Dynamic Analysis of Sounding Rocket Pneumatic System

Revision (-)

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Northrop Grumman Corporation

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Outline

Abstract

The recent fusion of decades of advancements in mathematical models, numerical algorithms and curve fitting techniques marked the beginning of a new era in the science of simulation. It is becoming indispensable to the study of rockets and aerospace analysis. In pneumatic system, which is the main focus of this paper, particular emphasis will be placed on the efforts of compressible flow in Attitude Control System of sounding rocket.

Key words: Sounding rocket, mathematical model of pneumatic system, pressure PDE, Euler finite difference, Cubic Bézier, MatLab

Introduction

The development of an Attitude Control System (ACS) is very complex due to the nonlinear nature of the expressions of the system. Pneumatic analysis is even more complicated by the fact that the operating fluid is extremely compressible. Most problems which occur with pneumatic applications become apparent in the dynamic operation. To analyze the system during dynamic operation it is necessary to develop component and system models. This paper discusses the modeling approach and reveals the principles upon which it is based. The technique unifies the various disciplines to effectively design and also modify existing design. It formulates the governing dynamic equations based upon the topological information presented by the schematic. The integrity of the output is cross validated by comparing to empirical data and real gas properties extracted from NIST (National Institute of Standards and Technology).

Pneumatic Schematic of Sounding Rocket

The pneumatic schematic consists of a high pressure reservoir, a manifold, pitch and roll blocks. It is divided into three segments, the rationale that schematic is divided into three is because of the provided experimental data at the end of each segment.

- Segment 1 (form tank to regulator)
- Segment 2 (from regulator transducer to pitch block)
- Segment 3 (from regulator transducer to roll block)

Mathematical Model of the Pneumatic System

Introduction

In this analysis, a descriptive model of the system is built as a hypothesis of how the system could operate, and to estimate how the design variables could affect the system.

The mathematical model describes a flow system by a set of fluid dynamics variables and a set of governing equations. Applied equations establish relationships between the flow variables. The variables represent fluid properties of the system. The equations are derived based on fundamental physical principles.

Roadmap for Deriving Nonlinear ODE for Pneumatic System of Sounding Rocket:

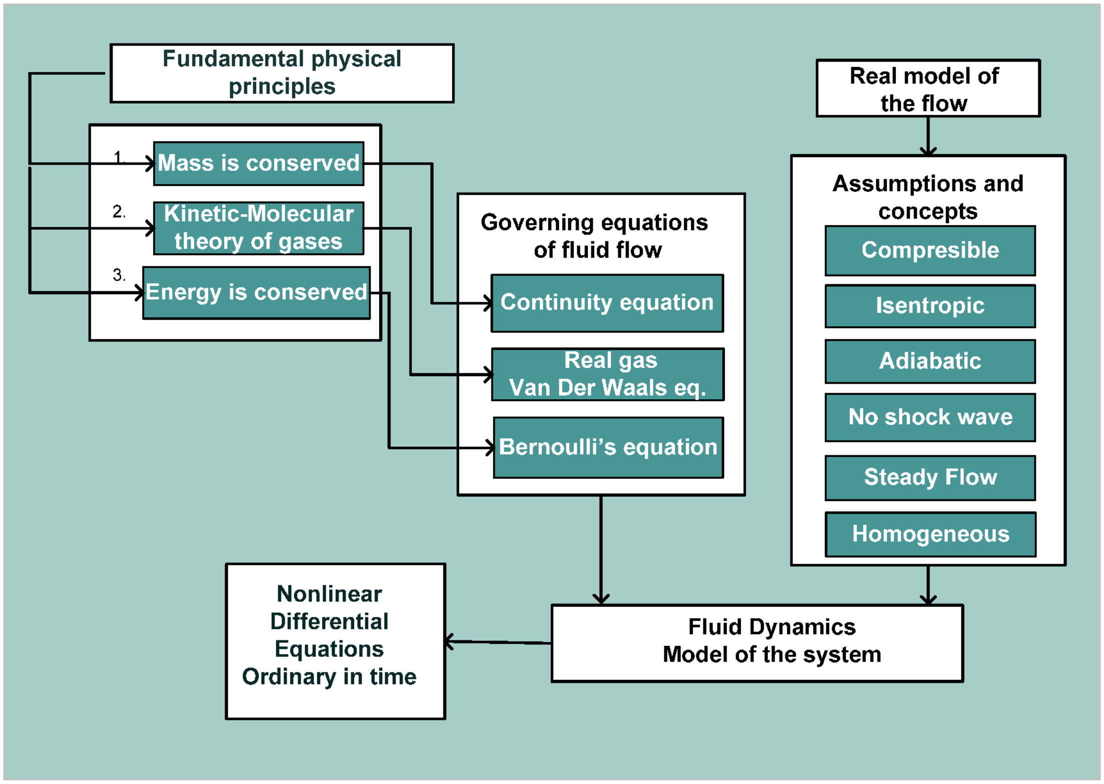


Fig 4.1

Modeling technique

Mathematical model of pneumatic system is subcategorized into blocks with inputs and outputs. Each block represents a pneumatic component in the assembly. A *posteriori* pressure behavior is provided from experimental data. Since there is no priori data available for critical blocks, the posteriori data is utilized to simulate the behavior. Therefore the model consists of a set of inputs, applied governing equations and a set of outputs.

Gas flow through the system (assumptions and concepts)

The analysis of gas flow in the system involves a number of assumption and concepts:

- The density of the fluid changes with respect to pressure ∵ flow is considered to be a compressible flow.

- The gas expands isentropically (i.e., at constant entropy) ∵ the flow is reversible (frictionless and no dissipative losses), and adiabatic (changes in temperature occur due to changes in pressure of a gas while not adding or subtracting any heat.)
- The gas is assumed to be a real gas (Van Der Waals equation) and homogeneous.
- The system of flow through the tubes could be in a steady state because there is a constant flow of fluid. Conversely, the tank which is being drained or filled with fluid would be a system in transient state, because the volume of fluid contained in it changes with time.

Symbols

Symbol	Definition
V	Tank Volume
U_t	Velocity of flow gets in
U_r	Velocity of flow when gets out (downstream)
U_i	Velocity of flow in component
A_t	Area flow gets in
A_r	Area flow gets out
L_i	Length of the tube
D_i	Diameter of the tube
Z	Elevation
ϵ	Surface roughness of the tube
K_i	Loss coefficient
f_i	Friction loss
ρ	Density of the fluid inside tank
$\dot{m}$	Mass flow rate
G	Gravity
R_s	Gas constant
T	Absolute temperature
P_t	Tank Pressure
P_r	Regulator Pressure
N	Number of moles
A	Measure of the attraction between particles
B	The volume excluded by a mole of particles
γ	Specific heat dimensionless ratio
Y_i	Expansion factor

Table 3.1

Governing equations:

In this section:

- The fundamental physical principles are written down
- Then applied to the suitable model of the flow.
- Obtain equations which represent the behavior of the flow.

1. Continuity equation

Consider the flow model i.e. (segment1 from tank to regulator), namely, a control volume of arbitrary shape and a finite size. Conservation of mass states:

> *"Net mass flow out of control volume through surface is equal the time rate of decrease of mass inside control volume."*

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0$$

Assume the gas is homogeneous ∵

$$\dot{m} = \frac{\partial}{\partial t}(\rho V)$$

Given an area *A*, and an incompressible fluid flowing through it with uniform velocity *U* with an angle θ away from the perpendicular direction to *A*, the flow rate is:

$$Q = A.U.\cos\theta$$

In the special case where the flow is perpendicular to the area *A*, that is, θ = 0, the volumetric flow rate is:

$$Q = AU = A_t U_t = A_r U_r \rightarrow U_t = \left(\frac{A_r}{A_t}\right) U_r \qquad \textit{eq. (a)}$$

Flow expands isentropically, the flow rate is derived for incompressible flow then it is modified by introducing the expansion factor Y_i to account for the compressibility of gases. ∵

$$\dot{m} = Y_i.\rho.\dot{V} = Y_i.\rho.Q \qquad \textit{eq. (b)}$$

From eq.(a) and eq.(b) ∵ $\dot{m}_r = Y_i.\rho.A_r.\overline{U}_r$ *eq. (c)*

The expansion factor Y_i , which allows for the change in the density of an ideal gas as it expands isentropically is given by: (Ref: Perry's Chemical Engineers' Handbook 7th edition)

$$Y_i = \sqrt{\left(\frac{P_i}{P_o}\right)^{\frac{2}{k}} \frac{k}{k-1} \frac{1-\left(\frac{P_i}{P_o}\right)^{\frac{k-1}{k}}}{1-\frac{P_i}{P_o}}}$$

2. Equation of state (Real Gas)

The **Van Der Waals** equation of state for a fluid such as Argon is:

$$\left(P + \frac{n^2 a}{V^2}\right).(V - nb) = n.R.T$$

By dividing the side of equation by the mass of the gas, the volume becomes the specific volume.

$$\left(P + \frac{n^2 a}{V^2}\right).\left(\frac{V}{m} - \frac{nb}{m}\right) = \frac{n.R.T}{m}$$

$$\left(P + \frac{n^2 a}{V^2}\right).\left(\frac{V}{m} - \frac{nb}{m}\right) = R_s.T$$

$$\left(P + \frac{n^2 a}{V^2}\right).\left(\nu - \frac{nb}{m}\right) = R_s.T$$

$$\left(P + \frac{n^2 a}{V^2}\right).\left(\rho^{-1} - \frac{nb}{m}\right) = R_s.T$$

$$\rho^{-1} - \frac{nb}{m} = \left(P + \frac{n^2 a}{V^2}\right)^{-1}.R_s.T$$

$$\rho^{-1} = \left(P + \frac{n^2 a}{V^2}\right)^{-1}.R_s.T + \frac{nb}{m}$$

$$\rho = \left(\left(P_t + \frac{n^2 a}{V^2}\right)^{-1}.R_s.T + \frac{nb}{m}\right)^{-1} \qquad \textit{eq.(d)}$$

Absolute temperature refers to use of the Rankine (°R) temperature scales, with zero being absolute zero. The value of new constant depends on the type of gas as opposed to the universal constant gas which is same for all gases.

$$\frac{\partial\rho}{\partial t}=\frac{\partial}{\partial t}\left[\left(\mathrm{R_s}.\mathrm{T_{(t)}}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)^{-1}+\frac{\mathrm{nb}}{\mathrm{m}}\right)^{-1}\right]$$

$$\frac{\partial\rho}{\partial t}=\frac{\partial}{\partial t}\left[\mathrm{R_s}^{-1}.\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]$$

$$\frac{\partial\rho}{\partial t}=\mathrm{R_s}^{-1}.\frac{\partial}{\partial t}\left[\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]\quad \text{eq.(e)}$$

3. Bernoulli's equation

In the present section, the third physical principle as itemized in the roadmap applied to the pneumatic system, namely,

"Energy is conserved; meaning rate of change of energy inside fluid element is equal to the total of net flux of heat into element and rate of work done on element due to body and surface forces."

