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Evolving Systems and Adaptive Key Component Control

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1. Introduction

We propose a new framework called Evolving Systems to describe the self-assembly, or autonomous assembly, of actively controlled dynamical subsystems into an Evolved System with a higher purpose. An introduction to Evolving Systems and exploration of the essential topics of the control and stability properties of Evolving Systems is provided. This chapter defines a framework for Evolving Systems, develops theory and control solutions for fundamental characteristics of Evolving Systems, and provides illustrative examples of Evolving Systems and their control with adaptive key component controllers.

Evolving Systems provide a framework that facilitates the design and analysis of self-assembling systems. The components of an Evolving System self-assemble, or mate, to form new components or the Evolved System. The mating of the subsystem components can be self-directed or agent controlled. The Evolving Systems framework provides a scalable, modular architecture to model and analyze the subsystem components, their connections to other components, and the Evolved System. Ultimately, once all the components of an Evolving System have joined together to form the fully Evolved System, it will have a new, higher purpose that could not have been achieved by the individual components collectively.

Autonomous assembly of large, complex structures in space, or on-orbit assembly, is an excellent application area for Evolving Systems. For example, the Solar Power Satellite (SPS) is a conceptual space structure that collects solar energy, which is then transmitted to Earth as microwaves (NASA, 1995). The solar array of the SPS, as envisioned in fig. 1, is a complex structure that could be assembled from many actively controlled components to form a new system with a higher purpose.

System stability is a trait that could be exhibited by an Evolving System or their components. We say that a subsystem trait is *inherited* by an Evolving System when the system retains the properties of the trait after assembly. The inheritance of subsystem traits, or genetics, such as controllability, observability, stability, and robustness, in Evolving Systems is an important research topic.

A critical element of successful on-orbit assembly of flexible space structures is the autonomous control of a structure during and after the connection of two or more subsystem

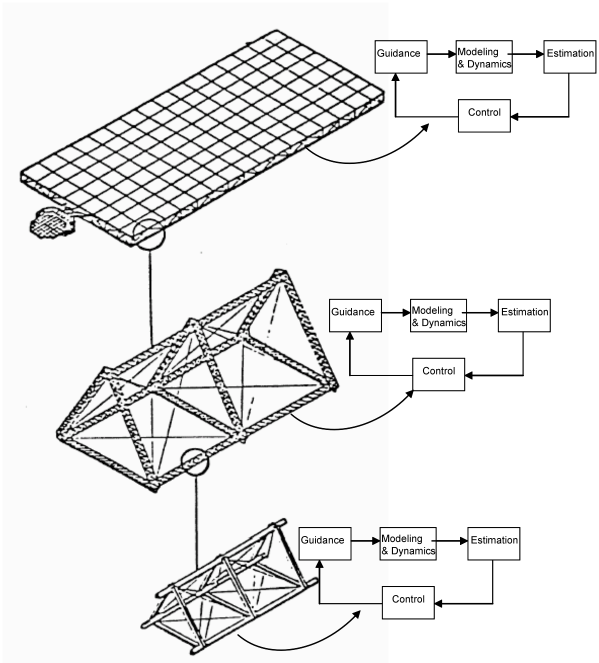

Fig. 1. Solar array component of a Solar Powered Satellite, image credit NASA.

components. The inheritance of stability in Evolving Systems is crucial in space applications due to potential damage and catastrophic losses that can result from unstable space systems. The subsystem components of an Evolving System are designed to be stable as free-fliers, or unconnected components, but the Evolving System might fail to inherit stability at any step of the assembly, resulting in an unstable Evolved System. The fundamental topic of stability in Evolving Systems has been a primary focus of our Evolving Systems research (Balas & Frost, 2007; Frost & Balas, 2007a;b; 2008b;a; Balas & Frost, 2008; Frost, 2008). In this chapter, we develop an adaptive key component control method to ensure that stability is inherited in flexible structure Evolving Systems.

1.1 Description of Evolving Systems

Evolving Systems are dynamical systems that are self-assembled from actively controlled subsystem components. Central to the concept of Evolving Systems is the idea that an Evolved System has a higher functioning purpose than that of its subsystem components. For instance, the subsystem components might include a truss system, optical equipment, control systems, and communications equipment. If these components are assembled to form a space-based telescope, this would have a higher purpose than that of the individual components. Subsystems could consist of deployed components and self-assembled components. One could imagine that a space-based telescope, such as the Hubble Space Telescope, could be built as an Evolving System. The higher functioning purpose of the Evolving System would most likely come about not directly from the assembly of the subsystem components into a new system, but as a result of a new controller or agent taking over operation of the Evolving System after the subsystem components are fully assembled.

It is assumed that the components of an Evolving System would self-assemble, either through their own knowledge, or through the knowledge of an external agent. Note that the agent would not be a human, but an autonomous agent with knowledge of the assembly requirements of the Evolving System. In the Evolving Systems framework presented here, it is assumed that the positioning of the subsystem components in space and time would be handled by the agent or the components themselves. Once the components are positioned, they would be self-directed or agent-directed to assemble with the appropriate components.

The actual connections made between subsystem components in an Evolving System are envisioned as compliant connections, so no degrees of freedom would be lost as a consequence of two components joining together in a rigid manner. A key concept in Evolving Systems is an evolutionary connection parameter, ϵ, that enables the compliant connection to smoothly go from not existing at all $(\epsilon = 0)$, to the full compliance of the connection $(\epsilon = 1)$. The evolution of the connection parameter would occur independent of time. In Evolving Systems of flexible structures, the compliant connection might be modeled by a spring joining two components. Formation flying of imaging satellites to create synthetic apertures could be modeled as Evolving Systems with virtual forces representing the distance maintained between members of the satellite constellation.

In the formulation of Evolving Systems presented here, the evolution of the connection between components occurs independent of time. We are ignoring time in our formulation because it is assumed that the mating of the components is not time critical. We are interested in studying the joining of subsystem components to form an Evolved System, which is controlled by the evolution of the connection parameter going from zero to one. We say an Evolving System is *fully* evolved when all of the connection parameters joining the subsystem components equal one. An Evolving System is said to be *partially* evolved when at least one of its connection parameters never attains the value of 1 due to some event. In the case of a partially Evolved System, some of the components have failed to completely join together to form the prescribed configuration of the Evolving System.

Evolving Systems could be used for the design and analysis of self-assembling systems at all scales. Self-assembly occurs in nature and technology starting at the molecular or nanoscale (formation of crystals and nanostructures) to the macro-scale (formation of netted computer systems). See Whitesides & Grzybowski (2002) for an excellent survey of present and future applications of self-assembly.

The Evolving Systems framework is ideal for systems that are modular and can be scaled for complexity. If a system can be decomposed into modules, the detailed design process for each module needs to be performed only once. Parameter variations affecting the module can often be accommodated by the original design with significantly less effort than a new design would require. Once the design and validation of the module is complete, scaling the system to include more modules would be cost effective within the Evolving Systems framework.

1.2 Motivation for Evolving Systems

Future space missions will require on-orbit assembly of large aperture (greater than 10 meters) space systems, possibly at distant locations that prohibit astronaut intervention (Flinn, 2009). Historically, deployable techniques, sometimes in combination with astronaut assistance, have been used for fielding space systems. As the aperture size of the fielded space structure increases, deployable fielding techniques can become overly complex and unreliable. The increasing complexity of space structures, including such missions as the International Space Station (ISS) and the Hubble Space Telescope, often results in the need for extraordinary astronaut and ground crew assistance for assembly, servicing, and upgrades. Evolving Systems research could facilitate self-assembly and autonomous servicing of complex space systems (Saleh et al., 2002). Additionally, future space missions might entail systems where the scale, complexity, and distance preclude astronaut assistance due to the inherent risks and costs associated with direct human involvement in these missions. These considerations suggest the need for an Evolving Systems framework and methodologies to enable on-orbit autonomous assembly and servicing of space systems with little or no direct human involvement.

Once an autonomous assembly problem has been solved with the Evolving Systems approach, the same solution can be used repeatedly or scaled to solve a similar problem. For example, the assembly of a large truss structure can be broken down into the assembly of smaller components. These components might consist of a small number of beams that are assembled into certain configurations. The designer only needs to develop the methods to assemble a certain type of component once, then this solution can be repeated to create any number of similar components. One can envision the development of a repository of designs that could be reused in different platforms with small modifications or parameterization changes for new dimensions, configurations, or other characteristics of the components. Evolving Systems enables the scaling and reuse of subsystem components, allowing new platforms to leverage existing technologies or reuse demonstrated solutions.

Flexible structure Evolving Systems are actively controlled, self-assembling flexible structures. The autonomous assembly of space structures provides an efficient means to build very large space structures with the elimination of space walk missions. Removing the astronaut from the assembly of space structures removes the dependency on transportation of the astronaut to the structure, eliminates the risk to human life, and eliminates the high costs associated with transporting humans to space. On-orbit assembly also gives the capability to build and service space systems at distant locations in space that are inaccessible to humans. A key benefit of Evolving Systems is its ability to enable on-orbit servicing and upgrades to existing space systems, thereby leveraging our existing space assets to their fullest capability.

The Evolving Systems framework is ideal for exploiting the inherent modularity and scalability of flexible structure space systems to potentially deliver more reliable systems at lower costs. Space systems that are self-assembled from components can lead to greater launch packing efficiency than can be achieved in traditionally deployed systems. The component aspect of Evolving Systems aids in the mitigation of vibration damage associated with the launch environment by allowing subsystem components to be individually enclosed in energy absorbing packaging. The modular framework of Evolving Systems allows designers to easily add redundancy to systems, thereby mitigating risks. Evolving Systems has the potential to solve difficult autonomous assembly and on-orbit servicing missions of flexible structure space systems, hence, the framework and the control problems investigated here are tailored to the application of flexible structure Evolving Systems.

1.3 Previous Research

Decentralized control theory and analysis has been applied to the control of large interconnected systems; see the excellent survey paper by Nils Sandell (Sandell, Jr. et al., 1978) on this topic. Generally, decentralized control has been used to decrease the complexity of the control issues affecting large interconnected systems. Several researchers have proposed methods to decompose large interconnected systems into subsystems which can then be analyzed for stability properties and for the use of decentralized control methodologies (Michel, 1983; Willems, 1986; Corfmat & Morse, 1976b). These ideas are related, but not equivalent to the Evolving Systems viewpoint.

