Capability Extension to the Turbine Off-Design Computer Program AXOD With Applications to the Highly Loaded Fan-Drive Turbines

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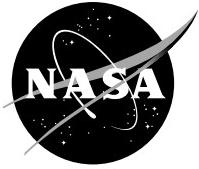
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Summary

The axial flow turbine off-design computer program AXOD has been upgraded to include the outlet guide vane (OGV) into its acceptable turbine configurations. The mathematical bases and the techniques used for the code implementation are described and discussed in lengths in this paper. This extended capability is verified and validated with two cases of highly loaded fan-drive turbines, designed and tested in the V/STOL Program of NASA. The first case is a 4 1/2-stage turbine with an average stage loading factor of 4.66, designed by Pratt & Whitney Aircraft. The second case is a 3 1/2-stage turbine with an average loading factor of 4.0, designed in-house by the NASA Lewis Research Center (now the NASA Glenn Research Center). Both cases were experimentally tested in the turbine facility located at the Glenn Research Center. The processes conducted in these studies are described in detail in this paper, and the results in comparison with the experimental data are presented and discussed. The comparisons between the AXOD results and the experimental data are in excellent agreement.

1.0   Introduction

AXOD is a well established, well tested preliminary axial-flow turbine off-design computer program. Its evolution and its usages in aircraft turbine off-design performance analysis are documented in Reference 1. A theoretical analysis and discussion on its mathematical foundation and formulations are described in Reference 2. AXOD has a matching design code, TD2-2, as its companion. The evolution and the usages of TD2-2 are documented in Reference 3. The performance of both AXOD and TD2-2 has been verified independently in Reference 2 with selected cases. The purpose of this work is to extend the current capability of AXOD to include an outlet guide vane into its acceptable turbine configurations, so that it can become functionally more complete and more valuable, and to be fully compatible with the design capability of the design code TD2-2, which already has the capability of including the outlet guide vane into its design practice (as described in Ref. 3). Since the bulk of work presented in this paper actually is related to the computer code construction, a more suitable reference for the preview is Reference 4, in which the detailed layout of the structure of AXOD is presented. Although the content of this paper is not exclusively about code construction, but will provide in depth descriptions as appropriate.

The arrangement of this paper is the following: The topology of the existing code structure and the rationale for the construction of an outlet guide vane are described; followed by the derivations of the analytical method applied to blank-out the unwanted rotor blade row. The logic of termination of the program execution is updated. And lastly this extended code capability is validated and verified through the studies of two cases on highly loaded fan-drive turbines, designed and tested in the V/STOL Program of NASA in the 1970's, in which the outlet guide vanes are an intricate part of the turbine configurations. The results from these studies, in comparison with the experimental data, are presented and discussed.

2.0   Topology

AXOD is constructed with the stator and the rotor as two separate entities. Stators are envisioned in the absolute frame-of-reference, while rotors are envisioned in the relative frame-of-reference. In between the stator and the rotor, an annulus region connects the two. Solutions are processed and obtained only at

the starting and the ending stations of each component: the stator, the annulus, the rotor, and the annulus that follows. The information of the component losses needed to resolve the flows at a station is obtained via loss-modeling, as was described in References 1 and 2; the annulus region, however, is treated as if there were no loss (isentropic) in the flow domain. The switching of the frame-of-references between the stator and the rotor is executed in the annulus region. The annulus region after the stator receives the flow conditions from the stator discharge in the absolute frame, and calculates the inflow conditions, in relative frame, for the rotor inlet. The annulus after the rotor receives discharge flow conditions in the relative frame from the rotor, and calculates the inflows needed, in absolute frame, for the following stator inlet.

