

Time-dependent Selection of an Optimal Set of Sources to Define a Stable Celestial Reference Frame

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Abstract

Temporal statistical position stability is required for VLBI sources to define a stable Celestial Reference Frame (CRF) and has been studied in many recent papers. This study analyzes the sources from the latest realization of the International Celestial Reference Frame (ICRF2) with the Allan variance, in addition to taking into account the apparent linear motions of the sources. Focusing on the 295 defining sources shows how they are a good compromise of different criteria, such as statistical stability and sky distribution, as well as having a sufficient number of sources, despite the fact that the most stable sources of the entire ICRF2 are mostly in the Northern Hemisphere. Nevertheless, the selection of a stable set is not unique: studying different solutions (GSF005a and AUG24 from GSFC and OPA from the Paris Observatory) over different time periods (1989.5 to 2009.5 and 1999.5 to 2009.5) leads to selections that can differ in up to 20% of the sources. Observing, recording, and network improvement are some of the causes, showing better stability for the CRF over the last decade than the last twenty years. But this may also be explained by the assumption of stationarity that is not necessarily right for some sources.

1. Introduction

The purpose of this study was primarily to apply methods developed in 2003 by M. Feissel-Vernier [1] to new sets of VLBI source position time series. In 2009, Lambert & Gontier [4] did some preliminary analysis in this sense, using a former set of source time series. This paper is inspired by these two papers, but extended to the second realization of the International Celestial Reference Frame (ICRF2) sources and seen with a new point of view. The goal is to select stable sources using the Allan variance as the statistical tool to judge source stability, taking into account their linear trend components.

2. Data

Three sets of VLBI position time series were analyzed, all produced with Calc/Solve software. The first two (from D. Gordon at GSFC) are series GSF005a, which used only global VLBI sessions from August 1979 through February 2009, and series AUG24, which used global, mobile, and some regional VLBI sessions through August 2009. Mobile and regional VLBI sessions used weaker stations and smaller networks, and thus will give noisier results. The third series, OPA (from S. Lambert at the Paris Observatory) used a different approach in its analysis strategy. The analysis was done over two different periods of time. The first covered the approximately twenty years of the best VLBI, from 1989.5—2009.5, and the second covered the last ten years from 1999.5 to 2009.5.

3. Allan Variance as a Tool to Qualify the Stability of VLBI Sources

3.1. Source Stability Index

The following is a quick reminder of the mathematical expression of the Allan variance. If $(x_i)_i$ are the measurements and T the sampling time, the Allan variance is $\sigma_A^2(T) = \frac{1}{2} \langle (\bar{x}_{i+1} - \bar{x}_i)^2 \rangle$. By analogy with the power spectral density, the slope of the Allan variance curve in a log-log plot indicates the type of noise. White noise will give a slope equal to -1 , flicker noise to 0 , and random walk to $+1$. More details can be found in [6].

The stability index SI used in this study is calculated as a combination of the normalized values of the slope coefficients (computed over all data studied) and the Allan variance at one-year sampling time, on both coordinates (right ascension and declination): $SI(source_k)$ is the sum of the normalized values of $slopeRA_k$, $slopeDEC_k$, $\sigma_{RA_k}^2(1yr)$, and $\sigma_{DEC_k}^2(1yr)$.

3.2. Celestial Reference Frame Stability Indicator



Figure 1. Stability test scheme. This figure illustrates the method used to judge the stability of a subset of sources by comparing yearly averaged Celestial Reference Frames $(CRF)_i$ to a mean CRF.

To judge the stability of a subset of chosen sources, we compare two Celestial Reference Frames (CRFs) realized by this subset (see Figure 1): one is the yearly mean realization $(CRF)_i$, while the other is the mean being computed over the full period. To do so, we process three rotations (A_1, A_2, A_3) and a fictitious declination bias dz (cf., [2], [3]) by the following equation:

$$\begin{cases} (\alpha_m - \alpha_i) \cos \delta_m &= A_1(i) \tan \delta_i \cos \alpha_i + A_2(i) \tan \delta_i \sin \alpha_i - A_3(i) \\ \delta_m - \delta_i &= -A_1(i) \sin \alpha_i + A_2(i) \cos \alpha_i + dz(i) \end{cases}$$

where i indicates the year of the average of the coordinates and m the mean coordinates computed over the entire studied period. The stability criteria is obtained by looking at the $(A_1(i), A_2(i), A_3(i), dz(i))_i$ parameter time series for different subsets of sources.