Formations or constellations of satellites, nano-satellites, or micro-spacecraft could be included in the Evolving Systems framework. These formations of multiple, low cost spacecraft enable missions to accomplish complex objectives with the benefit of greater redundancy, improved performance, and reduced cost. An especially challenging control problem for constellations having large numbers of satellites is the task of coordinating and controlling the relative distances and phases between members of the fleet (Mueller et al., 2001; Kapilal, 1999). The solutions proposed in this work are specific to the application of constellations of satellites, and so are not as general as the Evolving Systems framework we are presenting here.

On the experimental side, a research group at the Information Sciences Institute at the University of Southern California (USC) has been conducting research in self-reconfigurable, autonomous robots and systems. They have conducted experimental work to study the feasibility of techniques for assembling large space structures as part of their FIMER (Free-flying Intelligent MatchmakER robots) project (Suri et al., 2006; Shen et al., 2003). This group uses a distributed control method with simple proportional derivative control laws for the self-assembly of components.

2. Theoretical Formulation of Evolving Systems

This section provides the general theoretical formulation of Evolving Systems, expanding on work first presented in (Balas et al., 2006). In the previous section, we introduced the reader to the variety of dynamical systems that can be modeled by Evolving Systems and some of the benefits applications can obtain by using the Evolving Systems approach. Flexible structures are relatively simple, generally well understood mechanical dynamic systems, so they will be used to illustrate many ideas presented here. The state space representation developed in this section will be for general linear time-invariant (LTI) Evolving Systems, although the framework can be easily extended to account for nonlinear time-invariant and time varying Evolving Systems.

2.1 General Formulation of Evolving Systems

In this section we give the general mathematical formulation of Evolving Systems. Consider a system of L individually actively controlled components, where the components are given by

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}}_i & = & f_i(\mathbf{x}_i, \mathbf{u}_i); \quad \mathbf{x}_i(0) = \mathbf{x}_{i0} \\ \mathbf{y}_i & = & g_i(\mathbf{x}_i, \mathbf{u}_i) \end{array} \right. \tag{1}$$

where $i = 1, 2, \ldots, L$, $\mathbf{x}_i \equiv \left[x_1^i x_2^i \cdots x_{n_i}^i\right]^T$ is the component state vector, $\dot{\mathbf{x}}_i \equiv \left[\dot{x}_1^i \dot{x}_2^i \cdots \dot{x}_{n_i}^i\right]^T$, $\mathbf{u}_i \equiv \left[u_1^i u_2^i \cdots u_{m_i}^i\right]^T$ is the control input vector, $\mathbf{y}_i \equiv \left[y_1^i y_2^i \cdots y_{p_i}^i\right]^T$ is the vector of sensed outputs, and $\mathbf{x}_{i_0}$ is the vector of initial conditions. Note that n_i is the dimension of the state vector $\mathbf{x}_i$, m_i is the dimension of the control vector $\mathbf{u}_i$, and p_i is the dimension of the output vector $\mathbf{y}_i$. Each component has an objective to be satisfied by the perfomance cost function J_i. Local control that depends only on local state or local output information will be used to keep the components stable and to meet the component performance requirements, J_i. In general, the local controller for a Evolving System component would have the form given by

$$\begin{cases} \mathbf{u}_i &= h_i(\mathbf{z}_i) \\ \dot{\mathbf{z}}_i &= l_i(\mathbf{z}_i, \mathbf{y}_i) \end{cases} \tag{2}$$

where h_i and l_i are control operators and $\dot{\mathbf{z}}$ represents the dynamical part of the control law.

The components are the building blocks of the Evolving System. When these individual components join to form an Evolving System, the interconnections between components i and j are represented by the function $k_{ij}(\mathbf{x}, \mathbf{u})$. The connection parameter, ϵ_{ij}, multiplies the interconnections between components i and j.

The subsystem components of the Evolving System with the interconnections included is given by

$$\begin{cases} \dot{\mathbf{x}}_i &= f_i(\mathbf{x}_i, \mathbf{u}_i) + \sum_{j=1}^{L} \epsilon_{ij} k_{ij}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y}_i &= g_i(\mathbf{x}_i, \mathbf{u}_i) \end{cases} \tag{3}$$

where $\mathbf{x} = \left[\mathbf{x}_1 \mathbf{x}_2 \cdots \mathbf{x}_L\right]^T, \mathbf{u} = \left[\mathbf{u}_1 \mathbf{u}_2 \cdots \mathbf{u}_L\right]^T$, and $0 \leq \epsilon_{ij} \leq 1$.

The connection parameter, ϵ_{ij}, is a mathematical construct representing the evolutionary joining of components in an Evolving System. The connection parameter evolves continuously from zero to one as the components assemble. The connection parameter is zero when the components are *unconnected*, or free-fliers. In the free-flier configuration, the components are completely independent of each other. The concept of *partial* evolution versus *full* evolution is an important distinction in Evolving Systems. *Full* evolution of two components occurs when the evolution parameter controlling the connection of the components evolves completely, resulting in the connection reaching its full magnitude and the components being joined together. *Partial* evolution is the case where, for some reason, the connection parameter ϵ_{ij} joining two components fails to attain the value of 1, resulting in the failure of the two components to join together. An important characteristic of the Evolving Systems framework is that the *evolution process* of a system comprises the homotopies $0 \leq \epsilon_{ij} \leq 1$, not just the endpoints where $\epsilon_{ij} = 0$ or $\epsilon_{ij} = 1$. In Evolving Systems, the mating of components is independent of the evolution of time in the system. The time parameter and the connection parameter are uncoupled in Evolving Systems because the connection parameter completely defines the evolutionary joining of components.

When the subsystem components join to form an Evolved System, the new entity becomes

$$\begin{cases} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= g(\mathbf{x}, \mathbf{u}) \end{cases} \tag{4}$$

2.2 Finite Element Method Formulation of Evolving Systems of Linear Flexible Structures

A flexible structure Evolving System is a mechanical dynamical system consisting of actively controlled flexible structure components that are joined together by compliant forces, e.g., springs. A practical and well accepted representation of flexible structures is based on the finite element method (FEM), see Balas (1982); Meirovitch (2001). The fundamental law governing mechanical systems is Newton's second law, which we use to write the dynamical equations describing a flexible structure. The FEM of the lumped model in physical coordinates, $\mathbf{q}_i$, for an arbitrary actively controlled flexible structure component, i, with n elements,

m control inputs, and p outputs is given in matrix form as

$$\begin{cases} \mathbf{M}_i\ddot{\mathbf{q}}_i(t) + \mathbf{D}_i\dot{\mathbf{q}}_i(t) + \mathbf{K}_i\mathbf{q}_i(t) = \mathbf{B}_i\mathbf{u}_i(t); \mathbf{q}_i(0) = \mathbf{q}_{i0} \\ \mathbf{y}_i(t) = \mathbf{C}_i\mathbf{q}_i(t) + \mathbf{E}_i\dot{\mathbf{q}}_i(t) \end{cases} \tag{5}$$

where $\mathbf{M}_i \equiv \text{diag}(m_1, m_2, \ldots, m_n)$ is the nxn element mass matrix, $\mathbf{q}_i(t) \equiv \left[q_1(t)q_2(t)\cdots q_n(t)\right]^T$ is the displacement vector, $\dot{\mathbf{q}}_i(t) \equiv \left[\dot{q}_1(t)\dot{q}_2(t)\cdots\dot{q}_n(t)\right]^T$ is the velocity vector, $\ddot{\mathbf{q}}_i(t) \equiv \left[\ddot{q}_1(t)\ddot{q}_2(t)\cdots\ddot{q}_n(t)\right]^T$ is the acceleration vector, $\mathbf{D}_i$ is the nxn element damping matrix, $\mathbf{K}_i$ is the nxn element stiffness matrix, $\mathbf{B}_i$ is the nxm matrix of control input constants, $\mathbf{u}_i(t) \equiv \left[u_1(t)u_2(t)\cdots u_m(t)\right]^T$ are the control inputs, $\mathbf{y}_i(t) \equiv \left[y_1(t)y_2(t)\cdots y_p(t)\right]^T$ is the vector of sensed outputs of the component, and $\mathbf{C}_i$ and $\mathbf{E}_i$ are the pxn matrices of output constants.

The damping in space structures in orbit above the atmosphere is expected to be quite small and can be well modeled by Rayleigh damping (Balas, 1982) as given by

$$\mathbf{D}_i = \alpha_1\mathbf{M}_i + \alpha_2\mathbf{K}_i \tag{6}$$

Because the damping is quite small, it is customary to use the undamped generalized eigenproblem for eq. 5 given by

$$\left(\mathbf{K}_i - \omega_k^2\mathbf{M}_i\right)\phi_k = 0 \tag{7}$$

where $k = 1, 2, \ldots, H$, $\mathbf{M}_i$ is symmetric, positive definite, $\mathbf{K}_i$ is symmetric, positive semidefinite, and H is equal to the number of degrees of freedom (DOF) in the physical model. The mode shapes ϕ_k and the mode frequencies ω_k are calculated from the generalized eigenproblem. Modal coordinates, $\mathbf{z}$, are obtained from the transformation

$$\mathbf{q} = \Phi\mathbf{z} \tag{8}$$

where $\Phi = \begin{bmatrix}\phi_1 & \phi_2 & \cdots & \phi_H\end{bmatrix}$. Generally, the number of modes computed for design and analysis is much smaller than the number of DOF included in the physical model (Bansenauer & Balas, January-February 1995).

The active control of each flexible structure component is local in the sense that the controller only uses the input and output ports located on its component. In the examples presented here, the active component control is in the form of Proportional Derivative (PD) control or Proportional Integral Derivative (PID) control.

The flexible structure components are the building blocks of the Evolving System. Any number of components can join together in an arbitrary, but predetermined, configuration to form an Evolved System. The components of an Evolving System are joined by connection forces operating on the displacements of physical coordinates within the components. The connection forces joining the components are modeled by linear springs connecting two elements, one from each component. Note that the connections could also be made through the velocities of the physical coordinates, with dampers connecting the components.

For the flexible structure Evolving System being described here, each connection force, or spring, joining physical coordinates from two components will be multiplied by a connection parameter. The symbol ϵ_{ij} will denote the *connection parameter* that multiplies the forces joining the i^{th} and the j^{th} components. For simplicity, the formulation of Evolving System presented here will only allow one connection parameter to multiply the forces joining two components. However, it would be possible to construct more complex flexible structure Evolving Systems which have multiple, distinct connection parameters corresponding to the forces joining different physical coordinates of two subsystem components.