In general form the energy equation is derived from Green Gauss' theorem as following:

$$\frac{d}{dt}\iiint\left(u+\frac{U^2}{2}+gz\right)\rho dV+\oiint\rho(u+\frac{P}{\rho}+\frac{U^2}{2}+gz)\overline{V}.\hat{n}dA=\dot{Q}-\dot{W}_s$$

By assuming steady state for the pneumatic system:

$$\oiint\rho(u+\frac{P}{\rho}+\frac{U^2}{2}+gz)\overline{V}.\hat{n}dA=\dot{Q}-\dot{W}_s$$

Furthermore for compressible, adiabatic flow of perfect gas ($P\rho^{\gamma}=constant$), steady flow and negligible shock waves, the Bernoulli's equation represents the simplified energy equation as following:

$$\frac{U_r^{\,2}-U_t^{\,2}}{2}-\frac{\gamma}{\gamma-1}\frac{P_t}{\rho}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)+g(z_r-z_t)=0$$

$$\frac{U_r^{\,2}-U_t^{\,2}}{2}=\frac{\gamma}{\gamma-1}\frac{P_t}{\rho}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)-g(z_r-z_t)$$

$$U_r^{\ 2} = U_t^{\ 2} + 2 * \left[\frac{\gamma}{\gamma - 1} \frac{P_t}{\rho} \left(1 - \left(\frac{P_r}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) - g(z_r - z_t) \right]$$

$$U_r^{\ 2} = \frac{P_t}{\rho} \left(1 - \left(\frac{P_r}{P_t} \right)^2 \right) \left(\sum_{i=1}^{n} \frac{f_i L_i}{D_i} + \sum_{j=1}^{m} K_j - \frac{\gamma + 1}{2\gamma} \ln \left(\frac{P_r}{P_t} \right)^2 \right)^{-1} + \frac{2\gamma}{\gamma - 1} \frac{P_t}{\rho} \left(1 - \left(\frac{P_r}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) - 2g(z_r - z_t)$$

$$U_r^{\ 2} = \frac{P_t}{\rho} \left[\left(1 - \left(\frac{P_r}{P_t} \right)^2 \right) \left(\sum_{i=1}^{n} \frac{f_i L_i}{D_i} + \sum_{j=1}^{m} K_j - \frac{\gamma + 1}{2\gamma} \ln \left(\frac{P_r}{P_t} \right)^2 \right)^{-1} + \frac{2\gamma}{\gamma - 1} \left(1 - \left(\frac{P_r}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) \right] - 2g(z_r - z_t)$$

$$\overline{U}_r = \sqrt{\frac{P_t}{\rho} \left[\left(1 - \left(\frac{P_r}{P_t} \right)^2 \right) \left(\sum_{i=1}^{n} \frac{f_i L_i}{D_i} + \sum_{j=1}^{m} K_j - \frac{\gamma + 1}{2\gamma} \ln \left(\frac{P_r}{P_t} \right)^2 \right)^{-1} + \frac{2\gamma}{\gamma - 1} \left(1 - \left(\frac{P_r}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) \right] - 2g(z_r - z_t)}$$

By substituting velocity equation $\overline{U}_r$ and density ρ from real gas law (eq. d) into derived mass flow rate (eq. c) ∵

$$\dot{m}_r = Y_i . \rho . A_r . \overline{U}_r$$

$$\dot{m}_r = Y_i . \rho . A_r . \sqrt{\frac{P_t}{\rho} \left[\left(1 - \left(\frac{P_r}{P_t} \right)^2 \right) \left(\sum_{i=1}^{n} \frac{f_i L_i}{D_i} + \sum_{j=1}^{m} K_j - \frac{\gamma + 1}{2\gamma} \ln \left(\frac{P_r}{P_t} \right)^2 \right)^{-1} + \frac{2\gamma}{\gamma - 1} \left(1 - \left(\frac{P_r}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) \right] - 2g(z_r - z_t)}$$

eq.(f)

By assuming the tank as the control volume and the gas as homogenous object ∵

$$\dot{m}_t = Y_i \frac{\partial}{\partial t} (\rho V) = Y_i V \frac{\partial \rho}{\partial t} \qquad eq. (g)$$

By substituting changes of density in time eq.(e) into eq.(g) ∵

$$\dot{m}_t = Y_i \frac{\partial}{\partial t} (\rho V) = Y_i V \frac{\partial \rho}{\partial t} = Y_i . V . \mathrm{R_s}^{-1} . \frac{\partial}{\partial t} \left[\mathrm{T_{(t)}}^{-1} . \left(\mathrm{P_{(t)}} + \frac{\mathrm{n^2 a}}{\mathrm{V^2}} \right) \right] \qquad eq. (h)$$

Principle of mass/matter conservation states that the mass of a closed system (in the sense of a completely isolated system) will remain constant over time. ∵

$$\dot{m}_t = \dot{m}_r = \dot{m} \qquad \text{eq.(i)}$$

Therefore mass flow rates calculated from eq.(h) and eq(f) have to be equal. ∵

$$Y_i.V.\mathrm{R_s}^{-1}.\frac{\partial}{\partial t}\left[\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]$$

$$= Y_i.\rho.A_r.\sqrt{\frac{P_t}{\rho}\left[\left(1-\left(\frac{P_r}{P_t}\right)^2\right)\left(\sum_{i=1}^{n}\frac{f_iL_i}{D_i}+\sum_{j=1}^{m}K_j-\frac{\gamma+1}{2\gamma}\ln\left(\frac{P_r}{P_t}\right)^2\right)^{-1}+\frac{2\gamma}{\gamma-1}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]-2g(z_r-z_t)}$$

$$\frac{\partial}{\partial t}\left[\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]$$

$$= \mathrm{R_s}.\rho.\mathrm{V}^{-1}.A_r.\sqrt{\frac{P_t}{\rho}\left[\left(1-\left(\frac{P_r}{P_t}\right)^2\right)\left(\sum_{i=1}^{n}\frac{f_iL_i}{D_i}+\sum_{j=1}^{m}K_j-\frac{\gamma+1}{2\gamma}\ln\left(\frac{P_r}{P_t}\right)^2\right)^{-1}+\frac{2\gamma}{\gamma-1}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]-2g(z_r-z_t)}$$

Eq.(j) (Partial differential equation for first segment of the pneumatic system)

Classification of derived differential equation

The mathematical model of the pneumatic system is classified as of the following:

1. **Linear vs. Nonlinear:** Since the operators in this mathematical model exhibit nonlinearity, the resulting mathematical model is defined as nonlinear. Although linearity exists in the system.
2. **Deterministic vs. probabilistic (stochastic):** The applied model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables, therefore it is deterministic.
3. **Static vs. dynamic:** The curve of pressure drop is descretized in time, and it is dynamic since a dynamic model accounts for the element of time. Because it is a dynamic model, it is represented with differential equations.
4. **Lumped vs. distributed parameters:** Argon (Ar) is the operational fluid in this model. It is homogeneous (consistent state throughout the entire system), then the parameter is classified as lumped.

Empirical Data of the Pneumatic System of Sounding Rocket

Pressure test:

Transducers and gages are utilized to measure the pressure in the test article, for procedure please reference to appendix B.

Thermal Testing Procedure:

Thermal testing for the system began with calibrating each of the thermal couples. This was achieved using a calibrated thermal chamber along with two calibrated thermal couples. The calibration was done using three temperatures, 20 degrees C, -60 degrees C and 40 degrees C to achieve a calibration curve.

Once the calibration curve was known testing could begin on the pneumatic system. The thermal couples were placed in or around the flow of the system to determine the temperature of the flow. After placing the thermal couples in the desired location the system was pressurized. Once the system was stable the thermal data acquisition system was turned on and the pitch valves of the system were opened until the pressure transducer in the system read 3700 psi. The test was complete once the pressure stabilized at 3700 psi for nine-hundred seconds.

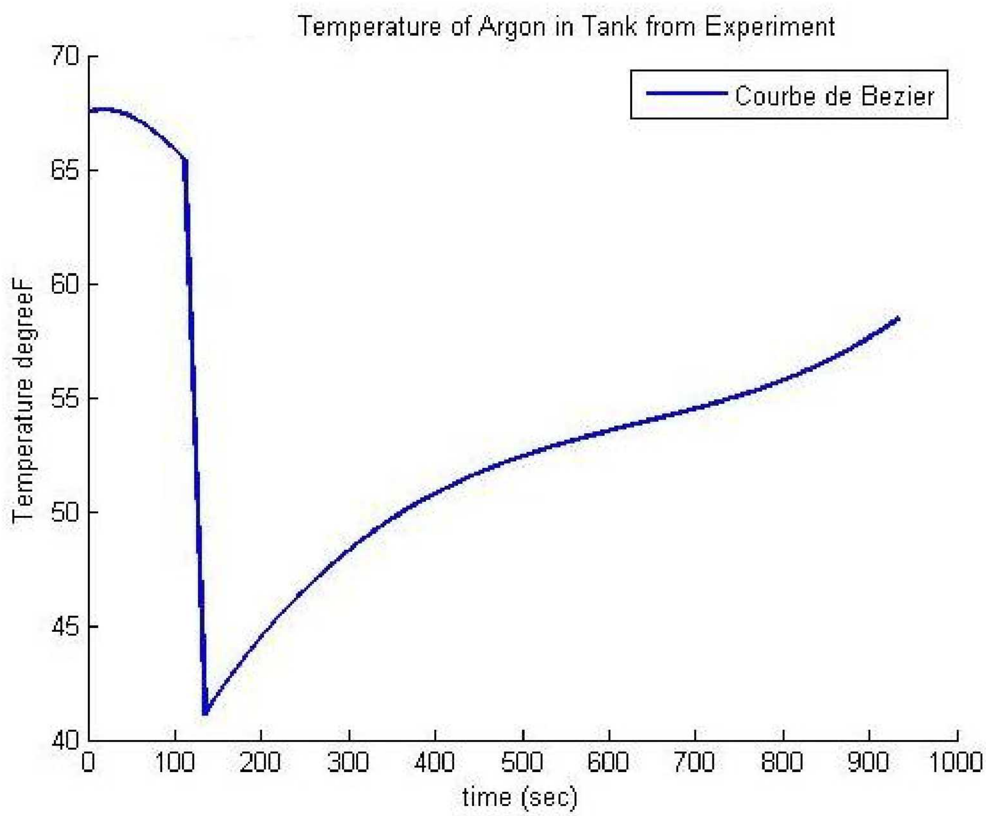


Fig 5.1

Computational Fluid Dynamics Approach

Introduction

In this section, computational fluid dynamics approach synergistically complements the other two approaches of pure theory and pure experiment.

