

# A Simplified Model for Detonation Based Pressure-Gain Combustors

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# A Simplified Model for Detonation Based Pressure-Gain Combustors

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## Abstract

A time-dependent model is presented which simulates the essential physics of a detonative or otherwise constant volume, pressure-gain combustor for gas turbine applications. The model utilizes simple, global thermodynamic relations to determine an assumed instantaneous and uniform post-combustion state in one of many envisioned tubes comprising the device. A simple, second order, non-upwinding computational fluid dynamic algorithm is then used to compute the (continuous) flowfield properties during the blowdown and refill stages of the periodic cycle which each tube undergoes. The exhausted flow is averaged to provide mixed total pressure and enthalpy which may be used as a cycle performance metric for benefits analysis. The simplicity of the model allows for nearly instantaneous results when implemented on a personal computer. The results compare favorably with higher resolution numerical codes which are more difficult to configure, and more time consuming to operate.

## Nomenclature

$a^*$	reference speed of sound
$a/f$	air to fuel ratio (by mass)
$ff$	fill fraction, fraction of tube filled with detonable mixture
$h_f$	fuel heating value
$m$	mass
$p$	pressure
$PR$	total pressure ratio of exit to inlet flow
$q_0$	non-dimensional heat addition parameter
$R_g$	real gas constant
$T$	temperature
$TR$	total temperature ratio of exit to inlet flow
$u$	gas velocity
$U$	wheel velocity
$W$	mass averaged work per cycle
$\gamma$	ratio of specific heats
$\eta_t$	non-uniform turbine efficiency
$\rho$	density

### Subscripts

3	pertaining to the compressor exit
4	pertaining to the turbine inlet
$CV$	constant volume
$e$	exit plane
$eff$	effective
init	initial post-combustion state
initial	initial pre-combustion state
$s$	static
$t$	total

### Superscripts

-	mass average
,	deviation from mean

## I. Introduction

Confined volume combustion cycles, implemented in non-positive displacement devices, are currently under investigation for use as pressure-gain combustors in gas turbines (Refs. 1 to 7). They are envisioned as replacements for the conventional, constant pressure combustors (which in reality always effect a total pressure loss). Pressure-gain combustion converts the operational basis of the gas turbine from a Brayton cycle to something approaching an Atkinson cycle which, for the same heat addition and mechanical compression, is thermodynamically superior.

One concept under consideration utilizes detonative combustion in a series of circumferentially arranged tubes, each undergoing a periodic cycle (aka, a multi-tube pulse detonation engine, or PDE). Each tube is filled, or partially filled, with a detonable mixture of fuel and air from the upstream compressor. The filling end is then closed, and detonation is initiated. The detonation propagates supersonically down the tube, followed immediately by a Taylor wave which brings the flow nominally to rest at an elevated pressure and temperature. The entropy produced during this portion of the cycle is theoretically only slightly less than what would be produced from a true (deflagrative) constant volume combustion event. In principle however, a detonation can propagate down the entire tube considerably faster than a deflagration. This may yield higher frequency operation, which in turn can yield greater throughflow for a given combustor size. When the detonation reaches the exhaust end of the tube, a blowdown period commences. During this time the fluid rapidly exhausts from the tube, and the pressure in the tube drops due to expansion. The high velocity exhaust gas, which is at a higher total pressure than the compressor discharge air, is directed into the downstream turbine. Eventually, the pressure at the inlet end of the tube drops to the level of the compressor outlet, and the tube inlet is opened. The tube is refilled with a fresh fuel/air charge and the cycle is completed. In principle, this cycle repeats at high frequency in each of the many tubes comprising the combustor. It is a matter of ongoing debate whether the tubes should be fixed or rotary, as well as whether the gas exhausted during the blowdown period, which is both temporally and spatially non-uniform, should be mixed and diffused prior to entry into a turbine nozzle ring, or sent directly into a turbine rotor. Much debate has also surrounded the question of whether the exhaust end of the tubes should be valved as is the inlet end. Theoretically, higher performance is achievable by doing so; however, considerable complexity is added. In this paper, only configurations that have unvalved (i.e., open) exhaust are considered.

In order to reasonably assess thermodynamic benefits, and to perform preliminary sizing for pressure-gain combustors in a variety of gas turbine engine classes, a performance model is needed. There are numerous approaches borrowed from the PDE community that may be considered for this task. These range from complex single or multi-dimensional, multi-species, reactive computational fluid dynamic (CFD) codes, to the relatively simple, algebraic, state-based models (Refs. 7 to 14). For preliminary benefits analyses, many of the CFD approaches are prohibitively costly in terms of configuration (gridding, boundary conditions, event timing, chemistry models, etc.), computational resources, and turnaround time. On the other hand, the simplest approaches do not always capture the essential physics required to provide realistic results. In particular, the algebraic models (Ref. 9) fail to account for the variation in the exhausted flow, which in turn affects the calculated availability of the gas supplied to the turbine. Even the so-called zero-dimensional (a.k.a. 0-D, or lumped volume) approaches (Refs. 12 and 13), which are time-dependent, can be unreliable for the analysis of many gas turbine pressure-gain combustors. In this application, the exhaust static pressure of the combustion device may be quite close to the inlet total pressure. In other words, there is no ram pressure to force flow through it, and it must therefore be nearly self-aspirating. Modeling a self-aspirated cycle requires a momentum equation, which the 0-D approaches generally lack.

