

into  $n$  independent equations and to express  $P$  as a product of  $n$  probabilities  $P_i$ :

$$\frac{\partial P_i'}{\partial t} = -\gamma_i \frac{\partial P_i'}{\partial x_i} \left( (x_i) - x_i \right)^{\frac{1}{3}} \text{ and}$$

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P_i'(x_i)$$

By use of these equations, it can be shown that the control forces create a powerful terminal attractor in probabilistic space that corresponds to occur-

rence of the target trajectory with probability one (see figure). In configuration space (space in the sense in which "space" is understood in casual conversation), the effect of the control forces is to suppress exponential divergence of close neighboring trajectories without affecting the target trajectory. As a result, the post-instability motion is represented by a set of functions that describe the evolution of such statistical invariants such as expectations, vari-

ances, and higher moments of the statistics of the state variables  $x_i$  as functions of time.

*This work was done by Michail Zak of Caltech for NASA's Jet Propulsion Laboratory. Further information is contained in a TSP (see page 1).*

*The software used in this innovation is available for commercial licensing. Please contact Karina Edmonds of the California Institute of Technology at (818) 393-2827. Refer to NPO-41519.*

## Two High-Temperature Foil Journal Bearings

These are prototypes of foil bearings for aircraft gas turbine engines.

John H. Glenn Research Center, Cleveland, Ohio

An enlarged, high-temperature-compliant foil bearing has been built and tested to demonstrate the feasibility of such bearings for use in aircraft gas turbine engines. At 150 mm in diameter, this is the largest foil bearing known to date. This bearing is a scaled-up version of a patented 100-mm-diameter foil bearing, augmented by coating the foil with a proprietary high-temperature material. In a companion development, a foil bearing as described above has been combined with a 150-mm-diameter active magnetic bearing to make a hybrid foil magnetic bearing.

Foil bearings are attractive for use in some machines in which (1) speeds of rotation, temperatures, or both exceed maximum allowable values for rolling-element bearings; (2) conventional lubricants decompose at high operating temperatures; and/or (3) it is necessary or desirable not to rely on conventional lubrication systems. In a foil bearing, the lubricant is the working fluid (e.g., air or a mixture of combustion gases) in the space between the journal and the shaft in the machine in which the bearing is installed. At no or low speed, the shaft is supported at by a spring-loaded foil journal lining. Once the shaft is rotating rapidly enough, the hydrodynamic and viscous forces exerted by the flow of working fluid between the foil and the shaft force the foil away from the shaft, so that the shaft becomes supported by a film of the working fluid.

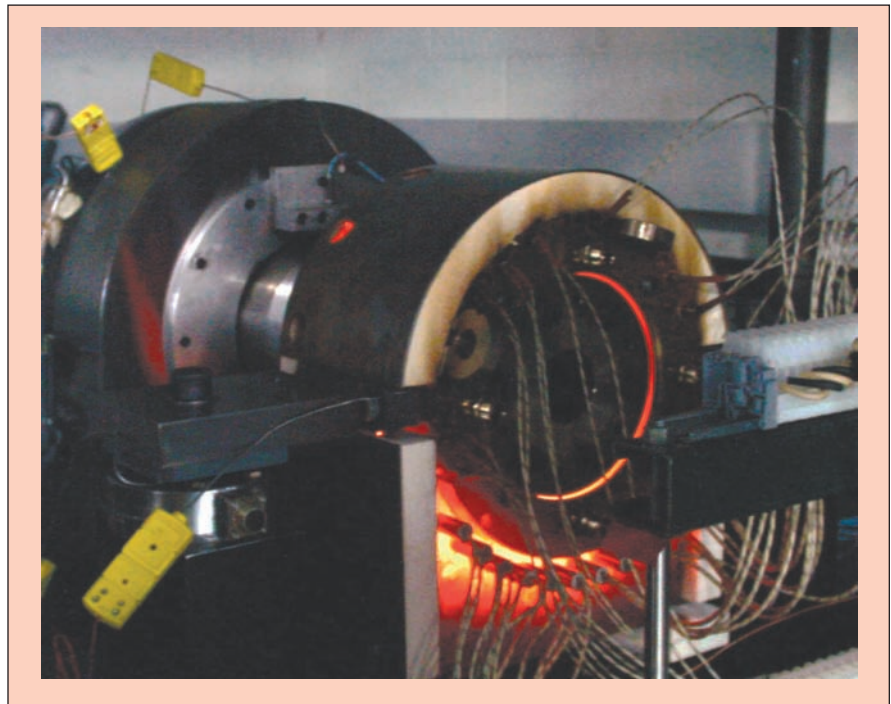
The present enlarged, high-temperature foil bearing has been tested at speeds up to 27,000 rpm (at 150 mm diameter, corresponding to a surface speed of 212 m/s) and at temperatures

in excess of 1,200 °F (>649 °C). These speed and temperature limits exceed those of rolling-element bearings by several fold.