The connection forces between components of an Evolving System are represented in the connection matrix, $\mathbf{K_E}(\epsilon_{ij})$, which multiplies the displacements of the component elements and has the form

$$\mathbf{K_E}(\epsilon_{ij}) = \left[\mathbf{K}_{ij}(\epsilon_{ij})\right] \tag{9}$$

where $i, j = 1, 2, \ldots L$. The connection parameter, ϵ_{ij}, multiplies the elements of the connection matrix corresponding to the connection forces joining physical coordinates of component i to coordinates of component j, where $i \neq j$, i.e., components i and j are separate components. There is only one connection parameter connecting component i to component j, so $\epsilon_{ij} \equiv \epsilon_{ji}$.

If there are no connections between any elements of components i and j, then $\epsilon_{ij} \equiv 0$ and $\mathbf{K}_{ij}(\epsilon_{ij}) \equiv \mathbf{K}_{ji}(\epsilon_{ij}) \equiv 0$. The connection matrix has zero entries for the elements of components that have nothing connected to them. There are no cyclic connections within components represented in the connection matrix, so $\epsilon_{ii} \equiv 0$. Since $\mathbf{K_E}(\epsilon_{ij})$ is a matrix of connection forces that are symmetric, $\mathbf{K}_{ji}(\epsilon_{ij}) \equiv \mathbf{K}_{ij}(\epsilon_{ij})^T$.

The off-diagonal elements of the connection matrix have the form

$$\mathbf{K}_{ij}(\epsilon_{ij}) = -\epsilon_{ij} \cdot \begin{bmatrix} k(i^1, j^1) & \cdots & k(i^1, j^{j_n}) \\ \vdots & \ddots & \vdots \\ k(i^{i_n}, j^1) & \cdots & k(i^{i_n}, j^{j_n}) \end{bmatrix} \tag{10}$$

where i^n represents the n^{th} element of the i^{th} component, j^m represents the m^{th} element of the j^{th} component, $k(i^n, j^m)$ is the connection force exerted by the n^{th} element of component i on the m^{th} element of component j, and the values i_n and j_n represent the number of elements in the i^{th} and the j^{th} component FEM representations, respectively.

The block diagonal elements, $\mathbf{K}_{ii}(\epsilon_{ij})$, of the connection matrix are more complex, since they represent the connection forces of all of the components in the Evolving System which connect to component i. If more than one component connects to a given component i, then $\mathbf{K}_{ii}(\epsilon_{ij})$ will include multiple connection parameters. A general form of the block diagonal elements of the connection matrix is given by

$$\mathbf{K}_{ii}(\epsilon_{ij}) = \operatorname{diag}\left(\sum_{j=1}^{L} \epsilon_{ij} \sum_{m=1}^{j_n} k(i^1, j^m),\ \sum_{j=1}^{L} \epsilon_{ij} \sum_{m=1}^{j_n} k(i^2, j^m), \ldots, \sum_{j=1}^{L} \epsilon_{ij} \sum_{m=1}^{j_n} k(i^{i_n}, j^m)\right) \tag{11}$$

We can write an individual component of the flexible structure Evolving System as

$$\begin{cases} \mathbf{M}_i \ddot{\mathbf{q}}_i(t) + \mathbf{D}_i \dot{\mathbf{q}}_i(t) + \mathbf{K}_i \mathbf{q}_i(t) + \displaystyle\sum_{j=1}^{L} \mathbf{K}_{ij}(\epsilon_{ij}) \mathbf{q}_j(t) = \mathbf{B}_i \mathbf{u}_i(t);\ \mathbf{q}_i(0) = \mathbf{q}_{i0} \\ \mathbf{y}_i(t) = \mathbf{C}_i \mathbf{q}_i(t) + \mathbf{E}_i \dot{\mathbf{q}}_i(t) \end{cases} \tag{12}$$

The Evolving System consisting of L interconnected components can now be written in matrix form as

$$\begin{cases} \mathbf{M_0} \ddot{\mathbf{q}}(t) + \mathbf{D_0} \dot{\mathbf{q}}(t) + \mathbf{K_0} \mathbf{q}(t) + \mathbf{K_E}(\epsilon_{ij}) \mathbf{q}(t) = \mathbf{B_0} \mathbf{u}(t); \quad \mathbf{q}(0) = \mathbf{q}_0 \\ \mathbf{y}(t) = \mathbf{C_0} \mathbf{q}(t) + \mathbf{E_0} \dot{\mathbf{q}}(t) \end{cases} \tag{13}$$

where $\mathbf{M_0} \equiv \operatorname{diag}(\mathbf{M_1}, \mathbf{M_2}, \ldots, \mathbf{M_L})$, $\mathbf{B_0} = \operatorname{diag}(\mathbf{B_1}, \mathbf{B_2}, \ldots, \mathbf{B_L})$, $\mathbf{q}(t) \equiv \left[\mathbf{q_1}(t) \mathbf{q_2}(t) \cdots \mathbf{q_L}(t)\right]^T$, $\dot{\mathbf{q}}(t) \equiv \left[\dot{\mathbf{q}}_\mathbf{1}(t) \dot{\mathbf{q}}_\mathbf{2}(t) \cdots \dot{\mathbf{q}}_\mathbf{L}(t)\right]^T$, $\ddot{\mathbf{q}}(t) \equiv \left[\ddot{\mathbf{q}}_\mathbf{1}(t) \ddot{\mathbf{q}}_\mathbf{2}(t) \cdots \ddot{\mathbf{q}}_\mathbf{L}(t)\right]^T$, $\mathbf{u}(t) \equiv \left[\mathbf{u_1}(t) \mathbf{u_2}(t) \cdots \mathbf{u_L}(t)\right]^T$,

$\mathbf{K_0} = \text{diag}(\mathbf{K_1},\mathbf{K_2},\ldots,\mathbf{K_L})$, $\mathbf{K_E}(\epsilon_{ij})$ is the connection matrix as described above, $\mathbf{y}(t) = \left[\mathbf{y_1}(t)\mathbf{y_2}(t)\cdots\mathbf{y_L}(t)\right]^T$, $\mathbf{C_0} \equiv \text{diag}(\mathbf{C_1},\mathbf{C_2},\ldots,\mathbf{C_L})$, and $\mathbf{E_0} \equiv \text{diag}(\mathbf{E_1},\mathbf{E_2},\ldots,\mathbf{E_L})$. For simplicity of notation, the time parameter, t, will be omitted henceforth.

The evolution of the different connection parameters, ϵ_{ij}, that enable the mating of components is controlled by the self-assembling components or the agent. The various connection parameters can evolve at different times during the assembly of the Evolving System, thereby orchestrating the assembly order of the components. It is also possible to envision the evolution of the connection parameters proceeding at different rates, effectively causing the assembly of some components to be accomplished more rapidly than others. However, we will not address the evolution rate of the connection parameter at this time. The Evolving Systems framework allows the evolution of two components to be undone, i.e., connected components can *devolve*, where their connection parameter goes from one down to zero.

The flexible structure Evolving Systems described above are some of the simplest examples of dynamic behavior where the nature of Evolving Systems can be investigated. The approaches developed here can be generalized rather easily and scaled for much more complex Evolving Systems.

The matrix notation which we have developed for flexible structure Evolving Systems is useful for understanding the formulation of Evolving Systems, however, a model based on a state space representation of Evolving Systems will prove essential as we analyze more complex Evolving Systems.

2.3 State Space Description of Flexible Structure Evolving Systems

A state space representation of linear time-invariant Evolving Systems is developed here. Suppose we have a flexible structure Evolving System consisting of L individual components as described in section 2.2 and given by the FEM

$$\begin{cases} \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{D}_i\dot{\mathbf{q}} + \mathbf{K}_i\mathbf{q}_i = \mathbf{B}_i^0\mathbf{u}_i; \quad \mathbf{q}_i(0) = \mathbf{q}_{i0} \\ \mathbf{y}_i = \mathbf{C}_i^0\mathbf{q}_i + \mathbf{E}_i\dot{\mathbf{q}}_i \end{cases} \tag{14}$$

We can represent the individual flexible structure components given by eq. 14 by the state-space description

$$\begin{cases} \dot{\mathbf{x}}_i &= \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_i\mathbf{u}_i;\ \mathbf{x}_i(0) \equiv \mathbf{x}_{i_0} \\ \mathbf{y}_i &= \mathbf{C}_i\mathbf{x}_i \end{cases} \tag{15}$$

where $i = 1,2,\ldots,L$, $\mathbf{x}_i \equiv \left[x_1^i x_2^i \cdots x_{n_i}^i\right]^T$ is the component state vector, $\dot{\mathbf{x}}_i \equiv \left[\dot{x}_1^i \dot{x}_2^i \cdots \dot{x}_{n_i}^i\right]^T$, $\mathbf{u}_i \equiv \left[u_1^i u_2^i \cdots u_{m_i}^i\right]^T$ is the control input vector, $\mathbf{y}_i \equiv \left[y_1^i y_2^i \cdots y_{p_i}^i\right]^T$ is the vector of sensed outputs, $\mathbf{x}_{i_0}$ is the vector of initial conditions, and $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ are constant matrices of dimension n_i x n_i, n_i x m_i, and p_i x n_i, respectively. Since the state space description comes from the dynamical equations given by eq. 14, we have that $\mathbf{A}_i = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}_i^{-1}\mathbf{K}_i & -\mathbf{M}_i^{-1}\mathbf{D}_i \end{bmatrix}$, $\mathbf{B}_i = \begin{bmatrix} 0 \\ \mathbf{M}_i^{-1}\mathbf{B}_i^0 \end{bmatrix}$ and $\mathbf{C}_i = \begin{bmatrix} \mathbf{C}_i^0 & \mathbf{E}_i^0 \end{bmatrix}$. Note that n_i is the dimension of the state vector $\mathbf{x}_i$, m_i is the dimension of the control vector $\mathbf{u}_i$, and p_i is the dimension of the output vector $\mathbf{y}_i$. The local controller on component i is given by

$$\mathbf{u}_i = l_i\mathbf{y}_i \tag{16}$$

where l_i is a linear control operator.

The subsystem components are the building blocks of the Evolving System. The connection forces between two components, i and j, of an Evolving System are represented in the connection matrix, $\mathbf{A}_{ij}$, which is multiplied by the connection parameter, ϵ_{ij}, where $i,j = 1,2,\ldots,L$, $i \neq j$, and $0 \leq \epsilon_{ij} \leq 1$. Even though connections may exist between the states of different components of the Evolving System, the component inputs and outputs are still local, i.e., there is no sharing of component inputs or outputs between components.