In this paper, a hybrid approach is presented for performance modeling of pressure-gain combustors. It essentially combines a non-reacting, one-dimensional (1-D) CFD approach for the continuous blowdown and refill portions of the cycle with an algebraic approach for the detonative (or otherwise constant volume) event. The result is a cycle simulation code that runs in fractions of a second on a personal computer, yet yields realistic estimates of performance, size, and operational frequency.

Difficulties associated with algebraic and lumped volume models will first be presented in order to motivate the present approach. This will be accomplished using an example gas turbine engine application for which a pressure-gain combustor is considered. The pressure-gain cycle is initially computed using a high-fidelity, reactive CFD code (Ref. 10), which for this work serves as the most accurate solution. The output is then compared to algebraic and lumped volume models. It is shown that neither model correctly captures details of the exhaust flow, the non-uniformity of which is proved to be essential in estimating performance. The hybrid model just introduced will then be described in detail. Results from it will also be presented and compared to those of the higher fidelity, reactive code. The assumption of a single, calorically perfect (i.e., fixed properties, fixed  $\gamma$ ) gas will be used throughout in order to simplify equations and comparisons. Though this is not a realistic assumption for a practical detonative

combustor environment, using it does not impact the concepts, or methodologies presented. The fixed value of  $\gamma = 1.3$  used herein represents an average value (between the maximum and minimum that would actually occur) chosen according to the methodology described in Reference 13.

## II. Application Example

Consider the notional combustor shown in Figure 1 at an operating point defined by the parameters and conditions of Table 1. These conditions are representative for sea level static operation of a modern regional turbofan engine with overall pressure ratio (OPR) 24. For any periodic cycle undergone by each combustor tube (e.g., detonative, Lenoir, etc.), the mass averaged total temperature of the exit flow may be obtained from the conservation of mass and energy as follows.

$$\frac{\bar{T}_{t4}}{\bar{T}_{t3}} = 1 + (\gamma - 1)q_0 \quad (1)$$

The non-dimensional heat addition parameter,  $q_0$ , is defined as

$$q_0 = \frac{h_f}{\gamma R_g \bar{T}_{t3} \left( \frac{a}{f} + 1 \right)} \quad (2)$$

Equation (1) must hold regardless of the cycle details or analysis method used. It results directly from the conservation of mass and energy applied to a control volume. The critical issue for gas turbine applications lies in assessing the associated total pressure,  $p_{t4}$ . For steady systems this is a straightforward process. It is essentially the inlet total pressure less some factor accounting for mixing losses, and those associated with heating at finite Mach number. For unsteady pressure-gain systems, assessing the exhaust total pressure is non-trivial, and therefore requires separate discussion.

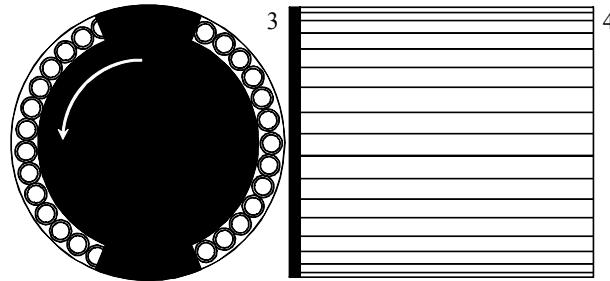


Figure 1.—Notional detonative combustor geometry.

TABLE 1.—COMBUSTOR PARAMETERS

$T_{t3}$ , R	1426
$p_{t3}$ , psia	352.8
$a/f$	42.2
$h_f$ , ft-lb <sub>f</sub> /lb <sub>m</sub>	$14.3 \times 10^6$
$\gamma$	1.3
$R_g$ , ft-lb <sub>f</sub> /lb <sub>m</sub> -R	53.57
$p_{s4}$ , psia	352.8
$a^*$ , ft/s	1787

### A. The Effect of Non-Uniformity

For ideally expanded, but fundamentally unsteady propulsion systems, the mass averaged kinetic energy emitted per period is always greater than the square of the gross specific impulse divided by two (Ref. 15). In steady systems, these two quantities are equal. Defining the instantaneous exit velocity as the sum of a mass averaged and a fluctuating component,  $u_e = \bar{u}_e + u'_e$ , this statement may be formally written as

$$\frac{1}{2m_e} \int_0^{m_e} u_e^2 dm_e = \frac{\bar{u}_e^2}{2} + \frac{1}{2m_e} \int_0^{m_e} u'_e^2 dm_e \quad (3)$$

or

$$\frac{\bar{u}_e^2}{2} = \frac{\bar{u}_e^2}{2} + \frac{\bar{u}'_e^2}{2} \quad (4)$$

The left hand side of Equation (4) is essentially the kinetic energy. The first term on the right hand side is the square of the specific impulse divided by two. The second term on the right hand side, the mass averaged kinetic energy of the fluctuating component, is only zero when the flow is steady. For all unsteady (and periodic) exhaust flows it is positive.