The hybrid foil magnetic bearing was conceived to take advantage of the strengths of the foil and the active magnetic bearing while utilizing each bearing to compensate for the weakness of the other, for the overall purpose of obtaining high load capacity at all speeds and temperatures (see figure). The active magnetic bearing exhibits excellent performance at low speed, where the surface coating on the foil bearing has limited load capacity. The foil bearing

exhibits excellent performance at high speed, where the active magnetic bearing can fail in response to shocks and other transient disturbances.

Unlike a conventional active magnetic bearing, the hybrid foil magnetic bearing can operate without need for a separate protective auxiliary/backup bearing. In case of failure of the active magnetic bearing in the hybrid foil magnetic bearing, the foil bearing plays the role of the backup bearing, so that a rotor can continue to run on the foil bearing alone and then come down to a safe stop. The hybrid foil magnetic bearing exhibits both the high load ca-



The Hybrid Foil Magnetic Bearing was photographed in operation at a speed of 15,000 rpm at a temperature of 1,200 °F (>649 °C)

capacity of the foil bearing and the high static stiffness and control versatility of the active magnetic bearing. Hence, the hybrid foil magnetic bearing system implements a significant advance in range of operation and reliability.

*This work was done by Hooshang Heshmat of Mohawk Innovative Technology, Inc. for Glenn Research Center. Further information is contained in a TSP (see page 1).*

*Inquiries concerning rights for the commercial use of this invention should be addressed*

*to NASA Glenn Research Center, Innovative Partnerships Office, Attn: Steve Fedor, Mail Stop 4-8, 21000 Brookpark Road, Cleveland, Ohio 44135. Refer to LEW-17643-1.*

## Using Plates To Represent Fillets in Finite-Element Modeling

**Structural deflections are approximated by use of simplified computational submodels of fillets.**

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A method that involves the use of fictitious plate elements denoted bridge plates has been developed for representing the stiffnesses of fillets in finite-element calculations of deflections, stresses, and strains in structures. In the absence of this method, it would be necessary to either neglect the effects of fillets to minimize the computational burden or else incur a large computational burden by using complex computational models to represent the fillets accurately. In effect, the bridge plates of the present method are reduced-order models of fillets that do not yield accurate stresses within fillets but do make it possible to accurately calculate the dynamic characteristics of the structure and to approximate the effects of fillets on stresses and strains elsewhere in a structure that contains the fillets. Such approximations are accurate enough for final modal analysis and preliminary stress analyses.

In a finite-element model according to this method, the model of a fillet includes bridge plates that connect the tangent lines of the fillets. For a given fillet, the bridge plates are characterized by a thickness ( $t_b$ ) and a pseudo Young's modulus ( $E_b$ ) to represent the mass and stiffness of the fillet as accurately as possible. It is necessary to calculate  $t_b$  and  $E_b$  in advance, by means of the procedure described in the next paragraph.

One generates two simultaneous nonlinear wide-beam-deflection equations for the rotation at the tangent lines: an equation applicable to the bridge-plate representation and an equation derived from an analytic representation of the fillet. These equations are formulated in terms of the independent variables  $r/t$  and  $t_{\text{wall}}/t$ , where  $r$  is the fillet radius,  $t_{\text{wall}}$  is the thickness of the non-filletted section of a wall adjacent to the filleted section,

and  $t$  is a thickness variable, the value of which one seeks. The equations are solved numerically to obtain  $t_b$  and  $E_b$ . In addition, surface fits of the solutions are obtained for use as the equivalent of closed-form equations for  $t_b$  and  $E_b$ .

The method has been verified in calculations pertaining to a representative filleted structure. The bridge-plate model yielded a level of accuracy for the calculation of natural frequencies and mode shapes better than or equal to that obtained by use of a high-fidelity solid model of the fillet, even though the bridge-plate model contained 90 percent fewer degrees of freedom.

*This work was done by Andrew Brown of Marshall Space Flight Center. For further information, access the Technical Support Package (TSP) free on-line at [www.techbriefs.com/tsp](http://www.techbriefs.com/tsp) under the Mechanics category. MFS-31992*