

Correlations Between the Contributions of Individual IVS Analysis Centers

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Abstract

Within almost all space-geodetic techniques, contributions of different analysis centers (ACs) are combined in order to improve the robustness of the final product. So far, the contributing series are assumed to be independent as each AC processes the observations in different ways. However, the series cannot be completely independent as each analyst uses the same set of original observations and many applied models are subject to conventions used by each AC. In this paper, it is shown that neglecting correlations between the contributing series yields too optimistic formal errors and small, but insignificant, errors in the estimated parameters derived from the adjustment of the combined solution.

1. Introduction

It is widely accepted that the results of the different space-geodetic techniques are combined in order to fully exploit the strengths of each technique and to overcome the technique-specific weaknesses. In recent years, it has been shown that also a combination of the results achieved by different analysis centers (ACs) using the observations of one technique leads to more robust results (see e.g., [4], [2]). Today, almost all products of the technique-specific services in the International Earth Rotation and Reference Systems Service (IERS) are determined in a so-called intra-technique combination.

As each AC processes the observations in a different way, their results are not identical. However, from a statistical point of view, these intra-technique combinations suffer from the fact that the contributions of different ACs are treated independently despite having been derived from almost the same set of original observations. Mathematically, these dependencies can be expressed by correlations. But, the determination of these correlations and their rigorous consideration presents a delicate problem within today's combination methodologies.

In the work of [5] a first concept is presented to take the impact of the identity, as well as the impact of differences due to independent analysis strategies, into account. So far, this idea has not been adapted for any operational intra-technique combination, most probably because this concept cannot be easily extended to more than two contributions.

Different from [5], in this paper, the dependency of two contributions of real IVS ACs is quantified and rigorously considered by combining directly the observation equations (Section 3). Moreover, the effects on the estimated combined parameters as well as on their formal errors are discussed, and the findings are applied to the operational IVS combination at the normal equation level (Section 4).

2. Mathematical Background

To introduce the basic concept, the most important formulas for the combination of linear equation systems are given for the contributions of two ACs, which can easily be extended for more. These equations are mainly based on [1]. An over-determined system of linear equations

$$\mathbf{l} + \mathbf{e} = \mathbf{A}\hat{\mathbf{x}} \quad (1)$$

with \mathbf{A} being the Jacobian matrix, \mathbf{l} the vector ‘observed minus computed’ observations and \mathbf{e} the vector of residuals is solved in a weighted least squares adjustment by minimizing $\|\mathbf{A}\hat{\mathbf{x}} - \mathbf{l}\|_P^2$. A combination of two systems simply means to stack the equations so that the elements of the combined system are:

$$\mathbf{l} = \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}, \quad (2)$$

if both systems coincide in the parameter vector and its a priori values. In the case of uncorrelated observations, the combined variance-covariance matrix reads as

$$\Sigma\left(\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2 \mathbf{P}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \sigma_2^2 \mathbf{P}_2^{-1} \end{bmatrix} \quad (3)$$

with \mathbf{P} being the weight matrix and σ the variance coefficients. Generally, a least squares solution $\hat{\mathbf{x}}$ can be computed by solving the normal equation system

$$\mathbf{N}\hat{\mathbf{x}} = \mathbf{n} \quad \text{with } \mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A} \quad \text{and } \mathbf{n} = \mathbf{A}^T \mathbf{P} \mathbf{l}. \quad (4)$$

For the computation of the combined normal equation system, this leads to the simple addition of the individual normal equations:

$$\underbrace{\left(\frac{1}{\sigma_1^2} \mathbf{N}_1 + \frac{1}{\sigma_2^2} \mathbf{N}_2\right)}_{\mathbf{N}} \hat{\mathbf{x}} = \underbrace{\frac{1}{\sigma_1^2} \mathbf{n}_1 + \frac{1}{\sigma_2^2} \mathbf{n}_2}_{\mathbf{n}}. \quad (5)$$

However, Eq. (3) and thus Eq. (5) only hold, if the observations \mathbf{l}_1 and \mathbf{l}_2 are uncorrelated. In the case of correlated observations, the combined variance-covariance matrix has to be extended by the cofactor matrices \mathbf{P}_{12} and the covariance coefficient σ_{12} :

$$\Sigma\left(\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2 \mathbf{P}_1 & \sigma_{12} \mathbf{P}_{12} \\ \sigma_{12} \mathbf{P}_{12} & \sigma_2^2 \mathbf{P}_2 \end{bmatrix}^{-1}. \quad (6)$$

The combined parameter vector $\hat{\mathbf{x}}$ can no longer be computed by simply accumulating the individual normal equation systems \mathbf{N}_1 and \mathbf{N}_2 as in Eq. (5). Instead the combined observation equation system has to be solved:

$$\underbrace{\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T \end{bmatrix}}_{\mathbf{N}} \Sigma^{-1} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \hat{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T \end{bmatrix}}_{\mathbf{n}} \Sigma^{-1} \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix}. \quad (7)$$

Thus, a fully rigorous consideration of the dependency of the individual contributions in an intra-technique combination can only be carried out at the observation equation level.