Besides applying to thrust production, Equation (4) is relevant for pressure-gain combustion systems in the context of work extraction by turbines. Consider an idealized, Pelton-style impulse turbine fed by a non-steady combustor tube, as shown schematically in Figure 2. The mass averaged specific work extracted (done by the gas to the wheel) per period is

$$\bar{W} = 2U(\bar{u}_e - U) \quad (5)$$

This has a maximum if  $U = \frac{\bar{u}_e}{2}$ , at which point  $\bar{W}_{\max} = \frac{\bar{u}_e^2}{2}$ . From Equation (4) therefore, it is evident that the maximum possible work extracted from a non-uniform flow is always less than the available kinetic energy. This result may be imposed on the system as either a mandatory efficiency loss applied to an unsteady turbine (i.e.,  $\eta_t = \frac{\bar{u}_e^2}{u_e^2}$ ) or as an availability loss applied to the combustor exhaust flow. In this paper, the latter is adopted. In

particular, the exhaust total pressure is assigned to be that which, when ideally expanded to the exhaust static pressure at the exhaust total temperature, yields the mass averaged specific thrust (not the kinetic energy (Ref. 9)), viz.

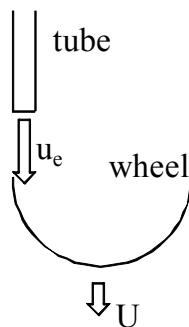


Figure 2.—Idealized Pelton type turbine schematic.

$$p_{t4} = \frac{p_{s4}}{\left[ 1 - \left( \frac{\gamma-1}{2} \right) \frac{(\bar{u}_e/a^*)^2}{(\bar{T}_{t4}/\bar{T}_{t3})} \right]^{\frac{\gamma}{\gamma-1}}} \quad (6)$$

Here, the velocity has been normalized by a reference speed of sound,  $a^*$ , calculated from the total temperature at the inlet (station 3).

Whether a turbine loss or Equation (6) is used, it is clear that an assessment of the mass averaged velocity is needed. This, in turn, implies the need for representative distributions of fluid states (i.e., pressure, density, velocity) in the exit plane of the combustor tubes. The mass averaged velocity is, after all, an integral evaluated over one period.

Figure 3 shows the computed, normalized exit velocity distribution from one tube of the Figure 1 notional combustor, subjected to the Table 1 parameters, and undergoing a detonative cycle. The results were obtained using a high resolution, reacting, computational fluid dynamic (CFD) code (Ref. 10). It is plotted as a function of the fraction of the initial mass ejected rather than as a function of time.

For reference, a complete wave diagram of the cycle is shown in Figure 4. Here, color contours of normalized pressure, temperature, velocity, and reactant fraction in the tube, over one period are plotted. The horizontal axis of each contour represents distance along the tube. The vertical axis is non-dimensional time. Next to each contour are numbers which represent the minimum and maximum value of the contoured variable plotted in the x-t space. These may be used to scale the spectrum shown. The black rectangle in the lower left of each contour represents temporal location of a wall boundary condition simulating a closed valve.

Returning to Figure 3, two velocity distributions are shown: the actual exit velocity  $u_e$ , and the effective exit velocity  $u_{e\_eff}$ . The latter is defined as

$$u_{e\_eff} = u_e + \frac{(p_e - p_{s4})}{\rho_e u_e} \quad (7)$$

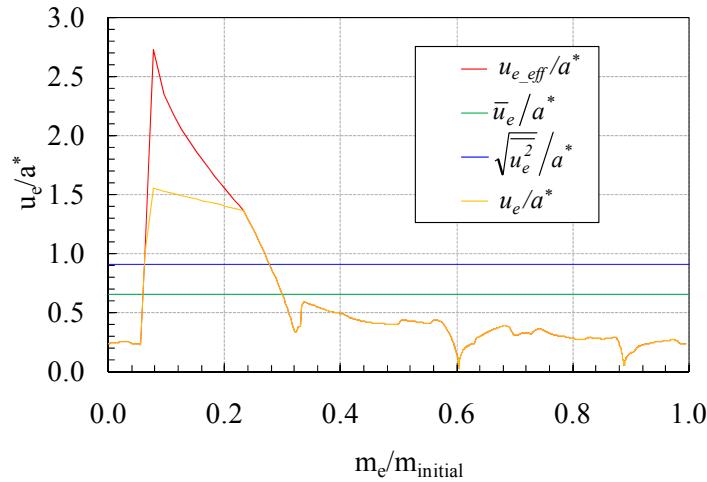


Figure 3.—CFD computed, normalized exit velocity distribution as a function of normalized ejected mass for a detonative cycle.

