{4}$ ] | E] [1b-in ${ }^{2}$ ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17 | taper | composite | 24 | 17,738,067 | 4.33426 | 76,881,383 |
| 18 | taper | composite | 25.5 | 17,605,278 | 4.78474 | 84,236,739 |
| 19 | taper | composite | 27 | 17,472,489 | 5.26190 | 91,938,418 |
| 20 | taper | composite | 28.5 | 17,339,700 | 5.76619 | 99,984,071 |
| 21 | taper | composite | 30 | 17,206,912 | 6.29811 | 108,371,101 |
| 22 | taper | composite | 31.5 | 17,074,123 | 6.85814 | 117,096,657 |
| 23 | taper | composite | 33 | 16,941,334 | 7.44674 | 126,157,639 |
| 24 | taper | composite | 34.5 | 16,808,546 | 8.06439 | 135,550,695 |
| 25 | taper | composite | 36 | 16,675,757 | 8.71158 | 145,272,220 |
| 26 | straight | composite | 37.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 27 | straight | composite | 39 | 16,631,175 | 8.93557 | 148,609,101 |
| 28 | straight | composite | 40.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 29 | straight | composite | 42 | 16,631,175 | 8.93557 | 148,609,101 |
| 30 | straight | composite | 43.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 31 | straight | composite | 45 | 16,631,175 | 8.93557 | 148,609,101 |
| 32 | straight | composite | 46.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 33 | straight | composite | 48 | 16,631,175 | 8.93557 | 148,609,101 |
| 34 | straight | composite | 49.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 35 | straight | composite | 51 | 16,631,175 | 8.93557 | 148,609,101 |
| 36 | straight | composite | 52.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 37 | straight | composite | 54 | 16,631,175 | 8.93557 | 148,609,101 |
| 38 | straight | composite | 55.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 39 | straight | composite | 57 | 16,631,175 | 8.93557 | 148,609,101 |
| 40 | straight | composite | 58.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 41 | straight | composite | 60 | 16,631,175 | 8.93557 | 148,609,101 |
| 42 | straight | composite | 61.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 43 | straight | composite | 63 | 16,631,175 | 8.93557 | 148,609,101 |
| 44 | straight | composite | 64.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 45 | straight | composite | 66 | 16,631,175 | 8.93557 | 148,609,101 |
| 46 | straight | MIDSPAN | 67.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 47 | straight | composite | 69 | 16,631,175 | 8.93557 | 148,609,101 |
| 48 | straight | composite | 70.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 49 | straight | composite | 72 | 16,631,175 | 8.93557 | 148,609,101 |
| 50 | straight | composite | 73.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 51 | straight | composite | 75 | 16,631,175 | 8.93557 | 148,609,101 |
| 52 | straight | composite | 76.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 53 | straight | composite | 78 | 16,631,175 | 8.93557 | 148,609,101 |
| 54 | straight | composite | 79.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 55 | straight | composite | 81 | 16,631,175 | 8.93557 | 148,609,101 |
| 56 | straight | composite | 82.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 57 | straight | composite | 84 | 16,631,175 | 8.93557 | 148,609,101 |
| 58 | straight | composite | 85.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 59 | straight | composite | 87 | 16,631,175 | 8.93557 | 148,609,101 |
| 60 | straight | composite | 88.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 61 | straight | composite | 90 | 16,631,175 | 8.93557 | 148,609,101 |
| 62 | straight | composite | 91.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 63 | straight | composite | 93 | 16,631,175 | 8.93557 | 148,609,101 |
| 64 | straight | composite | 94.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 65 | straight | composite | 96 | 16,631,175 | 8.93557 | 148,609,101 |
| 66 | straight | composite | 97.5 | 16,631,175 | 8.93557 | 148,609,101 |
| 67 | taper | composite | 99 | 16,675,757 | 8.71158 | 145,272,220 |
| 68 | taper | composite | 100.5 | 16,808,546 | 8.06439 | 135,550,695 |
| 69 | taper | composite | 102 | 16,941,334 | 7.44674 | 126,157,639 |
| 70 | taper | composite | 103.5 | 17,074,123 | 6.85814 | 117,096,657 |


| Node | Description | Material | Sta | E [psi] | ] [in ${ }^{4}$ ] | E] [1b-in $\left.{ }^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 71 | taper | composite | 105 | 17,206,912 | 6.29811 | 108,371,101 |
| 72 | taper | composite | 106.5 | 17,339,700 | 5.76619 | 99,984,071 |
| 73 | taper | composite | 108 | 17,472,489 | 5.26190 | 91,938,418 |
| 74 | taper | composite | 109.5 | 17,605,278 | 4.78474 | 84,236,739 |
| 75 | taper | composite | 111 | 17,738,067 | 4.33426 | 76,881,383 |
| 76 | taper | composite | 112.5 | 17,870,855 | 3.90997 | 69,874,449 |
| 77 | taper | composite | 114 | 18,003,644 | 3.51139 | 63,217,787 |
| 78 | taper | composite | 115.5 | 18,136,433 | 3.13805 | 56,913,005 |
| 79 | taper | composite | 117 | 18,269,221 | 2.78947 | 50,961,462 |
| 80 | taper | composite | 118.5 | 18,402,010 | 2.46518 | 45,364,279 |
| 81 | taper | composite | 120 | 18,534,799 | 2.16470 | 40,122,339 |
| 82 | taper | composite | 121.5 | 18,667,587 | 1.88757 | 35,236,292 |
| 83 | taper | composite | 123 | 18,800,376 | 1.63330 | 30,706,566 |
| 84 | taper | composite | 124.5 | 18,933,165 | 1.40142 | 26,533,372 |
| 85 | taper | composite | 126 | 19,065,954 | 1.19148 | 22,716,724 |
| 86 | taper | composite | 127.5 | 19,198,742 | 1.00301 | 19,256,453 |
| 87 | insert | composite | 129 | 19307452 | 0.86437 | 16,688,733 |
| 88 | insert | composite | 130.5 | 19307452 | 0.86437 | 16,688,733 |
| 89 | ftg/insert | composite | 132 | 19307452 | 0.86437 | 16,688,733 |
| 89 | ftg/insert | metal | 132 | 16000000 | 0.10730 | 1,716,766 |
| 90 | end fitting | metal | 133.5 | 16000000 | 0.10730 | 1,716,766 |
| 91 | end fitting | metal | 135 | 16000000 | 0.10730 | 1,716,766 |

Figure 2.3.4.1-1. 44K Strut Properties for Newmark Buckling Analysis
The Newmark buckling analysis was performed on the strut using the properties at each segment, using the procedure documented in the Boeing Design Manual, BDM-6238.

Step 1: An initial deflection is assumed for each node. To facilitate quicker convergence, a parabolic deflection pattern was used for the initial condition:
$\delta_{\mathrm{n}}=-4\left[\left(\mathrm{x}_{\mathrm{n}} / \mathrm{L}_{\text {strut }}\right)^{2}-\left(\mathrm{x}_{\mathrm{n}} / \mathrm{L}_{\text {strut }}\right)\right]$
Step 2: The moment is calculated at each node
$\mathrm{M}_{\mathrm{n}}=\mathrm{P} \delta_{\mathrm{n}}$
Step 3: $\alpha_{n}$ is calculated at each node
$\alpha_{n}=-\frac{M_{n}}{E_{n} I_{n}}$
using the values of (EI) tabulated above
Step 4: The concentrated nodal slope values are calculated at each node using the following parabolic fit equations:
$R_{a n}=\frac{h\left(7 \alpha_{n}+6 \alpha_{n+1}-\alpha_{n+2}\right)}{24}$
$R_{b n}=\frac{h\left(2 \alpha_{n-1}+20 \alpha_{n}+2 \alpha_{n+1}\right)}{24}$
$R_{c n}=\frac{h\left(-\alpha_{n-2}+6 \alpha_{n-1}+7 \alpha_{n}\right)}{24}$
At the far left node of the strut, the concentrated nodal slope value is $\bar{\alpha}_{1}=\mathrm{R}_{\mathrm{a} 1}$

At the far right node of the strut, the concentrated nodal slope value is
$\bar{\alpha}_{91}=R_{c 91}$
At the discontinuities between the end fitting and strut body,
$\bar{\alpha}_{3}=R_{a 3}+R_{c 3}$
$\bar{\alpha}_{89}=\mathrm{R}_{\mathrm{a} 89}+\mathrm{R}_{\mathrm{c} 89}$
At all other nodes,
$\bar{\alpha}_{n}=R_{b n}$
Step 5: The slope is calculated as a cumulative sum of the concentrated nodal slope values:
slope $_{1}=\bar{\alpha}_{1}$
slope $_{n}=$ slope $_{n-1}+\bar{\alpha}_{n}$
Step 6: Trial deflections are calculated using the slope and segment length:
$y_{t 1}=0$
$y_{t n}=y_{t(n-1)}+h\left(\right.$ slope $\left._{n-1}\right)$
Step 7: Linear correction factors are calculated for the deflections:
$y_{c 1}=0$
$y_{c n}=\frac{-h(n-1)\left(\text { right }- \text { most value of } y_{t}\right)}{L}$
Step 8: Corrected final deflections are calculated:
$y_{\mathrm{fn}}=\mathrm{y}_{\mathrm{tn}}+\mathrm{y}_{\mathrm{cn}}$
Step 9: The deflections are normalized to the maximum deflection of the previous iteration:
$\delta_{\mathrm{n}}=\frac{\mathrm{y}_{\mathrm{fn}}(\text { max imum deflection from prior iteration }(\operatorname{step} 1))}{\left.\text { maximum value of } \mathrm{y}_{\mathrm{fn}} \text { from this iteration (step } 8\right)}$
Step 10: The buckling load is calculated:
$P_{\mathrm{cr}}=\frac{\sum M_{n} y_{\mathrm{fn}}}{\sum y_{\mathrm{fn}}{ }^{2}}$
If the buckling load has converged sufficiently (i.e., shows acceptable agreement with the previous iteration), then the iteration is complete. If it has not converged, the deflections from Step 9 are used as the initial deflections (Step 1) in another iteration.