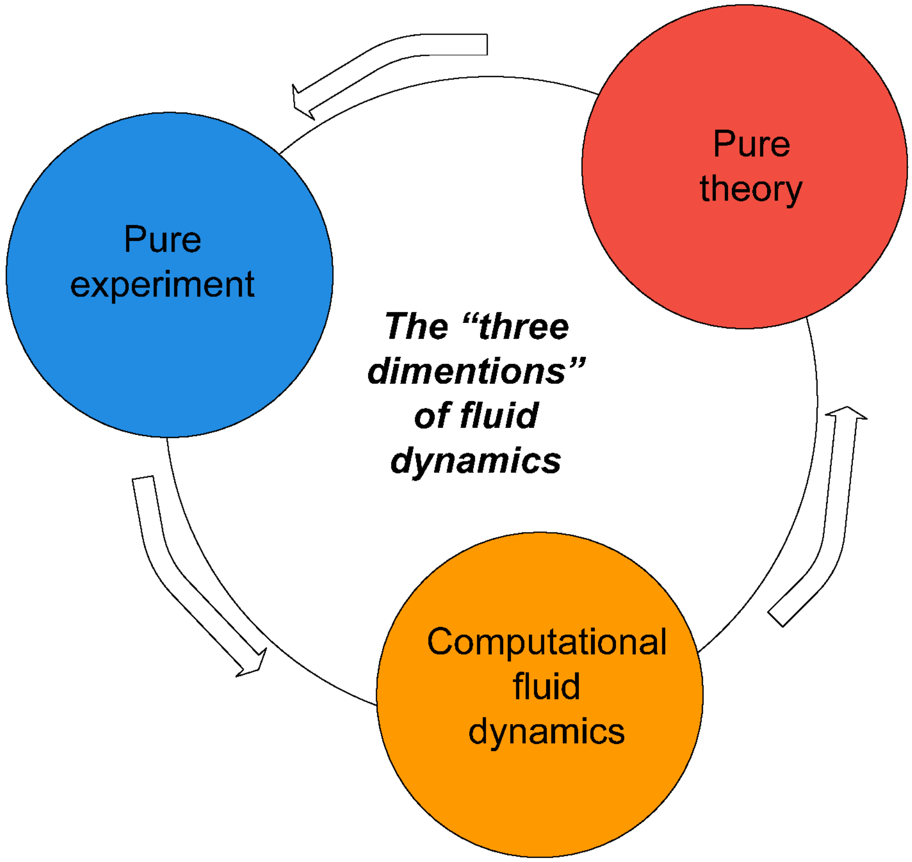


Fig 6.1

The system of differential equations for n segments that represents the pneumatic system:

$$\frac{d}{dt}\begin{bmatrix} P_t(t) \\ P_r(t) \\ \vdots \\ P_n(t) \end{bmatrix} = \begin{bmatrix} f_1(t, P_t, P_r, \ldots\ldots, P_n) \\ f_2(t, P_t, P_r, \ldots\ldots, P_n) \\ \vdots \\ f_n(t, P_t, P_r, \ldots\ldots, P_n) \end{bmatrix}$$

The *Jacobian* is an n-by-n matrix of partial derivatives:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial P_t} & \frac{\partial f_1}{\partial P_r} & \cdots & \frac{\partial f_1}{\partial P_n} \\ \frac{\partial f_2}{\partial P_t} & \frac{\partial f_2}{\partial P_r} & \vdots & \frac{\partial f_2}{\partial P_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial P_t} & \frac{\partial f_n}{\partial P_r} & \cdots & \frac{\partial f_n}{\partial P_n} \end{bmatrix}$$

The influence of the Jacobian on the local behavior is determined by the solution to the linear system of ordinary differential equations generated for each segment of pneumatic system (Eigenvalue problem):

$$\frac{dy}{dt} = Jy.$$

Derived pressure differential equation (eq. j) is a first order nonlinear partial differential equation. By application of temperature test results, and injective concept (If *Pressure* and *Temperature both* are injective, then *P* ∘ *T* is injective.) Therefore it converts to nonlinear ODE in time with initial value problem:

$$\begin{cases} \frac{dP_t(t)}{dt} = f(t, P_t(t), P_r(t)) \quad f: R * R \to R \\ P_{t_{time=0}}, P_{r_{time=0}} = given \end{cases}$$

Because of the non-linearity, it cannot be solved analytically and exactly. Since the regulator pressure is discretized in time domain, a MATLAB iterated program is developed for the solution. The solution is an approximation, based on forward difference explicit Euler method. It uses a step size and generates the approximate solution. The smaller the time step the more accurate the solution.

$$P_t'(t) = \frac{P_t(t+h) - P_t(t)}{h} \to P_t(t+h) = P_t(t) + hf(t, P_t) \qquad (numerical\ solution)$$

$$t(n+1) = t(n) + h;$$

However, the exact solution shall be calculated by Taylor series. Taylor series of $P_t(t+h)$ that is infinitely differentiable in a neighborhood of a real (t) or complex (t), is the power series:

$$P_t(t+h) = P_t(t) + hf(t, P_t) + \frac{h^2}{2!} P_t''(\zeta_k), \quad t_k \le \zeta_k \le t_{k+1} \qquad (exact\ solution)$$

But $P_c''(\zeta_k)$ is not known (no exact solution is available), by comparing the exact solution and numerical solution, the $\frac{h^2}{2!} P_t''(\zeta_k)$ is truncation error. It is second order of the associated time step $O(h^2)$. The other error is round-off calculation error arising from the use of floating point arithmetic.

The Bezier Curve Smoothing Technique:

Extracted upstream pressure data in time domain encountered truncation error and round off error. Based on iterated expectation law, we need to smooth the fitted curve. Parametric Bézier curves are important tools used to model smooth curves. In this study Cubic Bézier fitting method is applied to generated upstream pressure in attitude control system. Cubic Bézier has four control points and the parametric form of the curve is:

$$B(t) = (1-t)^n P_0 + \binom{n}{1}(1-t)^{n-1} t P_1 + \binom{n}{2}(1-t)^{n-2} t^2 P_2 + \cdots + \binom{n}{n} t^n P_n \; ; t \in [0,1]$$

$$n = 3 \; (Cubic\ Bezier) \rightarrow B(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t) t^2 P_2 + t^3 P_3 \; ;$$

$t \in [0,1]$ P_0, P_1, P_2, P_3 *are Bezier Control Points.*

Flow through Segment 1 (from tank to regulator)

In the pneumatic system, flow of gas from tank to the regulator is considered as segment 1. The objective is the calculation of pressure, mass flow rate and velocity at downstream. This segment consists of a tank, a high pressure block with expansion and contraction, a regulator and an adiabatic flow through constant diameter tubes with friction and bends, (assume volume, temperature and pressure of the tank, regulating pressure, tube bend angles, lengths diameter and surface roughness are known from experimental data.)

Procedure:

1. Calculation of friction factor for the tubes:
   - Estimate Velocity
   - Compute $Re = \frac{U.D}{\nu}$ and then f from Colebrook-White equation

$$f = \left[1.14 - 2\log\left(\frac{\epsilon}{D} + \frac{21.25}{Re^{.9}}\right)\right]^{-2}$$

   - Calculate U from $U = \left(\frac{2\Delta P.D}{f\rho L}\right)^{1/2}$
   - Iterate, returning to step 2, until desired accuracy is achieved.

   The coding in MatLab:

```
for U_est=.1:.25:1000
Reynold_comp=U_est*D_comp/Ar_neu;
friction1=(1.14-2*log10(epcilon/D_comp+21.25/(Reynold_comp^.9)))^-2;
delta_P_comp1=P_comp1-P_comp2;
```

```
U_comp=(2*delta_P_comp1*D_comp/(friction1*Rou_var_Ar*L_comp))^.5;
     if (U_comp <= (U_est + err_rate) & U_comp >= (U_est -err_rate))
     break
     end
end
```

2. Input loss coefficients for tube bends from Fluid Dynamics handbook.
3. Calculate losses in abrupt contraction and expansions from Applied Fluid Dynamics handbook.
4. Calculate velocity at upstream from:

$$U_t = \frac{\frac{P_t}{\rho}\left(1-\left(\frac{P_r}{P_t}\right)^2\right)}{\sum_{i=1}^{n}\frac{f_i L_i}{D_i}+\sum_{j=1}^{m}K_j-\frac{\gamma+1}{2\gamma}\ln\left(\frac{P_r}{P_t}\right)^2}$$

5. Solve the nonlinear partial pressure drop differential equation:

$$\frac{\partial}{\partial t}\left[\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]=\mathrm{R_s}.\rho.\mathrm{V^{-1}}.A_r.\sqrt{{U_t}^2+\frac{P_t}{\rho}\left[\frac{2\gamma}{\gamma-1}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]-2g(z_r-z_t)}$$

Pseudocode in Matlab for Euler method:

```
function
[Matrix]=Euler_ODE_solver(n,d_orifice,Area_Orifice,Volume_Tank,gr,Temp,R,A,Di
scharge_coeff,Rou_var,z_2,z_1,U_t,gamma)
for i=2:length(A)
    if (Tank_Pressure(i-1)-Tank_Pressure_differential(i-1)) > A(i,2)
       Tank_Pressure(i)=Tank_Pressure(i-1)-Tank_Pressure_differential(i-1);
    else
       Tank_Pressure(i)=A(i,2);
    end
if complex_root_checker(i)<0 | shock_critical==Ratio_Mach
    break
    end
Tank_Pressure_differential(i)=Tank_Pressure_dot(i).*(A(i,1)-A((i-1),1));
   Delta_P(i)=Tank_Pressure(i)-A(i,2);
end
[Bezier_generated_data]=NGC_All_Bezier_Interpolated_values(ctrlP0,ctrlP1,ctrl
P2,ctrlP3)
[ctrlP0,ctrlP1,ctrlP2,ctrlP3,fbi]=NGC_bezier(DeltaP_from_venting,Max_Square_d
istance);
[Vel,mdot]=mdot_calc(Pressure_vs_Time,M(:,2),Rou_var_Ar,Area_Orifice,U_t,gamm
a_Ar,z_1,z_2,gr)
```

6. Calculated mass flow rate is constant for the system, it is used for segment 2 and 3 to calculate the pressure drop at roll and pitch blocks.