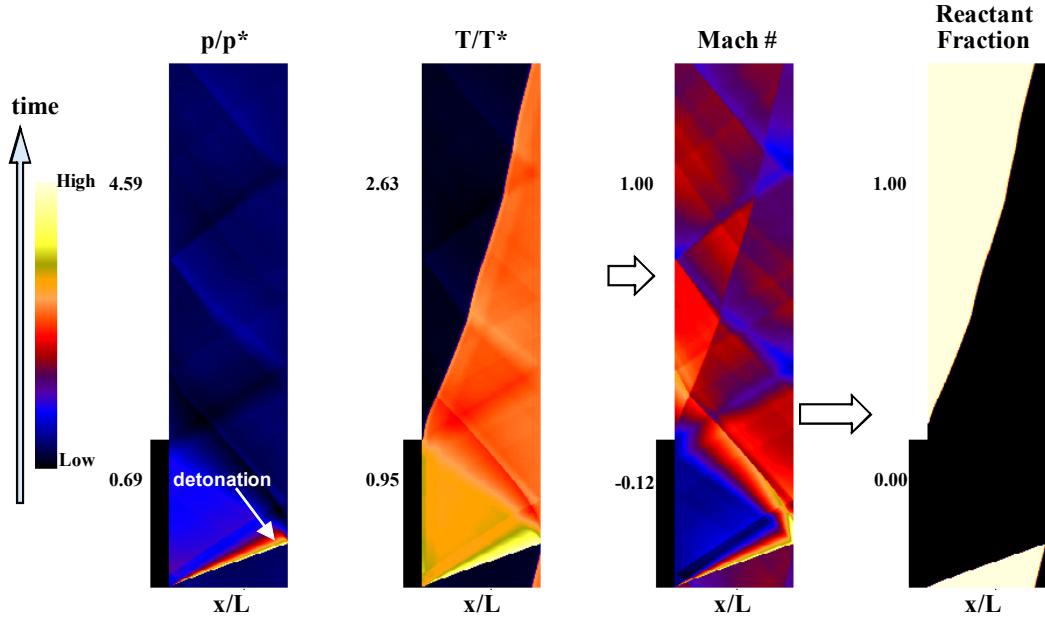


Figure 4.—Contours of normalized pressure, temperature, Mach number, and reactant fraction in a tube over the course of one computed detonative cycle.

The effective velocity accounts for the contribution to momentum from pressure forces when the flow is over or under expanded in the exhaust plane. In the present cycle, this situation arises immediately after the detonation reaches the exhaust end of the tube (see Fig. 4). Also shown in Figure 3 are  $\bar{u}_e$  (essentially the area under the  $u_{e\_eff}$  line), and  $\sqrt{\bar{u}_e^2}$ . Using Equations (1), (2), (6), and the computed value of  $\bar{u}_{e\_eff}/a^* = 0.665$ , it is found that  $TR \equiv \frac{\bar{T}_{t4}}{\bar{T}_{t3}} = 2.00$ , and  $PR \equiv \frac{P_{t4}}{P_{t3}} = 1.16$ <sup>1</sup>

CFD based calculations of  $\bar{u}_e$  are arguably the most accurate for obtaining pressure-gain combustor performance estimates such as those just presented. As mentioned however, they are cumbersome and time consuming to use, and may be considered impractical for performance assessments requiring many operating points or engine configurations. On the other hand, the much simpler algebraic methods cannot convey any information about  $\bar{u}_e$  since they are neither time nor mass dependent. Interestingly, with some simplifying assumptions about the nature of entropy production (Refs. 9 and 15), they can be used to find  $\bar{u}_e^2$ . However, Equation (4) and a glance at Figure 3 will show that if this is used in place of  $\bar{u}_e^2$  in Equation (6), it will result in a value of  $PR = 1.35$  for the example operating point and cycle under consideration here. This unreasonably high pressure ratio, combined with a lack of available information regarding flow rates (i.e., sizing information) or cycle periods makes the algebraic methods impractical for performance assessment.

## B. The Lumped Volume Approach

Lumped volume models represent a time-dependent approach; however, there is no spatial dependence (Ref. 10). The PDE tube is considered as a single vessel (lump) which, at any moment in the cycle to be described, can be represented by a single state. The exhaust nozzle is modeled in quasi-steady fashion meaning that, at each moment in time, the exit conditions can be found from the specified nozzle geometry, the exit static pressure, and the upstream PDE vessel state. They are relatively simple to construct, produce rapid results (compared to CFD), and can be quite accurate for certain configurations.

<sup>1</sup>For the detonative cycle,  $TR = 1.98$ . This is slightly below the value of 2.00 found with Equations (1) and (2) and Table 1. The reduction arises from the necessity of a small amount of unfueled buffer gas which must be inserted between the hot exhaust gas in the tube and the new detonable charge that is entering. The impact on PR is negligible.

In the model process, the vessel is first filled with a reactive mixture of known stoichiometry at the inlet state. A constant volume combustion process then proceeds instantaneously, leaving the vessel fluid in a uniform high pressure, high temperature state to begin the blowdown process. Blowdown is a time-dependent, isentropic expansion process. The exit plane quantities are determined by the specified nozzle geometry, the ambient static pressure and the instantaneous tube pressure and temperature. Blowdown continues until the tube pressure matches the inlet total pressure, after which the fill portion of the model cycle commences. During the fill portion of cycle, a new detonable mixture enters through the inlet while the remaining post-detonative gas exits through the nozzle. The fresh charge and hot gas are assumed to stay separated by an interface, across which the pressure is constant, but the density varies. Fill proceeds until the total mass which has flowed from the exit plane matches the mass originally in the tube when the cycle began.