The values calculated from the initial iteration are tabulated in Figure 2.3.4.1-2 as an example.
The buckling load from the first iteration is

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{cr}}=\frac{\sum \mathrm{M}_{\mathrm{n}} \mathrm{y}_{\mathrm{fn}}}{\sum \mathrm{y}_{\mathrm{fn}}{ }^{2}} \\
& \mathrm{P}_{\mathrm{cr}}=744.44 / 0.0116=64,451 \mathrm{lb}
\end{aligned}
$$

The iterated buckling loads are presented in Figure 2.3.4.1-3, along with the ultimate load of $61,600 \mathrm{lb}$.

| Node | Sta | $\delta_{n}$ | $\mathrm{M}_{n}$ | $\alpha_{n}$ | $\bar{\alpha}_{n}$ | Slope | $y_{\text {m }}$ | $\mathrm{Y}_{\text {cn }}$ | $y_{\text {m }}$ | $\delta_{n}$ | $M_{n} Y_{\text {fin }}$ | $\mathrm{yfin}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0.0000 | 0.0 | 0 | -6.436E-06 | -6.436E-06 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1.5 | 0.0440 | 44.0 | -2.560E-05 | -3.833E-05 | -4.477E-05 | -9.654E-06 | 0.0008 | 0.0008 | 0.0528 | 0.036 | 6.58E-07 |
| 3 | 3 | 0.0869 | 86.9 | -5.063E-05 | -3.175E-05 | 8.105E-05 |  |  |  |  |  |  |
| 3 | 3 | 0.0869 | 86.9 | -5.208E-06 | -4.538E-06 | -8.105E-05 | -7.680E-05 | 0.0016 | 0.0016 | 0.1019 | 0.136 | 2.45E-06 |
| 4 | 4.5 | 0.1289 | 128.9 | -7.723E-06 | -1.158E-05 | -9.263E-05 | -1.984E-04 | 0.0025 | 0.0023 | 0.1475 | 0.292 | 5.12E-06 |
| 5 | 6 | 0.1699 | 169.9 | -1.018E-05 | -1.505E-05 | -1.077E-04 | -3.373E-04 | 0.0033 | 0.0029 | 0.1919 | 0.500 | 8.68E-06 |
| 6 | 7.5 | 0.2099 | 209.9 | -1.090E-05 | -1.627E-05 | -1.239E-04 | -4.989E-04 | 0.0041 | 0.0036 | 0.2348 | 0.757 | $1.30 \mathrm{E}-05$ |
| 7 | 9 | 0.2489 | 248.9 | -1.096E-05 | -1.641E-05 | -1.404E-04 | -6.848E-04 | 0.0049 | 0.0042 | 0.2762 | 1.055 | $1.80 \mathrm{E}-05$ |
| 8 | 10.5 | 0.2869 | 286.9 | -1.081E-05 | -1.620E-05 | -1.566E-04 | -8.953E-04 | 0.0057 | 0.0048 | 0.3160 | 1.391 | $2.35 \mathrm{E}-05$ |
| 9 | 12 | 0.3240 | 324.0 | -1.055E-05 | -1.582E-05 | -1.724E-04 | -1.130E-03 | 0.0066 | 0.0054 | 0.3541 | 1.761 | $2.95 \mathrm{E}-05$ |
| 10 | 13.5 | 0.3600 | 360.0 | -1.022E-05 | -1.532E-05 | -1.877E-04 | -1.389E-03 | 0.0074 | 0.0060 | 0.3907 | 2.159 | 3.60E-05 |
| 11 | 15 | 0.3951 | 395.1 | -9.846E-06 | -1.477E-05 | -2.025E-04 | -1.670E-03 | 0.0082 | 0.0065 | 0.4259 | 2.582 | 4.27E-05 |
| 12 | 16.5 | 0.4291 | 429.1 | -9.460E-06 | -1.419E-05 | -2.167E-04 | -1.974E-03 | 0.0090 | 0.0071 | 0.4595 | 3.027 | $4.98 \mathrm{E}-05$ |
| 13 | 18 | 0.4622 | 462.2 | -9.070E-06 | -1.361E-05 | -2.303E-04 | -2.299E-03 | 0.0098 | 0.0075 | 0.4918 | 3.489 | $5.70 \mathrm{E}-05$ |
| 14 | 19.5 | 0.4943 | 494.3 | -8.686E-06 | -1.303E-05 | -2.433E-04 | -2.644E-03 | 0.0107 | 0.0080 | 0.5228 | 3.967 | 6.44E-05 |
| 15 | 21 | 0.5254 | 525.4 | -8.311E-06 | -1.247E-05 | -2.558E-04 | -3.009E-03 | 0.0115 | 0.0085 | 0.5525 | 4.456 | 7.19E-05 |
| 16 | 22.5 | 0.5556 | 555.6 | -7.951E-06 | -1.193E-05 | -2.677E-04 | -3.393E-03 | 0.0123 | 0.0089 | 0.5810 | 4.954 | $7.95 \mathrm{E}-05$ |
| 17 | 24 | 0.5847 | 584.7 | -7.605E-06 | -1.141E-05 | -2.791E-04 | -3.794E-03 | 0.0131 | 0.0093 | 0.6083 | 5.459 | $8.72 \mathrm{E}-05$ |
| 18 | 25.5 | 0.6128 | 612.8 | -7.275E-06 | -1.091E-05 | -2.900E-04 | -4.213E-03 | 0.0140 | 0.0097 | 0.6345 | 5.968 | $9.48 \mathrm{E}-05$ |
| 19 | 27 | 0.6400 | 640.0 | -6.961E-06 | -1.044E-05 | -3.005E-04 | -4.648E-03 | 0.0148 | 0.0101 | 0.6596 | 6.479 | 1.02E-04 |
| 20 | 28.5 | 0.6662 | 666.2 | -6.663E-06 | -9.996E-06 | -3.105E-04 | -5.099E-03 | 0.0156 | 0.0105 | 0.6837 | 6.991 | 1.10E-04 |
| 21 | 30 | 0.6914 | 691.4 | -6.380E-06 | -9.571E-06 | -3.200E-04 | -5.564E-03 | 0.0164 | 0.0108 | 0.7068 | 7.501 | $1.18 \mathrm{E}-04$ |
| 22 | 31.5 | 0.7156 | 715.6 | -6.111E-06 | -9.168E-06 | -3.292E-04 | -6.045E-03 | 0.0172 | 0.0112 | 0.7290 | 8.007 | 1.25E-04 |
| 23 | 33 | 0.7388 | 738.8 | -5.856E-06 | -8.785E-06 | -3.380E-04 | -6.538E-03 | 0.0181 | 0.0115 | 0.7503 | 8.508 | 1.33E-04 |
| 24 | 34.5 | 0.7610 | 761.0 | -5.614E-06 | -8.423E-06 | -3.464E-04 | -7.045E-03 | 0.0189 | 0.0118 | 0.7708 | 9.003 | 1.40E-04 |
| 25 | 36 | 0.7822 | 782.2 | -5.385E-06 | -8.107E-06 | -3.545E-04 | -7.565E-03 | 0.0197 | 0.0121 | 0.7904 | 9.489 | $1.47 \mathrm{E}-04$ |
| 26 | 37.5 | 0.8025 | 802.5 | -5.400E-06 | -8.114E-06 | -3.626E-04 | -8.097E-03 | 0.0205 | 0.0124 | 0.8092 | 9.967 | $1.54 \mathrm{E}-04$ |
| 27 | 39 | 0.8217 | 821.7 | -5.529E-06 | -8.293E-06 | -3.709E-04 | -8.641E-03 | 0.0213 | 0.0127 | 0.8272 | 10.434 | 1.61E-04 |
| 28 | 40.5 | 0.8400 | 840.0 | -5.652E-06 | -8.478E-06 | -3.794E-04 | -9.197E-03 | 0.0222 | 0.0130 | 0.8445 | 10.888 | $1.68 \mathrm{E}-04$ |
| 29 | 42 | 0.8573 | 857.3 | -5.769E-06 | -8.652E-06 | -3.880E-04 | -9.766E-03 | 0.0230 | 0.0132 | 0.8609 | 11.327 | $1.75 \mathrm{E}-04$ |
| 30 | 43.5 | 0.8736 | 873.6 | -5.878E-06 | -8.817E-06 | -3.969E-04 | -1.035E-02 | 0.0238 | 0.0135 | 0.8764 | 11.751 | 1.81E-04 |
| 31 | 45 | 0.8889 | 888.9 | -5.981E-06 | -8.971E-06 | -4.058E-04 | -1.094E-02 | 0.0246 | 0.0137 | 0.8911 | 12.157 | 1.87E-04 |
| 32 | 46.5 | 0.9032 | 903.2 | -6.078E-06 | -9.116E-06 | -4.149E-04 | -1.155E-02 | 0.0254 | 0.0139 | 0.9049 | 12.545 | 1.93E-04 |
| 33 | 48 | 0.9165 | 916.5 | -6.167E-06 | -9.250E-06 | -4.242E-04 | -1.217E-02 | 0.0263 | 0.0141 | 0.9178 | 12.911 | $1.98 \mathrm{E}-04$ |
| 34 | 49.5 | 0.9289 | 928.9 | -6.251E-06 | -9.375E-06 | -4.336E-04 | -1.281E-02 | 0.0271 | 0.0143 | 0.9298 | 13.257 | 2.04E-04 |
| 35 | 51 | 0.9402 | 940.2 | -6.327E-06 | -9.490E-06 | -4.431E-04 | -1.346E-02 | 0.0279 | 0.0144 | 0.9409 | 13.579 | 2.09E-04 |
| 36 | 52.5 | 0.9506 | 950.6 | -6.397E-06 | -9.594E-06 | -4.527E-04 | -1.413E-02 | 0.0287 | 0.0146 | 0.9511 | 13.877 | 2.13E-04 |


