Advances in Adaptive Control Methods

Nhan Nguyen (NASA Ames Research Center)

Integrated Resilient Aircraft Control

Constraint-Based Adaptive Control - Optimal Control Modification

Objective

- Introduces notion of **constraint-based adaptive control** that combines adaptive control with optimal control to achieve constrained error minimization.
- Develops robust **optimal control modification** adaptive law that **enforces linear quadratic constraints**.

Technical Challenges

- **Persistent excitation** (PE) can adversely affect robustness of adaptive control due to high-frequency input signals.
- **Nonlinear input-output mapping** of adaptive control can result in unpredictable performance.

Technical Approach

- Minimize LQ cost function $J = \lim_{t \to \to \infty} \frac{1}{2} \int_0^{t_f} \left[e(t) - \Delta(t)\right]^\top Q \left[e(t) - \Delta(t)\right] dt$

  subject to error dynamics $\dot{e}(t) = A_m e(t) + B\left[\tilde{\Theta}^\top(t)\Phi(x(t)) - \epsilon(x(t))\right]$
- Approach based on application of Pontryagin's Minimum Principle
- **Optimal Control Modification Adaptive Law**

$$\boxed{\dot{\Theta}(t) = -\Gamma\Phi(x(t))\left[e^\top(t)P - \nu\Phi^\top(x(t))\Theta(t)B^\top P A_m^{-1}\right]B}$$

  Lyapunov stability proof shows that the adaptive law is stable and tracking error is UUB.
- **Modification term proportional to persistent excitation** (PE) to counteract adverse effects of PE

Asymptotic Linearity for Linear Uncertainty

- Fast adaptation condition $\Phi^\top(x(t))\Gamma\Phi(x(t)) \gg \|A_m\| \gg 1$
- Asymptotic behavior $\boxed{B\Theta^\top(t)\Phi(x(t)) \to \frac{1}{\nu}P^{-1}A_m^\top P e(t)}$
- Linear tracking error dynamics for linear uncertainty

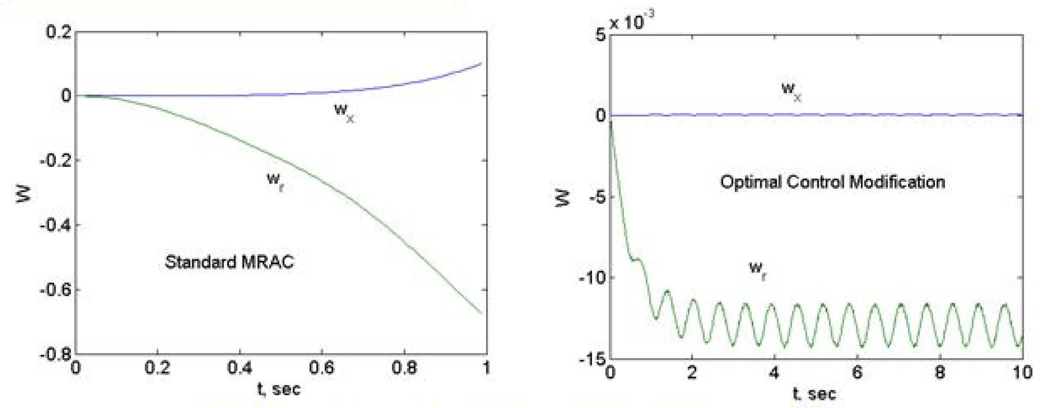
$$\boxed{\dot{e}(t) = -P^{-1}\left[\left(\frac{1+\nu}{2\nu}\right)Q - \left(\frac{1-\nu}{2\nu}\right)S\right]e(t) - B\Theta^{*\top}x(t)}$$

- **Adaptive law can be designed to guarantee stability** for given bound on Θ^* using projection operator such that $A_c = -P^{-1}\left[\left(\frac{1+\nu}{2\nu}\right)Q - \left(\frac{1-\nu}{2\nu}\right)S\right] + B\Theta^{*\top}$ is Hurwitz and satisfies linear stability margin requirements everywhere inside projection bound ⟹ **certifiable adaptive control**
- Note: e-modification or sigma-modification results in nonlinear error dynamics even for linear uncertainty