The FEM representation of a flexible structure Evolving System component is given by

$$\begin{cases} \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{D}_i\dot{\mathbf{q}}_i + \mathbf{K}_i\mathbf{q} + \sum_{j=1}^{L}\mathbf{K}_{ij}(\epsilon_{ij})\mathbf{q}_j = \mathbf{B}_i^0\mathbf{u}_i;\ \mathbf{q}_i(0) = \mathbf{q}_{i0} \\ \mathbf{y}_i = \mathbf{C}_i^0\mathbf{q}_i + \mathbf{E}_i\dot{\mathbf{q}}_i \end{cases} \tag{17}$$

The state space equations for an individual component including connections to other components in an Evolving System are given by

$$\begin{cases} \dot{\mathbf{x}}_i &=& \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_i\mathbf{u}_i + \sum_{j=1}^{L}\epsilon_{ij}\mathbf{A}_{ij}\mathbf{x};\ \mathbf{x}_i(0) \equiv \mathbf{x}_{i_0} \\ \mathbf{y}_i &=& \mathbf{C}_i\mathbf{x}_i \end{cases} \tag{18}$$

where $\mathbf{x} \equiv \left[\mathbf{x_1}\mathbf{x_2}\cdots\mathbf{x_L}\right]^T$ is the concatenated state vectors of the entire Evolving System, $\mathbf{A}_{ij}$ is the connection matrix, and $0 \leq \epsilon_{ij} \leq 1$ is the connection parameter. The connection matrix, $\mathbf{A}_{ij}$, has dimension n_i by $\dim(\mathbf{x})$, where n_i is the dimension of the state vector $\mathbf{x}_i$ corresponding to component i and $\dim(\mathbf{x}) \equiv \sum_{k=1}^{L} n_k$. In eq. 17, the matrix $\mathbf{K}_{ij}(\epsilon_{ij})$ multiplies the vector $\mathbf{q}_j$. The elements of the matrix $\mathbf{A}_{ij}$ are related to the elements of $\mathbf{K}_{ij}(\epsilon_{ij})$, except that they are mass normalized by $\mathbf{M}_i^{-1}$ and rearranged so that they multiply the elements of $\mathbf{x}$ corresponding to $\mathbf{q}_j$. The other elements of $\mathbf{A}_{ij}$ are set to zero.

The connection parameter, ϵ_{ij}, multiplies the forces connecting the physical coordinates of component i to physical coordinates of component j. The connection parameter is the same in the state space representation as in the FEM flexible structure model. If there is no connection between any states of components i and j, then $\epsilon_{ij} \equiv 0$ and the connection matrix $\mathbf{A}_{ij} \equiv 0$.

When a system of L individual components, as described by eq. 18, mate to form an Evolving System, the new entity becomes

$$\begin{cases} \dot{\mathbf{x}} &=& \mathbf{A}(\epsilon_{ij})\mathbf{x} + \mathbf{B}\mathbf{u};\ \ \mathbf{x}(0) \equiv \mathbf{x_0} \\ \mathbf{y} &=& \mathbf{C}\mathbf{x} \end{cases} \tag{19}$$

where $\mathbf{x} \equiv \left[\mathbf{x_1}\mathbf{x_2}\cdots\mathbf{x_L}\right]^T$, $\dot{\mathbf{x}} \equiv \left[\dot{\mathbf{x_1}}\dot{\mathbf{x_2}}\cdots\dot{\mathbf{x_L}}\right]^T$, $\mathbf{u} \equiv \left[\mathbf{u_1}\mathbf{u_2}\cdots\mathbf{u_L}\right]^T$, $\mathbf{y} \equiv \left[\mathbf{y_1}\mathbf{y_2}\cdots\mathbf{y_L}\right]^T$, $\mathbf{B} \equiv \mathrm{diag}(\mathbf{B_1},\mathbf{B_2},\ldots,\mathbf{B_L})$, $\mathbf{C} \equiv \mathrm{diag}(\mathbf{C_1},\mathbf{C_2},\ldots,\mathbf{C_L})$, $\mathbf{A}(\epsilon_{ij}) \equiv \mathrm{diag}(\mathbf{A}_i) + \left[\sum_{j=1}^{L}\epsilon_{1j}\mathbf{A}_{1j}\sum_{j=1}^{L}\epsilon_{2j}\mathbf{A}_{2j}\cdots\sum_{j=1}^{L}\epsilon_{Lj}\mathbf{A}_{Lj}\right]^T$ and $0 \leq \epsilon_{ij} \leq 1$. The system given by eq. 19 will also be represented by the standard state space notation of (A,B,C).

We can form the closed-loop Evolving System by taking the Evolving System given by eq. 19 and connecting each of the local component controllers, $\mathbf{u}_i$, to their corresponding input and output ports, i.e., close the loops in each of the components. The closed-loop Evolving System can be written as

$$\dot{\mathbf{z}} = \bar{\mathbf{A}}(\epsilon_{ij})\mathbf{z}; \;\; \mathbf{z}(0) \equiv \begin{bmatrix}\mathbf{x_0}\mathbf{0}\end{bmatrix}^T \tag{20}$$

where $\mathbf{z} = \begin{bmatrix}\mathbf{x} & \eta\end{bmatrix}^T$ is the augmented state vector, $\bar{\mathbf{A}}(\epsilon_{ij})$ is the closed-loop system, and ϵ_{ij} is the connection parameter. The closed-loop Evolving System given by eq. 20 will be used for stability analysis.

Flexible structure Evolving Systems can be written in a form that is mass normalized and the component state vectors can be rearranged to appear as one flexible structure, instead of multiple component state vectors concatenated together. The state space description of flexible structure Evolving Systems can be easily extended to describe other applications of Evolving Systems.

2.4 Impedance-Admittance Formulation of Contact Dynamics in Evolving Systems

In this section, we formulate the contact dynamics in Evolving Systems in terms of mechanical impedance and admittance, as first described in (Frost & Balas, 2007a). For many dynamical systems, the impedance-admittance form is a useful tool for modeling the contact dynamics of components, see Harris & Crede (1976).

Definition 2.1 *The impedance of a mechanical system is determined by the equation $f = Z(v)$, where f is the force exerted by the system, v is the velocity of the system, and Z is the impedance of the system.*

Definition 2.2 *The admittance of a mechanical system, Y, is the inverse of the impedance of the system, e.g., $Y \equiv Z^{-1}$ and $v = Y(f)$.*

Impedance and admittance can be seen as nonlinear operators describing the relationship between the output of a mechanical system, or the force it exerts at a contact point, with the input of the system, or the velocity at the contact point. When two components join at a point of contact, their velocities are equal and the forces exerted are equal and opposite. If the contact points of the two components are represented as (f_1, v_1) and (f_2, v_2) with displacements q_1 and q_2, then we can write

$$\begin{cases} f_1 & = & -f_2 \\ v_1 & \equiv & \dot{q}_1 = v_2 \equiv \dot{q}_2 \end{cases} \tag{21}$$

This formulation can also be seen as the feedback connection of two components in an Evolving System, where the admittance of component 1 is connected in feedback with the impedance of component 2, as shown in fig. 2. We introduce two nonlinear operators Y_1 and Z_2 that provide the admittance and impedance formulation of the contact dynamics of nonlinear Evolving Systems components. These operators relate the force and velocity at the contact point of two mating components as given by the equations $\mathbf{v}_1 = Y_1(\mathbf{f}_1)$ and $\mathbf{f}_2 = Z_2(\mathbf{v}_2)$. In linear time-invariant systems, these operators can be easily calculated using Laplace transforms. For nonlinear components, the admittance and impedance operators cannot be easily found. However, this does not invalidate the analysis provided in this chapter, which will provide a foundation for adaptive key component control.

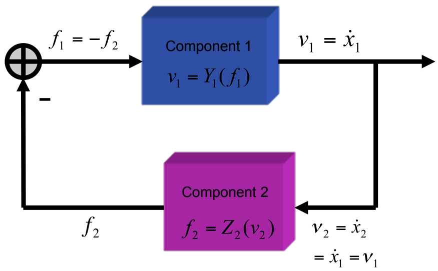

Fig. 2. Admittance-impedance feedback connection of two components.

3. Stability Inheritance in Evolving Systems

The application of Evolving Systems to self-assembly of structures in space imposes the need for the inheritance of stability. Many textbooks (Vidyasagar, 1993; Brogan, 1991; Ogata, 2002; Slotine & Li, 1991) give excellent discussions of linear and nonlinear systems stability analysis. For linear time-invariant Evolving Systems, we will examine the closed-loop poles of the system to evaluate the stability of the system as it evolves. In particular, we will examine the eigenvalues of the matrix $\bar{\mathbf{A}}(\epsilon_{ij})$ from the state space equation of the closed-loop Evolving System given by eq. 20 as ϵ_{ij} goes from $0 \rightarrow 1$. The system is unstable if any of the closed-loop poles cross the $j\omega$-axis.