| Node | Sta | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}}$ | $\alpha_{n}$ | $\bar{\alpha}_{n}$ | Slope | $y_{\text {m }}$ | $Y_{\text {cn }}$ | $y_{\text {fin }}$ | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}} \mathrm{Y}_{\text {fin }}$ | $\mathrm{ym}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37 | 54 | 0.9600 | 960.0 | -6.460E-06 | -9.689E-06 | -4.623E-04 | -1.480E-02 | 0.0295 | 0.0147 | 0.9603 | 14.150 | 2.17E-04 |
| 38 | 55.5 | 0.9684 | 968.4 | -6.516E-06 | -9.774E-06 | -4.721E-04 | -1.550E-02 | 0.0304 | 0.0149 | 0.9686 | 14.397 | 2.21E-04 |
| 39 | 57 | 0.9758 | 975.8 | -6.566E-06 | -9.849E-06 | -4.820E-04 | -1.621E-02 | 0.0312 | 0.0150 | 0.9759 | 14.617 | 2.24E-04 |
| 40 | 58.5 | 0.9822 | 982.2 | -6.609E-06 | -9.913E-06 | -4.919E-04 | -1.693E-02 | 0.0320 | 0.0151 | 0.9823 | 14.809 | $2.27 \mathrm{E}-04$ |
| 41 | 60 | 0.9877 | 987.7 | -6.646E-06 | -9.968E-06 | -5.018E-04 | -1.767E-02 | 0.0328 | 0.0152 | 0.9877 | 14.973 | $2.30 \mathrm{E}-04$ |
| 42 | 61.5 | 0.9921 | 992.1 | -6.676E-06 | -1.001E-05 | -5.119E-04 | -1.842E-02 | 0.0336 | 0.0152 | 0.9921 | 15.107 | 2.32E-04 |
| 43 | 63 | 0.9956 | 995.6 | -6.699E-06 | -1.005E-05 | -5.219E-04 | -1.919E-02 | 0.0345 | 0.0153 | 0.9956 | 15.213 | 2.33E-04 |
| 44 | 64.5 | 0.9980 | 998.0 | -6.716E-06 | -1.007E-05 | -5.320E-04 | -1.997E-02 | 0.0353 | 0.0153 | 0.9980 | 15.288 | 2.35E-04 |
| 45 | 66 | 0.9995 | 999.5 | -6.726E-06 | -1.009E-05 | -5.421E-04 | -2.077E-02 | 0.0361 | 0.0153 | 0.9995 | 15.334 | 2.35E-04 |
| 46 | 67.5 | 1.0000 | 1000.0 | -6.729E-06 | -1.009E-05 | -5.522E-04 | -2.158E-02 | 0.0369 | 0.0153 | 1.0000 | 15.349 | 2.36E-04 |
| 47 | 69 | 0.9995 | 999.5 | -6.726E-06 | -1.009E-05 | -5.623E-04 | -2.241E-02 | 0.0378 | 0.0153 | 0.9995 | 15.334 | 2.35E-04 |
| 48 | 70.5 | 0.9980 | 998.0 | -6.716E-06 | -1.007E-05 | -5.723E-04 | -2.325E-02 | 0.0386 | 0.0153 | 0.9980 | 15.288 | 2.35E-04 |
| 49 | 72 | 0.9956 | 995.6 | -6.699E-06 | -1.005E-05 | -5.824E-04 | -2.411E-02 | 0.0394 | 0.0153 | 0.9956 | 15.213 | $2.33 \mathrm{E}-04$ |
| 50 | 73.5 | 0.9921 | 992.1 | -6.676E-06 | -1.001E-05 | -5.924E-04 | -2.499E-02 | 0.0402 | 0.0152 | 0.9921 | 15.107 | $2.32 \mathrm{E}-04$ |
| 51 | 75 | 0.9877 | 987.7 | -6.646E-06 | -9.968E-06 | -6.024E-04 | -2.587E-02 | 0.0410 | 0.0152 | 0.9877 | 14.973 | $2.30 \mathrm{E}-04$ |
| 52 | 76.5 | 0.9822 | 982.2 | -6.609E-06 | -9.913E-06 | -6.123E-04 | -2.678E-02 | 0.0419 | 0.0151 | 0.9823 | 14.809 | $2.27 \mathrm{E}-04$ |
| 53 | 78 | 0.9758 | 975.8 | -6.566E-06 | -9.849E-06 | -6.221E-04 | -2.770E-02 | 0.0427 | 0.0150 | 0.9759 | 14.617 | $2.24 \mathrm{E}-04$ |
| 54 | 79.5 | 0.9684 | 968.4 | -6.516E-06 | -9.774E-06 | -6.319E-04 | -2.863E-02 | 0.0435 | 0.0149 | 0.9686 | 14.397 | $2.21 \mathrm{E}-04$ |
| 55 | 81 | 0.9600 | 960.0 | -6.460E-06 | -9.689E-06 | -6.416E-04 | -2.958E-02 | 0.0443 | 0.0147 | 0.9603 | 14.150 | 2.17E-04 |
| 56 | 82.5 | 0.9506 | 950.6 | -6.397E-06 | -9.594E-06 | -6.512E-04 | -3.054E-02 | 0.0451 | 0.0146 | 0.9511 | 13.877 | 2.13E-04 |
| 57 | 84 | 0.9402 | 940.2 | -6.327E-06 | -9.490E-06 | -6.607E-04 | -3.152E-02 | 0.0460 | 0.0144 | 0.9409 | 13.579 | $2.09 \mathrm{E}-04$ |
| 58 | 85.5 | 0.9289 | 928.9 | -6.251E-06 | -9.375E-06 | -6.700E-04 | -3.251E-02 | 0.0468 | 0.0143 | 0.9298 | 13.257 | $2.04 \mathrm{E}-04$ |
| 59 | 87 | 0.9165 | 916.5 | -6.167E-06 | -9.250E-06 | -6.793E-04 | -3.351E-02 | 0.0476 | 0.0141 | 0.9178 | 12.911 | 1.98E-04 |
| 60 | 88.5 | 0.9032 | 903.2 | -6.078E-06 | -9.116E-06 | -6.884E-04 | -3.453E-02 | 0.0484 | 0.0139 | 0.9049 | 12.545 | 1.93E-04 |
| 61 | 90 | 0.8889 | 888.9 | -5.981E-06 | -8.971E-06 | -6.974E-04 | -3.556E-02 | 0.0492 | 0.0137 | 0.8911 | 12.157 | 1.87E-04 |
| 62 | 91.5 | 0.8736 | 873.6 | -5.878E-06 | -8.817E-06 | -7.062E-04 | -3.661E-02 | 0.0501 | 0.0135 | 0.8764 | 11.751 | 1.81E-04 |
| 63 | 93 | 0.8573 | 857.3 | -5.769E-06 | -8.652E-06 | -7.148E-04 | -3.767E-02 | 0.0509 | 0.0132 | 0.8609 | 11.327 | 1.75E-04 |
| 64 | 94.5 | 0.8400 | 840.0 | -5.652E-06 | -8.478E-06 | -7.233E-04 | -3.874E-02 | 0.0517 | 0.0130 | 0.8445 | 10.888 | 1.68E-04 |
| 65 | 96 | 0.8217 | 821.7 | -5.529E-06 | -8.293E-06 | -7.316E-04 | -3.983E-02 | 0.0525 | 0.0127 | 0.8272 | 10.434 | $1.61 \mathrm{E}-04$ |
| 66 | 97.5 | 0.8025 | 802.5 | -5.400E-06 | -8.114E-06 | -7.397E-04 | -4.092E-02 | 0.0533 | 0.0124 | 0.8092 | 9.967 | 1.54E-04 |
| 67 | 99 | 0.7822 | 782.2 | -5.385E-06 | -8.107E-06 | -7.478E-04 | -4.203E-02 | 0.0542 | 0.0121 | 0.7904 | 9.489 | 1.47E-04 |
| 68 | 100.5 | 0.7610 | 761.0 | -5.614E-06 | -8.423E-06 | -7.563E-04 | -4.316E-02 | 0.0550 | 0.0118 | 0.7708 | 9.003 | $1.40 \mathrm{E}-04$ |
| 69 | 102 | 0.7388 | 738.8 | -5.856E-06 | -8.785E-06 | -7.650E-04 | -4.429E-02 | 0.0558 | 0.0115 | 0.7503 | 8.508 | 1.33E-04 |
| 70 | 103.5 | 0.7156 | 715.6 | -6.111E-06 | -9.168E-06 | -7.742E-04 | -4.544E-02 | 0.0566 | 0.0112 | 0.7290 | 8.007 | 1.25E-04 |
| 71 | 105 | 0.6914 | 691.4 | -6.380E-06 | -9.571E-06 | -7.838E-04 | -4.660E-02 | 0.0574 | 0.0108 | 0.7068 | 7.501 | 1.18E-04 |
| 72 | 106.5 | 0.6662 | 666.2 | -6.663E-06 | -9.996E-06 | -7.938E-04 | -4.777E-02 | 0.0583 | 0.0105 | 0.6837 | 6.991 | 1.10E-04 |
| 73 | 108 | 0.6400 | 640.0 | -6.961E-06 | -1.044E-05 | -8.042E-04 | -4.896E-02 | 0.0591 | 0.0101 | 0.6596 | 6.479 | $1.02 \mathrm{E}-04$ |
| 74 | 109.5 | 0.6128 | 612.8 | -7.275E-06 | -1.091E-05 | -8.151E-04 | -5.017E-02 | 0.0599 | 0.0097 | 0.6345 | 5.968 | 9.48E-05 |