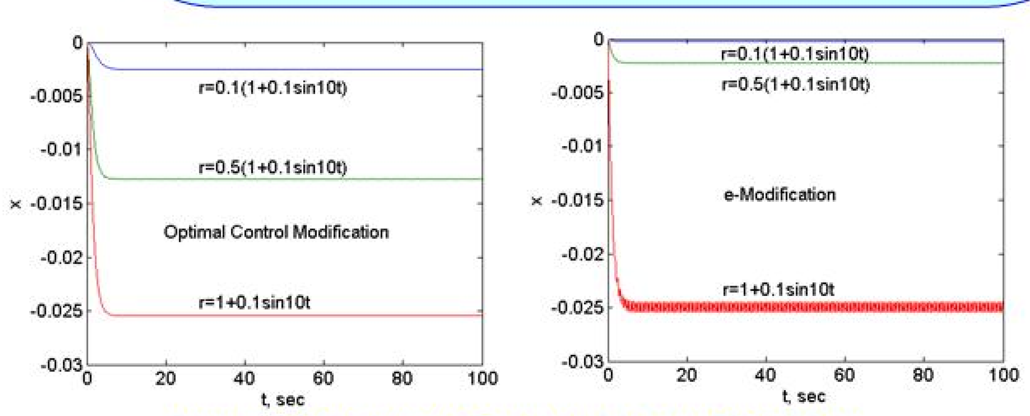
Example

$$\begin{aligned}
\dot{x} &= -x + 2u - 0.1y \\
\ddot{y} + 100\dot{y} + 100y &= 7x \\
r &= 1 + 0.1\sin 10t \\
u &= -0.5x + r - w_x x - w_r r
\end{aligned}$$

1st-order plant with 2nd-order unmodeled dynamics and input at the same frequency as that of unmodeled dynamics



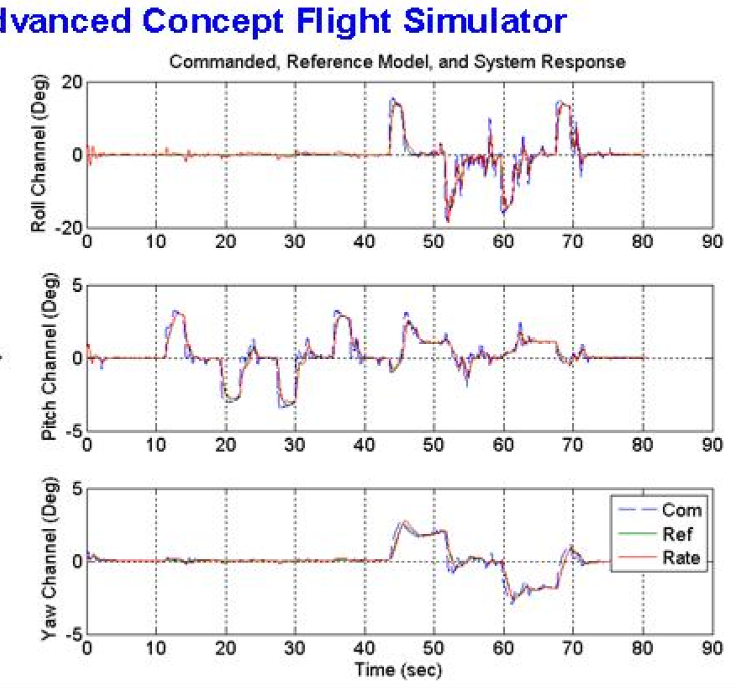
Robustness to Unmodeled Dynamics



Asymptotic Input-Output Linear Mapping

Pilot-in-the-Loop GTM Simulations

Pilot-in-the-Loop Advanced Concept Flight Simulator



10K ft, 250 Kn IAS
A = 0, B scaled
Doublet to capture flight director task

Summary

- Optimal Control Modification can provide stable fast adaptation to improve tracking
- Asymptotic linearity with fast adaptation can guarantee linear stability for linear structured uncertainty
- Pilot-in-the-loop simulations demonstrate effectiveness of the method

Adaptive Control of Time-Delay Systems - Time Delay Margin of MRAC

Objective

- Develops **stability analysis** for **time-delay adaptive system** and analytical tool to compute **time delay margin** (TDM) based on **Bounded Linear Stability Analysis**

Technical Challenges

- Currently no analytical tool exists to provide non-conservative and practical TDM estimate.

Technical Approach

- **Input-delay adaptive system**

$$\begin{aligned}
&\dot{x}(t) = Ax(t) + B\left[u(t - t_d) + \Theta^{*\top}\Phi(x(t))\right] && u_{ad}(t) = \Theta^\top(t)\Phi(x) \\
&u(t) = K_x x(t) + K_r r(t) - u_{ad}(t) && \dot{\Theta}(t) = -\Gamma\Phi(x(t))e^\top(t)PB
\end{aligned}$$

