Preliminary Multi-Variable Parametric Cost Model for Space Telescopes

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Parametric Cost Models

Parametric cost models have several uses:

- high level mission concept design studies,
- identify major architectural cost drivers,
- allow high-level design trades,
- enable cost-benefit analysis for technology development investment, and
- provide a basis for estimating total project cost.

In the past 12 months

Added JWST cost information for 2003, 2006, 2008 and 2009.

Published two peer reviewed cost model papers:

Stahl, H. Philip, Kyle Stephens, Todd Henrichs, Christian Smart, and Frank A. Prince, "Single Variable Parametric Cost Models for Space Telescopes", Optical Engineering Vol.49, No.06, 2010

Stahl, H. Philip, "Survey of Cost Models for Space Telescopes", Optical Engineering, Vol.49, No.05, 2010

Now working on developing multi-variable cost models.

Objectives for Today

- Review Data Collection Methodology
- Define Statistical Analysis Methodology
- Summarize Single Variable Results
- Test Historical Models
- Introduce Preliminary Multi-Variable Models

Methodology

Data on 59 different variables was acquired for **30** NASA, ESA, & commercial space telescopes using:

- NAFCOM (NASA/ Air Force Cost Model) database,
- RSIC (Redstone Scientific Information Center),
- REDSTAR (Resource Data Storage and Retrieval System),
- project websites, and interviews.

Table 1: Cost Model Missions Database

X-Ray Telescopes	Infrared Telescopes
Chandra (AXAF)	CALIPSO
Einstein (HEAO-2)	Herschel
	ICESat
UV/Optical Telescopes	IRAS
EUVE	ISO
FUSE	JWST
GALEX	SOFIA
HiRISE	Spitzer (SIRTF)
HST	TRACE
HUT	WIRE
IUE	WISE
Kepler	
Copernicus (OAO-3)	Microwave Telescopes
SOHO/EIT	WMAP
UIT	
WUPPE	Radio Wave Antenna
	TDRS-1
	TDRS-7

Missions

Of the 30 mission, we initially studied 21 'normal-incidence' UVOIR and Infrared telescopes.

Of these,

17 are 'Free Flying' and

4 are 'Attached'

To study wavelength diversity, we added microwave, radio wave and grazing incidence X-Ray/EUV.

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UV/Optical Telescopes	IRAS
EUVE	ISO
FUSE	JWST
GALEX	SOFIA
HiRISE	Spitzer (SIRTF)
HST	TRACE
HUT	WIRE
IUE	WISE
Kepler	
Copernicus (OAO-3)	Microwave Telescopes
SOHO/EIT	WMAP
UIT	
WUPPE	Radio Wave Antenna
	TDRS-1
	TDRS-7

Cost Variables

Total Cost is Phase A through D, it does not include:

- Phase E (post-launch) costs
- Launch related costs
- Civil servant costs (NASA employees)
- So our Total Cost is contract cost to make the system.

OTA Cost includes only:

- Primary mirror
- Secondary (and tertiary if appropriate) mirror(s)
- Related support structure

Total Mass and OTA Mass match the cost definitions

Technical Variables

Aperture Diameter

Mass (OTA and Total)

PM Focal Length

PM F/#

Field of View

Pointing Accuracy

Spectral Range Minimum

Wavelength of Diffraction Limit

Operating Temperature

Average Input Power

Data Rate

Design Life

Orbit

Programmatic Variables

Launch Year

Year of Development (or Start of Development)

Development Period

TRL (Technology Readiness Level)

Completeness of Data for 19 Variables

Table 2: Cost Model Variables Study and the completeness of data knowledge

Parameters	% of Data
OTA Cost	89%
Total Phase A-D Cost w/o LV	84%
Aperture Diameter	100%
Avg. Input Power	95%
Total Mass	89%
OTA Mass	89%
Spectral Range	100%
Wavelength Diffraction Limit	63%
Primary Mirror Focal Length	79%
Design Life	100%
Data Rate	74%
Launch Date	100%
Year of Development	95%
Technology Readiness Level	47%
Operating Temperature	95%
Field of View	79%
Pointing Accuracy	95%
Orbit	89%
Development Period	95%
Average	88%

Model Creation

Start with Correlation Matrix.

Look for Variables which are Highly Correlated with Cost.

- The higher the correlation the greater the Cost Variation which is explained by a given Variable.
- Sign of correlation is important and must be consistent with Engineering Judgment.

Important for Multi-Variable Models:

- We want Variables which Independently effect Cost.
- When Variables ‘cross-talk’ with each other it is called Multi-Collinearity.
- Thus, avoid Variables which are highly correlated with each other.

Goodness of Correlation, Fits and Regressions

‘Correlation’ between variables and ‘Goodness’ of single variable models is evaluated via Pearson’s r^2 standard percent error (SPE), and Student’s T-Test p-value.

‘Goodness’ of multivariable fits are evaluated via Pearson’s Adjusted r^2 which accounts for number of data points and number of variables.

Pearson’s r^2 coefficient describes the percentage of agreement between the fitted values and the actual data.

The closer r^2 is to 1, the better the fit.

SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit.

The closer SPE is to 0, the better the fit

Significance

The final issue is whether or not a correlation or fit is significant.

p-value is the probability that the fit or correlation would occur if the variables are independent of each other.

- The closer p-value is to 0, the more significant the fit or correlation.
- The closer p-value is to 1, the less significant.
- If the p-value for a given variable is small, then removing it from the model would cause a large change to the model.
- If p-value is large, then removing the variable will have a negligible effect

It is only possible to ‘test’ if the correlation between two variables is significant.

It is not possible to ‘test’ if two variables are independent.