| Node | Sta | $\delta_{n}$ | $M_{n}$ | $\alpha_{n}$ | $\bar{\alpha}_{n}$ | Slope | $y_{\text {en }}$ | $Y_{\text {cn }}$ | $y_{\text {fin }}$ | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}} \mathrm{Y}_{\text {fin }}$ | $\mathrm{yfn}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 75 | 111 | 0.5847 | 584.7 | -7.605E-06 | -1.141E-05 | -8.265E-04 | -5.139E-02 | 0.0607 | 0.0093 | 0.6083 | 5.459 | $8.72 \mathrm{E}-05$ |
| 76 | 112.5 | 0.5556 | 555.6 | -7.951E-06 | -1.193E-05 | -8.385E-04 | -5.263E-02 | 0.0616 | 0.0089 | 0.5810 | 4.954 | 7.95E-05 |
| 77 | 114 | 0.5254 | 525.4 | -8.311E-06 | -1.247E-05 | -8.509E-04 | -5.389E-02 | 0.0624 | 0.0085 | 0.5525 | 4.456 | 7.19E-05 |
| 78 | 115.5 | 0.4943 | 494.3 | -8.686E-06 | -1.303E-05 | -8.640E-04 | -5.517E-02 | 0.0632 | 0.0080 | 0.5228 | 3.967 | $6.44 \mathrm{E}-05$ |
| 79 | 117 | 0.4622 | 462.2 | -9.070E-06 | -1.361E-05 | -8.776E-04 | -5.646E-02 | 0.0640 | 0.0075 | 0.4918 | 3.489 | $5.70 \mathrm{E}-05$ |
| 80 | 118.5 | 0.4291 | 429.1 | -9.460E-06 | -1.419E-05 | -8.918E-04 | -5.778E-02 | 0.0648 | 0.0071 | 0.4595 | 3.027 | $4.98 \mathrm{E}-05$ |
| 81 | 120 | 0.3951 | 395.1 | -9.846E-06 | -1.477E-05 | -9.065E-04 | -5.912E-02 | 0.0657 | 0.0065 | 0.4259 | 2.582 | $4.27 \mathrm{E}-05$ |
| 82 | 121.5 | 0.3600 | 360.0 | -1.022E-05 | -1.532E-05 | -9.219E-04 | -6.048E-02 | 0.0665 | 0.0060 | 0.3907 | 2.159 | $3.60 \mathrm{E}-05$ |
| 83 | 123 | 0.3240 | 324.0 | -1.055E-05 | -1.582E-05 | -9.377E-04 | -6.186E-02 | 0.0673 | 0.0054 | 0.3541 | 1.761 | 2.95E-05 |
| 84 | 124.5 | 0.2869 | 286.9 | -1.081E-05 | -1.620E-05 | -9.539E-04 | -6.327E-02 | 0.0681 | 0.0048 | 0.3160 | 1.391 | 2.35E-05 |
| 85 | 126 | 0.2489 | 248.9 | -1.096E-05 | -1.641E-05 | -9.703E-04 | -6.470E-02 | 0.0689 | 0.0042 | 0.2762 | 1.055 | $1.80 \mathrm{E}-05$ |
| 86 | 127.5 | 0.2099 | 209.9 | -1.090E-05 | -1.627E-05 | -9.866E-04 | -6.615E-02 | 0.0698 | 0.0036 | 0.2348 | 0.757 | $1.30 \mathrm{E}-05$ |
| 87 | 129 | 0.1699 | 169.9 | -1.018E-05 | -1.505E-05 | -1.002E-03 | -6.763E-02 | 0.0706 | 0.0029 | 0.1919 | 0.500 | $8.68 \mathrm{E}-06$ |
| 88 | 130.5 | 0.1289 | 128.9 | -7.723E-06 | -1.158E-05 | -1.013E-03 | -6.914E-02 | 0.0714 | 0.0023 | 0.1475 | 0.292 | 5.12E-06 |
| 89 | 132 | 0.0869 | 86.9 | -5.208E-06 | -4.538E-06 | -1.049E-03 | -7.065E-02 | 0.0722 | 0.0016 | 0.1019 | 0.136 | $2.45 \mathrm{E}-06$ |
| 89 | 132 | 0.0869 | 86.9 | -5.063E-05 | -3.175E-05 |  |  |  |  |  |  |  |
| 90 | 133.5 | 0.0440 | 44.0 | -2.560E-05 | -3.833E-05 | -1.088E-03 | -7.223E-02 | 0.0730 | 0.0008 | 0.0528 | 0.036 | 6.58E-07 |
| 91 | 135 | 0.0000 | 0.0 | $0.000 \mathrm{E}+00$ | -6.436E-06 | -1.094E-03 | -7.386E-02 | 0.0739 | 0 | 0 | 0 | 0 |
|  |  |  |  |  |  |  |  |  |  | Sum: | 744.44 | 0.0116 |

Figure 2.3.4.1-2. Newmark Buckling Analysis Results of 44K Strut


Figure 2.3.4.1-3. Iterated Buckling Loads of 44K Strut

### 2.3.4.2 110K Strut

The 110 K strut is divided into 64 axial segments, corresponding to a segment length of 1.984 inches. Although this segment length divides evenly into the overall strut length, the end fitting length was adjusted from 4 inches to 3.969 inches in the analysis to ensure a node at the transition between the end fitting and strut body. The analysis approach is the same as that used for the 44 K strut.

## End Fitting Segments

$\mathrm{E}=160,000,000 \mathrm{psi}$
$\mathrm{I}=\pi / 4\left[(2.145 / 2)^{4}-(1.625 / 2)^{4}\right]=0.6969 \mathrm{in}^{4}$
$\mathrm{EI}=11,149,929 \mathrm{lb}-\mathrm{in}^{2}$

## Straight Segments (Midspan)

The diameter of the mandrel is 6.5 inches at the midspan, and it is wrapped with 180 -deg plies and $690-\mathrm{deg}$ plies.
$\mathrm{r}_{\mathrm{i} / \mathrm{midspan}}=6.5 / 2=3.25 \mathrm{in}$
$\mathfrak{t}_{0 / \text { midspan }}=18 * 0.00711=0.12798$ in
$\mathrm{t}_{90}=6 * 0.0055=0.033$ in
$\mathrm{t}_{\text {lam } / \text { midspan }}=\mathrm{t}_{0 / \text { midspan }}+\mathrm{t}_{90}=0.12798+0.033=0.1610 \mathrm{in}$
$\mathrm{r}_{\mathrm{o} / \mathrm{midspan}}=\mathrm{r}_{\mathrm{i} / \text { midspan }}+\mathrm{t}_{\text {lam } / \text { midspan }}=3.25+0.1610=3.411 \mathrm{in}$
$\mathrm{E}_{\text {lam } / \text { midspan }}=(0.12798 / 0.1610) * 21,000,000+(0.033 / 0.1610) * 1,100,000=16,920,611 \mathrm{psi}$ $\mathrm{I}=\pi / 4\left[(3.411)^{4}-(3.25)^{4}\right]=18.6939 \mathrm{in}^{4}$
$\mathrm{EI}=316,312,119 \mathrm{lb}-\mathrm{in}^{2}$

## Insert Segments

$\mathrm{r}_{\mathrm{i} / \text { insert }}=2.686 / 2=1.343 \mathrm{in}$
$\left.\mathrm{A}_{0}=\pi\left(\mathrm{r}_{\mathrm{i} / \text { midspan }}+\mathrm{t}_{0 / \text { midspan }}\right)^{2}-\mathrm{r}_{\mathrm{i} / \text { midspan }}{ }^{2}\right]=\pi *\left[(3.25+0.12798)^{2}-3.25^{2}\right]=2.665 \mathrm{in}^{2}$
$\mathrm{t}_{0 / \text { insert }}=\left[(2.665 / \pi)+1.343^{2}\right]^{1 / 2}-1.343=0.285$ in
$\mathrm{t}_{\text {lam } / \text { insert }}=\mathrm{t}_{0 / \text { insert }}+\mathrm{t}_{90}=0.285+0.033=0.318$ in
$\mathrm{r}_{0} /$ insert $=1.343+0.318=1.661$ in
$\mathrm{E}_{\text {lam } / \text { insert }}=(0.285 / 0.318) * 21,000,000+(0.033 / 0.318) * 1,100,000=18,937,920 \mathrm{psi}$
$\mathrm{I}=\pi / 4\left[(1.661)^{4}-(1.343)^{4}\right]=3.4298 \mathrm{in}^{4}$
$\mathrm{EI}=64,954,222 \mathrm{lb}-\mathrm{in}^{2}$

