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# Long-term Stability of Radio Sources in VLBI Analysis

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# Abstract

Positional stability of radio sources is an important requirement for modeling of only one source position for the complete length of VLBI data of presently more than 20 years. The stability of radio sources can be verified by analyzing time series of radio source coordinates. One approach is a statistical test for normal distribution of residuals to the weighted mean for each radio source component of the time series. Systematic phenomena in the time series can thus be detected. Nevertheless, an inspection of rate estimation and weighted root-mean-square (WRMS) variations about the mean is also necessary. On the basis of the time series computed by the BKG group in the frame of the ICRF2 working group, 226 stable radio sources with an axis stability of 10  $\mu as$  could be identified. They include 100 ICRF2 axes-defining sources which are determined independently of the method applied in the ICRF2 working group. 29 stable radio sources with a source structure index of less than 3.0 can also be used to increase the number of 295 ICRF2 defining sources.

# 1. General Information

Presently, complete global geodetic VLBI solutions with nearly 30 years of data are possible. Stable compact radio sources can be modeled with only one source position for the whole time span. Unstable radio sources known to have high-level source structure complexity are not suitable for this. They would cause a modeling error. So the question arises: which radio sources are stable? Possible approaches for the answer to this question are the estimation and analysis of time series of radio source positions or, independently of this statistical method, the use of the list of structure indices of radio sources determined by P. Charlot (2009) [1].

# 2. Time Series for Radio Source Positions

Our basis for deriving time series of radio source positions is a complete global VLBI solution with 24-hour VLBI sessions from January 1984 to July 2007. The main parameter types are globally estimated station coordinates and their velocities and 183 preliminarily stable radio source positions from former investigations [2]. The datum definition was realized by applying no-netrotation and no-net-translation conditions for 26 selected station positions and velocities with respect to VTRF2005 and a no-net-rotation condition for 183 preliminarily stable sources with respect to ICRF1. All other source positions were estimated in a local mode in each session for generating the time series. This solution is called base solution 1. But there is a problem. This solution does not produce time series for the 183 preliminarily stable radio sources. One possibility for solving this is to reduce the set of stable radio sources for the datum definition by one source. A separate solution with no other changes referring to the base solution 1 yields estimates of local source positions for this selected radio source in the respective sessions. An advantage of this procedure is that there is nearly no change in datum definition for generating time series for all radio source positions. A disadvantage is the separate calculation for each of the 183 preliminarily stable radio sources. After these additional computations time series for all radio sources are available.

# 3. Weighted Mean and Outlier Detection for Each Radio Source

On the condition that all radio sources are stable, the weighted mean for each radio source component can be estimated from the time series with locally determined source positions and their standard deviations. To increase the reliability of the used data, an outlier test [2] was applied, which tested the maximum amount of the residual w.r.t. the weighted mean. If an outlier was detected, the corresponding data point was not used in the new estimation of the weighted mean.

### 4. Test for Normal Distribution

Based on time series of radio source positions, residuals to the weighted mean of a radio source component can be tested for normal distribution. At least 12 sessions should be available for a radio source. Time series with less than 12 sessions are not considered for the test. The purpose of the test is to detect systematic phenomena in the time series of radio source positions. The basis is a smooth test between the empirically determined distribution and the theoretical normal distribution [2]. The parameters of the normal distribution are the mean value and the variance  $s^2$  computed from the sample (residuals). The test statistic  $\chi^2$  is computed by

$$\chi^{2} = \sum_{m=1}^{r} \frac{(h_{m} - np_{m})^{2}}{np_{m}}$$

where

r number of classes (r = 10), class width = s/2, m current number of classes, n number of values in the time series of a source component,  $h_m$  empirical absolute frequency of class m, and  $p_m$  theoretical probability of class m.

The confidence level  $\chi_s^2$  is listed in a table of the chi-square distribution with the significance level of 1 percent. The decision rule says here, if  $\chi^2 < \chi_s^2$  than the distribution of the sample (residuals) is in no contradiction with the assumption that it comes from a statistical universe. So systematic phenomena could not be proven. But if  $\chi^2 > \chi_s^2$  than the distribution of the sample (residuals) is in contradiction with the assumption that it comes from a statistical universe. So systematic phenomena could be proven: e.g., instabilities of radio source components exist.

