MESH-BASED ENTRY VEHICLE AND EXPLOSIVE DEBRIS RE-CONTACT PROBABILITY MODELING

Mark McPherson and Gavin Mendeck

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EXPLOSIVE BREAKUP

- Explosive breakup of jettisoned spacecraft segments during entry is a relatively poorly understood phenomenon

- ATV Jules Verne Breakup
http://www.youtube.com/watch?v=8os3Q0hZLfE

- Ejected fragments may collide with main entry vehicle and cause catastrophic damage that can result in mission failure

- Highly probabilistic event dependent upon jettisoned segment geometry, entry trajectory, component makeup, etc.
    - Difficult to predict breakup altitude or direction of ejected fragments
    - Makes deterministic assessment very challenging

PREVIOUS APPROACH TO DEBRIS RE-CONTACT

- Typically the re-contact problem has been approached using large set Monte Carlo runs that examine closest approaches to the entry vehicle
- However, re-contact events are rare, therefore even a large data set without conjunctions does not provide strong evidence for improbability
- Computationally expensive to generate very large sets of Monte Carlo data, especially when adding dispersions
- Hinders the ability to rapidly optimize separation sequences (jettison angles, separation burn magnitude, etc.)

MESH-BASED APPROACH

- Sought to develop a method that would aid in optimizing separation trajectories by minimizing the risk of debris re-contact to an entry vehicle in the case of an explosive breakup of a jettisoned segment

- Required a statistical approach to determine the likelihood of a debris strike given a certain spatial distribution of debris and separation trajectory

- Non-Homogeneous Poisson Process
  - Debris strike probability proportional to the cumulative debris flux along a particular trajectory

- Allows more robust analysis than simply considering the closest approach data from several thousand Monte Carlo runs

- Chosen because it is a relatively simple probabilistic model to explain and offers a reasonable approximation of the risk of re-contact

SIMULATION OF FRAGMENTS

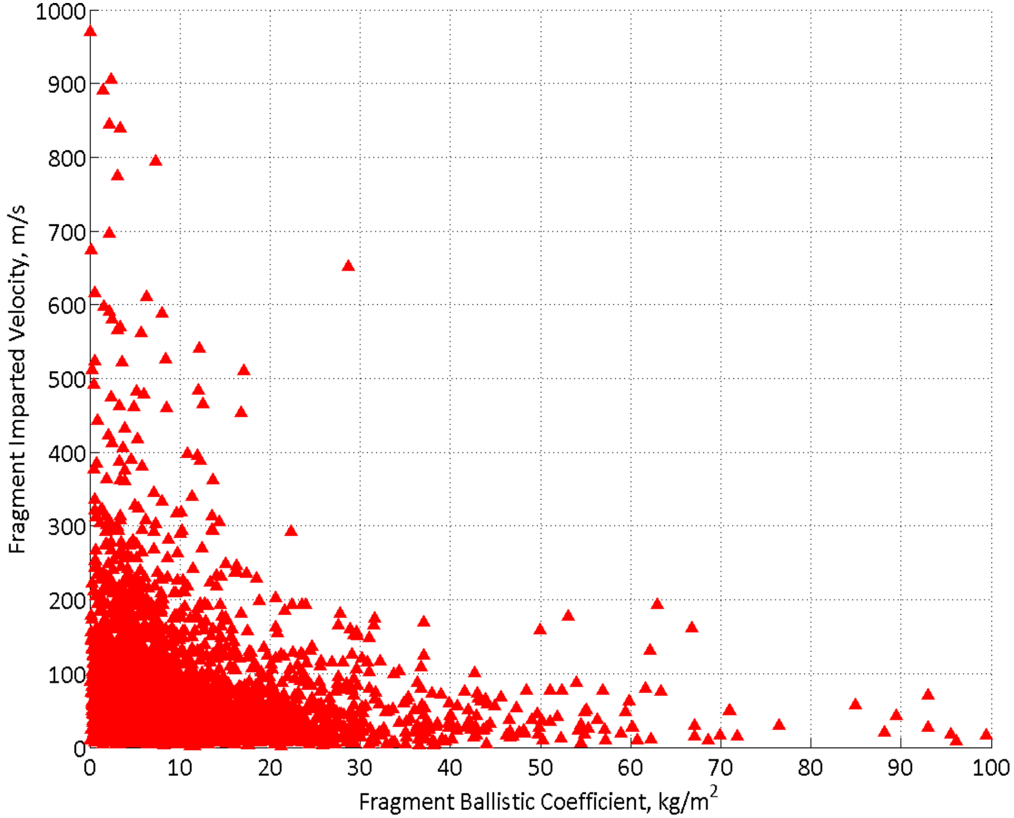
- Explosive breakup simulated by imparting an ejection velocity to each fragment in a random direction while assuming:
  - Instantaneous breakup
  - Constant drag model for debris pieces (no lift)
  - No collisions between fragments
  - No debris demise