- **Bounded Linearity Stability** approximates adaptive system as a locally bounded LTI system using time-window analysis

$$\begin{aligned}
&\dot{\Theta}^\top(t)\Phi(x(t)) = -B^\top P e(t)\Phi^\top(x(t))\Gamma\Phi(x(t)) \approx -\gamma B^\top P e(t) \\
&\gamma = \frac{1}{T_0}\int_{t_i}^{t_i+T_0} \Phi^\top(x(\tau))\Gamma\Phi(x(\tau))\,d\tau) > 0 \in \mathbb{R} \\
&t \in [t_i, t_i + T_0) \qquad i = 0, 1, \ldots, n
\end{aligned}$$

$$\boxed{\dot{u}_{ad}(t) =\approx -\gamma B^\top P e(t) + \Theta^\top(t)\dot{\Phi}(x(t))}$$

γ can be viewed as integral product of adaptive gain and PE value

- Locally LTI approximation of tracking error dynamics

$$\begin{bmatrix} \ddot{e}(t) \\ \dot{e}(t) \end{bmatrix} = C_i \begin{bmatrix} \dot{e}(t) \\ e(t) \end{bmatrix} - D_i \begin{bmatrix} \dot{e}(t - t_d) \\ e(t - t_d) \end{bmatrix} + \begin{bmatrix} d_1(t) + d_2(t - t_d) + d_3 \\ 0 \end{bmatrix}$$

$$C_i = \begin{bmatrix} A + B\Theta^{*\top}\Phi_i' & 0 \\ I & 0 \end{bmatrix} \qquad D_i = \begin{bmatrix} A - A_m + B\Theta_i^\top\Phi_i' & {}^\circ BB^\top P \\ 0 & 0 \end{bmatrix}$$

- **TDM estimation by matrix measure approach** - system is locally stable if time delay is less than TDM

$$\begin{aligned}
&\omega_i < \overline{\mu}(-jC_i) + \|D_i\| \\
&t_{d_i} < \frac{1}{\omega_i}\cos^{-1}\frac{\overline{\mu}(C_i) + \overline{\mu}(jD_i)}{\|D_i\|}
\end{aligned}$$

Matrix Measure Properties

$$\overline{\mu}(C) = \max_{1 \le i \le n} \lambda_i\left(\frac{C + C^*}{2}\right) = \lim_{\epsilon \to 0}\frac{\|I + \epsilon C\| - 1}{\epsilon}$$

$$\underline{\mu}(C) \le \mathrm{Re}\lambda_i(C) \le \overline{\mu}(C) \qquad \mathrm{Im}\lambda(C) \le \overline{\mu}(-jC)$$

$$\overline{\mu}(C) \le \|C\|$$

Given $\dot{x}(t) = Ax(t) - BKx(t - t_d),\ \lambda(A - BK) \in \mathbb{C}^-$

System is stable if $t_d < \frac{1}{\omega}\cos^{-1}\frac{\overline{\mu}(A) + \overline{\mu}(jBK)}{\|BK\|}$

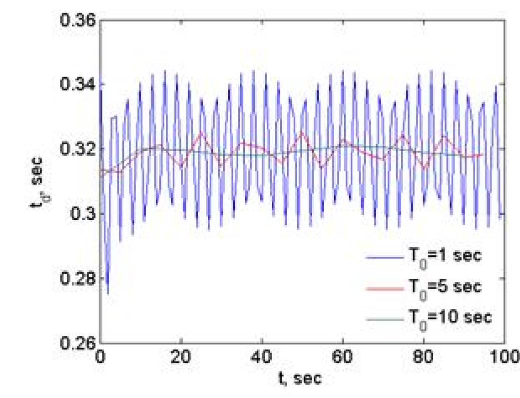
$$\omega < \overline{\mu}(-jA) + \|BK\|$$

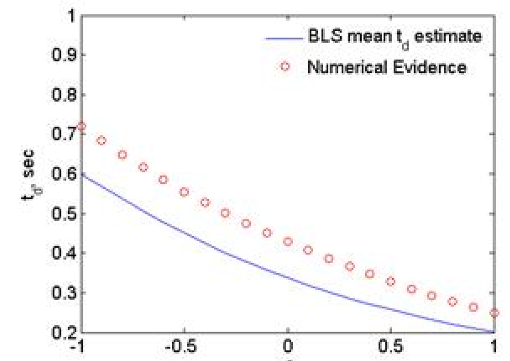
System is stable independent of time delay if

$$\overline{\mu}(A) < \|BK\| < -\underline{\mu}(A)$$

Example

$$\begin{aligned}
\dot{x} &= x + u + \theta^* x \\
\dot{x}_m &= -x_m + \sin t \\
u &= -2x + \sin t - \theta x
\end{aligned}$$





TDM estimate agrees well with simulation results

Summary

- New analytical method provides non-conservative TDM estimate
- Method can easily be extended to sigma-modification and optimal control modification