In the existing structure of AXOD, a dedicated subroutine is constructed for each of the four components, again, they are: the stator, the annulus that follows, the ensuing rotor, and the annulus that follows the rotor. The turbine can be a multi-stage turbine, but each of the turbine stages must be a full-stage turbine configuration. To implement a half-stage (an outlet guide vane) to the existing code structure, one can consider adding another dedicated subroutine just for the half-stage configuration, but that would be breaking the existing structure of code, and it is difficult and laborious. A better way to do this, which is adopted in this work, is to maintain the existing code structure but to analytically and computationally "blank-out" the last rotor-row (on request, by setting the parameter KOGV = 1 in the new code), such that the last turbine stage consists, essentially, of a stator (the outlet guide vane), the annulus that follows, the blanked-out rotor which is equivalent to another annulus region, and the actual annulus that would exist after the rotor. All existing program logic and output utilities are intact and are functional with only modest amount of modifications needed, effectively no new code construction, such as additional subroutine, is been added.

The blanking of the last rotor-row is done in the most generic fashion, which means one can just take a given turbine, be it a multi- or a single-stage turbine with the full-stage configuration, and simply blanks-out the existing last rotor-row on request. In fact, the functionality check-out of the code during the capability extension was done exactly this way, by taking a given turbine, with fully specified physical dimensions and blade angles, and simply blanked-out the existing last rotor-row, transforming it into an annulus-equivalent. The check-out tests were conducted on four different given turbine configurations, the GE-E[3] HPT (a 2-stage, air cooled, High Pressure Turbine), the GE-E[3] LPT (a 5-stage Low Pressure Turbine), a lift-fan turbine (with 4 full stages), and a single stage transonic turbine, the outcome of these tests were deemed fully successful. Since these are only check-out tests, the results will not be shown nor will these cases be referenced in this paper. However, the input-files of these cases are stored in the data bank of AXOD and are available, one can easily recreate these tests by simply adding a command-line "KOGV = 1" into the input-namelists. Physically however, blanking-out a given full-stage turbine results in ill-behaved outlet guide vane, because a turbine stator is physically a nozzle that accelerates the flow, while a properly designed outlet guide vane is actually a diffuser that decelerates the flow. Two validation cases conducted on two published fan-drive turbines, with properly designed outlet guide vanes, are provided in the "Results and Discussion" section of this paper.

3.0   Mathematical Formulations

In this section, the method of transforming a given rotor-row into an annulus-equivalent is discussed. The material in this section is best understood if read with References 2 and 4 together.

Computationally, a rotor-row is handled in the relative frame by specifying a speed of rotation of the turbine wheel. The solution algorithm implemented in AXOD is a marching scheme. All necessary inflow conditions are known from the calculations conducted in the preceding annulus region. To obtain the flow solution at the station of rotor discharge, typically a set of five first-principle equations are solved. For the rotor, these equations are casted into the mathematical form of the relative frame of reference.

The equation for the conservation of mass is:

$$\omega = \rho_2 W_{x2} * A_2 * (1 - YC_R),$$ (1)

where, ω is the mass flow rate through the rotor exit, W_{x2} is the relative axial velocity of the flow at discharge, A_2 is the sector area of the annulus at the rotor exit, and YC_R is the area blockage loss factor of the rotor.

The streamwise momentum balance is expressed as:

$$Y_R = (P'_{t2s} - P'_{t2})/(P'_{t2} - P_2),$$ (2)

where Y_R is the streamwise total pressure loss coefficient for the rotor-row, the P'_{t2s} and P'_{t2} are, respectively, the ideal and the actual relative total pressures at the rotor exit, while P_2 is the static pressure at the rotor exit. This is the symbolic equation used to represent the streamwise momentum loss. Actually in AXOD, the streamwise momentum loss across a blade row is casted into two separate mechanisms with two loss factors: YA_R and YB_R (the exact formulas are given in Reference 2). Since the five principal equations are inter-connected, when envisioned as the loss of relative total pressure $(P'_{t1} - P'_{t2})$ across the rotor blade row, all three loss factors, YA_R, YB_R, and YC_R, are contributing to this loss.