- Empirical evidence from observed breakups can be used to determine fragment ejection velocities and ballistic coefficients
  - NASA Standard Breakup Model

NASA STANDARD BREAKUP MODEL

- Uses empirical data from gathered from observed orbital breakups
- Gives the number, area to mass (A/M) ratio and corresponding delta-v for fragments from the explosive breakup of spacecraft and upper stage rocket bodies.
- Ballistic coefficient easily calculated from A/M ratio, assuming a $C_D = 1$

Delta- V vs. Ballistic Coefficient for Explosive Breakup



DEBRIS DISTRIBUTION MAPS

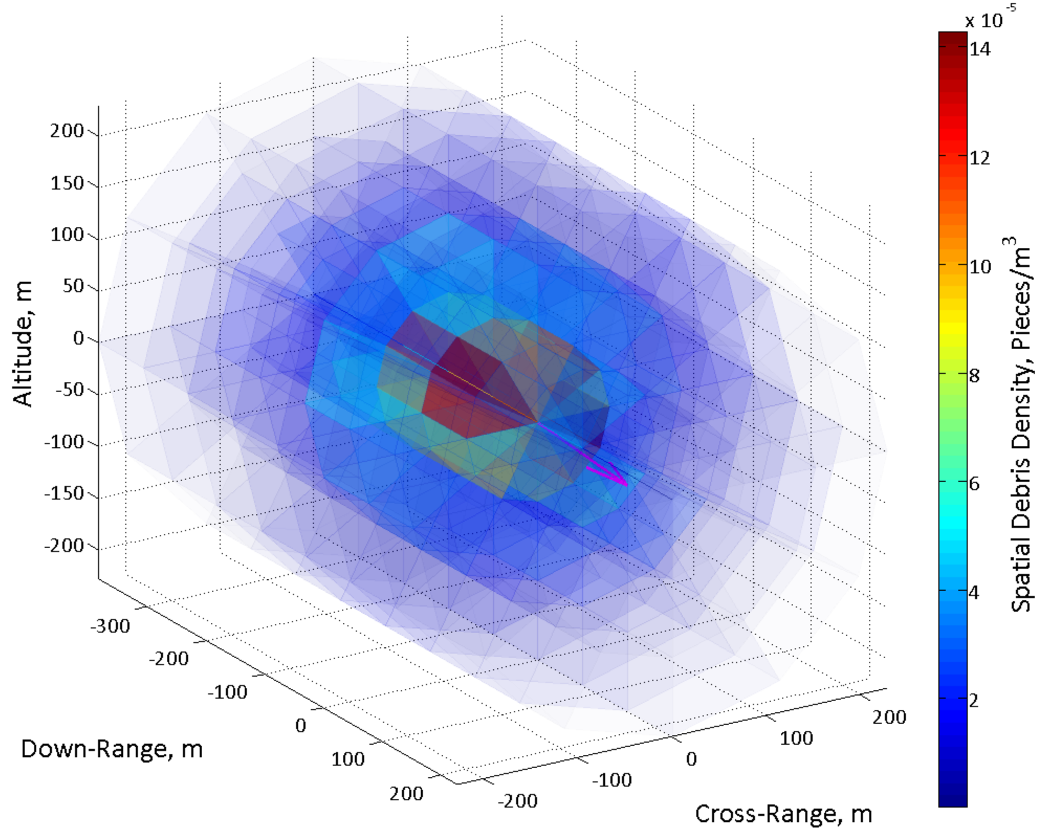
- Region of space around the jettisoned body divided up into a mesh grid describing volumetric cells