3.3. Statistical Analysis Scheme Used in This Study

The following describes the analysis of the entire set of sources:

1. Some sources can not be analyzed for some statistical reasons of consistency (time series shorter than five years, data gaps bigger than four years, needing more than 50% of gap filling for the regularization of the time series). These are eliminated from the analyzed set.
2. For each source remaining, the stability index SI explained previously is computed (combination of drift slopes and Allan variances at one-year for each coordinate). The sources are then sorted from the most stable (lowest SI value) to the least stable (largest SI value).

3. N bins B_j of sources are created (the 50 most stable sources in bin B_1 , then the 60 most stable sources in bin B_2 , then the 70 most stable in bin B_3, \dots until the entire set of analyzed sources is reconstructed in bin B_N).
4. A set of $(A_1(i, j), A_2(i, j), A_3(i, j), dz(i, j))_i$ where i is the year and j the number of the bin, is computed for each bin $(B_j)_{j=1..N}$.
5. For each bin $(B_j)_{j=1..N}$, we compute standard deviations and means of those four parameters time series, and we sum them together. Finally, we obtain one couple $(S_j, M_j)_{j=1..N}$ per bin $(B_j)_{j=1..N}$ where $S = \sigma(A_1) + \sigma(A_2) + \sigma(A_3) + \sigma(dz)$ and $M = \overline{A_1} + \overline{A_2} + \overline{A_3} + \overline{dz}$.

4. Results

4.1. Evaluation of the Defining Sources

In ICRF2, 295 sources are defining sources. They were chosen taking into account different criteria (cf., [5]), such as even sky distribution (the sky was divided into four divisions based on declination), the quality of the observations (computed by the positional stability of right ascension and declination, meaning a combination of WRMS, χ^2 , and formal error), and the compactness of the sources (source structure index).

In this section, only solution GSF005a is studied. The entire set of sources consists of 1206 sources, but only 519 passed the first step of elimination as showing good characteristics of consistency for statistical study, removing at this step 20% of the defining sources.

We looked at the sky distribution of the stable sources. If we stop the selection at the 50 most stable sources, 75% are defining sources. If we stop at 260, 60% are defining sources, and if we look at the overall sky distribution of those 260 sources, 80% of them are in the Northern Hemisphere. The less stable sources are then the ones in the Southern Hemisphere.

The full overview of the distribution of the most stable sources shows that the choice of the defining sources is finally a good compromise of statistical stability and good sky distribution.

4.2. Comparison Between Different Solutions

We compare here the three solutions (GSF005a, AUG24, and OPA) over two different periods (1989.5-2009.5 and 1999.5-2009.5). We plot $(S_j, M_j)_{j=1..N}$ as a function of the bin number considered as an increasing number of sources and we obtain the plots in Figure 2.

These plots show various information. First, solutions GSF005a and OPA show similar results, whereas AUG24 has the highest values of S_j per number of sources, implying that the non-global sessions are increasing the level of noise and decreasing the stability of the Celestial Reference Frame. Second, when we compare the two periods of time, the past ten years shows better stability than the last twenty years of VLBI observation. This can be attributed to various improvements in VLBI observing and data taking, such as receiver improvements, use of new strong stations (such as the ten VLBA antennas), increased spanned bandwidths, increased recorded bandwidths, and larger network sessions (such as the weekly 6-8 station R1 and R4 sessions). Finally, the comparison between GSF005a and OPA shows that the selection of a set of stable sources is not unique and depends on the analysis strategy. Indeed, if we take for example the bin containing 200 stable sources for GSF005a and the bin of 200 stable sources for OPA, the sources obtained are not the same: only 80% of the sources are common.