Cross Correlation Matrix

	Total Phase A-D Cost	OTA Cost	Areal Total Cost	Areal OTA Cost	Total Cost / Total Mass	OTA Cost / OTA Mass	Aperture Diameter	PM F Len.	PM F/N	OTA Volume	FOV	Pointing Accuracy	Total Mass	OTA Mass	OTA Areal Density	Spectral Range minimum	Wavelength Diffraction Limit	Operating Temperature	Avg. Input Power	Data Rate	Design Life	Technology Readiness Level	Year of Development	Development Period	Launch Date	Orbit
units	[FY09$M]	[FY09$M]	[FY09$M]	[FY09$M/m²]	[FY09$M/kg]	[FY09$M/kg]	[m]	[m]	unitless	[m³]	[°]	[Arc-Sec]	[kg]	[kg]	[kg/m²]	[μ]	[μ]	[K]	[Watts]	[Kbps]	[months]	TRL	[year]	[months]	[year]	[km]
Total Phase A-D Cost	1.00	0.70	0.09	-0.36	0.59	-0.05	0.64	0.80	0.38	0.83	0.26	-0.52	0.92	0.72	-0.48	-0.02	-0.40	-0.04	0.59	0.44	0.65	-0.41	-0.11	0.78	0.11	0.54
OTA Cost		1.00	-0.53	-0.30	0.32	0.21	0.87	0.82	0.39	0.84	0.00	-0.58	0.68	0.82	0.41	0.07	-0.23	0.01	0.14	0.15	0.46	-0.68	-0.31	0.45	-0.16	0.17
Areal Total Cost			1.00	0.68	0.20	0.22	-0.71	-0.40	0.03	-0.50	0.26	0.34	0.00	-0.47	0.38	-0.23	-0.18	-0.07	-0.05	-0.07	-0.20	-0.29	-0.28	0.14	-0.34	0.40
Areal OTA Cost				1.00	-0.34	0.58	-0.74	-0.62	-0.16	-0.71	-0.56	0.30	-0.34	-0.48	0.59	-0.20	-0.07	-0.03	-0.48	-0.48	-0.41	-0.43	-0.56	-0.22	-0.68	0.04
Total Cost / Total Mass					1.00	-0.16	0.27	0.18	-0.02	0.23	0.19	-0.24	0.22	0.13	-0.30	0.31	-0.12	-0.35	0.03	0.45	0.26	0.26	0.10	0.67	0.24	0.69
OTA Cost / OTA Mass						1.00	-0.15	-0.20	0.03	-0.26	-0.30	0.26	-0.03	-0.39	-0.31	-0.19	-0.42	0.26	-0.49	-0.01	0.11	-0.64	-0.26	-0.35	-0.37	-0.19
Aperture Diameter							1.00	0.88	0.27	0.98	-0.09	-0.58	0.63	0.86	-0.60	0.14	-0.11	0.05	0.42	0.38	0.53	-0.29	0.09	0.37	0.26	0.08
PM F Len.								1.00	0.69	0.96	0.34	-0.66	0.84	0.78	-0.44	-0.50	-0.19	0.28	0.49	0.31	0.50	-0.38	-0.07	0.50	0.10	0.28
PM F/N									1.00	0.45	0.57	-0.41	0.48	0.33	-0.02	-0.61	-0.43	0.32	0.06	0.20	0.25	-0.37	-0.32	0.21	-0.29	0.08
OTA Volume										1.00	0.08	-0.65	0.84	0.84	-0.54	-0.36	-0.08	0.21	0.65	0.34	0.52	-0.31	0.06	0.54	0.26	0.31
FOV											1.00	0.12	0.16	-0.05	0.01	0.05	-0.38	-0.06	-0.02	0.18	0.09	-0.27	0.08	-0.01	0.09	0.09
Pointing Accuracy												1.00	-0.48	-0.71	0.14	0.31	0.08	-0.38	-0.37	-0.29	-0.35	-0.15	0.13	-0.55	-0.02	-0.32
Total Mass													1.00	0.82	-0.42	-0.15	-0.49	0.03	0.55	0.17	0.65	-0.56	-0.27	0.64	-0.10	0.33
OTA Mass														1.00	-0.11	-0.06	0.06	-0.03	0.60	0.09	0.40	-0.29	-0.16	0.57	0.02	0.47
OTA Areal Density															1.00	0.05	0.28	-0.31	-0.16	-0.39	-0.55	0.07	-0.36	-0.20	-0.46	-0.09
Spectral Range minimum																1.00	0.76	-0.79	-0.09	-0.12	-0.25	-0.09	0.21	0.20	0.23	0.01
Wavelength Diffraction Limit																	1.00	-0.55	-0.07	-0.25	-0.75	0.51	0.35	0.31	0.28	0.14
Operating Temperature																		1.00	0.09	0.31	0.31	0.11	-0.01	-0.30	0.00	-0.30
Avg. Input Power																			1.00	0.34	0.64	0.05	0.35	0.27	0.45	0.04
Data Rate																				1.00	0.54	0.51	0.49	-0.06	0.52	0.21
Design Life																					1.00	-0.15	0.12	0.12	0.24	0.14
Technology Readiness Level																						1.00	0.68	-0.24	0.64	0.33
Year of Development																							1.00	-0.23	0.97	-0.05
Development Period																								1.00	-0.02	0.51
Launch Date																									1.00	0.04
Orbit																										1.00

Correlations which are at least 95% significant are **Bolded**, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Diameter appears to be the most significant cost driver. So, in addition to total cost and OTA cost we have examined OTA Areal Cost, i.e. OTA Cost per unit Area of Primary Mirror collecting aperture. Diameter is correlated with all three with a significance of greater than 99%.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
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TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Primary Mirror Focal Length is also a significant correlation, but as we will discover later, it is multi-collinear with Diameter. The assumed explanation is that all space telescopes tend to have the same basic PM F/#.

Correlation Significance Details

	Total Cost			OTA Cost			OTA Areal Cost		
Parameter	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
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TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Pointing Accuracy has reasonably correlation with cost. And, as expected from engineering judgment, it has significant correlation (99% confidence level) with diameter and OTA mass. Interesting, as will be discussed later, pointing is not multi-collinear with either.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

As expected, Total Mass correlates most significantly with Total Cost while OTA Mass correlates most significantly with OTA Cost.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
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Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Unexpectedly, Minimum Spectral Range Value and Operating Temperature do not have a significant correlation with any Cost. However, as we will show later, Spectral Minimum does have a role in multi-variable cost models.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
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OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

As expected Electrical Power, Design Life and Development Period have significant correlations (99% confidence) with Total Cost.

Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Also unexpected is that TRL and Launch Year do not have significant correlations. But, as we will discuss later, they both have roles in multi-variable cost models. One problem with TRL is there are only 8 data points.