Lumped volume models may be appropriate for pressure-gain combustor configurations where  $p_{s4}$  is substantially below  $p_{t3}$  and where the notional combustor contains a throat (constriction) at the exhaust. For other configurations, such as the one considered in this paper, the lumped volume approach is problematic. Without spatial resolution, or a momentum equation, critical gasdynamic phenomena, and features related to fluidic inertia are lost. The results are often poor predictions of cycle periods, inaccurate distributions of  $u_e$ , and therefore improper measures of  $p_{t4}$ .

Figure 5 shows the distribution of  $u_e$  for the same notional combustor tube, as predicted by a lumped volume simulation. Since the exit static pressure,  $p_{s4}$  is assumed to be the same as the inlet total pressure,  $p_{t3}$ , the lumped volume method will never fully exhaust all of the flow (there is no pressure to drive the flow out and it has no inertia). In order to avoid this,  $p_{s4}/p_{t3}$  has been set to 0.9995. The resulting cycle period is approximately 15 times longer than that predicted by the CFD based simulation. The distribution of  $u_e$  in Figure 5 looks quite different than that of Figure 3. The mass averaged value, at  $\bar{u}_e/a^* = 0.544$ , is also different (and lower). This results in a lower available total pressure ratio of  $PR \equiv \frac{p_{t4}}{p_{t3}} = 1.10$ .

The disparity between the lumped volume and CFD performance predictions can be attributed to two factors. First and foremost, the differences in  $u_{e\_eff}$  distribution alter  $\overline{u_e'^2}$  from Equation (4). Second, the available kinetic energy from the constant volume combustion assumption of the lumped volume model is less (entropy production is greater) than from detonation based combustion used in the reactive CFD model.

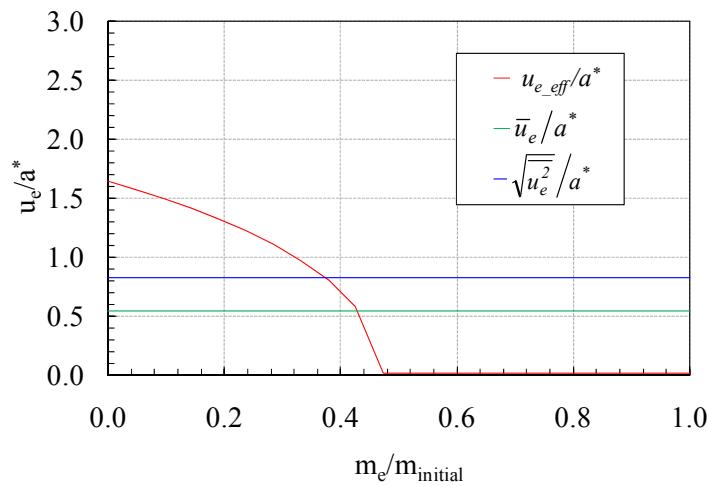


Figure 5.—Lumped volume computed exit velocity distribution as a function of normalized ejected mass for a constant volume cycle.

### III. The Hybrid Approach

In the present model, a sort of hybrid approach is taken to simulate the complete cycle. The tube is assumed to be initially filled with a stationary, uniform mixture of air and fuel at the inlet pressure and temperature. Like the lumped volume method, it is assumed to undergo an instantaneous, uniform, zero-velocity, constant volume reaction that results in a peak pressure and temperature as follows.

$$\frac{\bar{T}_{CV}}{\bar{T}_{t3}} = 1 + \gamma(\gamma - 1)q_0 \quad (7)$$

$$\frac{p_{CV}}{p_{t3}} = \frac{\bar{T}_{CV}}{\bar{T}_{t3}} \quad (8)$$

This state is then used as an initial condition for a quasi-one-dimensional (or in the case at hand, strictly one dimensional) Euler flow solver. Because the reaction has already taken place, the solver need not contain species equations or a reaction mechanism. Furthermore, with reactions removed from the computing domain, the potential for discontinuities (i.e., shocks) has largely been removed as well. With a continuous flowfield, the need for a computationally intense, high resolution, monotonicity preserving numerical scheme vanishes. As such, any number of relatively simple (though still second order in time and space) schemes can be used, and the grid spacing may be made quite large. Both the Euler equations and the numerical schemes for their solution are well documented in the literature (e.g., Ref. 16). As such, no details concerning them will be presented in this paper. For this work, MacCormack's integration method (Ref. 17) was used, with a computational grid comprising 50 cells. For reference, the grid for the detonative CFD simulation of Figure 3 contained 200 cells. Because of the Courant-Freidrich-Lowy (CFL) convergence criteria, fewer grid points also allows for larger time steps. The net result is a very rapid scheme for computing the gas state in the tube exit plane for the blowdown and fill portions of the cycle.

Starting from the post-combustion CV state, the left end of the tube has a solid wall boundary condition imposed, while the right end has a constant pressure boundary. Descriptions of these and other boundary conditions can readily be found in numerous CFD references, as well as in Reference 18. These conditions are imposed until the pressure at the left end of the tube matches  $p_{t3}$ . This point in time constitutes the beginning of the fill period and the left wall boundary condition is replaced with an open inflow condition. The temperature supplied for this boundary is  $T_{t3}$ . The simulation continues integrating until the total mass which has flowed from the exit plane is equal to the mass originally in the tube when the integration began.