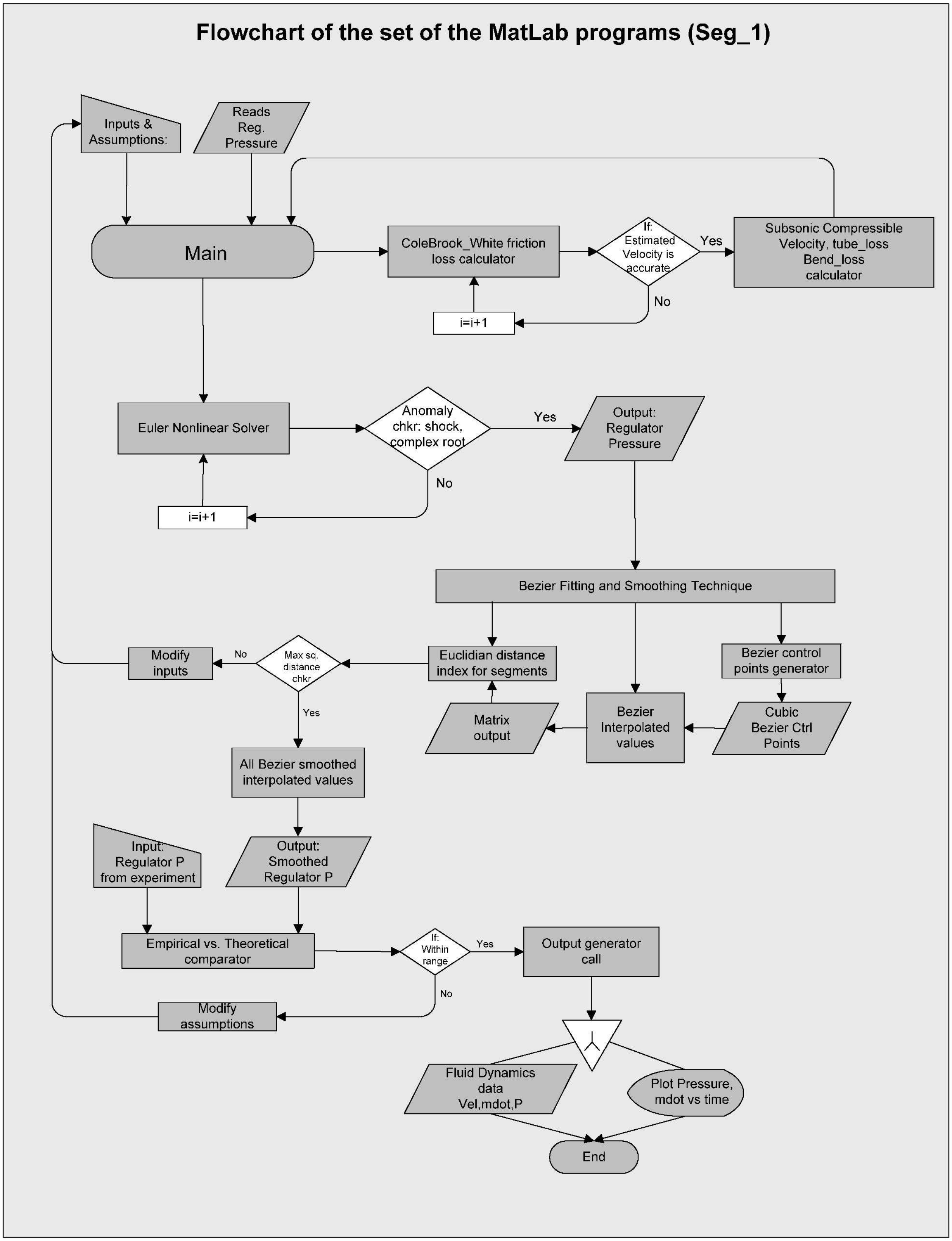

Flowchart of the set of the MatLab programs (Seg_1)
Inputs & Assumptions:
Reads Reg. Pressure
Main
ColeBrook_White friction loss calculator
If: Estimated Velocity is accurate
Yes
Subsonic Compressible Velocity, tube_loss Bend_loss calculator
No
i=i+1
Euler Nonlinear Solver
Anomaly chkr: shock, complex root
Yes
Output: Regulator Pressure
No
i=i+1
Bezier Fitting and Smoothing Technique
Modify inputs
No
Max sq. distance chkr
Euclidian distance index for segments
Bezier control points generator
Yes
Matrix output
Bezier Interpolated values
Cubic Bezier Ctrl Points
All Bezier smoothed interpolated values
Input: Regulator P from experiment
Output: Smoothed Regulator P
Empirical vs. Theoretical comparator
If: Within range
Yes
Output generator call
No
Modify assumptions
Fluid Dynamics data Vel,mdot,P
Plot Pressure, mdot vs time
End

Fig 8.1

Flow through segment 2 (from regulator to roll block)

In the pneumatic system, subsonic flow of gas from manifold to the roll block is considered as segment 2. This segment consists of an ideal adiabatic expansion or contraction at changes in manifold and adiabatic flow through constant diameter tubes with friction). Based on calculated mass flow rate and fluid dynamics properties at regulator (upstream) from segment 1, now the pressure, density, velocity and temperature can be calculated at roll block (downstream).

Procedure:

1. The mass rate of flow through each section is constant;
2. The isentropic stagnation temperature and isentropic stagnation speed of sound are constant for adiabatic flow regardless of friction.
3. The isentropic stagnation temperature, pressure, density and speed of sound are constant across isentropic expansion and these values varies only with Mach number and the ratio of specific heats.
4. Using conservation of mass:
$$\frac{\dot{m}R^{1/2}T_0^{1/2}}{P_0 A} = \gamma^{.5} M \left(1 + \frac{\dot{\gamma} - 1}{2} M^2\right)^{\frac{\gamma+1}{2(1-\gamma)}}$$
5. The ratios of static pressure, temperature, and speed of sound to their isentropic stagnation values are functions of Mach number and ratio of specific heats.
$$\frac{T}{T_0} = (\frac{c}{c_0})^2 = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$
$$\frac{P}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{1-\gamma}}$$
6. Based on solution to derived equation at regulator (segment 1), the followings are known:
   The mass flow rate
   Static pressure
   Absolute temperature
   Density
   Velocity
   Speed of sound (from NIST table)
   Ratio of specific heats(from NIST table)
7. This implies that $M = \frac{U_R}{c}$ and value of $\frac{fL_M}{D}$ is derived from following formula for adiabatic flow:
$$\frac{fL_M}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma - 1}{2\gamma} \ln \frac{(\gamma + 1)M^2}{2[1 + (\gamma - 1)M^2/2]}$$

8. Friction factor for the tube is calculated by Colebrook-White equation and as a function of surface roughness and Reynolds number:

$$f = \left[1.14 - 2\log(\frac{\epsilon}{D} + \frac{21.25}{Re^{.9}})\right]^{-2}$$

9. The frictional losses between two points are the sum of the straight pipe loss, bends loss, dividing T loss, valve loss, expansion loss and manifold loss.

$$\frac{fL_{M_2}}{D_1} = \frac{fL_{M_1}}{D_1} - \left(\frac{fL_{12}}{D_1} + K_{bends} + K_{mnfld} + K_v + K_T\right)$$

10. Then from $\frac{fL_{M_2}}{D_1}$ one can calculate Mach number by interpolation method:

```
function [ Mach2] =Mach_interp_calc(fL_M2overD,gamma)
```

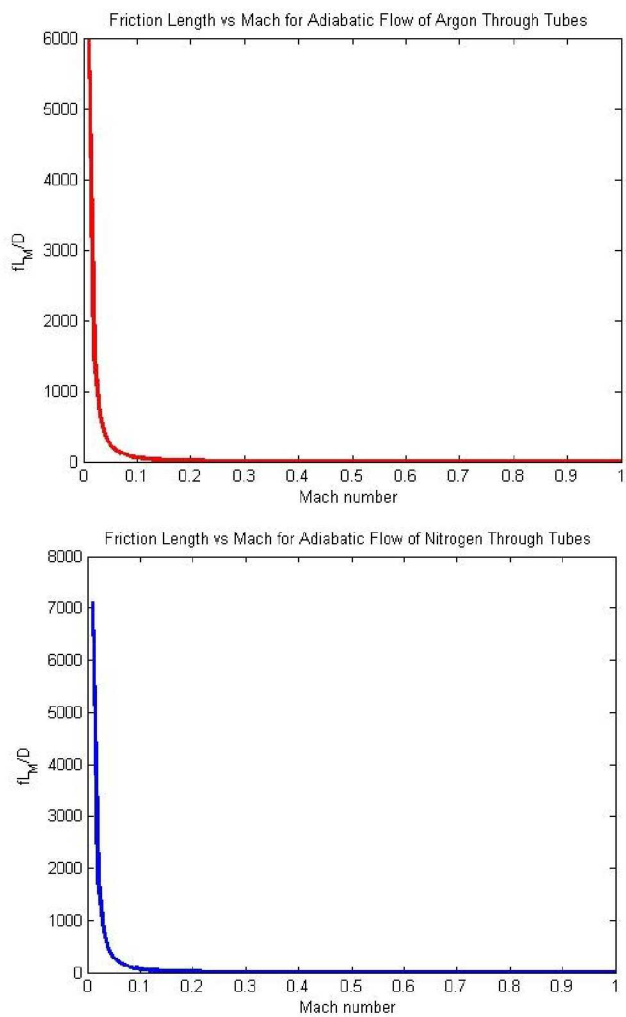


Fig 9.1

11. The ratios of $\frac{T}{T_0}$ and $\frac{c}{c_0}$ can be calculated from:

$$\frac{T}{T_0} = (\frac{c}{c_0})^2 = \left(1 + \frac{\gamma - 1}{2} {M_2}^2\right)^{-1}$$

12. Since the isentropic stagnation temperature and speed of sound are constant across the pipe:

$$T_2 = T_0\left({}^{T}/_{T_0}\right)$$
$$c_2 = c_0\left({}^{c}/_{c_0}\right)$$

13. From M_2 the flow velocity at 2 is known now $U_2 = M_2 c_2$.
14. Conservation of mass implies that the density must be $\rho_2 = \frac{\dot{m}}{U_2 A_2}$
15. The final step is the calculation of Pressure and Thrust at the end (Van Der Waals equation):

$$P_2 = P_1 \frac{\rho_2}{\rho_1}\frac{T_2}{T_1}$$
$$Thrust = \oint p.\hat{n}.dA + \oint \rho.\overrightarrow{V}.(\vec{V}.\hat{n})dA$$

Pseudo code in MatLab for roll segment:

```
fL_M1overD=(1-Mach1^2)/(gamma*Mach1^2)+(gamma-
1)/(2*gamma)*log(((gamma+1)*Mach1^2)/(2*(1+(gamma-1)*Mach1^2/2)))
ratT1overT_0=(1+(gamma-1)/2*Mach1^2)^-1
ratC1overC_0=sqrt(ratT1overT_0)
T_R_0=ratT1overT_0^-1*T_r
c_R_0=ratC1overC_0^-1*c
K_losses=K_mnfld+nT*K_T+nb90*K_90bend+K_roll_preece_valve+K_ctrction
fL_M2overD=fL_M1overD-(fLoverD+K_losses)
[Mach2]=Mach_interp_calc(fL_M2overD,gamma)
ratT2overT_0=(1+(gamma-1)/2*Mach2^2)^-1
T_2=ratT2overT_0*T_R_0
ratC2overC_0=sqrt(ratT2overT_0)
c2=ratC2overC_0*c_R_0
U_roll=Mach2*c2
rou2=mdot_system/(U_roll*A);
P_roll=P_r*T_2/T_r*rou2/rou1
Thrust=P*A+rou2* U_roll^2*A
```