The equation for the angular momentum conservation is:

$$PW = [U_2(W_{u2} + U_2) - U_1(W_{u1} + U_1)] / GJ,$$ (3)

where PW is the specific work-extract from the rotor. The U_1 and U_2 are the rotor blade velocities, and the W_{u1} and W_{u2} are the relative tangential flow velocities, at the rotor inlet and at the rotor exit, respectively. The G and J are the unit conversion factors. This equation is commonly known as the Euler equation, or the angular moment-of-momentum equation.

The radial equilibrium condition applied is:

$$\frac{G}{\rho_2} \frac{dP_2}{dr} = \frac{(W_{u2} + U_2)^2}{r}.$$ (4)

and lastly the energy equation is:

$$T'_{t2} = T'_{t1} + (U_2^2 - U_1^2)/2GJ\overline{C_p},$$ (5)

where T'_{t1} and T'_{t2} are the relative total temperatures at the rotor inlet and at the rotor exit, respectively. $\overline{C_p}$ is the specific heat at constant pressure, averaged off the two stations.

In addition to the five principal equations, a set of supplementary velocity component equations are needed. These are:

$$W = \sqrt{W_x^2 + W_u^2 + W_r^2}$$ (6a)

$$\beta' = \tan^{-1}(W_u / W_x)$$ (6b)

$$\upsilon' = \tan^{-1}(W_r / W_x),$$ (6c)

where, W is the relative total velocity of the flow, W_r is the radial velocity component, β' is the tangential flow angle at station of the rotor, and υ' is the meridional streamline slope angle of rotor.

The five principal equations are solved at the rotor discharge for the five primary unknowns that constitute the complete set of the flow solution. They are: the static pressure (P), the static temperature (T), and the three velocity components (W_x, W_u, W_r). How the equations are solved in AXOD is a rather complicated matter, this procedure is described in large in Reference 2. In order to blank-out a rotor, the issue of importance, and is discussed here is the well-posedness of the system equations. Although there are five principal equations, however, the radial equilibrium condition (Eq. (4)) is merely a differential equation, this condition has to be solved iteratively with the mass conservation (Eq. (1)) together until the continuity condition is satisfied, to obtain the absolute value of the static pressure. Thus Equations (1) and (4) together provide only one independent relation. Furthermore, the specific work-extract, PW, in AXOD is a resultant, not a designed quantity, since AXOD is solving the direct problem, not the inverse problem, thus the angular moment-of-momentum equation, Equation (3), is a redundant condition that provides no meaningful relation in solving the unknowns. This point becomes clearer when the wheel speed is set to zero as of the stator, in that case, Equation (3) is merely a $0 = 0$ identity. Therefore out of the five principal equations, only three independent relations are available. To solve for the system of five unknowns, two additional, supplementary conditions must be given. And the two conditions provided are Equations (6b) and (6c) of the supplementary velocity component equations. Essentially, the flow solution is obtainable if and only if one assigns the two flow angles: β' and υ'.

Between the two, the meridional streamline slope angle, υ', is of little impact to the present discussion, since it only provides the fraction of magnitude of the radial velocity component, acting solitarily. The standard procedure of defining υ' is to average off the end-wall slopes of the adjacent components, and interpolated onto the sectors. To blank-out a rotor, this needs not be changed. The tangential flow angle, β', on the other hand, is of great importance to the outcome of the flow solution. For subsonic flow, the β' at a real rotor discharge is assigned to be the same as the discharge rotor blade angle (deviation is not been accounted for in AXOD), and in the event when the sector outflow is supersonic (choked flow over-expansion at the sector discharge), the tangential flow angle at the rotor discharge is assigned to the value of supersonic flow-deflection angle, which maintains the sector mass flow rate to the value of the critical mass flow rate obtained from the 1-D flow function (see Reference 4 for this treatment). With this assignment, a set of corresponding turbine flow solution, (P, T, W_x, W_u, W_r), is obtained.