3. Pilot Study

Since the IVS intra-technique combination is based on datum-free normal equation systems, a rigorous combination taking into account the correlations between the input series is not possible, as the observation equations would be necessary. At the moment, these correlations are neglected. Therefore, the influence of neglecting the correlations on the estimated combined parameters and their formal errors is investigated in this pilot study.

Extensive modifications of the VLBI analysis software Calc/Solve have been carried out to extract the complete observation equations. With this modified version, the continuous 15-day campaign CONT02 has been analyzed by two different ACs using different analysis options as they are used for the contributions to the operational IVS combination as well. Using these observation equations, a combination according to Eq. (7) is carried out. In the first step, the correlations between the contributions are determined in a variance-covariance component estimation as, e.g., described in [3]. In Fig. 1 the correlation coefficients between the contributions of the two ACs estimated for each individual day of the two week period are shown. It is clearly visible that

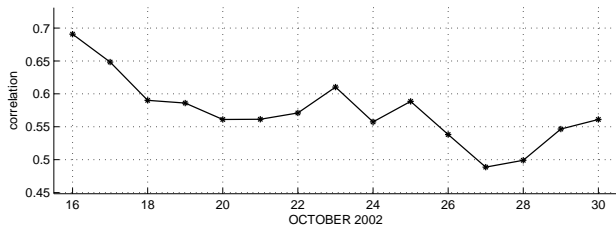


Figure 1. Level of correlations between the two contributions during the two-week CONT02 campaign.

the level of correlation is not constant over the whole campaign; it varies from session to session between 0.5 and 0.7 and naturally depends on the level of the agreement of the analysis options selected by the analysts as well as how much they practically affect the real observations.

In order to understand how much the negligence of these correlations influences the combined results, a combination is carried out twice—first, assuming independent contributions according to Eq. (3), and, secondly, taking the correlations (as presented in Fig. 1) into account with the stochastic model according to Eq. (6). The X-pole estimates and formal errors for the two individual solutions as well as for the combined solutions with and without taking the correlations into account are displayed in Fig. 2. The formal errors are (on average over the two weeks) too opti-

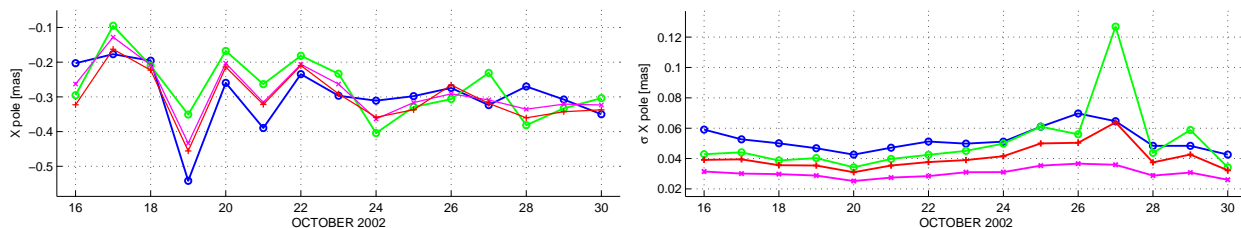


Figure 2. X-Pole estimates (left) and their formal errors (right) for the combined solutions neglecting correlations (magenta, x's) and considering correlations (red, +'s). In blue (dark circles) and green (light circles), the results of the individual solutions are displayed.

mistic by a factor of 1.2 if correlations are neglected. The differences of the combined parameters with and without considering correlation, are within their $1\text{-}\sigma$ formal errors (see Fig. 3) and, thus, insignificant.

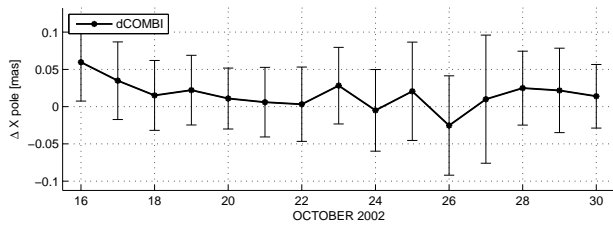


Figure 3. Differences between the estimates of the combined solution neglecting correlations and the estimates of the combined solution where correlations are considered. The error bars display the $1-\sigma$ formal errors of the differences.