Flowchart of the MatLab programs (Seg_2)

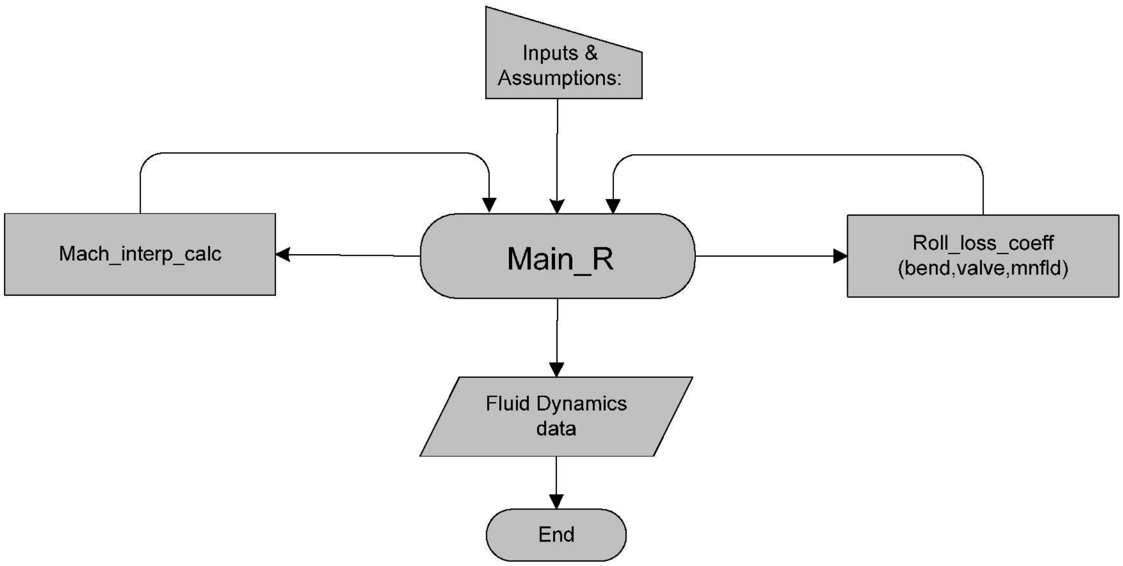


Fig 9.2

Flow through segment 3 (from regulator to pitch block)

In the pneumatic system, subsonic flow of gas from manifold to the pitch block is considered as segment 3. The procedure for pressure drop analysis is similar to segment 2, but the characteristics of the components are different.

Pseudocode in MatLab for pitch segment:

```
rou1=mdot_system/(U_r*A)

fL_M1overD=(1-Mach1^2)/(gamma*Mach1^2)+(gamma-
1)/(2*gamma)*log(((gamma+1)*Mach1^2)/(2*(1+(gamma-1)*Mach1^2/2)))

[K_mnfld, K_90bend]=pitch_loss_coeff(qty_mnfd, nb90)

K_losses=K_mnfld+nb90*K_90bend+K_ctrctionfL_M2overD=fL_M1overD-
(fLoverD+K_losses)

[Mach2]=Mach_interp_calc(fL_M2overD,gamma)

P_pitch=P_r*T_2/T_r*rou2/rou1

Thrust=P_pitch*A+rou2* U_roll^2*A
```

Flowchart of the MatLab programs (Seg_3)

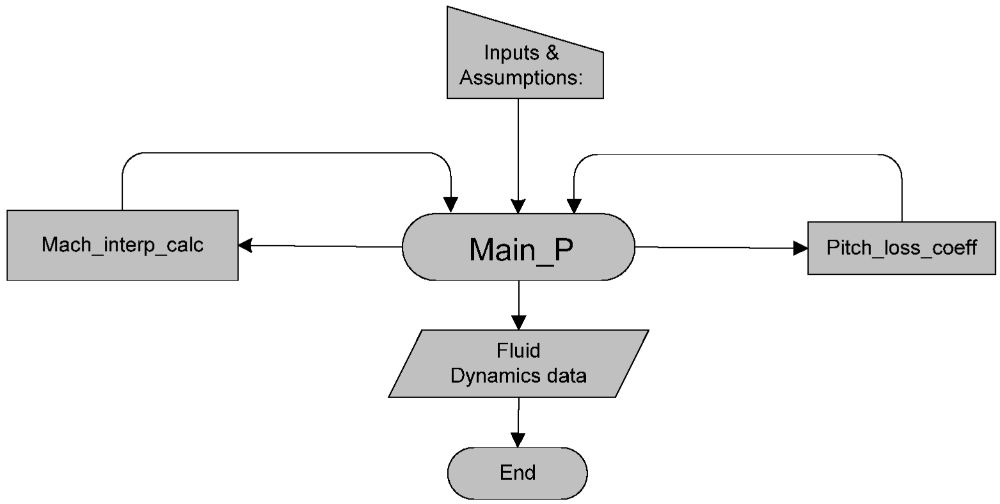


Fig 10.1

Evaluation of Applied Model

In this section the followings are addressed:

- ***How does the pneumatic analysis program work?***

A. Segment 1 (From tank to regulator)

1. Inputs:

   i. Characteristics of the components of the system (i.e. Volume, Diameter, Length, bend angle, surface roughness, etc.)

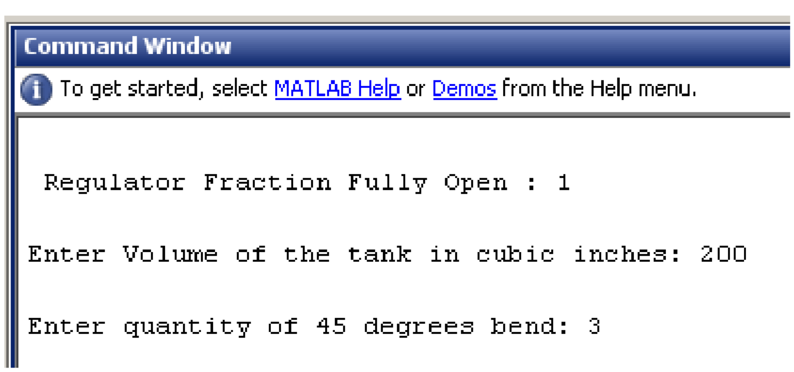
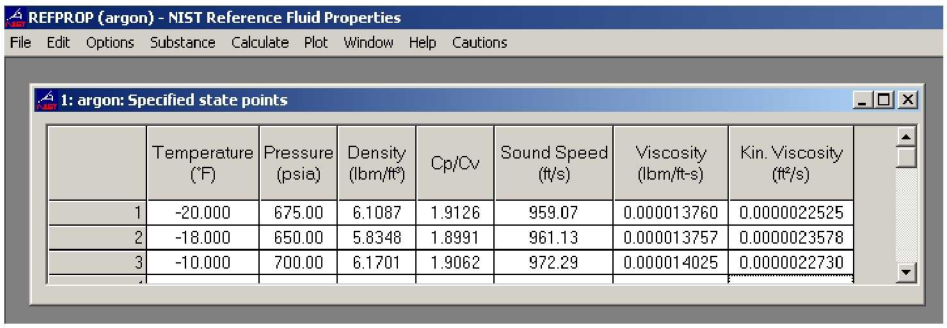


Fig 11.1 Command window in MatLab

ii. Operating temperature and regulating pressure

```
Pressure_vs_Time=xlsread('C:\hyekakan\jarmen\imput.xlsx)
```

iii. Properties of fluid for specified pressure and temperature (state of the gas) by referencing NIST software.



	Temperature (°F)	Pressure (psia)	Density (lbm/ft³)	Cp/Cv	Sound Speed (ft/s)	Viscosity (lbm/ft-s)	Kin. Viscosity (ft²/s)
1	-20.000	675.00	6.1087	1.9126	959.07	0.000013760	0.0000022525
2	-18.000	650.00	5.8348	1.8991	961.13	0.000013757	0.0000023578
3	-10.000	700.00	6.1701	1.9062	972.29	0.000014025	0.0000022730

Fig 11.2 Generated outputs in NIST

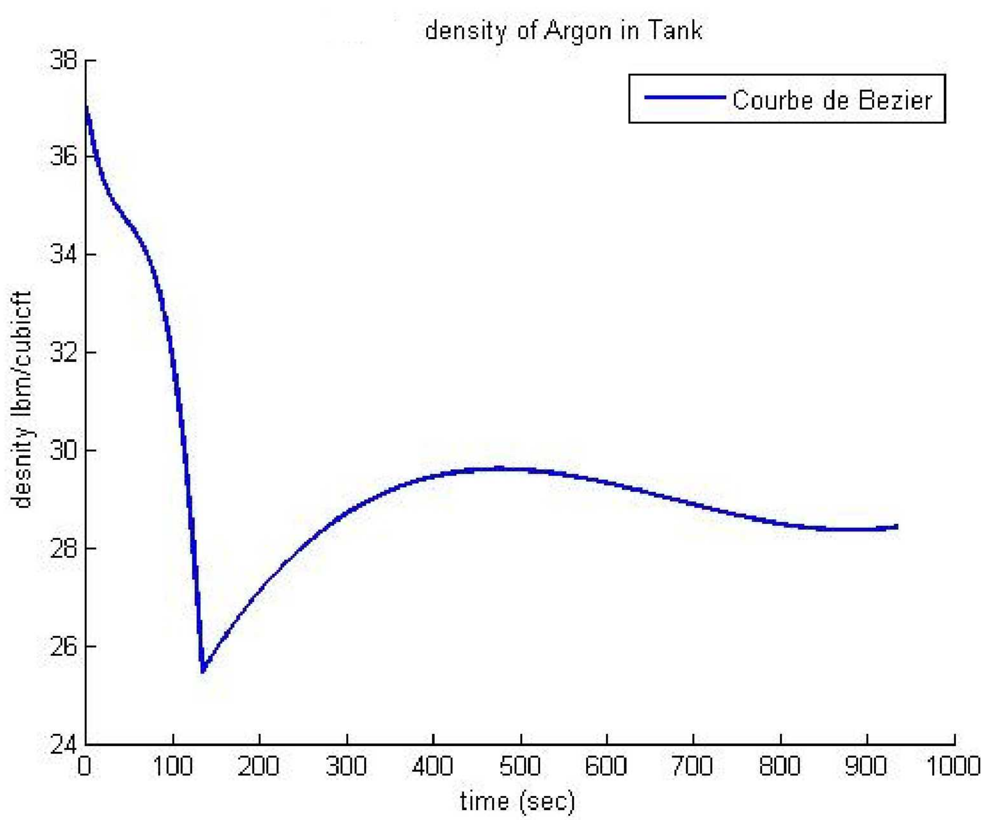


Fig 11.3

2. MatLab files:

A set of programs is developed to model the pneumatic system. By running main file and above inputs, it is linked to appropriate .m files (function call) and each .m file does its task and returns outputs to the main file.

