Implementation of CAPIO for Composite Adaptive Control of Cross-Coupled Unstable Aircraft

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This paper presents an implementation of a recently developed control allocation algorithm CAPIO (a Control Allocation technique to recover from Pilot Induced Oscillations) for composite adaptive control of an inertially cross coupled unstable aircraft. When actuators are rate-saturated due to either an aggressive pilot command, high gain of the flight control system or some anomaly in the system, the effective delay in the control loop may increase due to the phase shifting between the desired and the achieved system states. This effective time delay may deteriorate the performance or even destabilize the system in some cases, depending on the severity of rate saturation. CAPIO reduces the effective time delay by minimizing the phase shift between the commanded and the actual attitude accelerations. We present simulation results for an unstable aircraft with cross-coupling controlled with a composite adaptive controller in the presence of rate saturation. The simulations demonstrate the potential of CAPIO serving as an effective rate saturation compensator in adverse conditions.

I. Introduction

Actuator rate saturation introduces an effective time delay (see Fig. 1) which, in general, degrades the system performance. If the rate saturation persists, it may destabilize the system and cause drastic results such as a Pilot Induced Oscillation (PIO).[1] The authors' earlier work on CAPIO (a Control Allocation technique to recover from Pilot Induced Oscillations)[2,3] addressed this problem by proposing an algorithm in control allocation framework which enabled phase compensation without the need of actuator ganging. CAPIO had two modes: Synchronization and tracking. In synchronization mode the goal was minimizing the derivative of the error between the desired and the actual control inputs, resulting in minimization of the phase shift. In the tracking mode the error between the desired and the actual control inputs is minimized, resulting in the convergence of the actual control input to the desired control input. Thanks to this dual behavior, phase minimization was achieved without any constant bias formation. It was assumed in the earlier work on CAPIO[2] that there exists a PIO detector on board, which enables the switching between CAPIO modes depending on whether there a PIO is detected or not. In this paper, we demonstrate that CAPIO can be used as a general phase compensator or a rate saturation anti windup algorithm regardless of an occurrence of a PIO event. This is achieved by using error minimization and the derivative error minimization simultaneously using appropriate weighting.

There are a handful of approaches for the input saturation problem in adaptive control.[4–8] These approaches in general includes the modification of the adaptive laws or the adaptive control signal itself to ensure the stability of the overall system in the presence of actuator saturation. In this paper we approach the problem from a control allocation standpoint where we employ our phase compensator CAPIO without altering the original adaptive controller. A control allocation approach has several advantages. First, one can separate the controller design and the control input distribution which enables reconfiguration without changing the controller design in the case of a failure or an objective change.[9] Second, in the presence of

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redundant actuators, secondary objectives can be achieved together with a primary objective. For example, in flight control, drag shaping can be achieved together with trajectory following. For an introduction to control allocation, see Harkegard.[9]

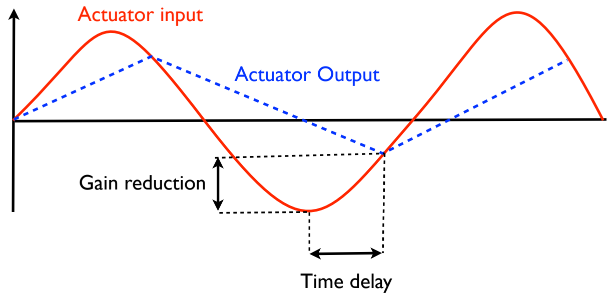

Figure 1. Input u and output δ of a rate saturated actuator.

To present the capabilities of CAPIO we use an unstable, inertially cross coupled aircraft model which is controlled by a combined/composite adaptive control architecture.[10–13] We use the architecture given in Stotsky.[12]

II. System Model

To show the advantages of CAPIO, a flight control example using a simplified[14] ADMIRE model[15,16] is used with some modifications to simulate inertial cross coupling.