- Spatial debris density computed using the weighted number of fragments inside each cell at any point in time

$$\bar{\rho}_D = \sum_{i=1}^{n} \eta_i \cdot \rho_{D,i}$$

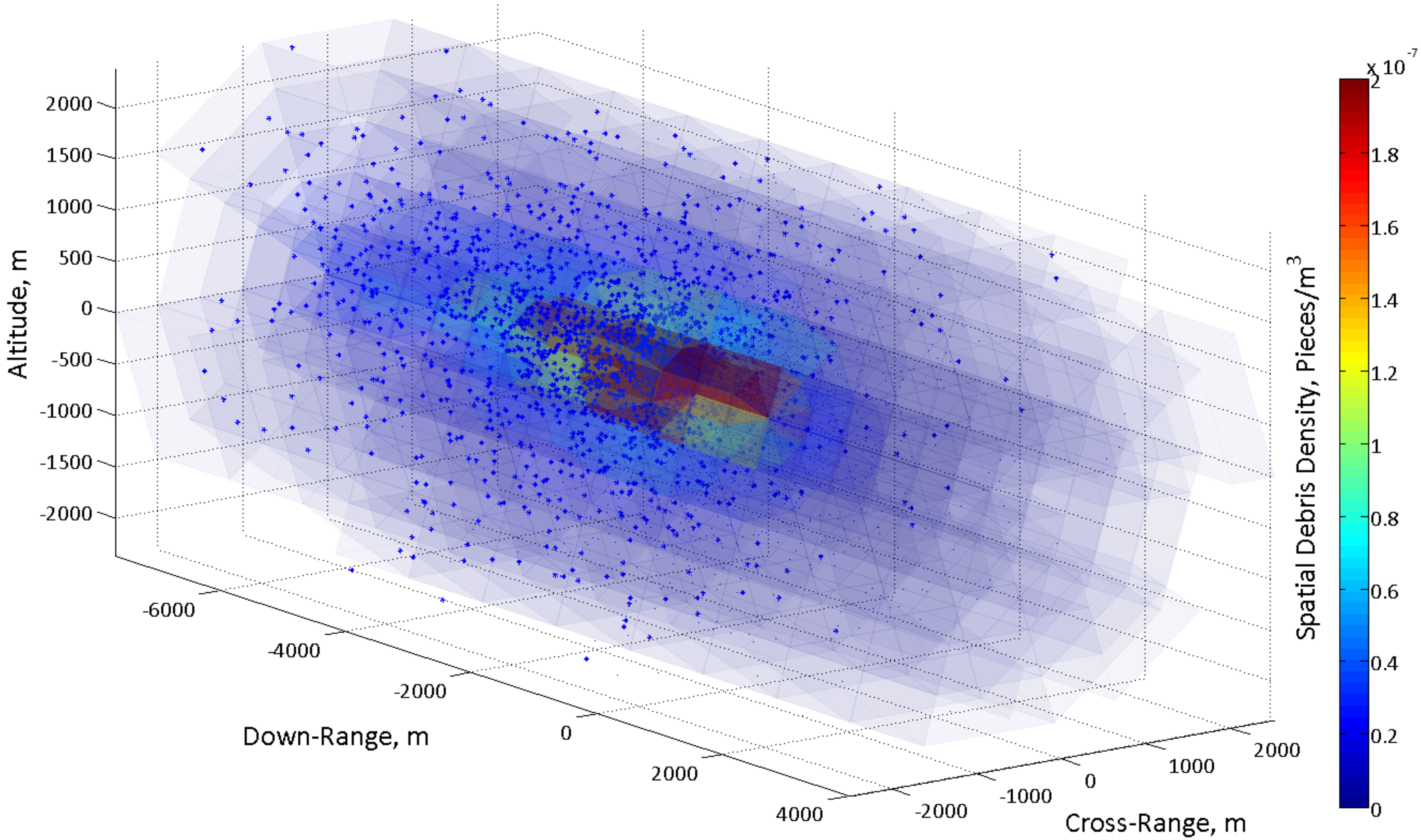
$$\eta_i = \frac{N_{A,i}}{N_{S,i}}$$

Debris Density Map T+3s after breakup
(*Have spherical and Cartesian equivalent as well*)