i. main.m (is where the program reads inputs and starts execution)
ii. VelocityandLoss_calc.m(iteratively calculates the velocity in upstream and losses associated with the system from tank until manifold)
iii. Euler_PDE_solver.m (solves derived partial differential pressure drop equation for the pneumatic system, output is either deltaP or pressure in upstream)
iv. NGC_bezier.m (Approximation of data by Cubic Bezier curves. Based on least square fit, uniform parameterization. Finds Control Point of Bezier Curve that approximates the given data up to specified squared distance limit
v. NGC_columnVec_output.m(changes row to column vector)
vi. NGC_Euclidean_distanceandIndex_getter.m (this algorithm is based on Euclidean distance)
vii. NGC_vector_checker.m (checks the argument is a vector)
viii. NGC_Bezier_AllCtrlPoints_generator.m(this program finds the control points of Bezier curve for each segment of the curve)
ix. NGC_All_Bezier_Interpolated_values.m (Bezier interpolation of control points based on segmentation values)
x. NGC_All_Segments_Euclidian_distance_index.m (finds maximum square distance)
xi. NGC_Bezier_Interpolator.m (Bezier interpolation for given four control points)
xii. NGC_Matrix_comparator.m (Compares and checks dimensions of the matrices)
xiii. NGC_Bezier_CtrlPoint_getter (Least Square Method using specified Parameterization)
xiv. plot_bezier_originaldata_controlP.m (plots curves and control points)
xv. mdot_calc.m (calculates mass flow rate of the system)

3. Outputs and generated curves from Matlab:

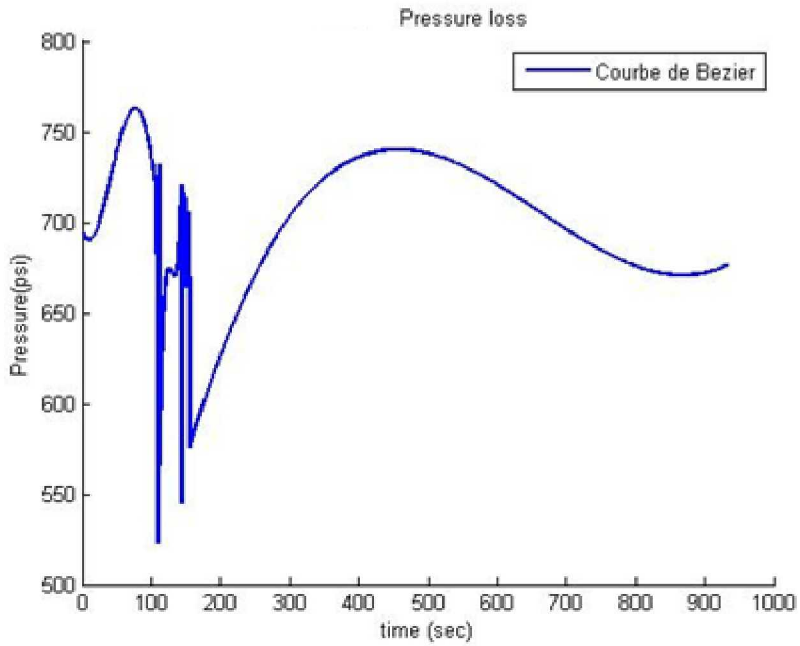


Fig 11.4

- *End-user studies and evaluation of outputs:*
  Pressure curve shows that the generated curves are converging, and mathematically it means the sequence has only one limit. In physics terms, it is attenuating (the gradual loss in intensity of flux through a medium). Converging results approves the validity of generated pressure curve from MATLAB program.

B. Segment 2 (From manifold to roll block)

1. Inputs:

   i. Characteristics of the components of the system (i.e. Manifold, dividing T, Diameter, Length, bend angle, surface roughness of the tube, etc.)

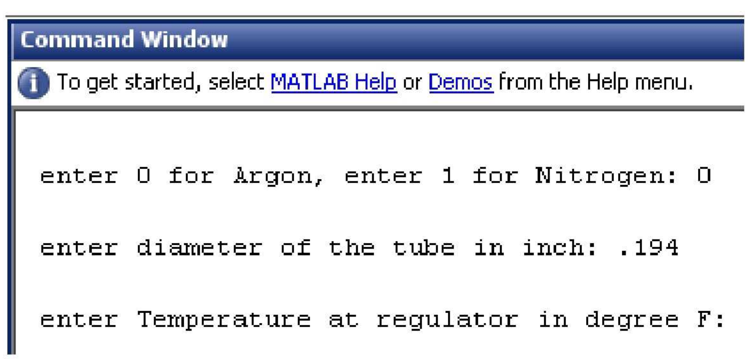


Fig 11.5 Command window in MatLab

ii. Mass flow rate, temperature, pressure and velocity of the flow at upstream calculated in segment 1.

iii. NIST inputs

2. MatLab files:

   i. mainR.m
   ii. DividingT_loss_coeff.m
   iii. Mach_interp_calc.m
   iv. Roll_loss_coeff.m

3. Outputs:

   Mach2

   ratT2overT_0

   T_Roll

   ratC2overC_0

   c2

   rou_R

   U_roll

   P_roll

   Thrust_roll

- *End-user studies and evaluation of outputs:* The analysis is finalized at this point and pressure and velocity are calculated at the roll block. For cross validation purposes the density calculated from the program, `rou2=mdot_system/(U_roll*A)` should match the density from state of the gas provided by the NIST table:

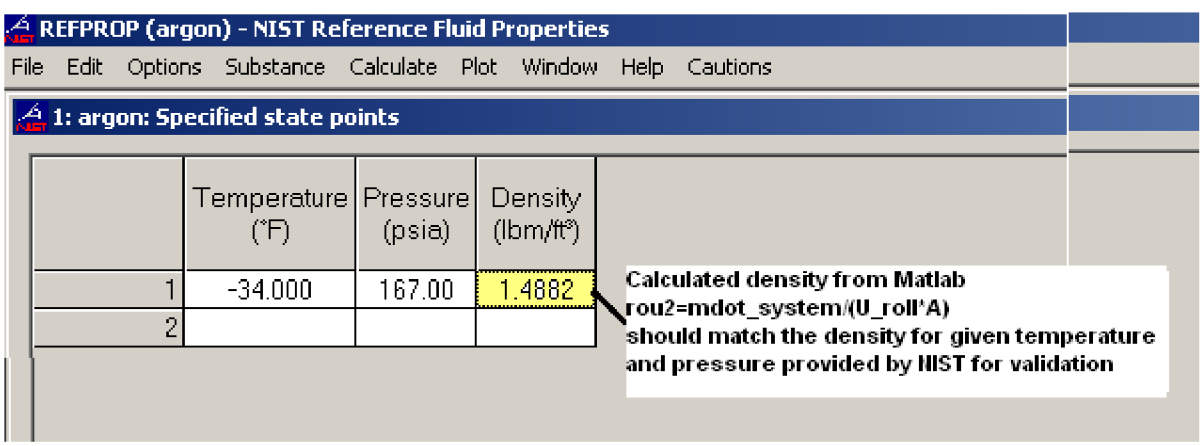


Fig 11.6 NIST program

C. Segment 3 (From manifold to pitch block)

Analysis is similar to segment 2, see section B for procedure.

Trends in Compressible adiabatic subsonic flow of gas

Static Pressure	**Decreases**
Total Pressure	**Decreases**
Velocity	**Increases**
Density	**Decreases**
Temperature	**Decreases**
Mach number	**Increases**
Reynolds number	**Increases**
Stagnation temperature	**Constant**

Table 11.1

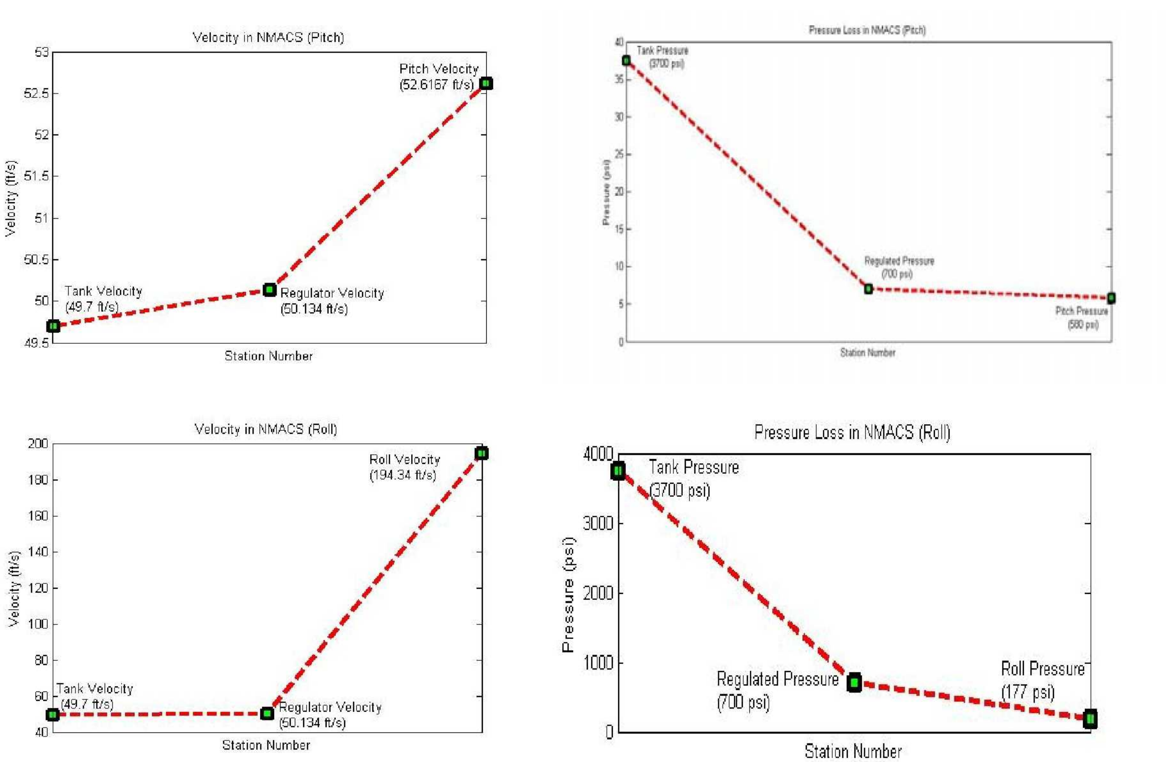


Fig 11.7

Synthesis of the Pneumatic System of Sounding Rocket

In this section:

- Break data into components of the pneumatic system by identifying causes and effects on overall efficiency of the system.
- Synthesis and compile data together in a different way by arranging elements in a new pattern or proposing alternative solution.