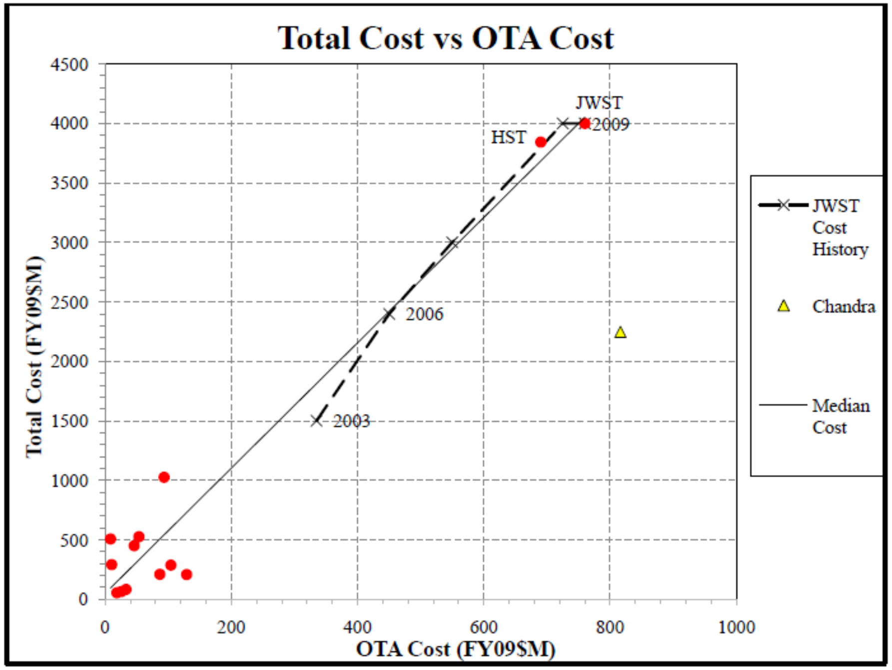
OTA Cost or Total Cost

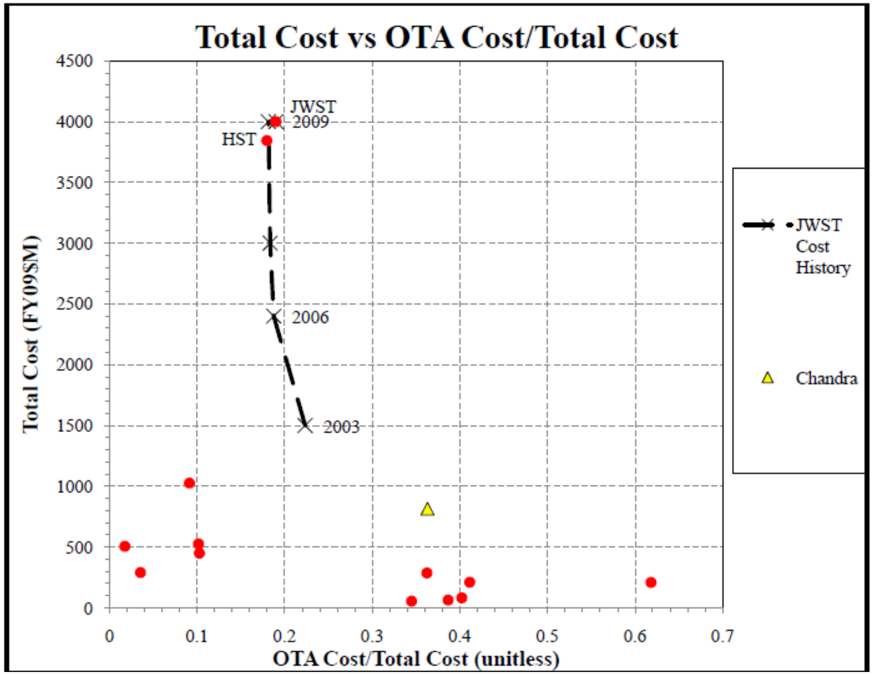
Engineering judgment says that OTA cost is most closely related to OTA engineering parameters. But, managers and mission planners are really more interested in total Phase A-D cost.

For 14 missions free flying missions,

OTA cost is ~20% of Phase A-D total cost ($R^2 = 96\%$)

with a model residual standard deviation of approximately $300M.

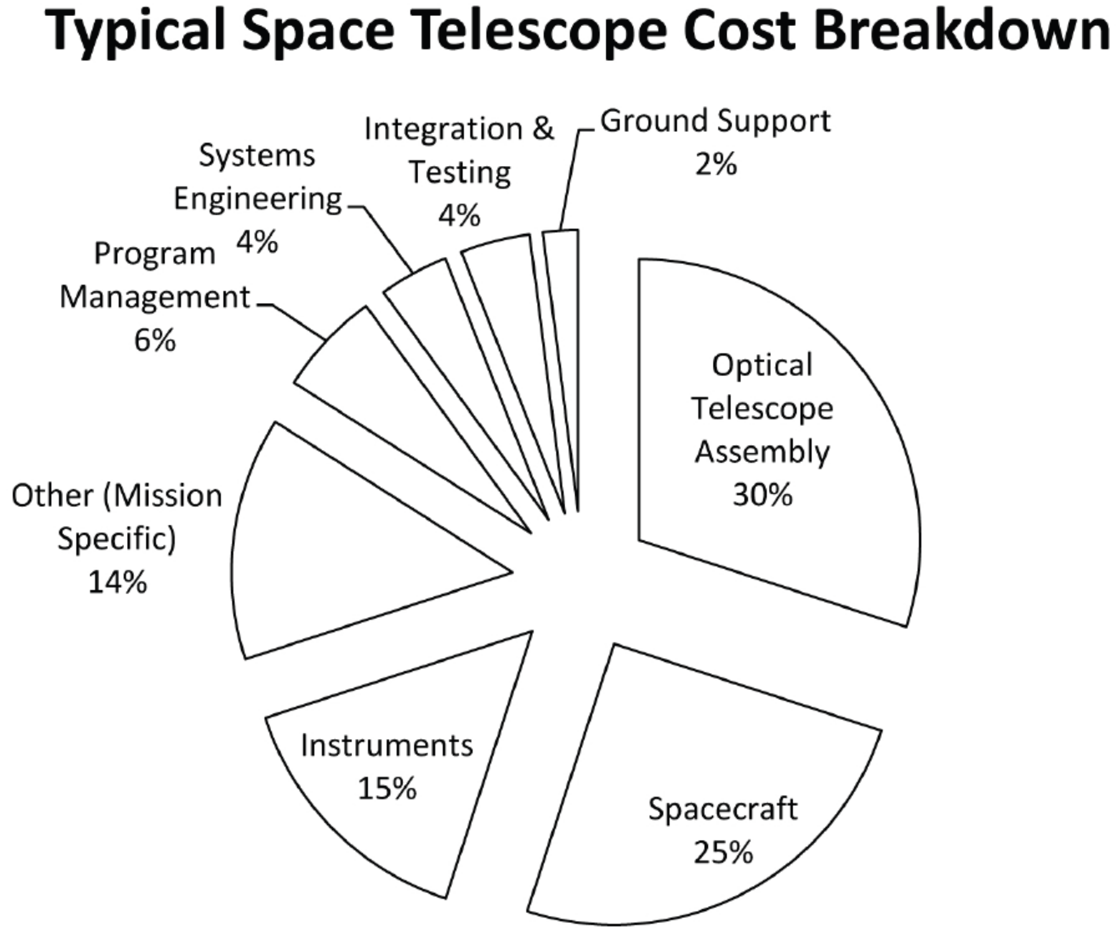




OTA Cost or Total Cost

We have detailed WBS data for 7 of the 14 free flying missions.