DEBRIS DISTRIBUTION MAPS

Evolution of Debris Density Map with Time (w/ debris fragments overlaid)

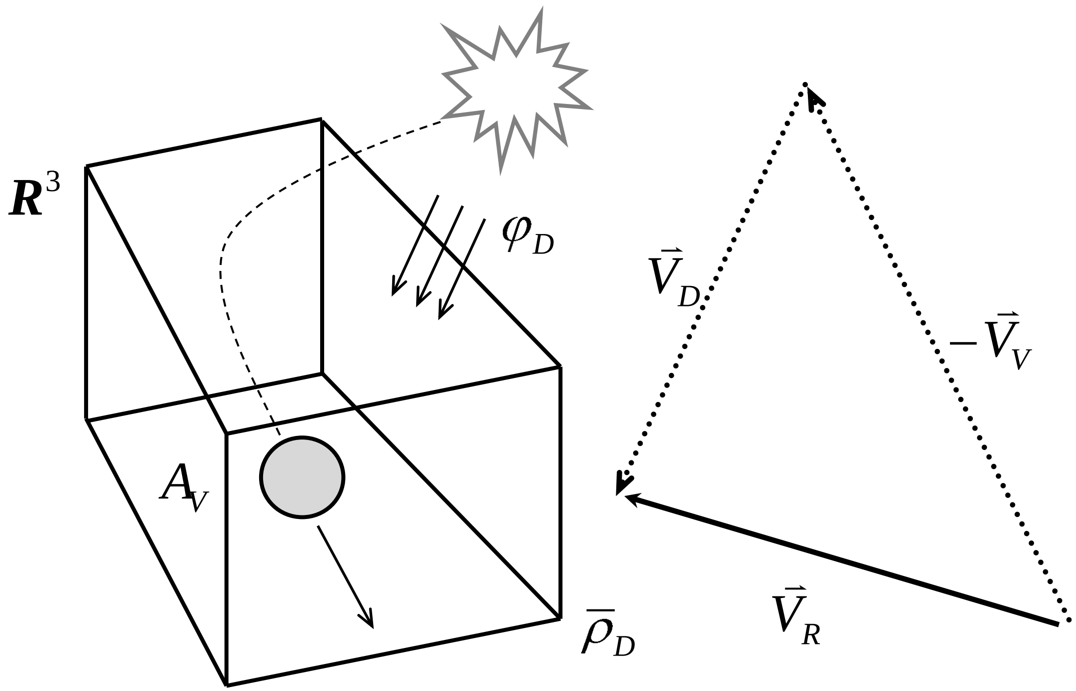


DEBRIS FLUX

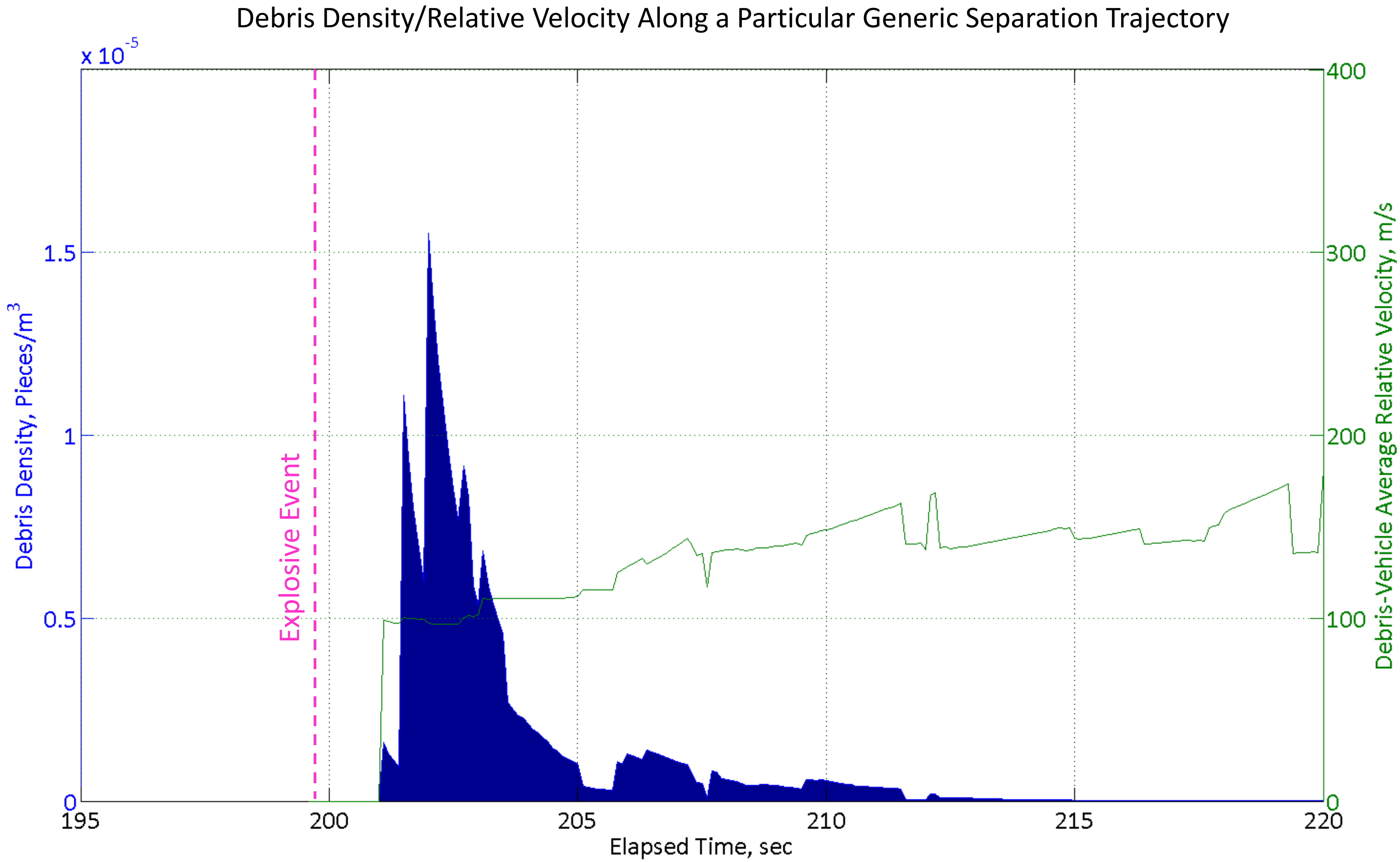
- As the entry vehicle moves along its trajectory relative to the jettisoned body and its fragments, it moves through the meshed region of space
- Use weighted spatial density inside a cell to calculate debris flux, φ_D at each point in time along a particular trajectory

$$\varphi_D(t) = \bar{\rho}_D(t) \cdot \left|\vec{V}_R(t)\right|$$

$$\vec{V}_R(t) = \vec{V}_D - \vec{V}_V$$



DEBRIS DENSITY ALONG A TRAJECTORY



APPLICATION OF STATISTICAL MODEL

- Using this debris flux we can determine the intensity parameter λ, as a function of time along a separation trajectory

- $\lambda(t)$ is the product of the debris flux and the cross-sectional area of the entry vehicle, taken as a constant equal to its maximum circular area, A_V

$$\lambda(t) = \varphi_D(t) \cdot A_V$$

- This can then be integrated over the interval $[t_1, t_2]$ to give the cumulative intensity parameter, Λ and therefore, the total probability of re-contact along a particular trajectory.