Effect of key characteristics on pressure loss:

Tube convergence, divergence, turns, surface roughness and other physical properties will affect the pressure drop. High flow velocities and / or high fluid viscosities result in a larger pressure drop across a section of pipe or a valve or elbow. Low velocity will result in lower or no pressure drop. Following table shows the effect of design factors on overall pressure drop of the system:

factor	Length increase by 1 foot	Diameter reduced to half	90 degree bend	Surface roughness increased by 25%	Manifold	Preece valve	Dividing tee	Tube
head loss	0.25% more drop	0.4% more drop	0.10%	0.05%	4.50%	73%	3%	0.38%

Table 12.1

Table 12.1 shows that maximum pressure drop happens in preece valves and in manifold. The pressure change is the sum of frictional losses and a pressure rise due to deceleration of the main pipe flow as some fluid is lost to the branch.

Dividing tee and losses:

The losses through a dividing tee are considerably reduced by rounding the edges of the T. in table below r is bend radius and D is diameter of tee. However the new manifold design eliminated the tee in the system.

r/D	0	0.1	0.2	0.3	0.5
T Loss coeff.	66%	38%	32%	30%	29%

Table 12.2

Tube surface roughness and material properties:

Three different types of tubes have been analyzed:

- Stainless Steel:
    a. Surface roughness is 40~70 micro inches.
    b. High yield strength preferable for vibration test and crack propagation criterion.
    c. Formability and bending issues. Stainless steel is hard to bend and the tube cross section issues.

d. Applicable for high pressure and temperature.

- Aluminum:
  a. Surface roughness is 15 micro inches.
  b. Lower yield strength inappropriate for vibration test and crack propagation criterion.
  c. Aluminum is considered as semi brittle and easier to bend but it ripples.
  d. Aluminum handles pressure and temperature to certain degree.
- Cupronickel (alloy 715 CuNi):
  a. Better surface roughness (10 micro inches).
  b. Copper Nickel alloy for high pressure and low temperature application.
  c. Better formability (copper tube, properly bent, will not collapse on the outside of the bend and will not buckle on the inside of the bend).
  d. Suitable for vibration test.

Temperature of the operating fluid and behavior of the O-rings at specified temperature:

Factors applying to all O-rings:

- Compatibility
- Temperature
- Pressure
- Extrusion

Viton is a brand of synthetic rubber and fluoropolymer elastomer used in O-rings for pneumatic design.

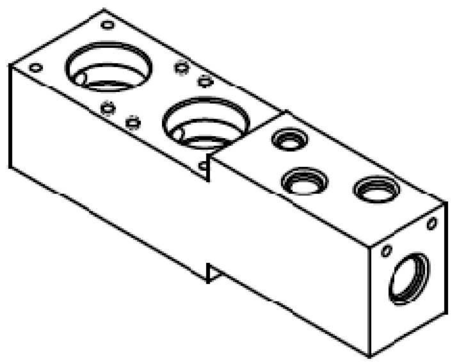
Redesign of the Manifold of the Pneumatic System (SolidWorks)

The following design criteria are considered for the new manifold:

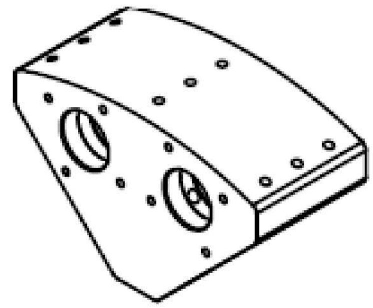
- For achieving uniform flow the main inlet is sufficiently large in diameter and the main pipe flow velocity sufficiently small so that the pressure changes along the main branch are small compared with the pressure loss for fluid exiting through the branches.
- Simple, robust design with standard mounting patterns for the valves, therefore; interchangeability becomes feasible for the system.
- Coned shaped design at the end of the roll valve to reduce velocity therefore increase pressure. (Low velocity will result in lower or no pressure drop.)
- Contains all the passages for an entire system with central shorter branches, and effective against external constraints. (Inlet and outlets are

located at appropriate surfaces for fewer bends in the tubes and it also eliminates the dividing T in the assembly.)

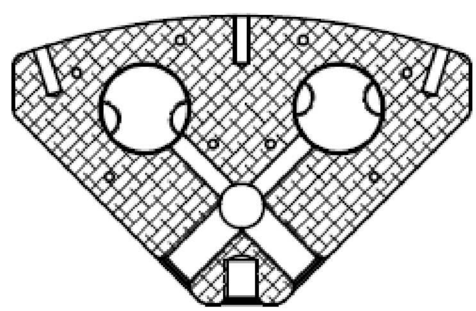
- Surface mounted compact polar design.

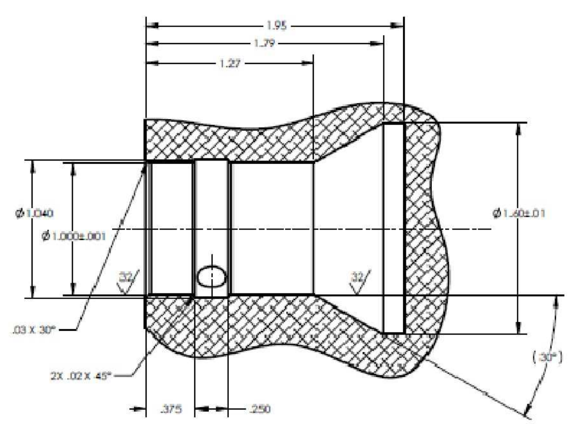


Before



After





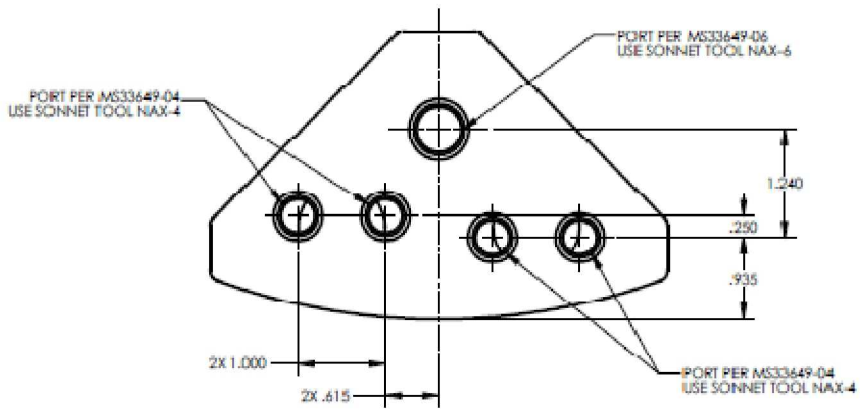


Fig. 13.1 Pneumatic system Manifold

Realization:

Actual implementation of dynamic analysis details when implementing architecture isn't totally straightforward. Certain constraints are worthy of consideration:

- **Refining of discharge factor.** C_d is assumed as "most probable value" based on experimental data. However, it varies for different flow regimes (laminar, transient and turbulent).
- **System constraints:** as mentioned in mathematical modeling section.
- **Locating of ACS system in the rocket** and affect of the gyroscopic moves to interchange of energy.
- **Pneumatic component behavior:** pneumatic components perform in a highly non-linear manner and the energy transfer medium is highly compressible. Both of these facts complicate the modeling and simulation of pneumatic system.

Summary and Future Work

Quite few analytical solutions exist for engineering problems. For instance in fluid mechanics, only simple potential flow has analytical solution. On the other hand, the real engineering problems usually are very complex and it's impossible to solve them analytically and exactly. Hence, modeling and simulation is very powerful technique for real engineering problems. Simulation is used in this study in order to gain insight into functioning of the physical system. Key issues in this simulation include:

- Acquisition of valid source information such as measured temperature and pressure about the relevant selection of key characteristics and behaviors mentioned in this study,
- The use of simplifying approximations and assumptions within the simulation,
- Fidelity and validity of the simulation outcomes.

For future trend, this project may conduct a further study to couple with application of the ***Kalman filter*** for the system discretized in the time domain (time response modal analysis and Eigen value problem).

References

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[4] J.D. Foley *et al.*: *Computer Graphics: Principles and Practice in C* (2nd ed., Addison Wesley, 1992) for Bezier Curve

[5] Van Wylen, G. J., and R. E. Sonntag, *Fundamentals of Classical Thermodynamics*, 2d Ed., John Wiley, New York,

[6] Lamb, Horace (1994). *Hydrodynamics* (6th ed.). Cambridge University Press. ISBN 0521458684. the 6th extended edition

Appendix A

Evaluation of derived equation by Buckingham π theorem:

$$\frac{\partial}{\partial t}\left[\mathrm{T_{(t)}}^{-1}.\left(\mathrm{P_{(t)}}+\frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)\right]$$

$$= \mathrm{R_s}.\rho.\mathrm{V}^{-1}.A_r.\sqrt{\frac{P_t}{\rho}\left[\left(1-\left(\frac{P_r}{P_t}\right)^2\right)\left(\sum_{i=1}^{n}\frac{f_iL_i}{D_i}+\sum_{j=1}^{m}K_j-\frac{\gamma+1}{2\gamma}\ln\left(\frac{P_r}{P_t}\right)^2\right)^{-1}+\frac{2\gamma}{\gamma-1}\left(1-\left(\frac{P_r}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]-2g(z_r-z_t)}$$

In mathematical terms, if we have a physically meaningful equation such as

$$f(x_1, x_{2,}, \dots, x_n) = 0.$$

The x_i are the *n* physical variables, and they are expressed in terms of *3* independent physical units, then the above equation can be restated as

$$F(\pi_1, \pi_2, \dots, \pi_p) = 0$$

Where the π_i are dimensionless parameters constructed from the x_i by $p = n - 3$ equations of the form

$$\pi_i = {x_1}^{m_1}{x_2}^{m_2}\dots{x_n}^{m_n}$$

Where the exponents m_i are rational numbers (they can always be taken to be integers: just raise it to a power to clear denominators).

$x_1 = P_{(t)} = ML^{-1}T^{-2}, x_2 = A_o = L^2, x_3 = V^{-1} = L^{-3}, x_4 = RT_c = \frac{P}{\rho} = L^2T^{-2}, \frac{dP_{(t)}}{dt} \cong \frac{\Delta P_{(t)}}{\Delta T} \rightarrow x_5 = ML^{-1}T^{-3}, x_6 = g = LT^{-2}, x_7 = (z_r - z_t) = \mathrm{L}$

$F(\pi_1, \pi_2, \dots, \pi_p) = 0 \rightarrow$

$$\pi_1 = ML^{-1}T^{-3} - ML^{-1}T^{-2}L^2L^{-3}L^1T^{-1} = ML^{-1}T^{-3} - ML^{-1}T^{-2}T^{-1} = ML^{-1}T^{-3} - ML^{-1}T^{-3} = 0$$

The conclusion is derived Nonlinear differential equation is valid based on Buckingham π theorem.