Mapping on common WBS indicates that OTA is ~30% of Total,

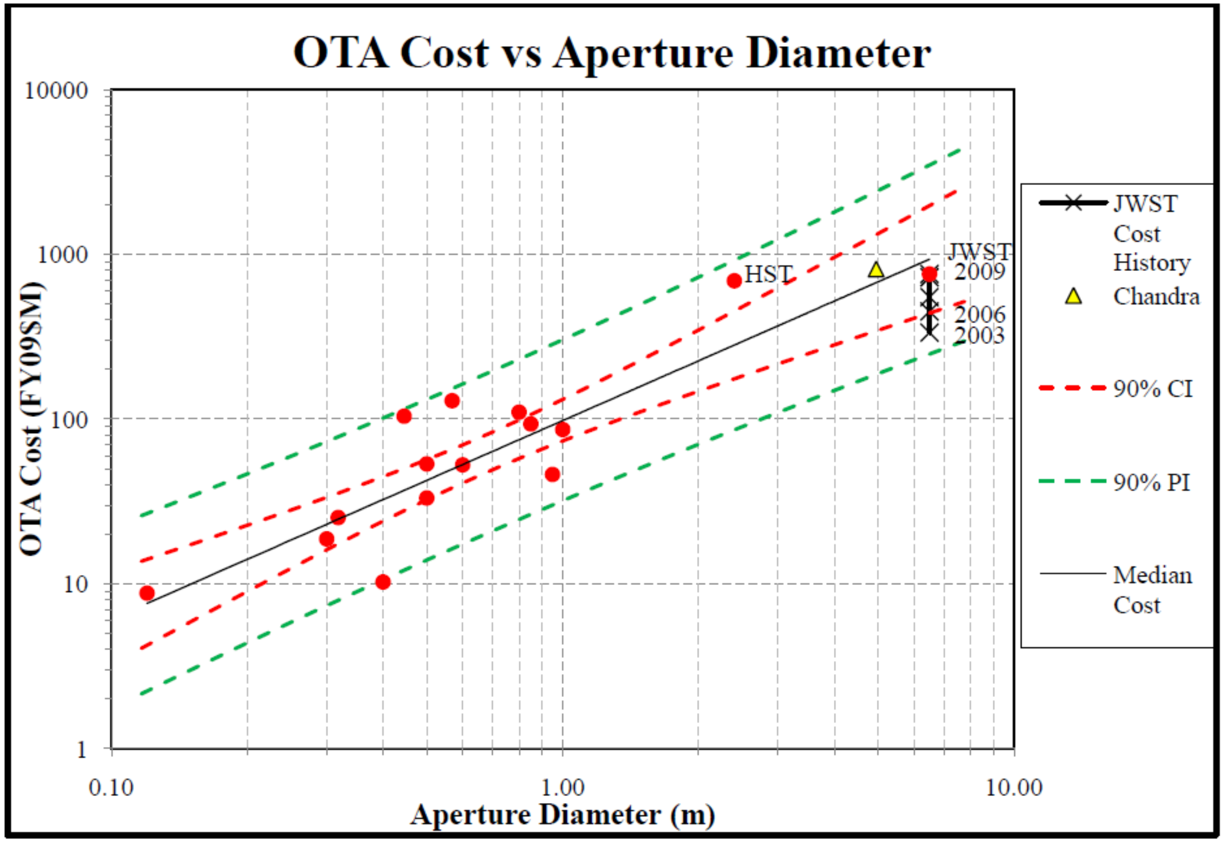


OTA Cost vs Aperture Diameter

For free-flying space telescopes:

OTA Cost ~ Aperture Diameter$^{1.28}$ **(N = 16; r2 = 84%) without JWST**

OTA Cost ~ Aperture Diameter$^{1.2}$ **(N = 17; r2 = 75%) with 2009 JWST**

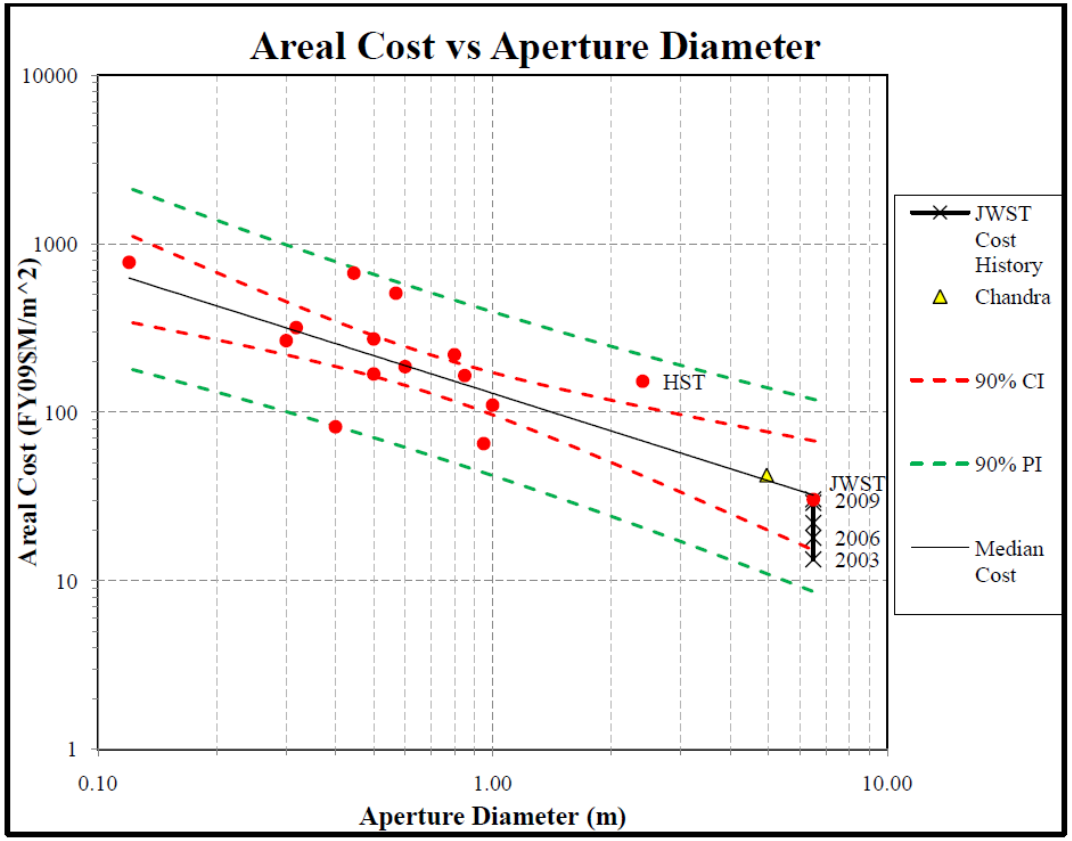


Area Cost

Total Cost is important, but Areal Cost might be more relevant.