Figure 6 shows contours of normalized pressure, temperature and Mach number for the example cycle using the present hybrid method. This should be qualitatively compared to the detonative cycle of Figure 4. It can be seen that several of the same gasdynamic features exist (with the obvious exception of the detonation itself). These include a large, left-running expansion fan at the commencement of the blowdown period, and a largely steady, rightward flow during the fill period, despite the existence of a weak adverse pressure gradient. Not surprisingly, the predicted period of the hybrid cycle is within 25 percent of the computed period of the detonative cycle. If the transit time of the detonation is accounted for, the predictions for total cycle period are even closer. Such accuracy is acceptable when preliminary performance and sizing estimates are being investigated. It represents substantial improvement over that obtained with either the algebraic or lumped volume modeling methods.

The computed distribution of exhaust for the notional combustor using the hybrid method is shown in Figure 7. For comparison, the detonative velocity distribution of Figure 3 is also shown. Although the hybrid velocity profile misses the detonation induced spike from  $0.06 < m_e/m_{\text{initial}} < 0.28$ , it remains at a higher level from  $0.28 < m_e/m_{\text{initial}} < 0.64$ . The two profiles are subsequently quite comparable during the fill phase which extends from  $0.64 < m_e/m_{\text{initial}} < 1.00$ . Most importantly however, the hybrid method value of  $\bar{u}_e/a^* = 0.710$  is quite close to the value of 0.665 obtained with the detonative CFD simulation. Consequently, the pressure ratio of  $PR = 1.18$  calculated from Equation (6) and  $\bar{u}_e$  from the hybrid method also nearly matches that obtained from the detonative CFD. Evidently then, the much simpler hybrid simulation method outlined here successfully captures the essential physics required for preliminary performance and combustor sizing at the example conditions of Table 1.

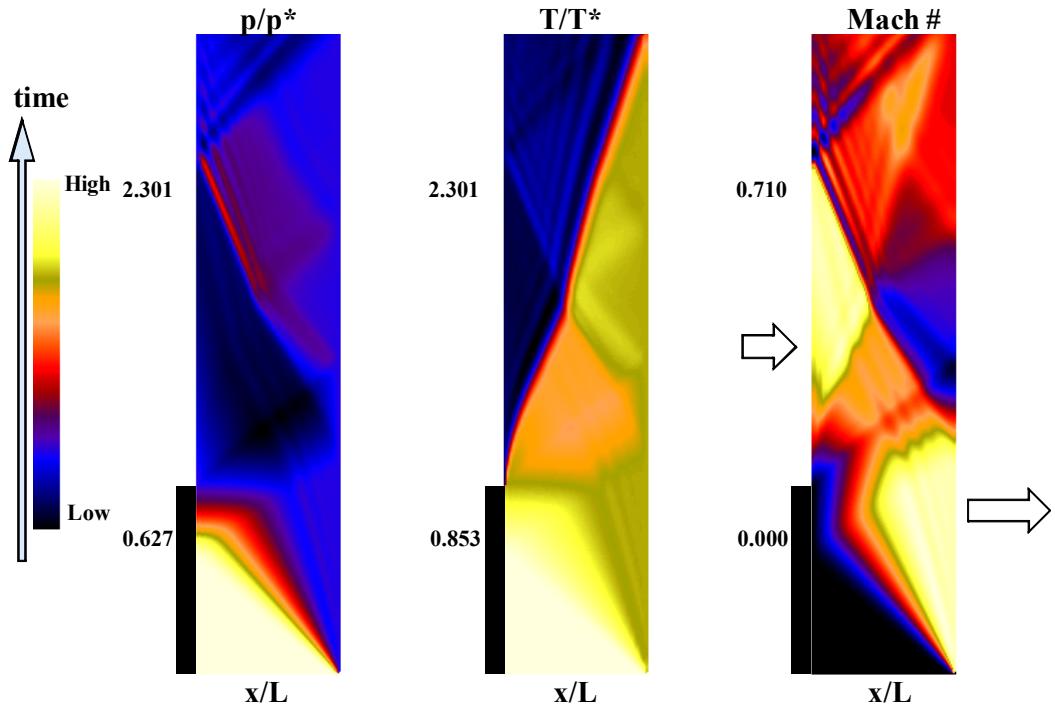


Figure 6.—Contour plots of normalized pressure, temperature, and Mach number, for the computed cycle using the hybrid method.

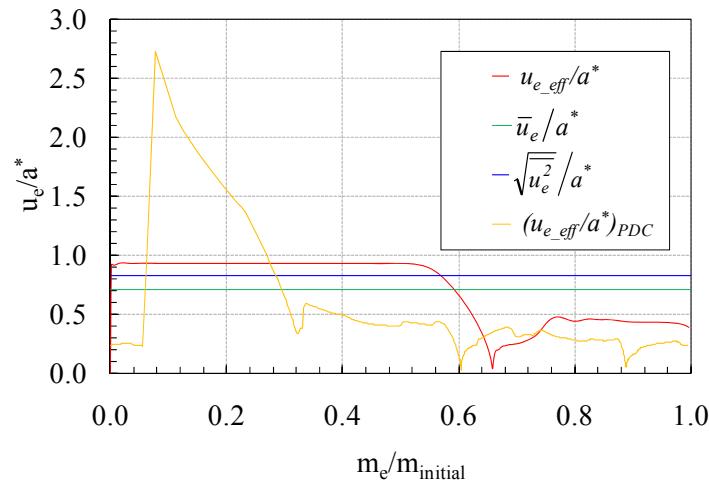


Figure 7.—Computed exit velocity distribution as a function of normalized ejected mass for the example cycle using the hybrid method.