Since the flow solution depends essentially on the value of β' assigned, in order to blank-out the rotor, one only has to over-write the built-in β' correlations and reassign them to a value that is consistent with the flow without the blades of a rotor.

The specific steps taken for transforming a rotor into an annulus-equivalent are the followings:

1. First and foremost, the wheel speed of the blanked-out rotor is set to zero individually. This makes all quantities of the relative frame become identical to the expressions of the absolute frame. The rotor is transformed essentially into a stationary stator.
2. All losses of the rotor are set to zero, since the flow in the annulus region is considered isentropic. No coolant flow injection is allowed in the blanked-out rotor, which is physically realistic since the exit annulus of a turbine is never cooled.
3. The incidence angle at the rotor inlet is set to zero. Since the flow deflection at inlet due to circulation (the loss factor YA_R) has been set to zero, this implies the rotor inlet blade angle should be inline with the approaching inflow angle, thus the given rotor inlet blade angle is over-written by the inflow angle.
4. The discharge flow angle is calculated by satisfying simultaneously the two equations: the mass conservation (Eq. (1)) and the angular momentum conservation in the absence of tangential blade force (deducible from Eq. (3)). In the absence of blockage loss YC_R, these equations read:

$$\rho_1 * A_1 * W_{x1} = \rho_2 * A_2 * W_{x2} \; , \tag{7}$$

$$(W_{u1} + U_1) * D_1 = (W_{u2} + U_2) * D_2 \; , \tag{8a}$$

D_1 and D_2 are the sector meanline diameters at the inlet and at the discharge, respectively.
Since the wheel speed is set to zero, Equation (8a) becomes

$$W_{u1} * D_1 = W_{u2} * D_2 \tag{8b}$$

From Equations (7) and (8b), and the definition of β' given in Equation (6b), the outflow angle of the blanked-out rotor is found to be:

$$\beta'_2 = \tan^{-1}(\tan(\beta'_1) * (D_1 / D_2) / (\rho_1 / \rho_2 * A_1 / A_2)) \tag{9}$$

In the code, β'_2 is adjusted iteratively with the continuity condition together, this outflow angle over-writes the built-in outflow angle correlations, until the overall convergence is reached.

This outflow angle (Eq. (9)) has been implemented into AXOD for the blanking-out of the rotor-row after the outlet guide vane. By examining a variety of flow variables from the stage-by-stage and the inter-stage printouts in the functionality check-out tests mentioned earlier, this outflow angle assignment has proven to be near perfect. There is one item stands out however, which is the tangential rotor blade force deduced in the inter-stage output-routine (printed under the "VCTD = 1" option) that caused some concern, and is discussed here. When the discharge vane angle of the last stator (the simulated outlet guide vane) is non-zero, the tangential blade force exerted by the blanked-out rotor at each of the sectors was found to be not exactly zero but small, in the order of 10^{-3} to the averaged tangential blade force exerted by the real rotor blades (the tolerance of convergence for the mass conservation was set at 10^{-6}), and they were coherent, meaning that they possessed the same sign, always negative to the blade force of the real rotor in these tests (the end-wall shapes are divergent in all these cases.) After careful considerations, this discrepancy (of not being exact zero) is believed to come from the truncation error of the discrete representation of the flow domain and the flow solutions adopted by AXOD. Put it simply, this non-zero sectorial blade force exerted by the blanked-out rotor is not an error in the mathematical formulation or the code implementation, but is the discrete approximation error introduced into the solution obtained, that some quantities are over-shooting while others are under-predicting. This numerical discretization error is inherent to all computational algorithms, and is practically unavoidable. It can be reduced, however, by increasing the number of sectors employed in representing the flow domain. In the event where the outlet guide vane (OGV) discharge vane angle is zero, as it should be in a properly designed outlet guide vane, no tangential blade force on the blanked-out rotor is observed, since the formula (Eq. (9)) disallow the creation of tangential velocity across the blade-row when the inflow tangential velocity is zero. In this situation, the appearance of this numerical error is suppressed, although its essence remains and may show up on some other deduced quantity if examined carefully.