Example 1 is a two component Evolving System where each of the components is actively controlled and stable, but the Evolving System fails to inherit the stability traits of the components. This particular system becomes unstable during the evolution process and remains unstable when the system is fully evolved. Consider the fully actuated, fully sensed three mass Evolving System shown in fig. 3. Component 1 contains only one mass with local control. The dynamical equations for component 1 are

$$\left\{ \begin{array}{rcl} m_1\ddot{q}_1 & = & u_1 \\ \mathbf{y_1} & = & \begin{bmatrix} q_1 & \dot{q}_1 \end{bmatrix}^T \end{array} \right. \tag{22}$$

where $m_1 = 30$ is the mass of mass 1, q_1 is the displacement of mass 1, and
$u_1 = -(0.9s + 0.1)q_1$ is the local controller for component 1 with the Laplace variable s.
The dynamical equations for component 2 are

$$\left\{ \begin{array}{rcl} m_2\ddot{q}_2 & = & u_2 - k_{23}(q_2 - q_3) \\ m_3\ddot{q}_3 & = & u_2 - k_{23}(q_3 - q_2) \\ \mathbf{y_2} & = & \begin{bmatrix} q_2 & \dot{q}_2 \end{bmatrix}^T \\ \mathbf{y_3} & = & \begin{bmatrix} q_3 & \dot{q}_3 \end{bmatrix}^T \end{array} \right. \tag{23}$$

where $m_2 = 1.0$ is the mass of mass 2, $m_3 = 1.0$ is the mass of mass 3, q_2 is the displacement of mass 2, q_3 is the displacement of mass 3, and $k_{23} = 1.0$. The controllers on component 2 are

$$\left\{ \begin{array}{rcl} u_2 & = & -\left(\frac{0.1}{s} + 0.2s + 0.5\right) q_2 \\ u_3 & = & -(0.6s + 1)\, q_3 \end{array} \right. \tag{24}$$

The controllers for components 1 and 2 have been designed to produce stable behavior when the components are unconnected. The two components are joined by a spring connecting mass 1 with mass 2. The Evolving System comprised of these two components can be written in the matrix form of eq. 13 as

$$\begin{cases} \mathbf{M_0}\ddot{\mathbf{q}} &= \mathbf{B_0}\mathbf{u} - \mathbf{K_0}\mathbf{q} - \mathbf{K_E}(\epsilon_{ij})\mathbf{q} \\ \mathbf{y} &= \mathbf{C_0}\begin{bmatrix}\mathbf{q} & \dot{\mathbf{q}}\end{bmatrix}^T \end{cases} \tag{25}$$

where $\mathbf{M_0} = \text{diag}(m_1, m_2, m_3)$, $\ddot{\mathbf{q}} = [\ddot{q_1}\ddot{q_2}\ddot{q_3}]^T$, $\mathbf{q} = [q_1 q_2 q_3]^T$ $\mathbf{u} = [u_1 u_2 u_3]^T$, $\mathbf{y} = [y_1 y_2 y_3]^T$, $\mathbf{B_0} = \mathbf{I_3}$, $\mathbf{C_0} = \mathbf{I_6}$, $\mathbf{K_0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_{23} & -k_{23} \\ 0 & -k_{23} & k_{23} \end{bmatrix}$, $\mathbf{K_E}(\epsilon_{ij}) = \begin{bmatrix} \epsilon_{12}k_{12} & -\epsilon_{12}k_{12} & 0 \\ -\epsilon_{12}k_{12} & \epsilon_{12}k_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $k_{12} = 1.0$, and $0 \leq \epsilon_{12} \leq 1$.

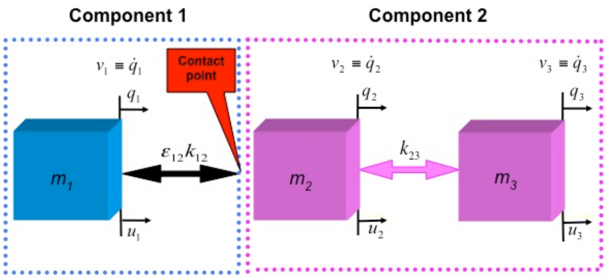

Fig. 3. Ex. 1: A two component flexible structure Evolving System.

Matlab and Simulink models of this system were created. To determine the stability of the Evolving System, we connect the local component controllers to their inputs and outputs, and we examine the closed-loop poles, or the eigenvalues, of the resulting composite system. Figure 4 shows the closed-loop poles of the Evolving System given by equation 25 as the system evolves, i.e., as ϵ_{12} goes from 0 to 1. Note that two of the closed-loop poles of ex. 1 cross the $j\omega$-axis for some $\epsilon_{12} > 0$, demonstrating that the Evolving System loses stability during evolution. When the system is fully evolved, i.e., $\epsilon_{12} = 1$, the Evolved System is unstable, i.e., it fails to inherit the stability of its components, as seen in fig. 4.

In the next section, we explore a method to restore stability to Evolving Systems that would otherwise fail to inherit the stability traits of their components.

4. Key Component Controllers

In this section we introduce the idea of controllers that stabilize flexible structure Evolving Systems during evolution. Often the design requirements for an Evolving System dictate that the individual components remain unchanged as much as possible. For situations where stability is not inherited during evolution, in many cases it would be advantageous to augment the controller on only one component to restore stability to the entire Evolving System, thereby leaving the other components and their controllers unmodified. Furthermore, it is desirable for the augmented controller to only use the input-output ports on the component on which it resides. In this section, we introduce the idea of key component controllers that restore stability to an Evolving System by augmenting the controller on a single subsystem component,

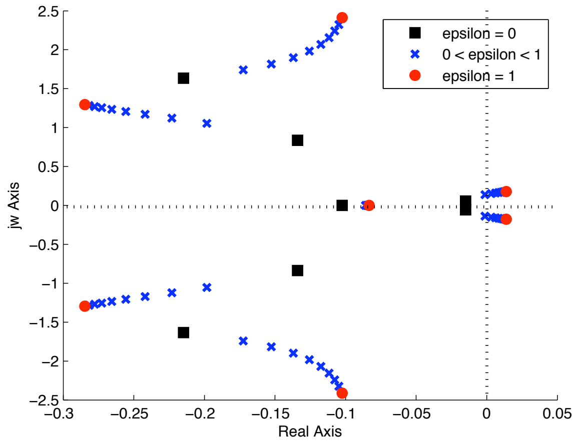

Fig. 4. Closed-loop poles of Ex. 1.

using only the input-output ports on that component. The key component controller using fixed gains was first proposed in (Frost & Balas, 2007b).

In the key component controller design approach, one key component is chosen from the Evolving System to have additional local control added to it with the objective of maintaining system stability during the entire evolution of the system. The control and sensing of the other subsystem components will be unaltered and remain local. The key component controller operates solely through a single set of input and output ports on the key component, see fig. 5. For components that lose stability when assembling, the individual components could mate with the key component one at a time. The key component would compensate for any component which caused instability, thereby restoring stability to the system.

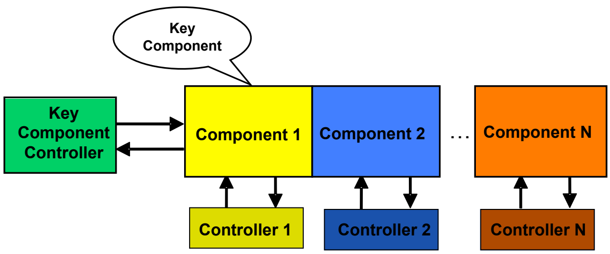

Fig. 5. Block diagram of key component controller and Evolving System.

A clear advantage of the key component design approach is that components can be reused in many different configurations of Evolving Systems without needing redesign from a stability point of view. Redesign of existing components is unnecessary because the key component will be responsible for maintaining overall system stability. The reuse of components that are space-qualified, or at least previously designed, built, and verified and validated, could reduce overall system development and validation time and could result in higher quality systems with potentially significant cost savings and risk mitigation.

The key component controller design requires the controllability and observability of the states of the Evolving System from a set of input and output ports on the key component. In the case of LTI Evolving Systems, we can use a method of applying local output feedback through specified input ports to obtain controllability and observability from a single set of input-output ports. Details of the method are given in (Corfmat & Morse, 1976a). Applying local output feedback on a component is seen as a minor modification that still preserves the idea of leaving the nonkey components mostly unmodified.

4.1 Adaptive Key Component Controllers for Restoring Stability in Evolving Systems

We present a key component controller that uses a direct adaptive control law to restore stability to an Evolving System. In many aerospace environments and applications, the parameters of a system are poorly known and difficult and costly to obtain. Control laws that use direct adaptation are a good design choice for systems where access to precisely known parametric values is limited, since these control laws adapt their gains to the system output. We propose the use of adaptive control laws in a key component controller to provide a practical solution to the problems described above. This approach was first proposed in (Frost & Balas, 2008b).

The adaptive key component controller adapts its gains based on the system outputs to ensure that the Evolving System remains stable during component assembly. The adaptive key component design has the same advantages as the fixed gain key component controller without the need to schedule the gains based on the value of ϵ.

We consider an Evolving System consisting of two components given by:

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}} & = & \mathbf{A}(\epsilon)\mathbf{x} + \mathbf{B}\mathbf{u};\ \mathbf{x}(0) \equiv \mathbf{x_0} \\ \mathbf{y} & = & \mathbf{C}\mathbf{x} \end{array} \right. \tag{26}$$

where $\mathbf{x} \equiv \left[\mathbf{x_1}\mathbf{x_2}\right]^T$, $\dot{\mathbf{x}} \equiv \left[\dot{\mathbf{x_1}}\dot{\mathbf{x_2}}\right]^T$, $\mathbf{u} \equiv \left[\mathbf{u_1}\mathbf{u_2}\right]^T$, $\mathbf{y} \equiv \left[\mathbf{y_1}\mathbf{y_2}\right]^T$, $\mathbf{B} \equiv \mathrm{diag}(\mathbf{B_1}, \mathbf{B_2})$, $\mathbf{C} \equiv \mathrm{diag}(\mathbf{C_1}, \mathbf{C_2})$, $\mathbf{A}(\epsilon) \equiv \mathrm{diag}(\mathbf{A_1}, \mathbf{A_2}) + \left[\sum_{j=1}^{2} \epsilon \mathbf{A}_{1j} \sum_{j=1}^{2} \epsilon \mathbf{A}_{2j}\right]^T$ and $0 \leq \epsilon \leq 1$.

Now we give the equations for an Evolving System with a key component controller. Without loss of generality, we can let component 1 be the key component since the system can be rewritten to switch component 1 with component 2. Also, we may think of component 2 as being the rest of the Evolving System to which the key component and its adaptive controller will be connected. The adaptive key component controller on component 1 is given by

$$\left\{ \begin{array}{lcl} \mathbf{u}_1^A & = & \mathbf{G}\mathbf{y}_1^A \\ \dot{\mathbf{G}} & = & -\mathbf{y}_1^A (\mathbf{y}_1^A)^T \mathbf{H};\ \mathbf{H} > 0 \end{array} \right. \tag{27}$$

The adaptive key component controller only uses the input and output ports located on component 1. Component 1, which is the key component of the system, can be written as

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}}_1 & = & \mathbf{A_1}\mathbf{x_1} + \mathbf{B_1}\mathbf{u_1} + \mathbf{B_1^A}\mathbf{u_1^A} + \sum_{j=1}^{2} \epsilon \mathbf{A}_{1j}(\mathbf{x}); \\ \mathbf{y_1} & = & \mathbf{C_1}\mathbf{x_1} \\ \mathbf{y_1^A} & = & \mathbf{C_1^A}\mathbf{x_1} \end{array} \right. \tag{28}$$

and component 2 can be written as

$$\begin{cases} \dot{\mathbf{x}}_2 &= \mathbf{A_2 x_2} + \mathbf{B_2 u_2} + \mathbf{B_2^A u_2^A} + \sum_{j=1}^{2} \epsilon \mathbf{A}_{2j}(\mathbf{x}) \\ \mathbf{y_2} &= \mathbf{C_2 x_2} \\ \mathbf{y_2^A} &= \mathbf{C_2^A x_2} \end{cases} \tag{29}$$

where the augmented control $\mathbf{u_2^A}$ would only be present if additional output feedback control were needed to satisfy sufficient condition for the adaptive controller. Next, we give some useful definitions.