The linearized aircraft model at Mach 0.22, altitude 3000m is given by

$$
\begin{aligned}
x &= [\alpha \quad \beta \quad p \quad q \quad r]^T - x_{\text{lin}}, \\
y &= Cx = [p \quad q \quad r]^T - y_{\text{lin}}, \\
\delta &= [\delta_c \quad \delta_{re} \quad \delta_{le} \quad \delta_r]^T - \delta_{\text{lin}}, \\
u &= [u_c \quad u_{\text{re}} \quad u_{le} \quad u_r]^T - u_{\text{lin}} \\
\begin{bmatrix} \dot{x} \\ \dot{\delta} \end{bmatrix} &= \begin{bmatrix} A & B_x \\ 0 & -B_\delta \end{bmatrix} \begin{bmatrix} x \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ B_\delta \end{bmatrix} u,
\end{aligned}
\tag{1}
$$

where α, β, p, q and r are the angle of attack, sideslip angle, roll rate, pitch rate and yaw rate, respectively. δ and u represent the actual and the commanded control surface deflections, respectively. Control surfaces are canard wings, right and left elevons and the rudder. $(.)_{\text{lin}}$ refers to values at the operating points where the linearization was performed. The actuators have the following position and rate limits

$$
\begin{aligned}
&\delta_c \in [-55, 25] \times \frac{\pi}{180}; \quad \delta_{re}, \delta_{le}, \delta_r \in [-30, 30] \times \frac{\pi}{180} \\
&\dot{\delta}_c, \dot{\delta}_{re}, \dot{\delta}_{le}, \dot{\delta}_r \in [-70, 70] \times \frac{\pi}{180}
\end{aligned}
\tag{2}
$$

and have first order dynamics with a time constant of 0.05 seconds. It is noted that the position limits given are the same as the ones given by Harkegard[14] but the rate limits are assumed to illustrate CAPIO properties.

To make this model suitable for control allocation implementation, the actuator dynamics are neglected and the control surfaces are viewed as pure moment generators and their influence on $\dot{\alpha}$ and $\dot{\beta}$ is neglected. It is noted that the actuators dynamics are present during the simulations, i.e. they are neglected only during the control allocation algorithm derivation. These assumptions lead to the following approximate model

$$
\begin{aligned}
\dot{x} &= Ax + B_u u = Ax + B_v v, \\
v &= Bu,
\end{aligned}
\tag{3}
$$

where

$$
B_u = B_v B, \quad B_v = \begin{bmatrix} 0_{2\times 3} \\ I_{3\times 3} \end{bmatrix},
$$

$$
A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix}.
$$

The virtual (total) control effort, v, consists of the angular accelerations in roll, pitch and yaw. To simulate the effects of inertial cross-coupling, we modify the A matrix so that a change in pitch angular velocity creates a moment in roll and yaw axes:

$$
A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 1 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0.1 & -0.2127 \end{bmatrix} \tag{4}
$$

In this flight control example the goal of the inner loop adaptive controller is to regulate the roll, pitch and yaw angles at given reference values. The overall system configuration is given in Fig. 2.

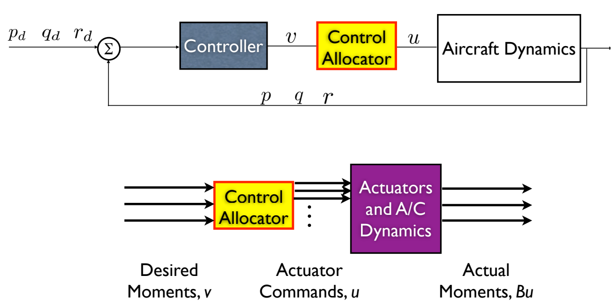

Figure 2. Overall MIMO system structure

The inner loop controller is a combined/composite adaptive controller[12] controller which uses p_d, q_d and r_d as references and produces the necessary attitude accelerations, $v \in \Re^3$, to track these references. This adaptive controller will be explained in more detail in the final manuscript.

The control allocator distributes this total control effort, v, to individual control surfaces via the actuator commands, $u \in \Re^4$. The control surfaces then produce actual attitude accelerations, Bu, where B is the control input matrix.

III. Simulation Results

III.A. Adaptive control with conventional control allocation

The conventional control allocation used in this example minimizes the following objective function

$$
J = ||Bu - v||_2^2 + \epsilon ||u||_2^2 \tag{5}
$$

subject to $max(\dot{u}_{min}T + u_{k-1}, u_{min}) \leq u \leq \min(\dot{u}_{max}T + u_{k-1}, u_{max})$, where T is the sampling interval. It is noted that norms, instead of square-norms, can be used in the objective function. Note that (5) is in the form of a typical objective function used in conventional control allocators,[17] where the main objective is to minimize the error between the desired and the actual total control efforts. As $\epsilon \to 0$, minimizing (5) becomes equivalent to achieving the main objective explained above and picking the solution that gives the minimum control surface deflection, among different solutions. In this example $\epsilon = 10^{-5}$.