#### IV. Additional Results

The preceding discussion has demonstrated that the hybrid simulation method reasonably matches a higher fidelity detonative CFD simulation at a single operating point. In this section, attention is turned to additional operating points. In particular, the same notional combustor configuration and inlet conditions are maintained, but the air/fuel ratio is varied. By Equation (2), a change in  $a/f$  alters  $TR$ , which alters  $\bar{u}_e$ , and thus  $PR$ . The computed results of  $a/f$  variations are shown in Figure 8. Here  $PR$  according to Equation (6) is plotted as a function of  $TR$  from Equation (2) using the various modeling methods described. The two lines corresponding to the algebraic methods use  $\bar{u}_e^2$  instead of  $\bar{u}_e^2$  in Equation (6), since the latter cannot be computed. The difference between the two algebraic lines is intended to illustrate the impact on available kinetic energy that arises from assuming constant volume versus true detonative combustion. It simply shows graphically that detonation generates less entropy. In either case, the predicted values of  $PR$  are well above those from the detonative CFD, which is considered the benchmark result for the purposes of this paper. As stated earlier, these results indicate that algebraic methods are not appropriate for performance analyses of pressure gain combustors.

Also shown in Figure 8 are results using the present hybrid, and the lumped volume model approaches. The former tends to match the detonative CFD at low  $TR$ , while over-predicting performance as  $TR$  increases. The lumped volume method under-predicts throughout the  $TR$  range examined (which spans the  $a/f$  range  $15.7 < a/f < 78.5$ ). This trend, combined with generally large over-predictions of cycle period, suggest that the lumped volume technique is also inappropriate for combustor configurations of the type described here.

##### A. Fill Fraction Variations

There is a second conceptual technique by which the overall  $a/f$  and related  $TR$  may be varied. The combustible mixture entering the tube is held at a low  $a/f$  (near stoichiometric), but the volume fraction of the tube actually filled with a combustible mixture is less than one. The remainder of the gas filling the tube is unfueled air. Such a scenario is often considered when fully filling the tube with a lean mixture would result in poor detonability or a generally slow combustion process. The post-combustion flow exiting the tube is thermally stratified, but the mass averaged total temperature still obeys Equation (2), provided all of the entering fuel and air are used in determining  $a/f$ . Examining the effects of this form of overall  $a/f$  variation is straightforward (though time consuming) using the detonative CFD code described above. It is also possible to account for it using in the present hybrid model. The key to doing so lies in choosing an appropriate post-reactive state with which to initiate the numerical blowdown process. There are several options available which will conserve mass and energy over the entire cycle. In this work,

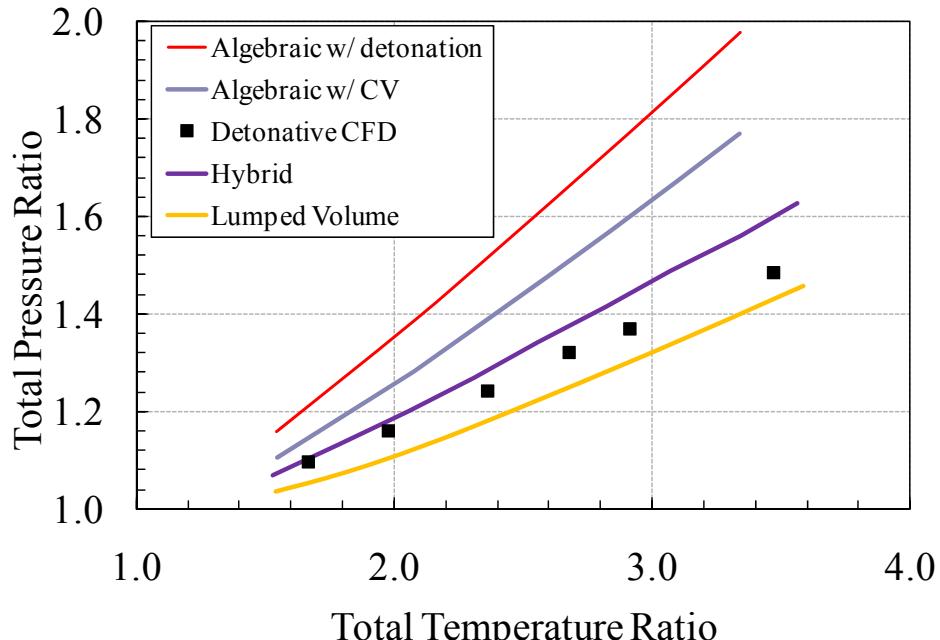


Figure 8.—Total pressure ratio as a function of total temperature ratio computed using various methods.  $TR$  is varied by directly varying  $a/f$ .

the state which provides the best match to the detonative CFD simulation is as follows. The density is held at the same value as the inlet (i.e., whatever processes occur during the instantaneous “combustion” event do so at constant overall volume). The pressure is uniform and chosen such that the total entropy generated is the same as what would be generated by constant volume combustion of only the reactant filled tube fraction.