Appendix B

Gas Pressure recording and experimental data from pneumatic laboratory

I. Non Flowing Gas recordings

- Tank pressure- The initial pressure noted in the tank to perform a specific task, whether it be testing or the actual flight on the rail. The purpose of recording the tank pressure is to know what potential energy the system has in the form of gas to complete a specific task. This pressure is recorded by a transducer in the tanks manifold block that turns resistance into voltage ranging from 0-5 volts to give the pressure inside the tanks at any given time.
- Manifold pressure- the supply of gas inside the manifold that incorporates the roll valves and feeds the pitch/yaw valves as well. The pressure here is important to the success of every mission and is directly controlled by the course regulator. To record this pressure there are two means of doing so, first is the transducer, similar to the transducer for the tanks reading and functions in the same manner, but for the final reading taking and for testing of the transducer. Readings are also taking from a direct gauge reading in which a gauge is screw into the main manifold to take reading during various phases in testing of pressure readings.

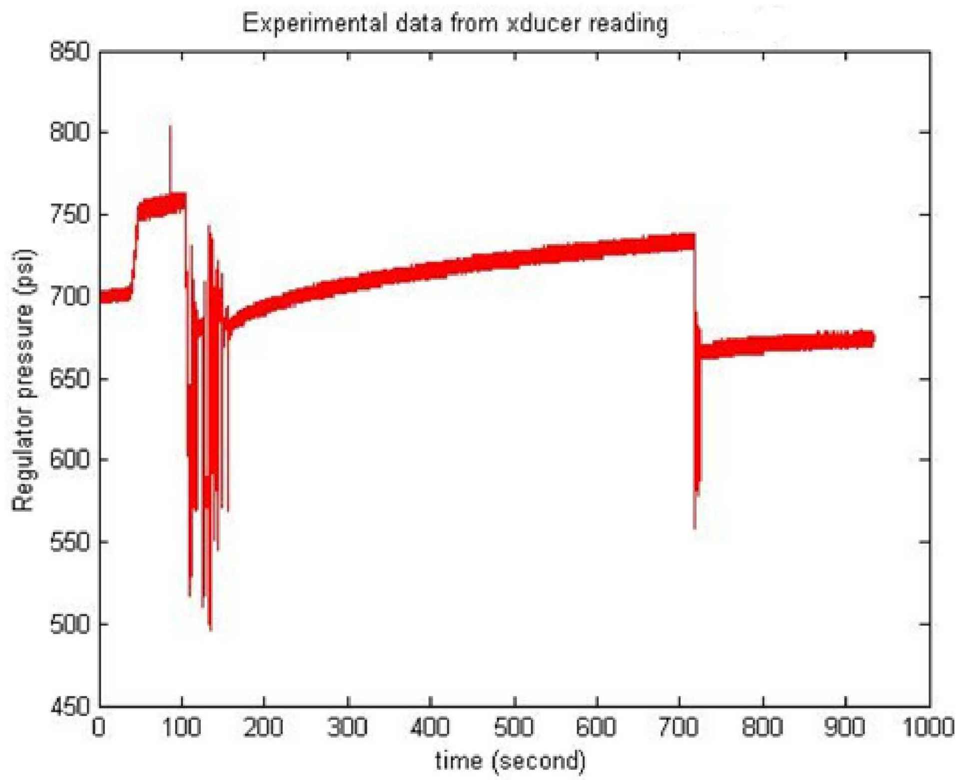
II. Flowing Gas recordings

- Tank pressure flowing is recorded by the transducer as in the non flowing readings and shows the supply of gas being used as well as the remaining gas. This is noted in most cases to check the efficiency, of the systems operation by the gas used at the end of the flight.
- Manifold pressure flowing is recorded when gas is flowing to the valve or valves being activated. This reading is taking in two ways as well as the non flowing manifold pressure reading by the direct reading of the gauge screwed into the main manifold as well as the transducer reading. This reading is always recorded in the log of the system for the final flight parameters.

- Roll flowing pressure is taken by direct reading only, in where there is a tee pipe screwed directly into the roll block being fired and on the other end of the tee is the actual nozzle being used. To the side of the tee is the gauge to record the reading of the gas flowing. This reading is always recorded, and is a vital part of setting up the system flight pressures and nozzle configurations.
- Pitch/Yaw pressure flowing- is the flow of gas from the pitch/yaw valve being fired, and is recorded in the same manner as the roll pressure. There is a tee installed in the block with nozzle on the other end and the gauge installed on the side of the tee. Along with the roll as well this plays a vital role in the flight pressure and nozzle installed into the system per flight requirements.

III. Lock up gas pressure

- Manifold Lock up pressure is the pressure recorded whenever the valve is deactivated. This pressure is important for the valves and components in the system to insure the limits are within the proper specs. Similar to the manifold, and the manifold flowing pressure, this reading is taken directly from the manifold by the gauge being screwed into the block. This reading is also recorded and maintained in the logbook.


Experimental data from xducer reading
Regulator pressure (psi)
850
800
750
700
650
600
550
500
450
0
100
200
300
400
500
600
700
800
900
1000
time (second)

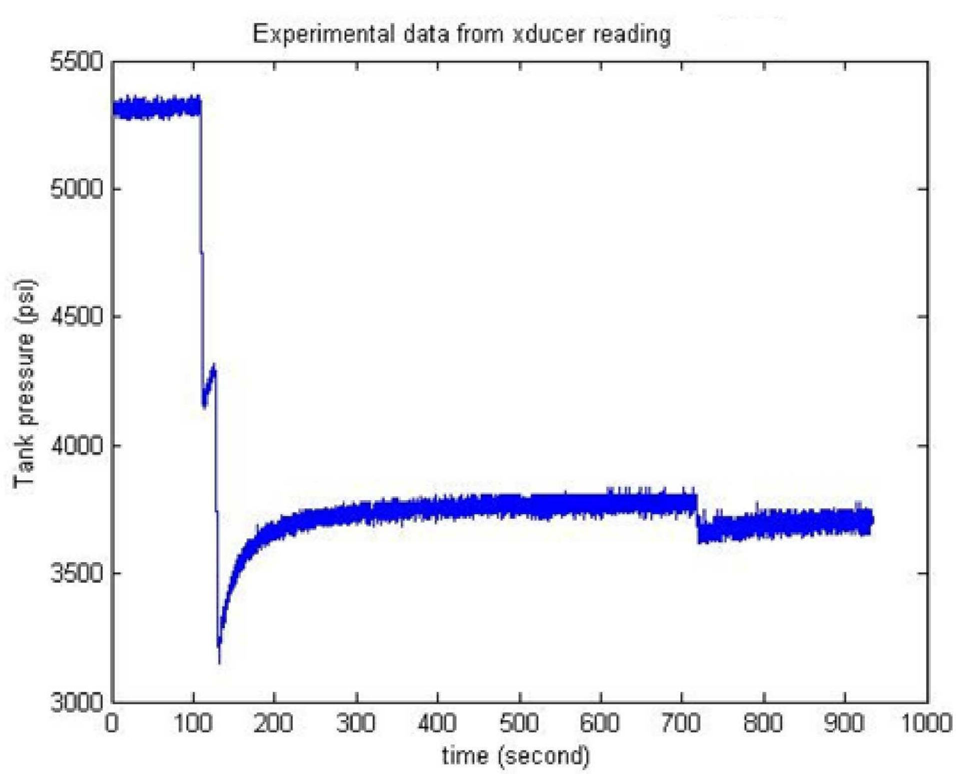

Experimental data from xducer reading
Tank pressure (psi)
5500
5000
4500
4000
3500
3000
0
100
200
300
400
500
600
700
800
900
1000
time (second)

Fig B.1

Appendix C

Bezier Interpolation Technique

Bezier curves were widely published in 1962 by the French engineer Pierre Bezier, who used them to design automobile bodies. These curves were first developed in 1959 by Paul de Casteljau using de Casteljau's algorithm, a numerically stable method to evaluate Bezier curves. Cubic Bezier curves defined by four control points P0, P1, P2 and P3 in either a plane or three-dimensional space were used for this study. The curve starts at P0 going toward P1 and arrives at P3 coming from the direction of P2. Usually, it will not pass through P1 or P2; these points are only there to provide directional information. The distance between P0 and P1 determines "how long" the curve moves into direction P2 before turning towards P3.

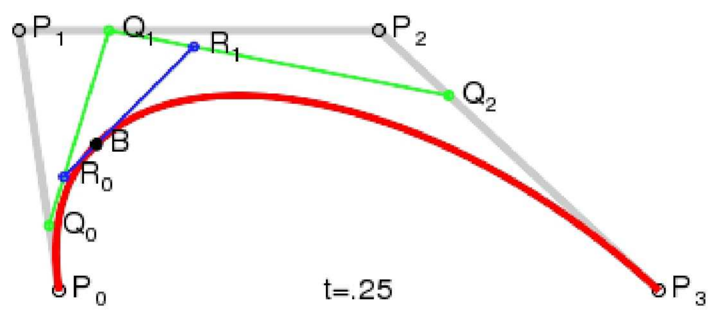


Fig C-1

The first and last control points of Bezier curve are the first and last points of the input data segment. The input data can be separated into segments or just one segment by specifying the initial set of break points. But the second and third control points are determined by "least square method" .

For fitting data, suppose we have a set of points (pressure drop in time domain extracted from Euler equation) and we want to approximate it using cubic Bezier. As an input we specify the value of limit of error (maximum allowed square distance between original and fitted data) and provide initial set of breakpoints. At least two breakpoints are required i.e., the first point and the last point of original data. Input data is divided into segments based on initial set of breakpoints. A segment is set of all points between two consecutive breakpoints. We have to fit each segment using cubic Bezier curve(s). Now the fitting process begins.

We generate *n* points (approximated data) $Q=\{q1,q2, . . . ,qn\}$ using cubic Bezier interpolation such that cubic Bezier curve(s) passes through breakpoints. Then we measure the error between original and approximated (fitted) data.

When approximated data is not close enough to original data i.e. limit of error bound is violated) then an existing segment is split (break) into two segments at a point called new breakpoint. After splitting the number of segments are increased by one (split segment is replaced by two new segments). Number of breakpoints are also increased by one (new breakpoint is added in the set of existing breakpoints). The point where the error is maximum between original and approximated data is selected as new breakpoint and this point is added in the set of breakpoints. After splitting, repeat the same fitting procedure using updated set of segments and breakpoints until error is less than or equal to limit of error.