Figure 3 represents the adaptive controller performance when a conventional control allocator used. System goes into sustained oscillations due to the effective time delay introduction of rate saturating actuators (see Fig. 4).

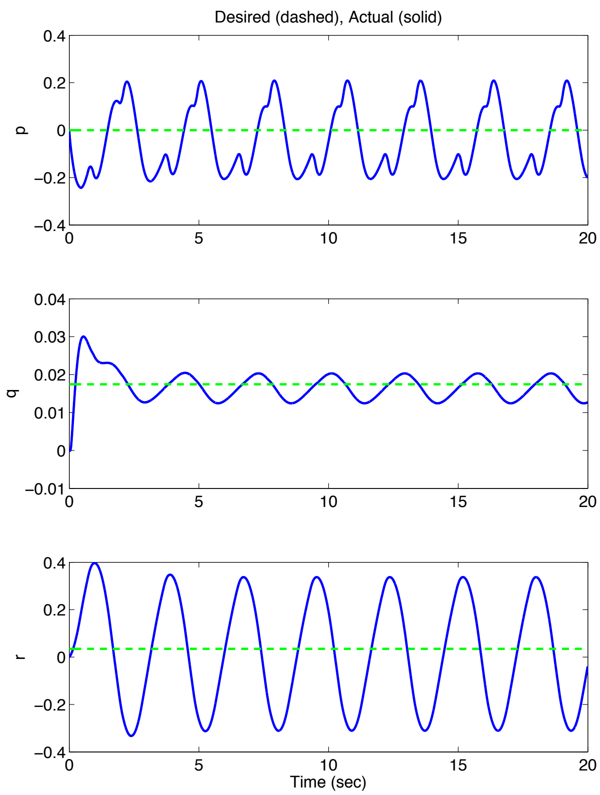

Figure 3. Regulation with adaptive controller + conventional control allocator

III.B. Adaptive control with CAPIO

To minimize the phase shift and thus effective time delay due to rate saturation, CAPIO forces the virtual (total) control effort v, to be in phase with the actual control effort Bu produced by the actuators. To achieve this, a derivative error term is added to objective function (5) to obtain the following CAPIO objective function

$$J^{'} = ||Bu - v||_2^2 + ||W_d(B\dot{u} - \dot{v})||_2^2 + \epsilon||u||_2^2 \tag{6}$$

where $W_d \in R^{3\times3}$ represents a weighting matrix on the derivative term. The cost function $J^{'}$ is minimized with respect to u, with $\dot{u} = (u - u^{-})/T$, where u^{-} denotes the value of u at the previous sampling instant. It is noted that with this modified objective function, the control allocator is trying to realize $\dot{v}$ as well as v.

Figure 5 represents the same adaptive controller performance when CAPIO is used as the control allocator. Since CAPIO is able to minimize the phase shift between the commanded and achieved total control input vectors, the system is stable and the controller can successfully force the aircraft attitude angular rates follow their desired references.

IV. Summary

In this extended-abstract, it was presented by simulations that the recently proposed control allocation scheme CAPIO is a reasonable candidate to be used as an effective phase compensator for an adaptively

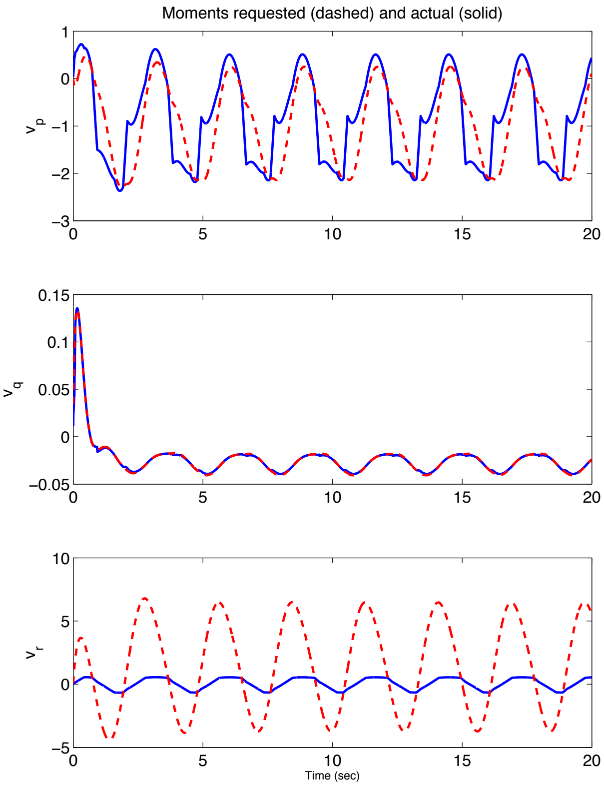

Figure 4. Requested and actual control inputs with adaptive controller + conventional control allocator