Definition 4.1 *Consider a linear system $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ with closed-loop transfer function, $T_c(s) \equiv \mathbf{C}(s\mathbf{I} - \mathbf{A})\mathbf{B}$. We say the system $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is strict positive real (SPR) when for all ω real and for some $\sigma > 0$*

$$Re[T_c(-\sigma + j\omega)] \geq 0 \tag{30}$$

Definition 4.2 *We say a linear system $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is almost strict positive real (ASPR) when it can be made strict positive real by adding output feedback.*

Remark: A linear system $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is ASPR if it has no nonminimum phase zeros and $\mathbf{CB} > \mathbf{0}$.

The following result from (Fuentes & Balas, 2000) gives the sufficient condition for a linear time-invariant system with an adaptive control law as described above, to be guaranteed to have bounded gains and asymptotic output tracking.

Theorem 4.3 *Assume the linear time-invariant system given by*

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t);\ \mathbf{x}(\mathbf{0}) = \mathbf{x_0} \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{cases} \tag{31}$$

is ASPR. Then the direct adaptive control law

$$\begin{cases} \mathbf{u}(t) &= \mathbf{G}\mathbf{y}(t); \\ \dot{\mathbf{G}} &= -\mathbf{y}(t)\mathbf{y}(t)^T\mathbf{H};\ \mathbf{H} > 0 \end{cases} \tag{32}$$

produces bounded adaptive gains, $\mathbf{G}$, and $\mathbf{y} \rightarrow 0$ as $t \rightarrow \infty$

This result suggests that the sufficient condition for an Evolving System with an adaptive key component controller to have guaranteed bounded gains and asymptotic tracking is that the system be ASPR. This idea will be developed further in a subsequent section. Note that the theory developed in (Fuentes & Balas, 2000) could also be applied to design the key component adaptive controller to track a desired reference model and reject disturbances.

4.2 Results of Restoring Stability to Ex. 1 with Adaptive Key Component Controllers

A Simulink model was created to implement the adaptive key component controller for ex. 1. Simulations were run with the connection parameter, ϵ_{12}, ranging from 0 to 1, allowing the system to go from unconnected components to a fully Evolved System. The key component controller was able to maintain system stability during the entire evolution process when it used the input-output ports on mass 1 of component 1, see fig. 6. When component 1 was the key component, the Evolving System is ASPR.

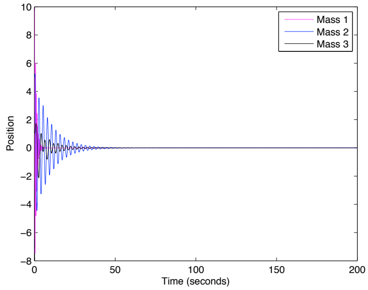

Fig. 6. Nondimensional position displacements of Ex. 1 with adaptive key component controller on component 1.

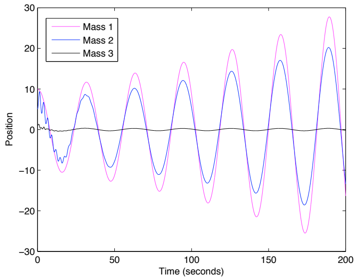

Fig. 7. Nondimensional position displacements of Ex. 1 with adaptive key component controller on mass 3 of component 2.

When the key component controller was located on component 2 and used the input-output ports on mass 3, stability was not maintained, see fig. 7. The adaptive key component controller was not able to restore stability on mass 3 because that system had nonminimum phase zeros at $0.00515 \pm 0.2009i$, i.e., the system was not ASPR.

5. Inheritance of Passivity Properties in Evolving Systems

In this section we explore the inheritance of different types of passivity in Evolving Systems. First we give some theorems on the inheritance of these traits in systems connected in feedback. Then we use the admittance-impedance formulation of Evolving Systems developed in Section 2.4 to determine the condition under which passivity traits are inherited in Evolving Systems. We use these results to determine the sufficient condition for LTI Evolving Systems with an adaptive key component controller to be guaranteed to have bounded gains and asymptotic state tracking.

Intuitively, a system is passive if the energy stored by the system is less than or equal to the energy supplied. Physical systems satisfy energy conservation equations of the form

$$\frac{d}{dt}[\text{Stored Energy}] = [\text{External Power Input}] + [\text{Internal Power Generation}] \tag{33}$$

Definition 5.1 *We say that a nonlinear system of the form*

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}} & = & f(\mathbf{x},\mathbf{u});\ \ \mathbf{x}(0) = \mathbf{x_0} \\ \mathbf{y} & = & \mathbf{h}(\mathbf{x},\mathbf{u}) \end{array} \right. \tag{34}$$

is passive if it has an positive definite energy storage function, $V(\mathbf{x})$, that satisfies

$$\dot{V}(\mathbf{x}) = \langle \mathbf{y},\mathbf{u} \rangle - S(\mathbf{x}) = \mathbf{y}^T\mathbf{u} - S(\mathbf{x}) \tag{35}$$

where $S(\mathbf{x})$ is a positive semi-definite function, i.e., $S(\mathbf{x}) \geq 0$.

The term $\dot{V}(\mathbf{x})$ in eq. 35 represents the energy storage rate of the system. The external power input term in eq. 33 is represented by the inner product of the input and the output of the system, i.e., $\mathbf{y}^T\mathbf{u}$. Note that $V(\mathbf{x})$ can also be seen as a Lyapunov candidate function. Excellent references exist on passivity in linear and nonlinear systems, see Vidyasagar (1993); Wen (1988); Slotine & Li (1991); Isidori (1995).

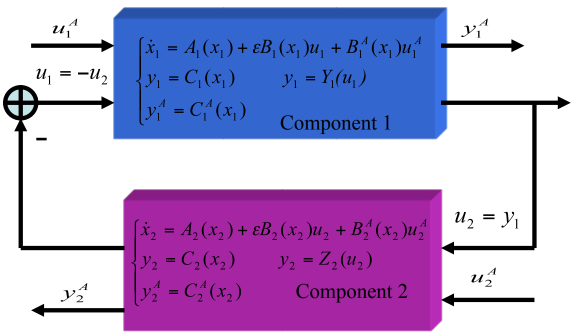

Fig. 8. Admittance-Impedance feedback connection of two nonlinear subsystems.

We can use the nonlinear impedance and admittance operators introduced in section 2.4 to find the state space representation of the impedance and admittance of Evolving System components. The nonlinear state space representation of the admittance of one component connected to the impedance of a second component is shown in fig. 8. We use the following representation for components that are nonlinear in state

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}}_i & = & \mathbf{A}_i(\mathbf{x}_i) + \epsilon \mathbf{B}_i(\mathbf{x}_i)\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x}_i)\mathbf{u}_i^A \\ \mathbf{y}_i & = & \mathbf{C}_i(\mathbf{x}_i) \\ \mathbf{y}_i^A & = & \mathbf{C}_i^A(\mathbf{x}_i) \end{array} \right. \tag{36}$$

Definition 5.2 *Consider a system that is nonlinear in state and is given by*

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}} & = & \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \\ \mathbf{y} & = & \mathbf{C}(\mathbf{x}) \end{array} \right. \tag{37}$$

We say that this system is Strictly Passive when $\exists V(\mathbf{x}) > 0\ \forall \mathbf{x} \neq 0$ such that

$$\dot{V}(\mathbf{x}) = \langle \mathbf{u},\mathbf{y} \rangle - S(\mathbf{x}) \tag{38}$$

with $S(\mathbf{x}) > 0\ \forall \mathbf{x} \neq 0$.

Definition 5.3 *Consider a nonlinear system of the form given by*

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})u \\ \mathbf{y} &= \mathbf{C}(\mathbf{x}) \end{cases} \tag{39}$$

We say that this system is <u>*Almost Strictly Passive*</u> *(ASP) when there is some output feedback,* $\mathbf{u} = \mathbf{G}\mathbf{y} + \mathbf{u_r}$*, that makes it strictly passive.*

We can state the following result about the inheritance of strict passivity in systems connected in feedback.

Theorem 5.4 *Suppose we have a pair of subsystems of the form*

$$\begin{cases} \dot{\mathbf{x}}_i &= \mathbf{A}_i(\mathbf{x}_i) + \mathbf{B}_i(\mathbf{x}_i)\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x}_i)\mathbf{u}_i^A \\ \mathbf{y}_i &= \mathbf{C}_i(\mathbf{x}_i) \\ \mathbf{y}_i^A &= \mathbf{C}_i^A(\mathbf{x}_i) \end{cases} \tag{40}$$

where $i = 1,2$ *and both subsystems* $\left(\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_1^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_1^A\end{bmatrix}\right)$ *and* $\left(\begin{bmatrix}\mathbf{u}_2\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_2\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *are strictly passive with energy storage functions* $V_1(\mathbf{x}_1)$ *and* $V_2(\mathbf{x}_2)$*. Then the feedback connection of the two subsystems, where* $(\mathbf{y}_1 = \mathbf{u}_2)$ *and* $(\mathbf{u}_1 = -\mathbf{y}_2)$*, will leave the resulting composite system* $\left(\mathbf{u}_A \equiv \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \mathbf{y}_A \equiv \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *strictly passive.*

Proof: Form the composite system by connecting $(\mathbf{u}_1, \mathbf{y}_1)$ in feedback with $(\mathbf{u}_2, \mathbf{y}_2)$, which gives us $\mathbf{y}_1 = \mathbf{u}_2$ and $\mathbf{u}_1 = -\mathbf{y}_2$. Let the energy storage function for the composite system be $V(\mathbf{x}) = V_1(\mathbf{x}_1) + V_2(\mathbf{x}_2)$. Using the fact that both components are strictly passive, and making the substitutions for the feedback connection, we have