## Taper Segments

$\mathrm{x}_{0 / \text { taper }}=\mathrm{L}_{\text {end fitting }}+\mathrm{L}_{\text {insert }}=4.061+3.969=8.030$ in
$\mathrm{L}_{\text {taper }}=(3.25-1.343) / \tan 4^{\circ}=27.271$ in
$\mathrm{r}_{\mathrm{i} / \text { taper }}=1.343+\tan \left(4^{\circ}\right) *(\mathrm{x}-8.030)$
$r_{0} /$ taper $=0.033+\left[(2.665 / \pi)+r_{i / \text { taper }}\right]^{2 / 2}$
$\mathrm{E}_{\text {lam } / \text { taper }}=18,937,920-[(\mathrm{x}-8.030) / 27.271] *(18,937,920-16,920,611)$
$\mathrm{E}_{\text {lam } / \text { taper }}=18,937,920-73972.68(\mathrm{x}-8.030)$
$\mathrm{I}=\pi / 4\left(\mathrm{r}_{\mathrm{o}} /\right.$ taper $\left.{ }^{4}-\mathrm{r}_{\mathrm{i} / \text { taper }}{ }^{4}\right)$
The values of $\mathrm{E}, \mathrm{I}$, and (EI) are tabulated for each segment of the strut in Figure 2.3.4.2-1. The Newmark buckling analysis is performed using the procedure described in the section for the 44 k strut. The values calculated from the initial iteration are tabulated in Figure 2.3.4.2-2 as an example.

| Node | Description | Material | Sta | $E[p s[]$ | $\\|\left[n^{4}\right]$ | 0.69687 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | end fitting | metal | 0.00 | 16000000 | 11149929 |  |
| 2 | end fitting | metal | 1.98 | 16000000 | 0.69687 | 11149929 |
| 3 | ftg $/ \mathrm{insert}$ | metal | 3.97 | 16000000 | 0.69687 | 11149929 |
| 3 | ftg /insert | composite | 3.97 | 18937920 | 3.42985 | 64954222 |
| 4 | insert | composite | 5.95 | 18937920 | 3.42985 | 64954222 |
| 5 | insert | composite | 7.94 | 18937920 | 3.42985 | 64954222 |
| 6 | taper | composite | 9.92 | 18797956 | 4.02630 | 75686269 |
| 7 | taper | composite | 11.91 | 18651169 | 4.71967 | 88027297 |
| 8 | taper | composite | 13.89 | 18504381 | 5.48406 | 101479205 |
| 9 | taper | composite | 15.88 | 18357593 | 6.32112 | 116040554 |
| 10 | taper | composite | 17.86 | 18210806 | 7.23247 | 131709100 |
| 11 | taper | composite | 19.84 | 18064018 | 8.21975 | 148481752 |
| 12 | taper | composite | 21.83 | 17917231 | 9.28461 | 166354540 |
| 13 | taper | composite | 23.81 | 17770443 | 10.42870 | 185322588 |
| 14 | taper | composite | 25.80 | 17623656 | 11.65366 | 205380100 |
| 15 | taper | composite | 27.78 | 17476868 | 12.96115 | 226520347 |
| 16 | taper | composite | 29.77 | 17330081 | 14.35283 | 248735658 |
| 17 | taper | composite | 31.75 | 17183293 | 15.83034 | 272017411 |
| 18 | taper | composite | 33.73 | 17036506 | 17.39535 | 296356032 |

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| Node | Description | Material | Sta | E [psi] | [ [in ${ }^{4}$ ] | E] [\|b-in ${ }^{2}$ ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | straight | composite | 35.72 | 16920611 | 18.69389 | 316312119 |
| 20 | straight | composite | 37.70 | 16920611 | 18.69389 | 316312119 |
| 21 | straight | composite | 39.69 | 16920611 | 18.69389 | 316312119 |
| 22 | straight | composite | 41.67 | 16920611 | 18.69389 | 316312119 |
| 23 | straight | composite | 43.66 | 16920611 | 18.69389 | 316312119 |
| 24 | straight | composite | 45.64 | 16920611 | 18.69389 | 316312119 |
| 25 | straight | composite | 47.63 | 16920611 | 18.69389 | 316312119 |
| 26 | straight | composite | 49.61 | 16920611 | 18.69389 | 316312119 |
| 27 | straight | composite | 51.59 | 16920611 | 18.69389 | 316312119 |
| 28 | straight | composite | 53.58 | 16920611 | 18.69389 | 316312119 |
| 29 | straight | composite | 55.56 | 16920611 | 18.69389 | 316312119 |
| 30 | straight | composite | 57.55 | 16920611 | 18.69389 | 316312119 |
| 31 | straight | composite | 59.53 | 16920611 | 18.69389 | 316312119 |
| 32 | straight | composite | 61.52 | 16920611 | 18.69389 | 316312119 |
| 33 | straight | MIDSPAN | 63.50 | 16920611 | 18.69389 | 316312119 |
| 34 | straight | composite | 65.48 | 16920611 | 18.69389 | 316312119 |
| 35 | straight | composite | 67.47 | 16920611 | 18.69389 | 316312119 |
| 36 | straight | composite | 69.45 | 16920611 | 18.69389 | 316312119 |
| 37 | straight | composite | 71.44 | 16920611 | 18.69389 | 316312119 |
| 38 | straight | composite | 73.42 | 16920611 | 18.69389 | 316312119 |
| 39 | straight | composite | 75.41 | 16920611 | 18.69389 | 316312119 |
| 40 | straight | composite | 77.39 | 16920611 | 18.69389 | 316312119 |
| 41 | straight | composite | 79.38 | 16920611 | 18.69389 | 316312119 |
| 42 | straight | composite | 81.36 | 16920611 | 18.69389 | 316312119 |
| 43 | straight | composite | 83.34 | 16920611 | 18.69389 | 316312119 |
| 44 | straight | composite | 85.33 | 16920611 | 18.69389 | 316312119 |
| 45 | straight | composite | 87.31 | 16920611 | 18.69389 | 316312119 |
| 46 | straight | composite | 89.30 | 16920611 | 18.69389 | 316312119 |
| 47 | straight | composite | 91.28 | 16920611 | 18.69389 | 316312119 |
| 48 | taper | composite | 93.27 | 17036506 | 17.39535 | 296356032 |
| 49 | taper | composite | 95.25 | 17183293 | 15.83034 | 272017411 |
| 50 | taper | composite | 97.23 | 17330081 | 14.35283 | 248735658 |
| 51 | taper | composite | 99.22 | 17476868 | 12.96115 | 226520347 |
| 52 | taper | composite | 101.20 | 17623656 | 11.65366 | 205380100 |
| 53 | taper | composite | 103.19 | 17770443 | 10.42870 | 185322588 |
| 54 | taper | composite | 105.17 | 17917231 | 9.28461 | 166354540 |
| 55 | taper | composite | 107.16 | 18064018 | 8.21975 | 148481752 |
| 56 | taper | composite | 109.14 | 18210806 | 7.23247 | 131709100 |
| 57 | taper | composite | 111.13 | 18357593 | 6.32112 | 116040554 |
| 58 | taper | composite | 113.11 | 18504381 | 5.48406 | 101479205 |
| 59 | taper | composite | 115.09 | 18651169 | 4.71967 | 88027297 |
| 60 | taper | composite | 117.08 | 18797956 | 4.02630 | 75686269 |
| 61 | insert | composite | 119.06 | 18937920 | 3.42985 | 64954222 |
| 62 | insert | composite | 121.05 | 18937920 | 3.42985 | 64954222 |
| 63 | ftg/insert | composite | 123.03 | 18937920 | 3.42985 | 64954222 |
| 63 | ftg /insert | metal | 123.03 | 16000000 | 0.69687 | 11149929 |
| 64 | end fitting | metal | 125.02 | 16000000 | 0.69687 | 11149929 |
| 65 | end fitting | metal | 127.00 | 16000000 | 0.69687 | 11149929 |