Areal Cost decreases with aperture size, therefore, larger telescopes provide a better ROI

OTA Areal Cost ~ Aperture Diameter $^{-0.74}$ ***(N = 17; r^2 = 55%) with JWST***



Mass Models

While aperture diameter is the single most important parameter driving science performance.

Total system mass determines what vehicle can be used to launch.

Significant engineering costs are expended to keep a given payload inside of its allocated mass budget.

Such as light-weighting mirrors and structure.

Space telescopes are designed to mass

Mass Models

Our data shows that

Total Mass is ~ 3.3X OTA Mass (r^2 = 92%), and

Total Cost is ~3.3X to 5X OTA Cost.

3.3X comes from WBS analysis

5X comes from regression analysis

Mission	Mass Ratio	Cost Ratio
JWST	~2.6X	~5.3X
Hubble	4.6X	5.5X
Chandra	6.2X	2.8X

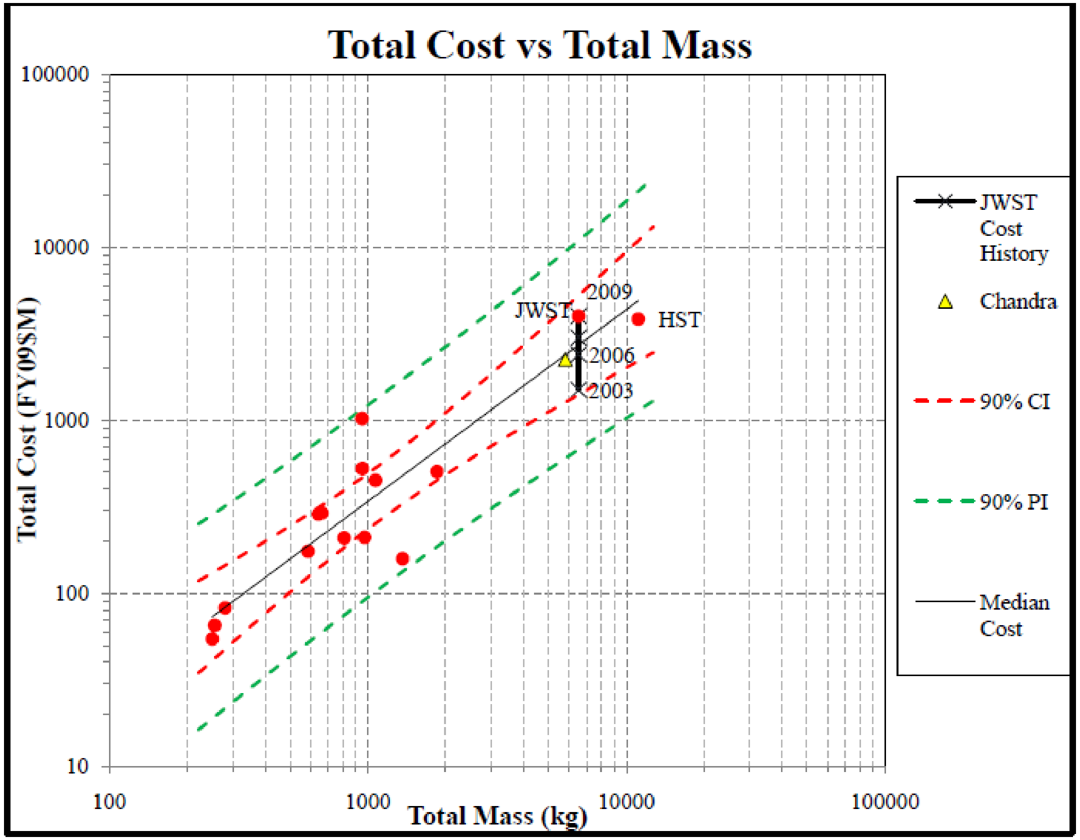
For Chandra, science instruments were massive and optics expensive

Total Cost vs Total Mass

Based on 15 free-flying OTAs

Total Cost ~ Total Mass $^{1.12}$ *(N = 15; r^2 = 86%) with JWST*

Total Cost ~ Total Mass $^{1.04}$ *(N = 14; r^2 = 95%) without JWST*

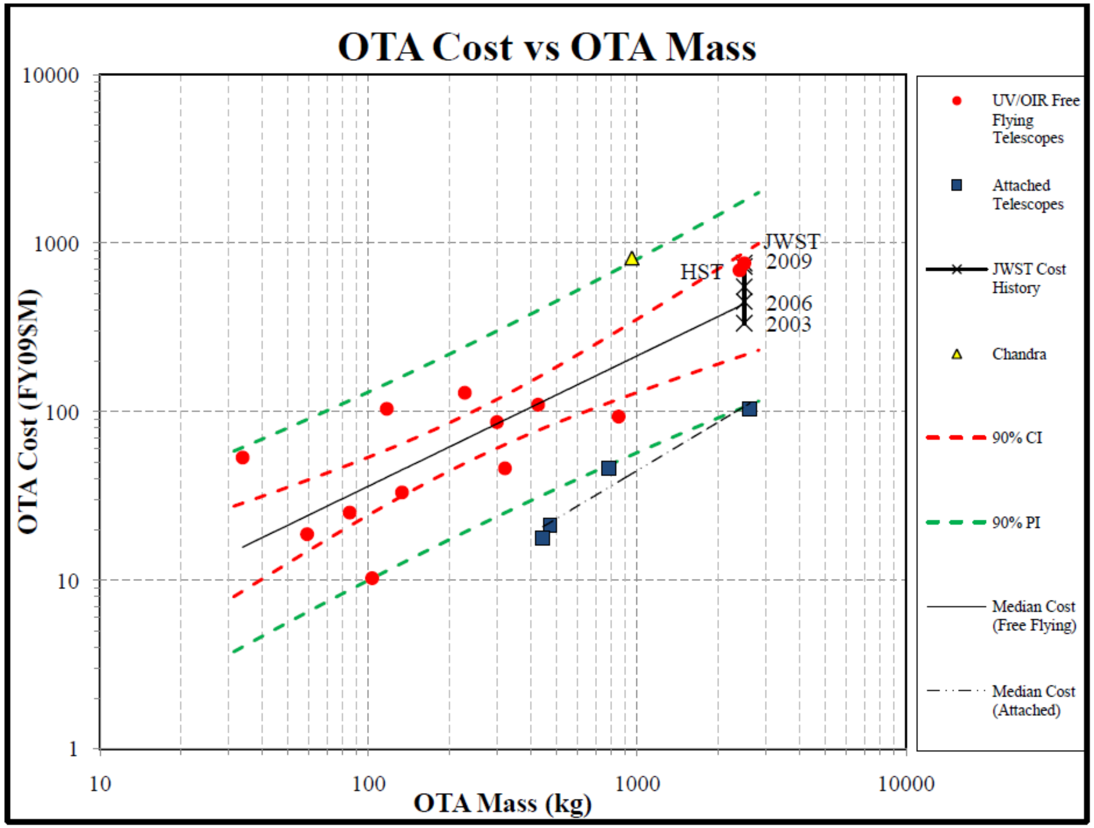


OTA Cost vs OTA Mass

Based on 15 free-flying OTAs

OTA Cost ~ OTA Mass $^{\mathbf{0.69}}$ ***(N = 14;*** r^2 ***= 84%) without JWST***