4. Impacts on the Operational IVS Combination

Since the dependency of the ACs' contributions to the operational IVS combination cannot be considered directly in the combination process, at least the knowledge from the pilot study regarding the multiplication factors for the formal errors can be transferred. In the case of two contributions these are too optimistic by a factor of approximately 1.2. Since six and not only two ACs contribute to the operational IVS combination at the moment, the formal errors are expected to be even less realistic. A simplified error propagation assuming six contributions, all correlated amongst each other with 0.6, leads to too optimistic formal errors of the combined parameters by a factor of 2.

In order to investigate if this is realistic, comparisons with independent EOP series can serve as a simple empirical approach. Here, the IGS EOP series (igs00p03.erp) is used as a reference. If the WRMS of the differences of the two series is within the formal errors, the formal errors can be assumed to be realistic. Therefore, the differences of all individual AC's solutions as well as of the combined solution w.r.t. IGS are computed. For the data of one month each, WRMS values and median formal errors of these differences are calculated and compared for the period 1996.0 to 2009.0.

As displayed in Fig. 4 for the X-pole, the ratio of the median formal errors and the WRMS, both computed over the data of one month with a 7-day sliding window, are less than one for all polar motion components, which indicates that the formal errors are too optimistic. For the individual VLBI series the ratios are about 0.6, while the ratio for the combined solution (without considering correlations) is less than 0.4. However, it is not possible to attribute unambiguously the too optimistic formal errors to the IGS or to the VLBI series. But, here, the goal is that the ratio of the combined series w.r.t. IGS equals the level of the ratio of the individual VLBI solutions. This can be achieved by scaling the formal errors of the combined solution by 2 (see Fig. 4) which is exactly the number derived from the pilot study.

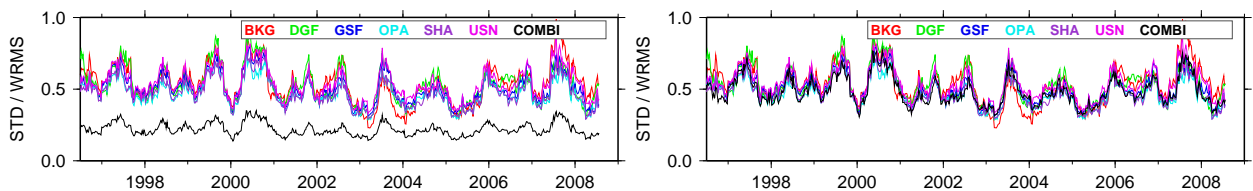


Figure 4. Ratio of the formal errors and the WRMS values of each individual series computed from the normal equation systems and the combined series w.r.t. IGS—(left) without scaling of the formal errors, (right) after scaling the formal errors of the combined solution by a factor of 2.

Figure 5 shows, as an example, the formal errors for the X-pole component between 1996 and 2009 computed from the individual normal equation systems and the combined system itself before

and after scaling. After scaling, the formal errors of the combined solution are clearly closer to the level of the individual series. However, they are still slightly smaller which is realistic because the combined solution contains more information than the individual solution.

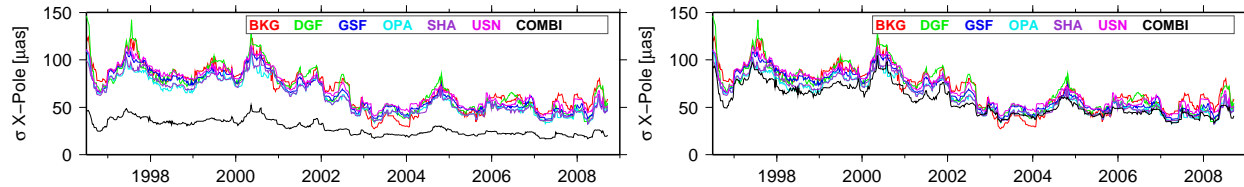


Figure 5. Median smoothed formal errors of the X-pole component between mid-1996 and 2009 for each individual series and the combined series itself, (left) before scaling the combined series, (right) after scaling the combined series by a factor of 2.

5. Summary and Conclusions

Up to now, in intra-technique combinations the dependency of contributions among each other has completely been neglected. Since most of the contributions to the IVS combination uses the same software package for the analysis of the VLBI observations, quite high correlations are expected. In this paper, a study is carried out where the observation equations are used directly for the combination. This allows determination of the level of correlations and investigation of the influence of neglecting the correlations on the estimated combined parameters as well as their formal errors. For the CONT02 data, it turned out that a realistic level of correlations for the contributions to the IVS combination is between 0.5 and 0.7. However, it has to be taken into account that correlations might be smaller using the contributions of different software packages or higher if more similar analysis options are chosen by the analysts. Moreover, it is shown that the negligence of correlations primarily impacts the formal errors of the estimated parameters and not the parameters themselves. For the IVS combined products with six contributing ACs, these are too optimistic by a factor of approximately 2 if the correlations are neglected.

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