Figure 2.3.4.2-1. 110K Strut Properties for Newmark Buckling Analysis

| Node | Sta | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}}$ | $\alpha_{n}$ | $\bar{\alpha}_{n}$ | Slope | Yt | $\mathrm{Y}_{\text {cn }}$ | $y_{\text {fn }}$ | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}} \mathrm{y}_{\text {f }}$ | $\mathrm{yfn}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.00 | 0.0000 | 0.0 | $0.000 \mathrm{E}+00$ | -1.839E-06 | -1.839E-06 | $0.000 \mathrm{E}+00$ | 0.0000 | 0.0000 | 0.0000 | 0.000 | $0.00 \mathrm{E}+00$ |
| 2 | 1.98 | 0.0615 | 61.5 | -5.518E-06 | -1.092E-05 | -1.276E-05 | -3.650E-06 | 0.0004 | 0.0004 | 0.0662 | 0.025 | 1.67E-07 |
| 3 | 3.97 | 0.1211 | 121.1 | -1.086E-05 | -9.023E-06 | -2393E-05 | -2897E-05 | 0.0008 | 0.0008 | 0.1288 | 0.096 | 6.33E-07 |
| 3 | 3.97 | 0.1211 | 121.1 | -1.864E-06 | -2.146E-06 | - | -875-05 | 0.0008 |  |  |  | 6.33E-07 |
| 4 | 5.95 | 0.1787 | 178.7 | -2.751E-06 | -5.455E-06 | -2.938E-05 | -7.645E-05 | 0.0012 | 0.0012 | 0.1879 | 0.207 | 1.35E-06 |
| 5 | 7.94 | 0.2344 | 234.4 | -3.608E-06 | -7.051E-06 | -3.643E-05 | -1.348E-04 | 0.0016 | 0.0015 | 0.2453 | 0.355 | $2.29 \mathrm{E}-06$ |
| 6 | 9.92 | 0.2881 | 288.1 | -3.806E-06 | -7.529E-06 | -4.396E-05 | -2.071E-04 | 0.0021 | 0.0019 | 0.3003 | 0.534 | 3.44E-06 |
| 7 | 11.91 | 0.3398 | 339.8 | -3.861E-06 | -7.649E-06 | -5.161E-05 | -2.943E-04 | 0.0025 | 0.0022 | 0.3530 | 0.741 | 4.75E-06 |
| 8 | 13.89 | 0.3896 | 389.6 | -3.840E-06 | -7.611E-06 | -5.922E-05 | -3.967E-04 | 0.0029 | 0.0025 | 0.4031 | 0.970 | $6.20 \mathrm{E}-06$ |
| 9 | 15.88 | 0.4375 | 437.5 | -3.770E-06 | -7.477E-06 | -6.670E-05 | -5.142E-04 | 0.0033 | 0.0028 | 0.4509 | 1.218 | 7.75E-06 |
| 10 | 17.86 | 0.4834 | 483.4 | -3.670E-06 | -7.280E-06 | -7.398E-05 | -6.466E-04 | 0.0037 | 0.0031 | 0.4962 | 1.481 | 9.39E-06 |
| 11 | 19.84 | 0.5273 | 527.3 | -3.552E-06 | -7.046E-06 | -8.103E-05 | -7.934E-04 | 0.0041 | 0.0033 | 0.5392 | 1.756 | 1.11E-05 |
| 12 | 21.83 | 0.5693 | 569.3 | -3.422E-06 | -6.791E-06 | -8.782E-05 | -9.542E-04 | 0.0045 | 0.0036 | 0.5799 | 2.039 | $1.28 \mathrm{E}-05$ |
| 13 | 23.81 | 0.6094 | 609.4 | -3.288E-06 | -6.525E-06 | -9.434E-05 | -1.128E-03 | 0.0049 | 0.0038 | 0.6185 | 2.327 | $1.46 \mathrm{E}-05$ |
| 14 | 25.80 | 0.6475 | 647.5 | -3.153E-06 | -6.256E-06 | -1.006E-04 | -1.316E-03 | 0.0054 | 0.0040 | 0.6549 | 2.619 | $1.64 \mathrm{E}-05$ |
| 15 | 27.78 | 0.6836 | 683.6 | -3.018E-06 | -5.989E-06 | -1.066E-04 | -1.515E-03 | 0.0058 | 0.0043 | 0.6894 | 2.910 | 1.81E-05 |
| 16 | 29.77 | 0.7178 | 717.8 | -2.886E-06 | -5.727E-06 | -1.123E-04 | -1.727E-03 | 0.0062 | 0.0045 | 0.7219 | 3.200 | $1.99 \mathrm{E}-05$ |
| 17 | 31.75 | 0.7500 | 750.0 | -2.757E-06 | -5.472E-06 | -1.178E-04 | -1.950E-03 | 0.0066 | 0.0046 | 0.7526 | 3.485 | 2.16E-05 |
| 18 | 33.73 | 0.7803 | 780.3 | -2.633E-06 | -5.233E-06 | -1.230E-04 | -2.183E-03 | 0.0070 | 0.0048 | 0.7815 | 3.765 | $2.33 \mathrm{E}-05$ |
| 19 | 35.72 | 0.8086 | 808.6 | -2.556E-06 | -5.099E-06 | -1.281E-04 | -2.428E-03 | 0.0074 | 0.0050 | 0.8087 | 4.038 | 2.49E-05 |
| 20 | 37.70 | 0.8350 | 835.0 | -2.640E-06 | -5.237E-06 | -1.334E-04 | -2.682E-03 | 0.0078 | 0.0052 | 0.8343 | 4.302 | $2.65 \mathrm{E}-05$ |
| 21 | 39.69 | 0.8594 | 859.4 | -2.717E-06 | -5.390E-06 | -1.387E-04 | -2.946E-03 | 0.0082 | 0.0053 | 0.8583 | 4.555 | 2.81E-05 |
| 22 | 41.67 | 0.8818 | 881.8 | -2.788E-06 | -5.531E-06 | -1.443E-04 | -3.222E-03 | 0.0087 | 0.0054 | 0.8804 | 4.794 | $2.96 \mathrm{E}-05$ |
| 23 | 43.66 | 0.9023 | 902.3 | -2.853E-06 | -5.660E-06 | -1.499E-04 | -3.508E-03 | 0.0091 | 0.0056 | 0.9008 | 5.020 | 3.09E-05 |
| 24 | 45.64 | 0.9209 | 920.9 | -2.911E-06 | -5.776E-06 | -1.557E-04 | -3.806E-03 | 0.0095 | 0.0057 | 0.9194 | 5.228 | $3.22 \mathrm{E}-05$ |
| 25 | 47.63 | 0.9375 | 937.5 | -2.964E-06 | -5.880E-06 | -1.616E-04 | -4.115E-03 | 0.0099 | 0.0058 | 0.9362 | 5.420 | 3.34E-05 |
| 26 | 49.61 | 0.9521 | 952.1 | -3.010E-06 | -5.972E-06 | -1.676E-04 | -4.435E-03 | 0.0103 | 0.0059 | 0.9510 | 5.592 | 3.45E-05 |
| 27 | 51.59 | 0.9648 | 964.8 | -3.050E-06 | -6.052E-06 | -1.736E-04 | -4.768E-03 | 0.0107 | 0.0060 | 0.9639 | 5.743 | 3.54E-05 |
| 28 | 53.58 | 0.9756 | 975.6 | -3.084E-06 | -6.119E-06 | -1.797E-04 | -5.112E-03 | 0.0111 | 0.0060 | 0.9749 | 5.873 | 3.62E-05 |
| 29 | 55.56 | 0.9844 | 984.4 | -3.112E-06 | -6.174E-06 | -1.859E-04 | -5.469E-03 | 0.0115 | 0.0061 | 0.9839 | 5.981 | 3.69E-05 |
| 30 | 57.55 | 0.9912 | 991.2 | -3.134E-06 | -6.217E-06 | -1.921E-04 | -5.838E-03 | 0.0120 | 0.0061 | 0.9909 | 6.065 | 3.74E-05 |
| 31 | 59.53 | 0.9961 | 996.1 | -3.149E-06 | -6.248E-06 | -1.984E-04 | -6.219E-03 | 0.0124 | 0.0062 | 0.9960 | 6.126 | $3.78 \mathrm{E}-05$ |
| 32 | 61.52 | 0.9990 | 999.0 | -3.158E-06 | -6.266E-06 | -2.046E-04 | -6.613E-03 | 0.0128 | 0.0062 | 0.9990 | 6.163 | 3.81E-05 |
| 33 | 63.50 | 1.0000 | 1000.0 | -3.161E-06 | -6.272E-06 | -2.109E-04 | -7.019E-03 | 0.0132 | 0.0062 | 1.0000 | 6.175 | $3.81 \mathrm{E}-05$ |
| 34 | 65.48 | 0.9990 | 999.0 | -3.158E-06 | -6.266E-06 | -2.172E-04 | -7.437E-03 | 0.0136 | 0.0062 | 0.9990 | 6.163 | 3.81E-05 |
| 35 | 67.47 | 0.9961 | 996.1 | -3.149E-06 | -6.248E-06 | -2.234E-04 | -7.868E-03 | 0.0140 | 0.0062 | 0.9960 | 6.126 | 3.78E-05 |
| 36 | 69.45 | 0.9912 | 991.2 | -3.134E-06 | -6.217E-06 | -2.296E-04 | -8.312E-03 | 0.0144 | 0.0061 | 0.9909 | 6.065 | 3.74E-05 |
| 37 | 71.44 | 0.9844 | 984.4 | -3.112E-06 | -6.174E-06 | -2.358E-04 | -8.767E-03 | 0.0148 | 0.0061 | 0.9839 | 5.981 | 3.69E-05 |