controlled unstable aircraft with inertial cross-coupling in the presence of input saturation. CAPIO achieves this by its dual-behavior: It minimizes both the errors and the derivatives of the errors between the commanded and achieved total control input vectors. By minimizing the derivative errors, it prevents effective delay introduction and by minimizing the errors themselves, it helps realizing the commanded control inputs. More details about the combined/composite adaptive controller and control allocator CAPIO will be provided in the final manuscript with supporting figures.

Acknowledgments

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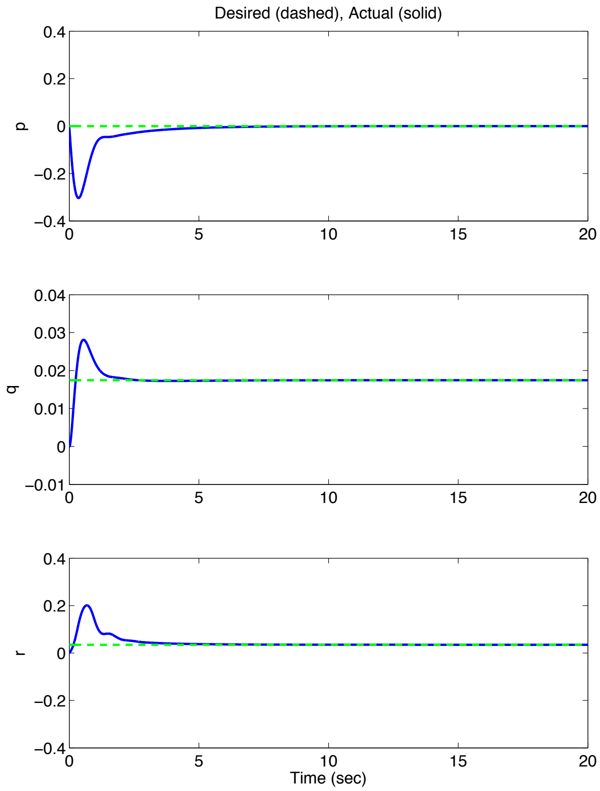

Figure 5. Regulation with adaptive controller + CAPIO

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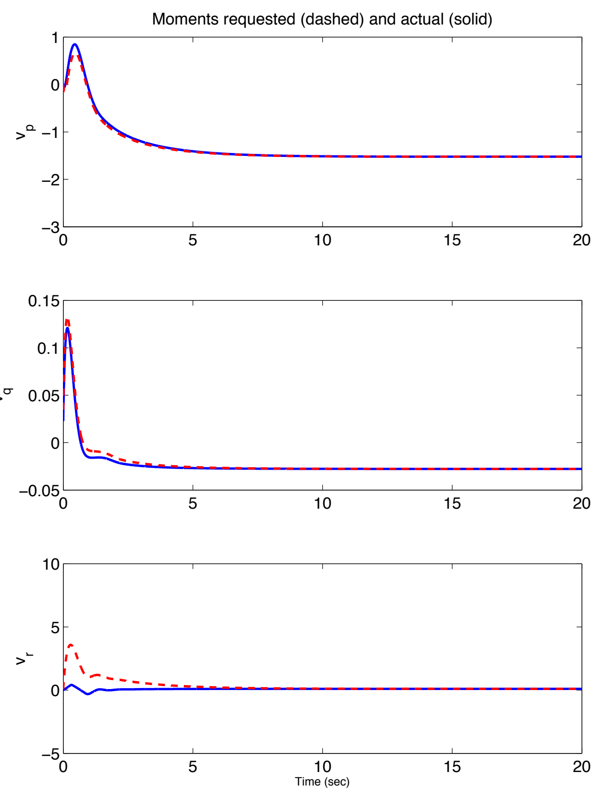

Figure 6. Requested and actual control inputs with adaptive controller + CAPIO