OTA Cost ~ OTA Mass $^{\mathbf{0.72}}$ ***(N = 15;*** r^2 ***= 92%) with JWST***



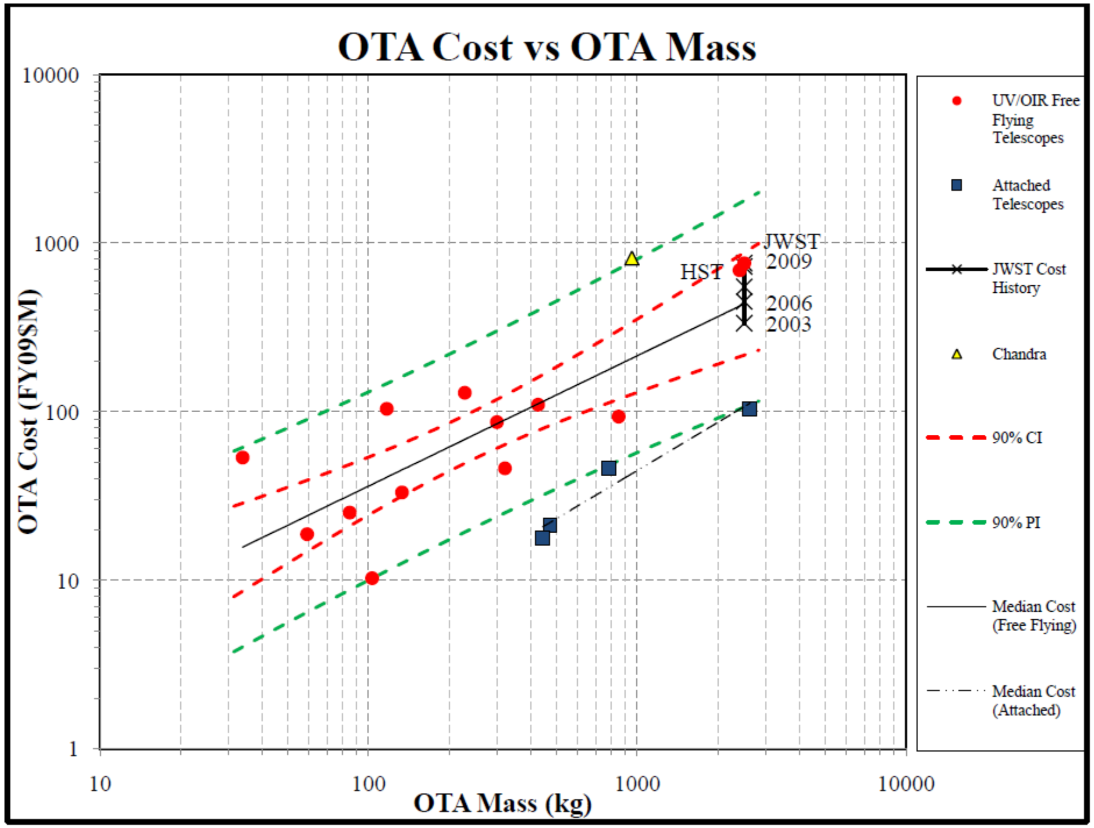
It costs more to make a Lightweight Telescope

For 15 free-flying and 4 attached missions

(3 to Space Shuttle Orbiter and SOFIA to Boeing 747)

‘Attached’ OTAs are ~10X more massive than ‘free-flying’

‘Attached’ OTAs cost ~60% less than ‘free-flying’



Problem with Mass

Mass may have a high correlation to Cost.

And, Mass may be convenient to quantify.

But, Mass is not an independent variable.

Mass depends upon the size of the telescope.

Bigger telescopes have more mass and Aperture drives size.

And, bigger telescopes typically require bigger spacecraft.

The correlation matrix says that Mass is highly correlated with:

Aperture Diameter, Focal Length, F/# ,Volume, Pointing and Power

<u>But in reality it is all Aperture, the others all depend on aperture.</u>

Single Variable Model Statistical Summary

While Mass regression has the highest correlation (Pearson’s r^2), it also has the highest uncertainty (SPE).

Table 4: Summary of Single Variable Cost Model Statistics

	OTA Cost		OTA Areal Cost		OTA Cost		Total Cost	
Variable	OTA Diameter		OTA Diameter		OTA Mass		Total Mass	
includes JWST	yes	no	yes	no	yes	no	yes	no
Exponent	1.2	1.28	-0.74	-0.72	0.72	0.69	1.12	1.04
Coefficient	98.5	103.5	122.0	133.6	1.03	1.58	0.16	0.24
$s_{log\$}$	0.62	0.64	0.62	0.64	0.70	0.70	0.53	0.54
Pearson's r^2	75%	84%	55%	52%	92%	84%	86%	95%
SPE	79%	79%	78%	79%	93%	91%	71%	77%
n	17	16	17	16	15	14	15	14

Multivariable Model required to increase r^2 and decrease SPE.

Testing Historical Models

Of all the historical models, the Horak model is the easiest to test.

- Our database has parameters equal to the Horak database.
- Horak published statistical fit details.

CER: $T1 = 0.357\,(\mathrm{Matl})(\mathrm{Dsn})(\mathrm{Apr})^{0.705}(\#\mathrm{Elem})^{0.473}(\lambda)^{-0.178}(K°)^{-0.191}\,e^{-0.033(\mathrm{Yr}-80)}$

T Statistics: (1.74) (8.80) (2.55) (-2.04) (-2.61) (-2.31)

Statistics $R^2 = 97.0\%$

$s = 0.212$ (17 Data Points)

For this effort, we will ignore the Material (glass vs metal) and Design (on vs off-axis) multiplier factors.

Testing the Horak Model

Horak model with p-value significance analysis:

$$T1 \sim Apr^{0.705} \# Elem^{0.473} \lambda^{-0.178} K^{o\,-0.191} e^{-0.033(Yr-80)}$$

T Statistics:	8.80	2.55	-2.04	-2.61	-2.31
p-values:	0.00	0.022	0.059	0.020	0.036

Based on 17 data points, all variables in model are significant and the fit has a good $R^2 = 97\%$.