| Node | Sta | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}}$ | $\alpha_{n}$ | $\bar{\alpha}_{n}$ | Slope | $Y_{\text {en }}$ | $\mathrm{Y}_{\text {cn }}$ | $y_{\text {fn }}$ | $\delta_{n}$ | $\mathrm{M}_{\mathrm{n}} \mathrm{y}_{\text {f }}$ | $y_{\text {fn }}{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 38 | 73.42 | 0.9756 | 975.6 | -3.084E-06 | -6.119E-06 | -2.419E-04 | -9.235E-03 | 0.0153 | 0.0060 | 0.9749 | 5.873 | 3.62E-05 |
| 39 | 75.41 | 0.9648 | 964.8 | -3.050E-06 | -6.052E-06 | -2.480E-04 | -9.715E-03 | 0.0157 | 0.0060 | 0.9639 | 5.743 | 3.54E-05 |
| 40 | 77.39 | 0.9521 | 952.1 | -3.010E-06 | -5.972E-06 | -2.540E-04 | -1.021E-02 | 0.0161 | 0.0059 | 0.9510 | 5.592 | 3.45E-05 |
| 41 | 79.38 | 0.9375 | 937.5 | -2.964E-06 | -5.880E-06 | -2.598E-04 | -1.071E-02 | 0.0165 | 0.0058 | 0.9362 | 5.420 | 3.34E-05 |
| 42 | 81.36 | 0.9209 | 920.9 | -2.911E-06 | -5.776E-06 | -2.656E-04 | -1.123E-02 | 0.0169 | 0.0057 | 0.9194 | 5.228 | 3.22E-05 |
| 43 | 83.34 | 0.9023 | 902.3 | -2.853E-06 | -5.660E-06 | -2.713E-04 | -1.175E-02 | 0.0173 | 0.0056 | 0.9008 | 5.020 | $3.09 \mathrm{E}-05$ |
| 44 | 85.33 | 0.8818 | 881.8 | -2.788E-06 | -5.531E-06 | -2.768E-04 | -1.229E-02 | 0.0177 | 0.0054 | 0.8804 | 4.794 | 2.96E-05 |
| 45 | 87.31 | 0.8594 | 859.4 | -2.717E-06 | -5.390E-06 | -2.822E-04 | -1.284E-02 | 0.0181 | 0.0053 | 0.8583 | 4.555 | 2.81E-05 |
| 46 | 89.30 | 0.8350 | 835.0 | -2.640E-06 | -5.237E-06 | -2.874E-04 | -1.340E-02 | 0.0186 | 0.0052 | 0.8343 | 4.302 | 2.65E-05 |
| 47 | 91.28 | 0.8086 | 808.6 | -2.556E-06 | -5.099E-06 | -2.925E-04 | -1.397E-02 | 0.0190 | 0.0050 | 0.8087 | 4.038 | 2.49E-05 |
| 48 | 93.27 | 0.7803 | 780.3 | -2.633E-06 | -5.233E-06 | -2.978E-04 | -1.455E-02 | 0.0194 | 0.0048 | 0.7815 | 3.765 | 2.33E-05 |
| 49 | 95.25 | 0.7500 | 750.0 | -2.757E-06 | -5.472E-06 | -3.032E-04 | -1.514E-02 | 0.0198 | 0.0046 | 0.7526 | 3.485 | $2.16 \mathrm{E}-05$ |
| 50 | 97.23 | 0.7178 | 717.8 | -2.886E-06 | -5.727E-06 | -3.090E-04 | -1.575E-02 | 0.0202 | 0.0045 | 0.7219 | 3.200 | 1.99E-05 |
| 51 | 99.22 | 0.6836 | 683.6 | -3.018E-06 | -5.989E-06 | -3.150E-04 | -1.636E-02 | 0.0206 | 0.0043 | 0.6894 | 2.910 | 1.81E-05 |
| 52 | 101.20 | 0.6475 | 647.5 | -3.153E-06 | -6.256E-06 | -3.212E-04 | -1.698E-02 | 0.0210 | 0.0040 | 0.6549 | 2.619 | 1.64E-05 |
| 53 | 103.19 | 0.6094 | 609.4 | -3.288E-06 | -6.525E-06 | -3.277E-04 | -1.762E-02 | 0.0214 | 0.0038 | 0.6185 | 2.327 | 1.46E-05 |
| 54 | 105.17 | 0.5693 | 569.3 | -3.422E-06 | -6.791E-06 | -3.345E-04 | -1.827E-02 | 0.0219 | 0.0036 | 0.5799 | 2.039 | 1.28E-05 |
| 55 | 107.16 | 0.5273 | 527.3 | -3.552E-06 | -7.046E-06 | -3.416E-04 | -1.893E-02 | 0.0223 | 0.0033 | 0.5392 | 1.756 | 1.11E-05 |
| 56 | 109.14 | 0.4834 | 483.4 | -3.670E-06 | -7.280E-06 | -3.489E-04 | -1.961E-02 | 0.0227 | 0.0031 | 0.4962 | 1.481 | 9.39E-06 |
| 57 | 111.13 | 0.4375 | 437.5 | -3.770E-06 | -7.477E-06 | -3.563E-04 | -2.031E-02 | 0.0231 | 0.0028 | 0.4509 | 1.218 | 7.75E-06 |
| 58 | 113.11 | 0.3896 | 389.6 | -3.840E-06 | -7.611E-06 | -3.639E-04 | -2.101E-02 | 0.0235 | 0.0025 | 0.4031 | 0.970 | 6.20E-06 |
| 59 | 115.09 | 0.3398 | 339.8 | -3.861E-06 | -7.649E-06 | -3.716E-04 | -2.173E-02 | 0.0239 | 0.0022 | 0.3530 | 0.741 | 4.75E-06 |
| 60 | 117.08 | 0.2881 | 288.1 | -3.806E-06 | -7.529E-06 | -3.791E-04 | -2.247E-02 | 0.0243 | 0.0019 | 0.3003 | 0.534 | 3.44E-06 |
| 61 | 119.06 | 0.2344 | 234.4 | -3.608E-06 | -7.051E-06 | -3.862E-04 | -2.322E-02 | 0.0247 | 0.0015 | 0.2453 | 0.355 | 2.29E-06 |
| 62 | 121.05 | 0.1787 | 178.7 | -2.751E-06 | -5.455E-06 | -3.916E-04 | -2.399E-02 | 0.0252 | 0.0012 | 0.1879 | 0.207 | 1.35E-06 |
| 63 | 123.03 | 0.1211 | 121.1 | -1.864E-06 | -2.146E-06 | -4.028E-04 | -2.477E-02 | 0.0256 | 0.0008 | 0.1288 | 0.096 | 6.33E-07 |
| 63 | 123.03 | 0.1211 | 121.1 | -1.086E-05 | -9.023E-06 |  |  |  |  |  |  |  |
| 64 | 125.02 | 0.0615 | 61.5 | -5.518E-06 | -1.092E-05 | -4.137E-04 | -2.557E-02 | 0.0260 | 0.0004 | 0.0662 | 0.025 | 1.67E-07 |
| 65 | 127.00 | 0.0000 | 0.0 | $0.000 \mathrm{E}+00$ | -1.839E-06 | -4.156E-04 | -2.639E-02 | 0.0264 | 0.0000 | 0.0000 | 0.000 | 0.00E+00 |
|  |  |  |  |  |  |  |  |  |  | Sum: | 211.43 | 0.00131 |

Figure 2.3.4.2-2. Newmark Buckling Analysis Results for 110K Strut

The buckling load from the first iteration is
$\mathrm{P}_{\mathrm{cr}}=\frac{\sum \mathrm{M}_{\mathrm{n}} \mathrm{y}_{\mathrm{fn}}}{\sum \mathrm{y}_{\mathrm{fn}}{ }^{2}}$
$\mathrm{P}_{\mathrm{cr}}=211.43 / 0.00131=160,623 \mathrm{lb}$
The iterated buckling loads are presented in Figure 2.3.4.2-3, along with the ultimate load of $154,000 \mathrm{lb}$.


149764-023.1
Figure 2.3.4.2-3. Iterated buckling loads of 110 K strut

### 2.3.5 Natural Frequency

The natural frequency of the first mode of a uniform beam with simply supported ends is:
$\mathrm{f}_{1}=\frac{9.87}{2 \pi} \sqrt{\frac{E l g}{w L^{4}}}$
where:
E is the modulus of elasticity
I is the moment of inertia
g is gravitational acceleration (in consistent units)
w is the weight per unit length of the beam
L is the length of the beam (Ref: Roark \& Young, 6th edition, Table 36, case 1b)
The Park struts have a significant mass located near the ends in the form of the titanium inserts and end fittings. Since the ends are simply-supported (pinned), these masses will not significantly affect the frequency of the first mode. As an approximation, they will be ignored. Similarly, the taper and overwrap sections will have less effect on the dynamic response than the midspan. Therefore, for this approximation, the midspan properties will be used in the natural frequency equation.

### 2.3.5.1 44K Strut

The cross-sectional area at the midspan is
$\mathrm{A}=\pi\left(\mathrm{r}_{0} /\right.$ midspan $\left.{ }^{2}-\mathrm{r}_{\mathrm{i} / \text { midspan }}{ }^{2}\right)=\pi\left(3.10021^{2}-3^{2}\right)=1.920 \mathrm{in}^{2}$
and the density of the composite is $0.0555 \mathrm{lb} / \mathrm{in}^{3}$
The weight per unit length is
$\mathrm{w}=\rho \mathrm{A}=(0.0555)(1.920)=0.10656 \mathrm{lb} / \mathrm{in}$
The natural frequency of the first mode is then
$\mathrm{f}_{1}=\frac{9.87}{2 \pi} \sqrt{\frac{(16,631,175)(8.9356)(386.4)}{(0.10656)(135)^{4}}}$
$\mathrm{f}_{1}=63 \mathrm{~Hz}$

### 2.3.5.2 110K Strut

$\mathrm{A}=\pi\left(\mathrm{r}_{0} /\right.$ midspan $\left.^{2}-\mathrm{r}_{\mathrm{i} / \text { midspan }}{ }^{2}\right)=\pi\left(3.411^{2}-3.25^{2}\right)=3.369 \mathrm{in}^{2}$
$\mathrm{w}=\rho \mathrm{A}=(0.0555)(3.369)=0.18698 \mathrm{lb} / \mathrm{in}$
$\mathrm{f}_{1}=\frac{9.87}{2 \pi} \sqrt{\frac{(16,920,611)(18.6939)(386.4)}{(0.18698)(127)^{4}}}$
$\mathrm{f}_{1}=79 \mathrm{~Hz}$

### 2.3.6 Compression Stress

The 0 -deg plies are assumed to carry the entire axial load; the contribution by the 90 -deg plies is neglected.

### 2.3.6.1 44K Strut

$$
\begin{aligned}
& \sigma_{\mathrm{comp}}=\mathrm{P}_{\mathrm{ULT}} / \mathrm{A}_{0}=1.4 * 44,000 / 1.493=41259 \mathrm{psi} \\
& \mathrm{MS}=191,000 / 41,259-1=3.63
\end{aligned}
$$

### 2.3.6.2 110K Strut

$\sigma_{\text {comp }}=\mathrm{P}_{\mathrm{ULT}} / \mathrm{A}_{0}=1.4 * 110,000 / 2.665=57786 \mathrm{psi}$
$\mathrm{MS}=191,000 / 57,786-1=2.31$

### 2.3.7 Crippling/Local Instability

Park applies an empirical equation derived from its own testing to determine the crippling cutoff stress:

$$
\sigma_{\mathrm{cr}}=\frac{456075}{\sqrt{\mathrm{D}_{\mathrm{i} / \text { midspan }} / \mathrm{t}_{0 / \text { midspan }}}}
$$

### 2.3.7.1 44K Strut

$\sigma_{\mathrm{cr}}=\frac{456075}{\sqrt{6 / 0.07821}}=52071$
$\mathrm{MS}=52,071 / 41,259-1=0.26$

### 2.3.7.2 110K Strut

$$
\begin{aligned}
& \sigma_{\mathrm{cr}}=\frac{456075}{\sqrt{6.5 / 0.12798}}=63996 \\
& \mathrm{MS}=63,996 / 57,786-1=0.11
\end{aligned}
$$

### 3.0 MANUFACTURING DEMONSTRATION ARTICLE

### 3.1 Background

The Shuttle Orbiter program uses aluminum struts (Figure 3.1-1) to replace easily damaged midfuselage boron/aluminum struts. However, these aluminum struts add significant weight to the vehicles. In response, Boeing conducted a preliminary development program for lightweight graphite composite struts that would replace the relatively heavy aluminum struts.