Lastly, we should mention that AXOD adopted an unconventional sign convention. The tangential velocity is assigned "positive" at the rotor inlet when its direction is pointing to the direction of the shaft rotation; while the tangential velocity is "positive" at the rotor discharge when its direction is pointing "opposite to" the direction of the shaft rotation. Therefore when implementing the β'_2 derived by Equation (9) into AXOD, one should add a "minus" sign to it. The mathematical formulas given in this paper, and those have listed in Reference 2, follow the conventional sign convention that the tangential velocity is "positive" whenever it is pointing towards the direction of the shaft rotation.

4.0   Logics for the Program Termination

In the original code of AXOD, in which the turbine contains only the full stages, termination of the code execution come from two separate criteria:

    (a) Code execution terminates when the last rotor-row is choked, when "DELA = 0" is specified.
    (b) Code execution terminates when the limit-loading condition is reached, if "DELA" is not zero.

The definition of the Limit-Loading condition applied here, and is somewhat different from the original coding in AXOD, is when the supersonic flow over-expansion after the choked last-rotor-row has reached the level, that the lowest sector flow Mach number of the total velocity in the ensuring annulus region reached the sonic condition of Mach 1. In this situation, one declares that the downstream disturbances of the annulus flow will not be able to propagate upstream to further influence the exit flow of the choked rotor, thus the condition of the limit-loading is reached.

For the full-stage turbines, conditions (a) and (b) still apply in this extended version of code. And in a full-stage turbine, condition (a) always occurs first before condition (b).

When the last stage is a half-stage without a rotor (i.e., an outlet guide vane (OGV)), the logic of program termination is as follow:

    (a) Code execution terminates when the last rotor-row (before the OGV) is choked, if "DELA = 0" is specified.
    (b) Code execution terminates when the limit-loading condition is reached, if "DELA" is not zero.
    (c) Code execution terminates when the OGV is found choked on any of its sectors.

The conditions (a) and (b) at above are the same as those applied to the full-stage turbine, the condition (c) however, is a competing mechanism to both the conditions (a) and (b), in the sense that whenever condition (c) is found exists, the code execution is terminated regardless the status of condition (a) or (b). The reason for this is that, if the outlet guide vane is found choked (even just partially), the mass flow rate that has flown through all the preceding full stages of the turbine will not pass the outlet guide vane in full. Thus an inconsistency in the law of mass conservation occurred and the solution can not be accepted.

In the functionality check-out tests mentioned, where the outlet guide vane is simply a regular turbine nozzle (a stator) with rather large discharge vane angles, we have found that the termination of the code often comes from condition (c), before the last rotor can even reach the choke-point. In the cases with properly designed outlet guide vanes, as those presented in the "Results and Discussion" section of this paper, we have found that always, condition (a) is reached first, while conditions (b) and (c) are competing, in that, although more often condition (b) is reached before (c), but on some occasion condition (c) prevails before (b).

5.0   Results and Discussion

Two case-studies were conducted to demonstrate the extended capability of AXOD, that the outlet guide vane can now be included as a part of the overall turbine configuration; and to verify the validity of the mathematical formulation and the code implementation described. The two cases are the highly loaded fan-drive turbines, design and tested in the V/STOL Program of NASA in the 1970's.

The first case is a 4 1/2-stage turbine with an average stage loading factor of 4.66, designed by Pratt & Whitney Aircraft (Ref. 5); the second case is a 3 1/2-stage turbine with an average stage loading factor of 4.0 (Ref. 7), designed in-house by the NASA Lewis Research Center (now the NASA Glenn Research Center). Both cases were experimentally tested in the turbine facility located at the Glenn Research Center. The test results were published as NASA technical memorandums (TM-X-3498 and TM-X-3482, respectively; Refs. 6 and 8) and are publicly available.