Testing Horak against our Data Base yields:

$$OTA\,Cost \sim Diam^{1.39} \# Elem^{-1.11} \lambda^{-0.024} K^{o\,-0.045} e^{-0.0369(Yr-80)}$$

T Statistics:	9.34	-1.03	-0.22	-0.38	-2.80
p-values:	0.00	0.320	0.829	0.710	0.014

Based on 16 data points, only Diameter and Launch Year are significant and the fit has a good $R^2=90.8\%$ and $r^2_{adj}=86.2\%$.

Testing the Horak Model

Eliminating the insignificant variables yields:

$$OTA\,Cost \sim Diam^{133} e^{-0.0434(LY-80)}$$

T Statistics:	10.61	-4.22
p-values:	0.00	0.001

Based on 17 data points, both Diameter and Launch Year are significant and the fit has a good R^2=89.2% and r^2_{adj}=87.6%.

The explanation is in the databases.

Horak’s database consisted mostly of DoD strategic systems most of which were laboratory experiments that were never deployed. Of the systems which were flown, most were airframe or missile systems.

Our database consists entirely of NASA space telescope missions.

Multi-Variable Models

Starting with Single Variable Model for OTA Cost vs Diameter:

OTA Cost ~ Aperture Diameter$^{1.28}$ (N = 16; r2 = 84%) without JWST

OTA Cost ~ Aperture Diameter$^{1.2}$ (N = 17; r2 = 75%) with 2009 JWST

Perform multi-variable regression to add a second variable.

Select multi-variable model based on:

- Change in Significance of Diameter to Fit
- Significance of Variable #2 to Fit
- Increase in r^2_{adj}
- Decrease in SPE
- Multi-Collinearity

Some variables may increase r^2_{adj} and/or decrease SPE, but they are not significant or their coefficients are not consistent with engineering judgment or they are multi-collinear.

Multi-Variable Models

There are two second variables with best meet all the critieria:

Year of Development, and

Launch Year

Launch Year has the advantage that it is a definite date, but it also has the disadvantage that a launch can be delayed. And, while a launch delay tends to increase the total mission cost, it may or may not increase the OTA cost.

Year of Development yields a slightly better regression, but its exact date is subject to definition. Is it the Start of Phase A or B or C?

TRL has a significant result that yields an improved r^2, but it does not reduce SPE. This is probably because of the relatively few data points.

OTA Cost versus Diameter and V2

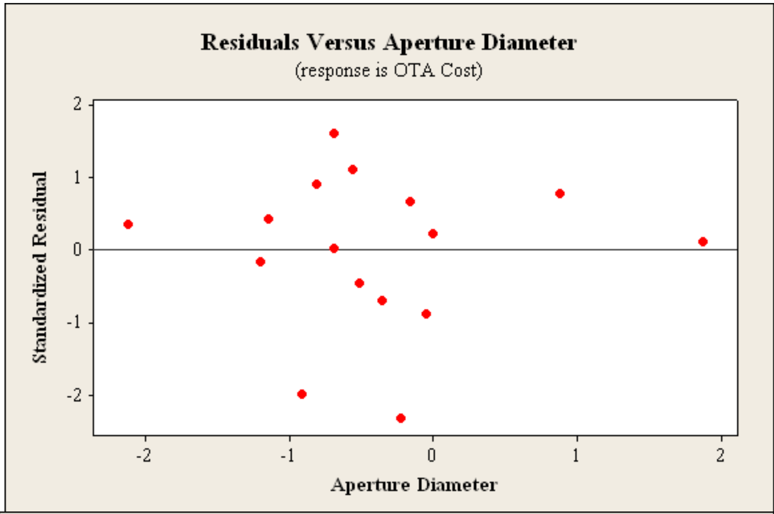
	coef	p																		
Second Variable	PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
Diameter	0.68	0.27	1.05	0.00	-0.02	0.99	1.16	0.01	1.14	0.00	0.76	0.12	1.45	0.00	1.22	0.00	1.19	0.00	1.21	0.00
Second Variable	0.35	0.45	0.26	0.57	0.35	0.45	-0.26	0.18	-0.05	0.45	0.35	0.26	0.35	0.26	-0.04	0.63	-0.10	0.55	-0.01	0.96
Adjusted r2	71%		71%		71%		14%		73%		83%		83%		73%		75%		71%	
SPE	77%		78%		77%		73%		78%		83%		83%		84%		95%		82%	
n	13		13		13		13		16		15		15		17		11		16	
Multicollinearity?	Yes		No		Yes		No		No		Yes		No		No		No		No	

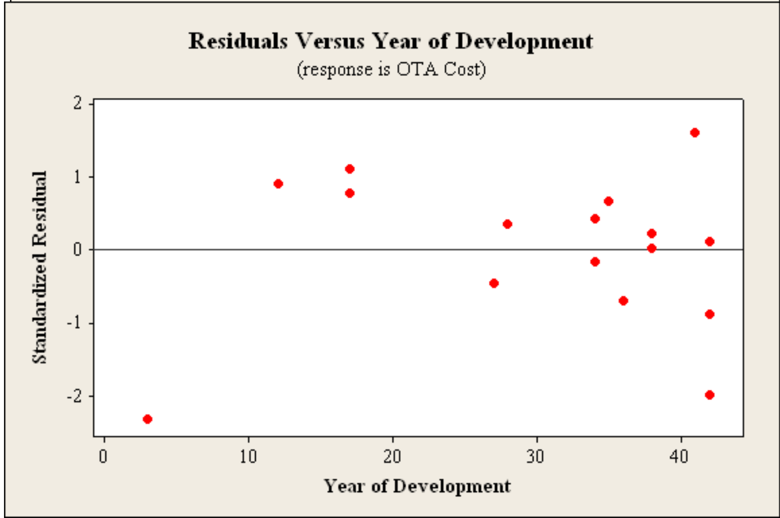
Second Variable	Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Diameter	1.41	0.00	1.40	0.00	1.21	0.00	1.13	0.00	1.31	0.00	1.27	0.00	1.19	0.00	1.20	0.00	1.34	0.00	1.23	0.00
Second Variable	-0.15	0.23	-0.08	0.28	-0.01	0.98	0.00	0.51	-0.09	0.02	-0.04	0.00	0.23	0.60	0.00	0.73	-0.04	0.00	0.02	0.62
Adjusted r2	70%		91%		71%		84%		97%		95%		71%		71%		93%		66%	
SPE	58%		59%		83%		81%		83%		39%		77%		78%		39%		85%	
n	16		12		17		17		8		16		16		16		17		15	
Multicollinearity?	No		No		No		No		No		No		No		No		No		No	