$$\Lambda = \int_{t1}^{t2} \lambda(\tau) d\tau$$

$$P\{X \geq 1\} = 1 - e^{-\Lambda}$$

ASSUMPTIONS OF STATISTICAL MODEL

- A debris strike is a rare event such that,

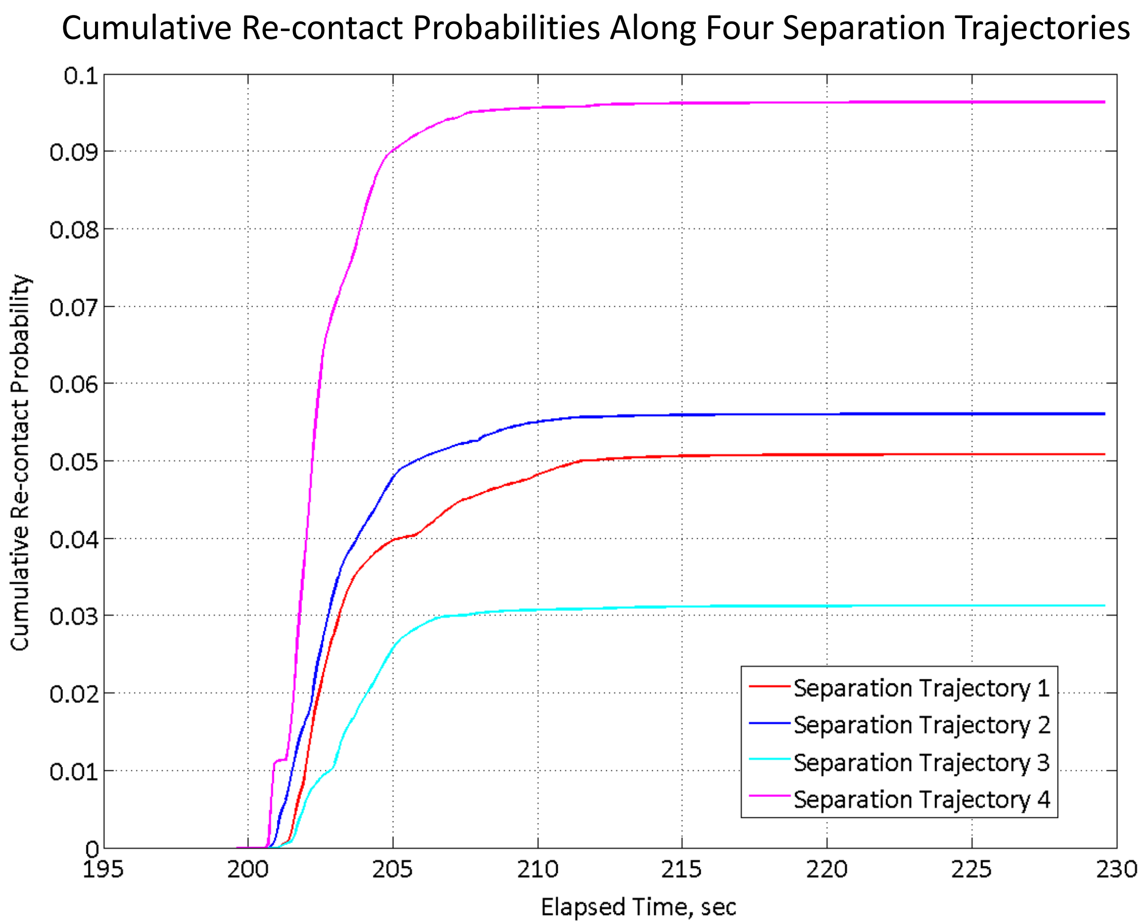
$$P\{X > 1\} \approx 0$$

- The probability of a debris strike at any time is an independent random variable,

$$P\{X_1 \mid X_2\} = P\{X_1\}$$

- Then frequency with which a strike is expected to occur is dependent only upon the debris flux (φ_D) in a region of space or volumetric cell
- Debris fragments are small relative to the entry vehicle
- The entry vehicle can be modeled as a sphere with uniform cross-sectional area in all directions