#### 5. Further Inspections

The sources with a successful test for normal distribution of the residuals to the weighted mean were also checked concerning the following empirically determined criteria. The computed rate in the time series of right ascension and declination of the radio source should be less than or equal to |0.1| mas per year, the ratio of rate and error of rate should be less than |3.5|, the WRMS should be less than or equal to 1 mas, and the total time span of observations per source should be greater than 3 years.

## 6. Results

Altogether 1189 radio sources with position time series were available, 482 of which were investigated with at least 12 sessions. The normal distribution was not rejected for 226 radio sources ( $\chi^2 < \chi_s^2$ ), i.e. 194 sources in both components and 32 sources in only one component. The further inspections of rate, sigma of rate, WRMS, and the total time span per source were successfully tested as well. The distribution of the 226 stable radio sources are depicted in Figure 1. The identified 226 stable sources can be compared with the mean source structure index (SI) values determined by Charlot (2009) [1]. SI values for 215 sources are available. 116 sources could be found with SI less than 3.0. That is the limit for the selection of the ICRF2 defining sources [3]. 99 sources have an SI greater than or equal to 3.0.



Figure 1. Distribution of 226 stable radio sources from BKG analysis 2009.

Another comparison can be made with the official 295 ICRF2 defining sources [3]. The number of identical sources is 100. The number of sources with insufficient data in the BKG analysis is 46 and there are no data for 31 sources. The number of sources which did not comply with the BKG stability criteria is 118. An important fact is that 29 stable sources from the BKG analysis with SI less than 3.0 are stable enough to increase the official 295 ICRF2 defining sources (Figure 2).

Time series and results are available at http://ivs.bkg.bund.de/vlbi or ftp://ivs.bkg.bund.de/pub/analysis/radio\_source\_positions\_time\_series.



Figure 2. Distribution of the 295 ICRF2 defining sources and 29 additional stable sources from the BKG analysis 2009.

# 7. Axis Stability

To assess the axis stability by a set of radio sources the following procedure was applied. Radio source position time series are the basis for the computation of annual reference frames. The next step is the estimation of the relative orientation between the annual reference frames and a reference catalog. The scatter (WRMS) of the rotation parameters from the annual reference frames is then a measure for the stability of the axes of the ICRF. The equation for the estimation of the rotation parameters  $\epsilon$  for one source position is

$$\begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix}_{time \ series} = \begin{pmatrix} 1 & \epsilon_Z & \epsilon_Y \\ -\epsilon_Z & 1 & \epsilon_X \\ \epsilon_Y & -\epsilon_X & 1 \end{pmatrix} \begin{pmatrix} X_{ref} \\ Y_{ref} \\ Z_{ref} \end{pmatrix}_{reference \ catalog}$$

with previous source position transformation from right ascension and declination to X, Y, Z. Least squares estimation in consideration of source position errors was applied to get the yearly rotation parameters. Table 1 shows the scatter (WRMS) of the rotation parameters derived from different sets of sources.

The accuracy of  $\epsilon_{XYZ}$  for the sets with 226 stable BKG sources and 295 ICRF2 defining sources is nearly the same, but it is almost two times better compared to the set of the 212 ICRF1 defining sources.

Set of	Number of	WRMS	WRMS	WRMS	WRMS
sources	sources	$\epsilon_X$	$\epsilon_Y$	$\epsilon_Z$	$\epsilon_{XYZ}$
	available	$\mu as$	$\mu as$	$\mu as$	$\mu as$
212 ICRF1 defining sources	211	15.2	7.4	24.3	16.5
226 stable BKG sources	226	3.4	6.2	14.6	10.4
295 ICRF2 defining sources	264	8.3	8.8	9.1	8.8

Table 1. Scatter (WRMS) of rotation parameters derived from different sets of sources with input time series bkg000c.ts and reference catalog gsf008a.cat

# 8. Conclusions

Generating and statistically analyzing time series of radio source positions are useful to get more information about long-term stability of radio sources. After a successful test for normal distribution of residuals to the weighted mean of both radio source components also an inspection of rate, sigma of rate, WRMS, and total time span per source are necessary to identify stable sources.

A set of 226 stable radio sources could be identified by this method with an axis stability of 10  $\mu as$ . 100 ICRF2 axes-defining sources could be verified independently of the method applied in the ICRF2 working group. 29 stable sources from the BKG analysis 2009 with an SI value less than 3.0 can be used to increase the number of the official 295 ICRF2 defining sources.

# References

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