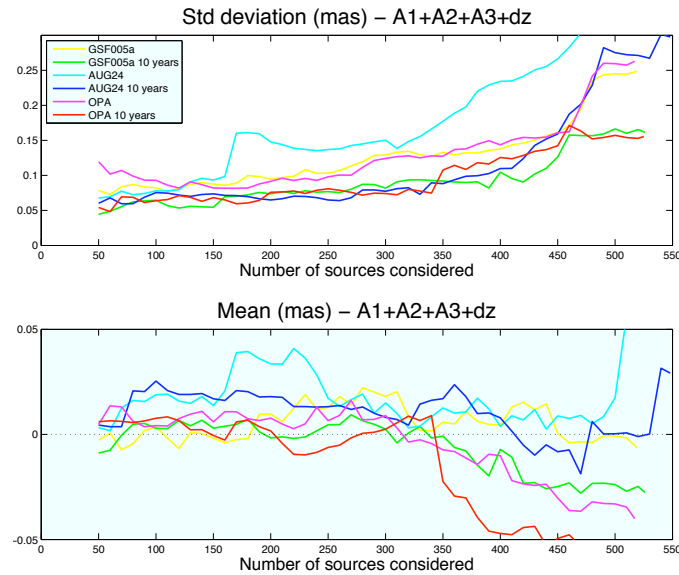


Figure 2. ICRF2 stability comparison. Three solutions are studied over two different periods of time (1989.5 to present and 1999.5 to present): GSF005a and AUG24, processed at GSFC (AUG24 contains the bad and the mobile sessions), and OPA at the Paris Observatory. These graphs compare the stability of the subsets of stable sources selected in each solution, by computing the standard deviation and the mean of the sum of the parameters of transformation between a yearly and a mean CRF realization of these subsets. These parameters are three rotations (A_1, A_2, A_3) and a fictitious declination bias dz .

5. Discussion on the Source Noise Stationarity Assumption

To explain why a set of stable sources is not unique and by noticing the better stability over the ten last years, we investigated the noise stationarity assumption in the different time series. For a set of twenty sources, the statistical study over two different periods of the time series show two completely different conclusions. The example of 3C418 is given in Figure 3. A first study processing the data from 1989 to 1993 shows white noise at the level of $100\mu\text{as}$ for both coordinates (Allan variance plots on the left side of the figure) and another study processing 1997 to 2009.5 shows a combination of white noise and flicker noise, with a level for the flicker noise as low as $50\mu\text{as}$ for both coordinates (Allan variance plots on the right side of the figure).

6. Conclusion

The statistical analysis of this study shows the 295 ICRF2 defining sources are a good compromise of statistical stability and good sky distribution. When looking at the full set of ICRF2 sources, we noticed that the selection of a set of stable sources is not unique and depends on the analysis strategy used for the processing (sessions used, analysis strategy). It may be of interest to compare in detail the impact of differences in the analysis strategy as well as the software used.

The investigation over the last ten years shows better stability, pointing to various VLBI improvements in the last decade, but also that the noise of the source time series is not a stationary

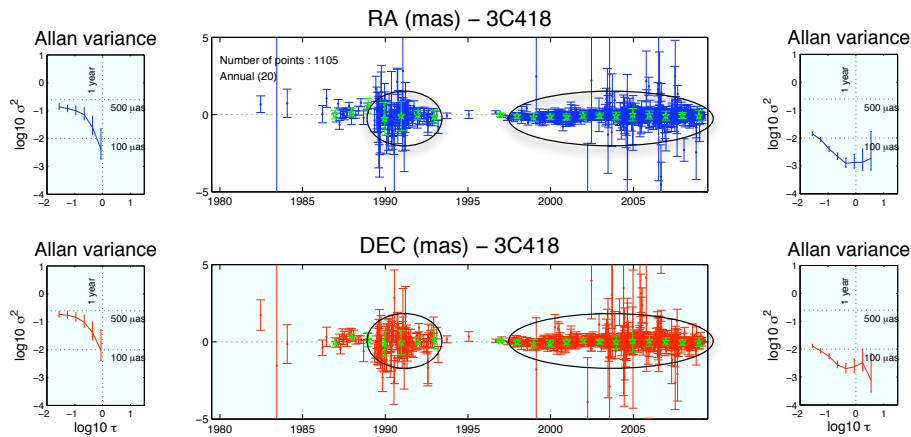


Figure 3. Example of non-stationarity: 3C418. The central part of the figure is the time series of the ICRF2 coordinates of 3C418 in mas (Up: Right ascension \times cos (declination); Bottom: Declination). On the left side, the Allan variances are computed from the 1989-1993 time series, on the right side, from the 1997 to 2009.5 time series. The Allan variance plots show the Allan variance (σ^2) function of the sampling time (τ) in a log10-log10 base.

process. An alternative in the selection of stable sources could be to consider only the stable periods of observation of the sources, instead of keeping only as a base for the Celestial Reference Frame the sources showing good statistical stability over their entire observation period.

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