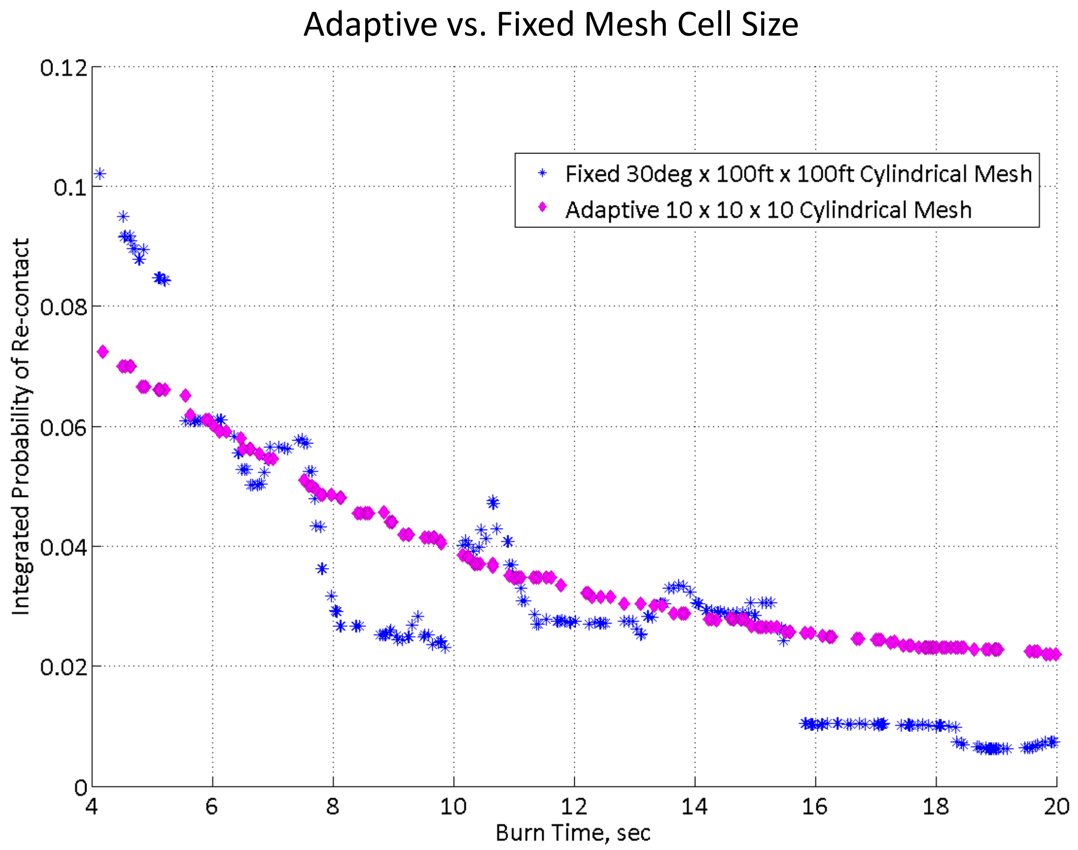
CUMULATIVE RE-CONTACT PROBABILITY



- Useful information from an operations perspective, to predict when an entry vehicle is 'under threat' from debris

SELECTION OF MESH CELL SIZE

- Adaptive method for selection of cell size allows cells to grow with the debris cloud
- Reduces discretization errors owing to the sparsity of fragments, particularly at greater distances from the blast
- More effective at capturing the evolution of the fragments with time
  - Volume of the cells grow as the uncertainties in fragment position grow



ADVANTAGES OF THIS APPROACH

- Intuitive explanation of the likelihood of a debris strike - ‘good’ and ‘bad’ regions of space are easily visualized
- Computationally efficient for optimizing entry vehicle separation trajectories
  - Only requires one set of debris data to run several trajectory cases
- Robust enough to handle a wide variety of separation maneuvers
  - Posigrade, retrograde, out-of-plane burns
- Allows capturing of additional data including relative velocity and ballistic coefficients of debris pieces
  - Capability to then use this to additionally calculate a consequence of impact metric, and hence the risk of a debris strike