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \left\langle \begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_1^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_1^A\end{bmatrix} \right\rangle + \left\langle \begin{bmatrix}\mathbf{u}_2\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_2\\ \mathbf{y}_2^A\end{bmatrix} \right\rangle - (S_1(\mathbf{x}_1) + S_2(\mathbf{x}_2)) \\ &= (\langle \mathbf{u}_1, \mathbf{y}_1\rangle + \langle \mathbf{u}_1^A, \mathbf{y}_1^A\rangle) + (\langle \mathbf{u}_2, \mathbf{y}_2\rangle + \langle \mathbf{u}_2^A, \mathbf{y}_2^A\rangle) - S(\mathbf{x}) \\ &= (\langle -\mathbf{y}_2, \mathbf{y}_1\rangle + \langle \mathbf{u}_1^A, \mathbf{y}_1^A\rangle) + (\langle \mathbf{y}_1, \mathbf{y}_2\rangle + \langle \mathbf{u}_2^A, \mathbf{y}_2^A\rangle) - S(\mathbf{x}) \\ &= \left\langle \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix} \right\rangle - S(\mathbf{x}) \end{aligned} \tag{41}$$

with $S(\mathbf{x}) \equiv S_1(\mathbf{x}_1) + S_2(\mathbf{x}_2) > 0 \;\; \forall \mathbf{x} \equiv \begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} \neq 0$. Therefore, by the definition of strict passivity, the composite system, given by $\left(\mathbf{u}_A \equiv \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \mathbf{y}_A \equiv \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix}\right)$ remains strictly passive. $\square$

We have shown that the feedback connection of two strictly passive systems results in a composite system that is strictly passive. Hence, strict passivity is inherited by systems connected in feedback. We now give a result on the inheritance of almost strict passivity.

Theorem 5.5 *Suppose we have a pair of subsystems of the form*

$$\begin{cases} \dot{\mathbf{x}}_i &= \mathbf{A}_i(\mathbf{x}_i) + \epsilon\mathbf{B}_i(\mathbf{x}_i)\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x}_i)\mathbf{u}_i^A \\ \mathbf{y}_i &= \mathbf{C}_i(\mathbf{x}_i) \\ \mathbf{y}_i^A &= \mathbf{C}_i^A(\mathbf{x}_i) \end{cases} \tag{42}$$

where $i = 1,2$ and both subsystems $\left(\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_1^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_1^A\end{bmatrix}\right)$ *and* $\left(\begin{bmatrix}\mathbf{u}_2\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_2\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *are almost strictly passive with energy storage function* $V_1(\mathbf{x}_1)$ *and* $V_2(\mathbf{x}_2)$. *Then the feedback connection of the two subsystems, where* $(\mathbf{y}_1 = \mathbf{u}_2)$ *and* $(\mathbf{u}_1 = -\mathbf{y}_2)$, *will leave the resulting composite system* $\left(\mathbf{u}_A \equiv \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \mathbf{y}_A \equiv \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *almost strictly passive.*

Proof: By the definition of almost strict passivity, there exists output feedback control that makes each of the subsystems strictly passive. Output feedback of the form $\mathbf{u} = \mathbf{G}\mathbf{y} + \mathbf{u}_r$ can be added to a system $(\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}), \mathbf{C}(\mathbf{x}))$ to obtain

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{G}\mathbf{C}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}_r \\ \mathbf{y} = \mathbf{C}(\mathbf{x}) \end{cases} \tag{43}$$

Let $\mathbf{u}_i^A = \mathbf{G}_i^A \mathbf{y}_i^A + \mathbf{u}_i^r$, where $i = 1,2$, be the output feedback that makes the subsystems given by eq. 42 strictly passive. The subsystems with $\mathbf{u}_i^A$ defined as above are now both strictly passive. We can connect the two subsystems in feedback, with $(\mathbf{y}_1 = \mathbf{u}_2)$ and $(\mathbf{u}_1 = -\mathbf{y}_2)$. By Theo. 5.4, the composite system resulting from the feedback connection of two strictly passive systems is strictly passive. Thus, the composite system $(\mathbf{u}_A, \mathbf{y}_A)$ is strictly passive. Now let $\begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix} \equiv \begin{bmatrix}\mathbf{G}_1^A & 0\\ 0 & \mathbf{G}_2^A\end{bmatrix}\begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix} + \begin{bmatrix}\mathbf{u}_1^r\\ \mathbf{u}_2^r\end{bmatrix}$. Adding this output feedback to the composite system, $(\mathbf{u}_A, \mathbf{y}_A)$ formed from the original subsystem components without the output feedback, is equivalent to adding the output feedback to the components and then connecting them in feedback. Since the two methods of adding output feedback are equivalent, we can add output feedback to the composite system, resulting in a strictly passive system. Hence by the definition of almost strict passivity, the composite system is almost strictly passive. Thus, almost strict passivity is inherited, and the result is true. □

Theorem 5.6 *A LTI system given by* $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ *is strict positive real iff it is strictly passive.*

Proof: First we show that if $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is SPR, then it is strictly passive. Since $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is SPR, the Kalman-Yacubovic Lemma (Vidyasagar, 1993) implies that $\exists\, \epsilon > 0$ such that

$$\begin{cases} (\mathbf{A} + \epsilon\mathbf{I})^T\mathbf{P} + \mathbf{P}(\mathbf{A} + \epsilon\mathbf{I}) = -\mathbf{Q} \\ \mathbf{P}\mathbf{B} = \mathbf{C}^T \end{cases} \tag{44}$$

with $\mathbf{Q} \geq 0$ and $\mathbf{P} > 0$. We can rearrange eq. 44 to obtain

$$\begin{cases} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -(\mathbf{Q} + 2\epsilon\mathbf{P}) \\ \mathbf{P}\mathbf{B} = \mathbf{C}^T \end{cases} \tag{45}$$

Since $\mathbf{P} > 0$ and $\mathbf{Q} \geq 0$, then $\mathbf{W}(\epsilon) \equiv \mathbf{Q} + 2\epsilon\mathbf{P} > 0$. Choose $V(\mathbf{x}) \equiv \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x}$ with $\mathbf{P}$ chosen as in eq. 44 and $\mathbf{P} > 0$. The time derivative along any state trajectory of $V(\mathbf{x})$ is given by

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \tfrac{1}{2}\left(\mathbf{x}^T\mathbf{P}\mathbf{A}\dot{\mathbf{x}} + \dot{\mathbf{x}}^T\mathbf{P}\mathbf{B}\mathbf{u}\mathbf{x}\right) \\ &= \tfrac{1}{2}\mathbf{x}^T(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A})\mathbf{x} + \mathbf{x}^T\mathbf{P}\mathbf{B}\mathbf{u} \\ &= -\tfrac{1}{2}\mathbf{x}^T\mathbf{W}(\epsilon)\mathbf{x} + (\mathbf{C}\mathbf{x})^T\mathbf{u} \\ &= -S(\mathbf{x}) + \mathbf{y}^T\mathbf{u} \end{aligned} \tag{46}$$

Therefore $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is strictly passive.

We now show that if $(\mathbf{A},\mathbf{B},\mathbf{C})$ is strictly passive, then it is SPR. Since $(\mathbf{A},\mathbf{B},\mathbf{C})$ is strictly passive, we have $V(\mathbf{x}) = -S(\mathbf{x}) + \mathbf{y}^T\mathbf{u}$ with $S(\mathbf{x}) > 0$. Choose $S(\mathbf{x}) \equiv \mathbf{W}(\epsilon)$, with $\mathbf{P}$ and $\mathbf{Q}$ as in eq. 44. Then all the previous arguments can be reversed, giving the desired result. $\square$

In Section 2.4 we showed that the physical connection of two Evolving System components is equivalent to the feedback connection of the admittance of one component and the impedance of the other component. Consequently, if the subsystem components of an Evolving System are in admittance-impedance form, then by Theo. 5.4 and Theo. 5.5 we see that strict passivity and almost strict passivity are traits that are always inherited in nonlinear Evolving Systems. Therefore, if the impedance of one component and the admittance of the other component are both strictly passive, then their feedback connection will be strictly passive. The same is true for almost strict passivity.

The following result gives the sufficient condition for an LTI Evolving System with an adaptive key component controller to be guaranteed to have bounded gains and asymptotic output tracking.

Theorem 5.7 *Consider a two component linear time-invariant Evolving System given by*

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}}_i & = & \mathbf{A}_i\mathbf{x}_i + \epsilon\mathbf{B}_1\mathbf{u}_1 + \mathbf{B}_1^A\mathbf{u}_1^A \\ \mathbf{y}_i & = & \mathbf{C}_i\mathbf{x}_i \end{array} \right. \tag{47}$$

where $i = 1,2$. Let component 1 have an adaptive key component controller with the following direct adaptive control law

$$\left\{ \begin{array}{lcl} \mathbf{u}_1^A & = & \mathbf{G}\mathbf{y}_1^A \\ \dot{\mathbf{G}} & = & -\mathbf{y}_1^A(\mathbf{y}_1^A)^T\mathbf{H};\ \ \mathbf{H} > 0 \end{array} \right. \tag{48}$$

If both components of the Evolving System are almost strictly passive from an admittance-impedance point of view, then the adaptive gains, $\mathbf{G}$, are bounded and $\mathbf{y} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$.

Proof: By Theo. 5.5, since both components are almost strictly passive, then the composite system resulting from the feedback connection of the components is almost strictly passive. In Theo. 5.6, we showed that for linear time-invariant systems, strict passivity is equivalent to the strict positive real property. A system that is almost strict positive real (ASPR) is one that can be made strict positive real with output feedback. Hence, for LTI systems, almost strict positive real is equivalent to almost strict passivity. Since the Evolving System given by eq. 47 is an almost strictly passive LTI system, it is an almost strict positive real system. Theorem 4.3 states that the sufficient condition for a LTI system with an adaptive control law given by eq. 48 to be guaranteed to have bounded gains and asymptotic output tracking is that the system be almost strict positive real. Therefore, by Theo. 4.3, the adaptive gains, $\mathbf{G}$, are bounded and $\mathbf{y} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$. $\square$

We think of the ports $\mathbf{u}_1$ and $\mathbf{u}_2$ as being the admittance-impedance ports through which the components make contact. For an Evolving System that has an adaptive key component controller, one of the ports $\mathbf{u}_1^A$ or $\mathbf{u}_2^A$ would be used for the key component controller to augment the system to restore stability if necessary. The ports $\mathbf{u}_1^A$ or $\mathbf{u}_2^A$ could also be used to add output feedback to make the Evolving System strictly passive.