TABLE 1.—COMPARISON OF THE DESIGN POINT PERFORMANCE INDICES BETWEEN TD2-2 AND TM-X-3498

	Total efficiency	Total-to-total pressure ratio	Total-to-static pressure ratio	Equivalent flow (lbm/s)
TM-X-3498	0.855	6.745	NA	13.3
TD2-2	0.854	6.785	8.386	13.4

TABLE 2.—COMPARISON OF THE DESIGN POINT PERFORMANCE INDICES BETWEEN TD2-2 AND TM-X-3482

	Total efficiency	Total-to-total pressure ratio	Total-to-static pressure ratio	Equivalent flow (lbm/s)
TM-X-3482	0.855	2.290	NA	44.12
TD2-2	0.858	2.281	2.548	42.17

The two cases were first studied using the design code TD2-2, which already has the capability of designing with the outlet guide vane included (see Ref. 3 for the description and the usage of this functionality.) The results in comparison with the experimental data are presented in Tables 1 and 2.

In Table 1, the equivalent mass flow rate measured at the design condition was reported as 13.3 lb/s, which is 0.992 to the design intent of 13.4 lb/s. TD2-2 prediction was made with the given flow rate of 13.4 lb/s that matched the design intent. In Table 2, the equivalent mass flow rate measured at the design condition was reported as 44.12 lb/s while the design intent was 42.17 lb/s, a factor of 1.046 too high. Again, TD2-2 prediction was conducted at the flow rate of the analytical design-intent. Neither TM-X-3498 nor TM-X-3482 had reported the overall total-to-static pressure ratio, which is marked as NA (as not available) in these tables.

Because of the concern with the high discrepancy in the mass flow rate reported by TM-X-3482, it was decided that the off-design performance analyses will not be conducted on the TD2-2 designs, in which the design code has produced the row-by-row blade angle designs and can be used for the study of the off-design performance by AXOD. Instead, the off-design performance analyses were conducted on the actual designs, with the design-point performance indices matched directly to the experimental findings. This process of matching by AXOD, which in essence is for the purpose of determining the loss coefficients, is described in depth in Reference 2.

As discussed in Reference 2, there are three loss mechanisms specified in AXOD, with three un-assigned loss coefficients (YA, YB, YC). In order to obtain the values of these loss coefficients, and thereby closing the loss model, one matches the design-point performance indices by adjusting the values of (YA, YB, YC) systematically, until the best match is made between the calculated performance indices by AXOD and the given design-point performance indices, in this paper, those experimental data reported.

In both of the two cases studied, however, the total-to-static pressure ratio is not available for matching, we therefore applied the matching on only the two available indices, the total-to-total pressure ratio and the total efficiency, with the stagnation region streamline deflection angle, λ, assigned to 3°, which is equivalent as saying that (YA) is being assigned directly.

The results of the two cases are presented and discussed in detail here.

5.1 NASA/PW Fan-Drive Turbine

5.1.1 The Matching Process for the Loss Coefficients in AXOD on TM-X-3498

In the case of NASA/PW Fan-Drive Turbine (TM-X-3498), there is an additional complication in the process of matching, and that is the design-point mass flow rate conceived by the experiment actually has choked the turbine at the given design-point pressure ratio, according to AXOD. The mass flow rate, then, can not be an imposed quantity assigned through input, but is obtained through the choke-iteration process implemented in AXOD, which ensures the continuity condition is satisfied across the choked component, and the correct choked-flow mass flow rate is obtained. This choke-iteration algorithm is activated only when the code is operating in the sweeping mode, with the varying pressure ratios. The process of matching, in this case, was then changed to:

1. Systematically adjust the set of (YB, YC) values with λ assigned to the fixed value of 3°. Execute AXOD in the sweeping mode; print the total efficiency and the mass flow rate under the sweeping total-to-total pressure ratio.
2. At the point where the total-to-total pressure ratio reads 6.745, which is the design-point pressure ratio conceived by the experiment of TM-X-3498, calculate and minimize the RMSE (root-mean-square-error) of

$$\text{RMSE} = \sqrt{\left[\left(\frac{\eta_t - \eta_t^d}{\eta_t^d}\right)^2 + \left(\frac{\omega - \omega^d}{\omega^d}\right)^2\right] \Big/ 2} = \text{Min.} \qquad (10)$$

In here, η_t is the total efficiency and ω is the mass flow rate. The superscript (d) refers to the reported experimental data at the design-point conceived.