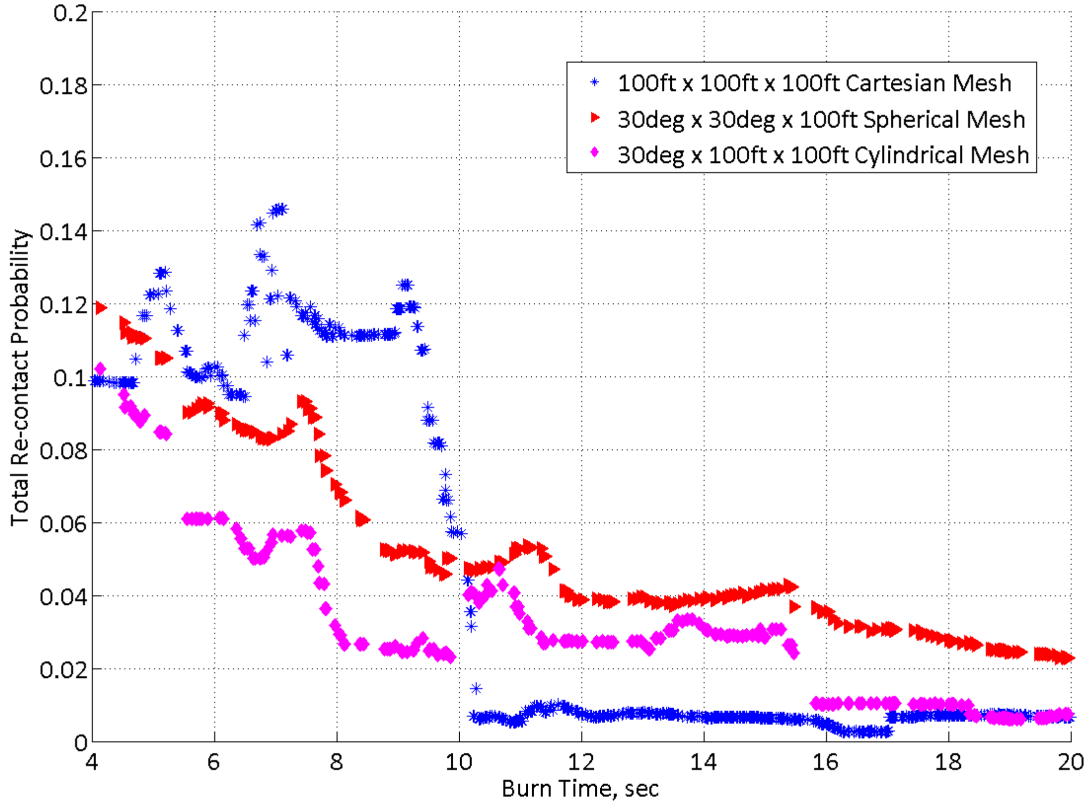
QUESTIONS

BACK-UP SLIDES

SELECTION OF MESH TYPE

- Mesh type can lead to discretization error where debris cloud is not well described by the mesh
  - Boundaries of cells don't match the debris cloud
  - Insufficient fragments modeled for the size of the cells
- Cartesian mesh tends to exaggerate these errors due to fixed volume cells
- Careful selection of size and shape must be made to avoid these errors in the model

Total Re-contact Probability for Three Mesh Types as Separation Burn Time is Varied



DERIVATION OF POISSON PROCESS PROBABILITY

The probability of k events occurring can be modeled using a Non-Homogeneous Poisson Process according to the following formula:

$$P\{X = k\} = \frac{(\Lambda)^k e^{-\Lambda}}{k!}$$

where,

$$\Lambda = \int_{t_2}^{t_1} \lambda(\tau) d\tau$$

If we consider the probability of k = 0 (no strikes), then the probability of one or more strikes occurring is given by:

$$P\{X = 0\} = \frac{(\Lambda)^0 e^{-\Lambda}}{0!}$$

$$\Rightarrow P\{X = 0\} = e^{-\Lambda}$$

$$\Rightarrow P\{X \geq 1\} = 1 - e^{-\Lambda}$$