6. Inheritance of Dissipativity Properties in Evolving Systems

In this section we briefly present several results that were presented in (Frost & Balas, 2010).

Definition 6.1 *Consider a nonlinear system of the form given by*

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\mathbf{x}) \end{cases} \tag{49}$$

We say that this system is <u>*Strictly Dissipative*</u> *when* $\exists V(\mathbf{x}) > 0\ \forall \mathbf{x} \neq 0$ *such that* $\forall \mathbf{x}$

$$\begin{cases} \nabla V \mathbf{A}(\mathbf{x}) \leq -S(\mathbf{x}) \\ \nabla V \mathbf{B}(\mathbf{x}) = \mathbf{C}^T(\mathbf{x}) \end{cases} \tag{50}$$

where $\nabla V \equiv$ gradient V and $S(\mathbf{x}) > 0\ \forall \mathbf{x} \neq 0$.
The function $V(\mathbf{x})$ is the Lyapunov candidate function for eq. 49. The function, $V(\mathbf{x})$, is related to ∇V by the following

$$\dot{V}(\mathbf{x}) \equiv \nabla V[\mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}] \tag{51}$$

The above says that the storage rate is always less than the external power. This can be seen by using eq. 50 to obtain

$$\begin{aligned} \dot{V}(\mathbf{x}) &\equiv \nabla V[\mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}] \\ &\leq -S(\mathbf{x}) + \mathbf{C}^T(\mathbf{x})\mathbf{u} \\ &= -S(\mathbf{x}) + \langle \mathbf{y}, \mathbf{u} \rangle \end{aligned} \tag{52}$$

Taking $\mathbf{u} \equiv \mathbf{0}$, it is easy to see that eq. 52 implies eq. 50(a), but not necessarily eq. 50(b). So eq. 50 implies eq. 52 but not conversely. The two are only equivalent if eq. 50(a) is an equality. If the inequalities in eq. 50 and eq. 52 are equalities, then the property is called Strict Passivity, which was defined in section 5.

Definition 6.2 *Consider a nonlinear system of the form given by*

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\mathbf{x}) \end{cases} \tag{53}$$

We say that this system is <u>*Almost Strictly Dissipative*</u> *(ASD) when there is some output feedback,* $\mathbf{u} = \mathbf{G}\mathbf{y} + \mathbf{u_r}$*, that makes it strictly dissipative.*

Theorem 6.3 *If a nonlinear system given by* $(\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}), \mathbf{C}(\mathbf{x}))$ *is strictly passive, then it is strictly dissipative.*

Theorem 6.4 *A LTI system given by* $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ *is strictly dissipative iff it is strictly passive.*

Theorem 6.5 *Suppose we have a pair of subsystems of the form*

$$\begin{cases} \dot{\mathbf{x}}_i &= \mathbf{A}_i(\mathbf{x}_i) + \epsilon \mathbf{B}_i(\mathbf{x})\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x})\mathbf{u}_i^A \\ \mathbf{y}_i &= \mathbf{C}_i(\mathbf{x}_i) \\ \mathbf{y}_i^A &= \mathbf{C}_i^A(\mathbf{x}_i) \end{cases} \tag{54}$$

where $i = 1,2$ and both subsystems $\left(\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_1^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_1^A\end{bmatrix}\right)$ *and* $\left(\begin{bmatrix}\mathbf{u}_2\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_2\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *are almost strictly dissipative with energy storage function* $V_1(\mathbf{x}_1)$ *and* $V_2(\mathbf{x}_2)$*, and*

$$\nabla V_i \epsilon \mathbf{B}_i(\mathbf{x}_i) = \epsilon \mathbf{C}_i^T(\mathbf{x}_i) \tag{55}$$

Then the feedback connection of the two subsystems, where $(\mathbf{y}_1 = \mathbf{u}_2)$ *and* $(\mathbf{u}_1 = -\mathbf{y}_2)$*, will leave the resulting composite system* $\left(\mathbf{u}_A \equiv \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \mathbf{y}_A \equiv \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *almost strictly dissipative.*

A corollary of Theo. 6.5 is that strict dissipativity is inherited by systems connected in feedback.

Corollary 6.6 *Suppose we have a pair of subsystems of the form*

$$\left\{\begin{array}{rcl} \dot{\mathbf{x}}_i &=& \mathbf{A}_i(\mathbf{x}_i) + \epsilon \mathbf{B}_i(\mathbf{x})\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x})\mathbf{u}_i^A \\ \mathbf{y}_i &=& \mathbf{C}_i(\mathbf{x}_i) \\ \mathbf{y}_i^A &=& \mathbf{C}_i^A(\mathbf{x}_i) \end{array}\right. \tag{56}$$

where $i = 1,2$ and both subsystems $\left(\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_1^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_1^A\end{bmatrix}\right)$ *and* $\left(\begin{bmatrix}\mathbf{u}_2\\ \mathbf{u}_2^A\end{bmatrix}, \begin{bmatrix}\mathbf{y}_2\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *are strictly dissipative with energy storage function* $V_1(\mathbf{x}_1)$ *and* $V_2(\mathbf{x}_2)$*, and*

$$\nabla V_i \epsilon \mathbf{B}_i(\mathbf{x}_i) = \epsilon \mathbf{C}_i^T(\mathbf{x}_i) \tag{57}$$

Then the feedback connection of the two subsystems, where $(\mathbf{y}_1 = \mathbf{u}_2)$ *and* $(\mathbf{u}_1 = -\mathbf{y}_2)$*, will leave the resulting composite system* $\left(\mathbf{u}_A \equiv \begin{bmatrix}\mathbf{u}_1^A\\ \mathbf{u}_2^A\end{bmatrix}, \mathbf{y}_A \equiv \begin{bmatrix}\mathbf{y}_1^A\\ \mathbf{y}_2^A\end{bmatrix}\right)$ *strictly dissipative.*

Theorem 6.5 and Cor. 6.6 can both be used to show that two component nonlinear Evolving Systems with components that are either both almost strictly dissipative or strictly dissipative from an admittance-impedance point of view inherit the properties of their subsystem components. Thus strict dissipativity and almost strict dissipativity are traits that are always inherited in nonlinear Evolving Systems.

Theorem 6.7 *Consider a two component nonlinear time-invariant Evolving System given by*

$$\left\{\begin{array}{rcl} \dot{\mathbf{x}}_i &=& \mathbf{A}_i(\mathbf{x}_i) + \epsilon \mathbf{B}_i(\mathbf{x}_i)\mathbf{u}_i + \mathbf{B}_i^A(\mathbf{x}_i)\mathbf{u}_1^A \\ \mathbf{y}_i &=& \mathbf{C}_i(\mathbf{x}_i) \end{array}\right. \tag{58}$$

where $i = 1,2$ with energy storage functions $V_1(\mathbf{x}_1)$ and $V_2(\mathbf{x}_2)$. Let component 1 have an adaptive key component controller with the following direct adaptive control law

$$\left\{\begin{array}{rcl} \mathbf{u}_1^A &=& \mathbf{G}\mathbf{y}_1^A \\ \dot{\mathbf{G}} &=& -\mathbf{y}_1^A(\mathbf{y}_1^A)^T\mathbf{H};\ \mathbf{H} > 0 \end{array}\right. \tag{59}$$

Assume that V_1 and V_2 are positive $\forall \mathbf{x} \neq 0$ and radially unbounded, and $(\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}), \mathbf{C}(\mathbf{x}))$ are continuous functions of $\mathbf{x}$ and $S(\mathbf{x})$ is positive $\forall \mathbf{x} \neq 0$ and has continuous partial derivatives in $\mathbf{x}$. Furthermore, assume:

1. *Component 2, given by $(\mathbf{u}_2, \mathbf{y}_2)$, is strictly dissipative and in impedance form;*
2. *Component 1, given by $(\mathbf{u}_1^A, \mathbf{y}_1^A)$, is almost strictly dissipative;*
3. *Component 1, given by $(\mathbf{u}_1, \mathbf{y}_1)$, is in admittance form.*

Then the adaptive key component controller given by eq. 59 produces global asymptotic state stability, i.e., $\mathbf{x} \equiv \left[\mathbf{x}_1 \mathbf{x}_2\right]^T \to 0$ as $t \to \infty$ with bounded adaptive gains when component 1 is joined with component 2 into an Evolved System and the outputs $\mathbf{y}_i = \mathbf{C}_i(\mathbf{x}_i) \to 0$ as $t \to \infty$.

The above results assume that the Lyapunov function, $V(\mathbf{x})$, is defined on the entire domain, $\mathbb{R}^n$, of the system. Thus all the stability and dissipativity results are global results. The same is true for the other results given in this chapter. For instance, Theo. 6.7 says that a nonlinear Evolving System with an adaptive key component controller as given by eq. 48 will have bounded gains and globally asymptotic state tracking. However, the Lyapunov function, $V(\mathbf{x})$, might only be defined on a neighborhood $N_i(0, r_i) \equiv \left\{ \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|} < r_i \right\}$ of the origin, in which case the results could only be local at the best.

Using Lemma 1 from (Balas et al., 2008), $\exists \delta > 0$ such that if the initial conditions of the system are close enough to the origin, i.e., within $N_\delta = (0, \delta)$, then the trajectories are guaranteed to stay in the neighborhood of the origin for which the Lyapunov function is defined. In such a case, then the results would be local. For instance, if the Lyapunov function $V(\mathbf{x})$ in Theo. 6.7 only has the assumed properties on a neighborhood $N_i(0, r_i) \equiv \left\{ \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|} < r_i \right\}$ of the origin and the trajectories all remain inside the neighborhood, then the stability is locally asymptotic to the origin. In that case, Theo. 6.7 gives the result that a nonlinear Evolving System with an adaptive key component controller as given by eq. 59 will have bounded gains and locally asymptotic state tracking.

7. Conclusions

In this chapter, we presented the motivation and the framework for Evolving Systems, a new area of aerospace research. We developed the adaptive key component controller approach to maintain stability in Evolving Systems that would otherwise fail to inherit the stability traits of their components. We showed that strict passivity, almost strict passivity, strict dissipativity, and almost strict dissipativity are inherited by systems connected in feedback. Using the impedance-admittance formulation of contact dynamics between components of Evolving Systems, we showed that these traits are also always inherited in nonlinear Evolving Systems. Finally, we gave sufficient conditions for the use of the adaptive key component controller with linear and nonlinear Evolving Systems.

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