$$\frac{p_{\text{init}}}{p_{t3}} = \left( ff \left[ \left( \frac{p_{CV}}{p_{t3}} \right)^{\frac{1}{\gamma}} - 1 \right] + 1 \right)^{\gamma} \quad (9)$$

Here, the term  $ff$  is the fraction of the tube which is filled with the air and fuel mixture. The total internal energy of the system is also preserved, which, unlike Equations (7) and (8), requires the imposition of an initial velocity. This is written as follows.

$$\frac{u_{\text{init}}}{a^*} = \sqrt{\left[ ff \left( \frac{T_{CV}}{T_{t3}} - 1 \right) + 1 - \frac{T_{\text{init}}}{T_{t3}} \right] \frac{2}{\gamma(\gamma-1)}} \quad (10)$$

Clearly, the imposition of a uniform post-combustion state on the hybrid method will not result in the appropriate thermal stratification of the exit flow that is observed in true partial fueling scenarios. Nevertheless, the computed mass averaged velocity appears to match the detonative CFD quite well. This is shown in Figure 9 where  $PR$  is plotted as a function  $TR$  for the two methods. The variation in  $TR$  for this figure represents a range in fill fraction from  $0.2 < ff < 1.0$ .

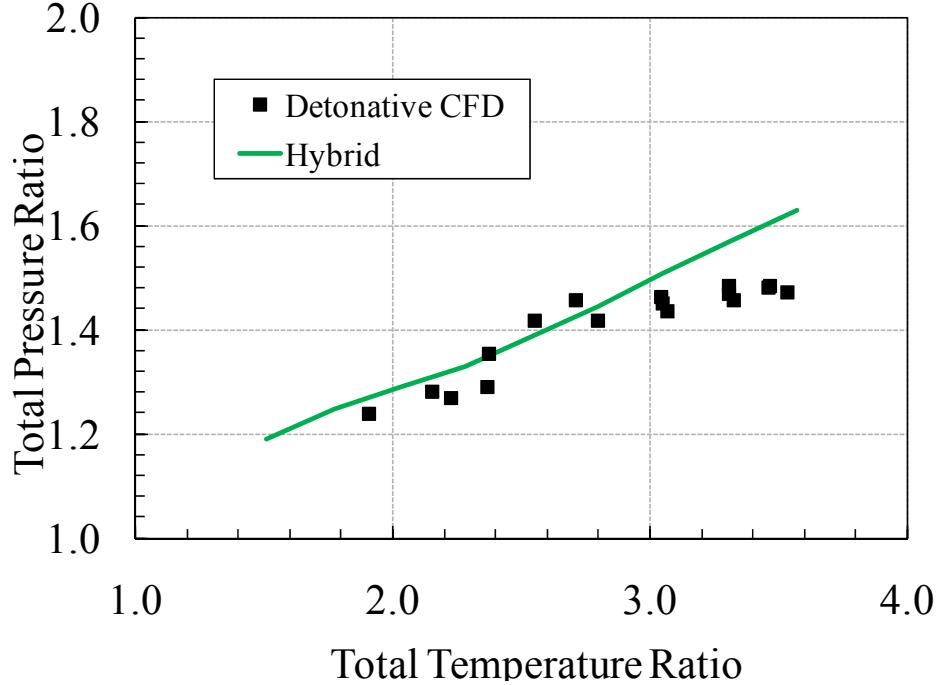


Figure 9.—Total pressure ratio as a function of total temperature ratio computed using detonative CFD and hybrid simulation methods.  $TR$  is varied by varying fill fraction.

## V. Discussion

The hybrid pressure-gain combustor model described in this paper reasonably matches the performance predictions and cycle times of a higher fidelity, detonative, CFD model over a range of operating conditions. The results are rapidly generated (fractions of a second, versus minutes), and require less input data and progress monitoring than detonative CFD. These features make the hybrid approach well suited for insertion as a pressure-gain module in a gas-turbine cycle deck. Furthermore, the fact that the hybrid approach does use a form of CFD suggests that real world loss models are straightforward to apply. The model presented is ideal in the sense that the only entropy generated comes from combustion and non-uniformity. However, many unavoidable losses, (heat transfer, viscous, leakage, etc.) are relatively easy to include as source terms in the governing equations (Refs. 19 and 20). This can enhance the real-world accuracy of the simulation without appreciably altering the cycle computing time.

Though the present model is strictly one-dimensional, modification to compute quasi-one dimensional flows is a trivial exercise. This would extend the model capability to include combustors with exhaust nozzles.

## VI. Conclusion

An efficient model for computing the performance of detonative pressure gain combustor configurations has been described. The approach focuses on correctly capturing the exhaust velocity distribution, while conserving overall mass and energy flows. Using a hybrid combination of algebraic combustion models and simple CFD algorithms appropriate for smooth, non-reacting flows, the new model can compute closed cycle combustor performance in fractions of a second on a typical personal computer. The results, including flow rates, cycle times, and pressure ratios, compare favorably with higher fidelity, reacting CFD simulations. The code is appropriate, and intended, for use in cycle decks so that pressure-gain benefits assessments may be performed on a variety of gas turbine platforms.

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