149764-018
Figure 3.1-1. Shuttle Aluminum Replacement Strut Configuration
Boeing and its subcontractor Park Aerospace Structures (previously Nova Composites) conducted design, development, and testing of a lightweight graphite composite replacement strut starting in 2001 (Figure 3.1-2). A preliminary, yet comprehensive, series of development tests included damage tolerance, tension, compression, and extreme environment testing. One concept that was investigated to visualize impact damage was to bond a single ply of fiberglass/epoxy to the outer tube surface. A detailed draft specification was also prepared.


Figure 3.1-2. Shuttle Replacement Strut Development
The objective of the development program was to verify that composite struts could satisfy the design requirements for a human-rated spacecraft while saving weight compared to the aluminum struts. The design requirements included strength, stiffness, and fatigue loading criteria. The composite struts were designed to withstand launch, ascent, on-orbit, descent, and landing loads. Other design goals and guidelines include (1) minimize strut configurations, (2) no overlapping of strut length between configurations, (3) maintain existing strut geometry
envelope, (4) minimize cost per pound of weight savings, (5) use existing orbiter attachment hardware, (6) exhibit improved damage tolerance, and (7) reduce replacement cycle time.

A comprehensive set of composite strut designs was developed to replace existing aluminum replacement struts currently on the Shuttle fleet. The end fitting design consists of a clevis that is threaded into a Park insert.

The demo strut fabrication activity provided an understanding of the Park strut fabrication process, and determined the improvements necessary to transition the Park current strut manufacturing process to a fully traceable and repeatable process capable of producing certified space flight hardware.

### 3.2 Demonstration Strut Design

The strut body is about one half the length of the full-scale analytical struts and has a sufficient number of plies to demonstrate process optimization for a full-scale strut. The selected end fitting features a clevis that is integral with the Park insert (Figures 3.2-1 and 3.2-2). Compared to the adjustable design, advantages of the integral end fitting include (1) approximately the same cost, (2) $33 \%$ less weight, and (3) more accurate dimensional control on assembly (holes are drilled in the clevis on assembly). One possible disadvantage is less adjustability.

The representative end fittings are threaded so that can be removed from the inserts. This modification was chosen to preclude changing the Park fabrication process, and to demonstrate the ability to incorporate various end fittings configurations. The 11- ply layup ( $90,0_{4}, 90,04,90$ ) uses IM7/8552 tow and tape prepreg.


Figure 3.2-1. Selected Composite Strut Body and Integral End Fitting Configuration


Figure 3.2-2. Integral End Fitting Configuration

### 3.3 Demonstration Strut Fabrication

Each step of the strut manufacturing process is documented in the final report for his task. An outline of the process is included herein. All materials used in these struts are commercially available and nonproprietary.

Figure 3.3-1 summarizes the strut fabrication process. A plaster mandrel is cast onto a steel rod. Machined titanium inserts with center holes are placed on the rod at either end of the plaster mandrel. Collars are placed onto the rod to secure the inserts during strut body fabrication. The strut body is layed up with $90-\mathrm{deg}$ prepreg tow and $0-\mathrm{deg}$ prepreg tape plies onto the plaster mandrel and titanium insert, with intermittent debulks to minimize fiber wrinkling. The plies are also layed onto the insert. The strut body is cured in an autoclave, and the plaster mandrel is washed out. In the improved Boeing process, the end fitting is then final machined from a blank that is an integral part of the insert.


Figure 3.3.1-1. Overview of Strut Fabrication Process
Composite tubular structures are often bagged from the inside with an inflatable or expanding bladder that compacts the laminate against an outer mold line clamshell-type tool. This approach eliminates the tendency of the material to wrinkle as its thickness reduces during consolidation from the layup thickness to the laminate thickness. However, because the Park strut design relies on an interlocking contact of the end fitting with the composite strut body, the strut must use an outer mold line bag against an inner mold line washout tool.

Figure 3.3.1-2 summarizes the strut body fabrication processes. These processes are generally applicable to both demo struts \#1 and \#2.


149764-055.1
Figure 3.3.1-2. Strut Body Fabrication Processes
All materials used in the fabrication of the demo struts were procured commercially, and prepreg material certifications were per Hexcel internal specifications (HS-AD-971A Rev 4 in the case of the demo struts).

Park inserts are machined from commercially procured annealed $6 \mathrm{Al}-4 \mathrm{~V}$ titanium. No passivation, penetrant inspection, or cleaning processes are performed. Insert fabrication was per Park standard process described above. Figure 3.3.1-3 shows one of the resulting inserts.


Figure 3.3.1-3. Titanium Insert

Completed demonstration parts are shown in figure 3.3.1-4.


Figure 3.3.1-4. Demo Struts \#1 and \#2

### 4.0 CONCLUSION

The objective of this task order was to perform an analytical study of two structurally efficient, full-scale, tapered composite struts, and to fabricate a subscale strut demonstration article. The approach was to leverage and extend recent experience on a Space Shuttle composite strut development program.

The first step of the analytical study identified design requirements and considerations as applicable to the analytical study, a production program, and the demonstration strut. Using these requirements and considerations, detailed design and stress analysis determined the optimum (minimum) weight of the full-scale struts. One full-scale strut was required to carry a 44,000-lb compression load and have a 135 -inch pin-to-pin length. The second strut was required to carry a compression load of $110,000 \mathrm{lb}$ and have a pin-to-pin length of 127 inches.

The design of the demonstration strut was selected from a set of existing designs created during the Shuttle composite strut program. Two approximately half-scale demo struts were fabricated by Park Aerospace Structures, which participated in the Shuttle composite strut program. The first demo strut was fabricated using a higher-cost single-ply autoclave debulk schedule with the objective of achieving the highest possible laminate quality. Thermography
inspection was performed on the first demo strut and on a strut fabricated using the Park standard nonautoclave process. The results were used to select the debulk schedule for the second demo strut. Consequently, the second strut was fabricated using a lower-cost debulk schedule that combined autoclave and shrink tape debulking with the objective of balancing fabrication cost and laminate quality. The two demo struts and Park standard strut were nondestructively inspected with an ultrasonic scan. The results identified distinctive indications in the Park standard strut that were less pronounced or absent in the demo struts.

Based on the experience from the fabrication of the demo struts, various process improvements were identified and are recommended to be implemented during the fabrication of a future full-scale strut test article. These improvements will likely improve the traceability and repeatability of the full-scale strut fabrication process, and the resulting quality of the strut laminate. These improvements may in turn improve laminate mechanical properties and further reduce strut weight.

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| 1. REPORT DATE (DD-MM-YYYY)01-05-2010 |  |  | 2. REPORT TYPE <br> Contractor Report |  |  |  | 3. DATES COVERED (From - To) August 7, 2008 - November 21, 2008 |
| 4. TITLE AND SUBTITLE <br> Design of Structurally Efficient Tapered Struts |  |  |  |  |  | 5a. CONTRACT NUMBER NNL04AA11B 5b. GRANT NUMBER |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | 5c. PROGRAM ELEMENT NUMBER |  |
| 6. AUTHOR(S) <br> Messinger, Ross |  |  |  |  |  | 5d. PROJECT NUMBER |  |
|  |  |  |  |  |  | 5e. TASK NUMBER <br> NNL08AD08T <br> 5f. WORK UNIT NUMBER <br> 727950.04 .05 .23 |  |
|  |  |  |  |  |  |  |  |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  <br> NASA Langley Research Center The Boeing Company <br> Hampton, VA 23681-2199 5302 Bolsa Avenue <br>  Huntington Beach, CA 92647-2099 |  |  |  |  |  |  | 8. PERFORMING ORGANIZATION REPORT NUMBER <br> PWDM08-0021 |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) <br> National Aeronautics and Space Administration <br> Washington, DC 20546-0001 |  |  |  |  |  |  | 10. SPONSOR/MONITOR'S ACRONYM(S) <br> NASA |
|  |  |  |  |  |  |  | 11. SPONSOR/MONITOR'S REPORT NUMBER(S) |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT <br> Unclassified - Unlimited <br> Subject Category 39 <br> Availability: NASA CASI (443) 757-5802 |  |  |  |  |  |  |  |
| 13. SUPPLEMENTARY NOTES <br> Langley Technical Monitor: Dawn C. Jegley |  |  |  |  |  |  |  |
| 14. ABSTRACT <br> This report describes the analytical study of two full-scale tapered composite struts. The analytical study resulted in the design of two structurally efficient carbon/epoxy struts in accordance with NASA-specified geometries and loading conditions. Detailed stress analysis was performed of the insert, end fitting, and strut body to obtain an optimized weight with positive margins. Two demonstration struts were fabricated based on a well-established design from a previous Space Shuttle strut development program. |  |  |  |  |  |  |  |
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| 15. SUBJECT TERMS <br> Composites; Graphite; Structural efficiency; Buckling |  |  |  |  |  |  |  |
| 16. SECURITY CLASSIFICATION OF: |  |  |  | 17. LIMITATION OF ABSTRACT <br> UU | 18. OF PAGES | 19a. NAME OF RESPONSIBLE PERSON STI Help Desk (email: help@sti.nasa.gov) |  |
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