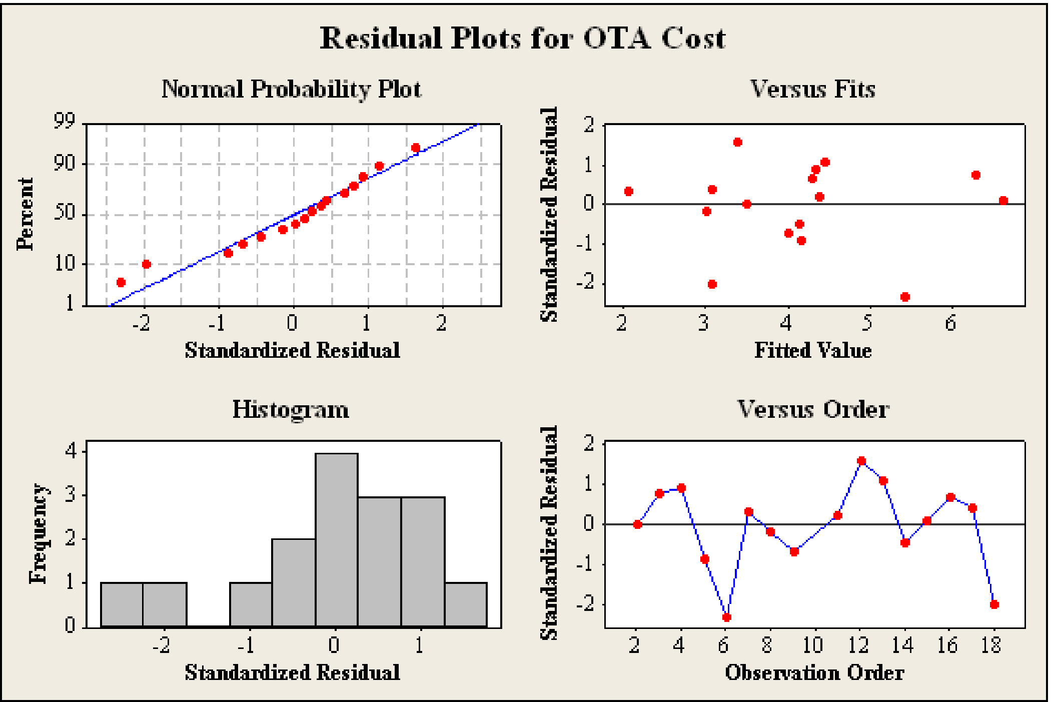
Diameter and Year of Development

$$OTACost = \$332M * Diameter^{1.27} * e^{-0.038*(YoD-1960)}$$

Pearson’s r^2_{adj}=95%
SPE=39%
All coefficients are significant (p-values all <0.01).
No evidence of non-constant variance or non-normality.


Residuals Versus Aperture Diameter
(response is OTA Cost)


Residuals Versus Year of Development
(response is OTA Cost)


Residual Plots for OTA Cost

OTA Cost vs Diameter, YoD and V3

Third Variable	PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit	
	coef	p																
Diameter	1.62	0.01	1.29	0.00	2.29	0.07	1.27	0.00	1.27	0.00	1.45	0.00	1.22	0.00	1.27	0.00	1.34	0.00
YoD (exp)	-0.04	0.01	-0.04	0.01	-0.04	0.01	-0.05	0.01	-0.04	0.00	-0.04	0.00	-0.04	0.00	-0.04	0.00	-0.05	0.01
Third Variable	-0.33	0.34	-0.33	0.30	-0.33	0.34	-0.01	0.96	0.00	0.97	-0.11	0.63	-0.11	0.63	0.01	0.81	-0.01	0.90
Adjusted r2	95%		95%		95%		82%		95%		92%		92%		94%		99%	
SPE	42%		41%		42%		45%		41%		42%		42%		40%		49%	
n	12		12		12		12		16		14		14		16		10	
Multicollinearity?	Yes		No		Yes		No		No		Yes		No		No		No	

Third Variable	Operating Temperature		Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		Development Period		Dev Per (exp)		Orbit	
Diameter	1.27	0.00	1.31	0.00	1.22	0.00	1.22	0.00	1.20	0.00	1.27	0.00	1.29	0.00	1.32	0.00	1.28	0.00
YoD (exp)	-0.04	0.00	-0.04	0.01	-0.03	0.02	-0.04	0.00	-0.04	0.00	-0.01	0.29	-0.04	0.00	-0.04	0.00	-0.04	0.00
Third Variable	0.00	0.98	-0.05	0.59	0.01	0.82	0.10	0.45	0.00	0.38	-0.71	0.09	-0.11	0.72	0.00	0.59	0.02	0.42
Adjusted r2	94%		86%		90%		97%		97%		99%		95%		95%		92%	
SPE	41%		41%		39%		40%		40%		33%		41%		40%		35%	
n	15		15		12		16		16		8		16		16		15	
Multicollinearity?	No		No		No		No		No		No		No		No		No	

Three-variable Models

None of the three-variable models are better than the base model.

While TRL looks promising, more data is needed. Also, adding TRL reduces the significance of YoD.

But what if we add more high and low wavelength telescopes to gain some wavelength diversity?

WMAP,

TDRS-1, TDRS-7,

EUVE,

Chandra and Einstein

OTA Cost vs Diameter, YoD and Spectral Minimum

	coef	p						
	Diam		Diam*spct		Diam*YoD		all 3	
Diameter	0.84	0.00	1.03	0.00	0.78	0.00	1.15	0.00
YoD (exp)	-	-	-	-	-0.03	0.12	-0.03	0.04
Spct Min	-	-	-0.13	0.00	-	-	-0.17	0.00
Adjusted r2	43%		69%		18%		92%	
SPE	126%		88%		97%		76%	
n	23		23		20		20	
Multicollinearity?	N/A		No		No		No	

Interestingly, adding wavelength diversity to the regression yields coefficients similar to the Horak model:

$$T1 \sim Apr^{0.705} \# Elem^{0.473} \lambda^{-0.178} K^{\circ -0.191} e^{-0.033(Yr-80)}$$

Conclusions

From engineering & science perspective, Aperture Diameter is the best parameter for a space telescope cost model.

But, the single variable model only predicts 84% of OTA Cost:

OTA Cost ~ $D^{1.3}$ (N = 16; r2 = 84%; SPE=79%) without JWST

OTA Cost ~ $D^{1.2}$ (N = 17; r2 = 75%; SPE=79%) with 2009 JWST

Two Variable Models provide better estimates

OTA Cost ~ $D^{1.3}\ e^{-0.04(LYr-1960))}$ ***(N = 17, r^2 = 93%; SPE=39%)***

OTA Cost ~ $D^{1.3}\ e^{-0.04(YoD-1960))}$ ***(N = 16, r^2 = 95%; SPE=39%)***

A potential Three Variable Model is:

OTA Cost ~ $D^{1.15}\ \lambda^{-0.17}\ e^{-0.03(YoD-1960))}$ ***(N = 20, r^2 = 92%; SPE = 76%)***

Finally, OTA mass is not a good CER

OTA mass is multi-collinear with diameter, and

more massive telescopes actually cost less to make.