The outcome of this best-match practice can be shown as:

TABLE 3.—COMPARISON OF THE DESIGN POINT PERFORMANCE INDICES BETWEEN AXOD AND TM-X-3498

	Total efficiency	Equivalent flow (lbm/s)	Total-to-total pressure ratio
TM-X-3498	0.855	13.30	6.745
AXOD	0.8545	13.303	6.745

and the optimum loss coefficients obtained are:

TABLE 4.—THE OPTIMUM LOSS COEFFICIENTS OBTAINED BY AXOD ON TM-X-3498

	λ on the 1st stator (degrees)	Blade row efficiency (1st stator) (1-YB)	Blockage factor (1st stator) (1-YC)
AXOD	3.0	0.956	0.984

5.1.2 The Comparison of Results Between AXOD and the Experiment of TM-X-3498

With the loss model closed and the loss coefficients found, AXOD was then executed by sweeping over two different speed-lines. The results in comparison with the experimental data reported in TM-X-3498 were plotted in Figures 1 to 4.

NASA/PW FAN-DRIVE TURBINE (4 +OGV stages)
FLOW MAP
Lines -- Prediction by AXOD
Symbols -- Experimental Data
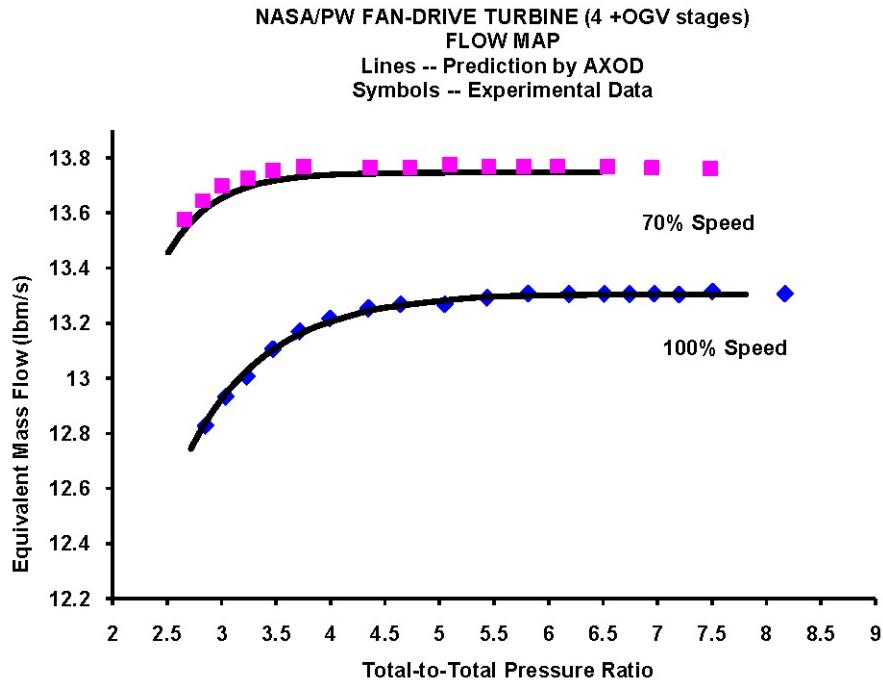
Figure 1.—Comparison of the equivalent mass flow rate (TM-X-3498).

NASA/PW FAN-DRIVE TURBINE (4 +OGV stages)
TORQUE MAP
**Lines -- Prediction by AXOD**
Symbols -- Experimental Data
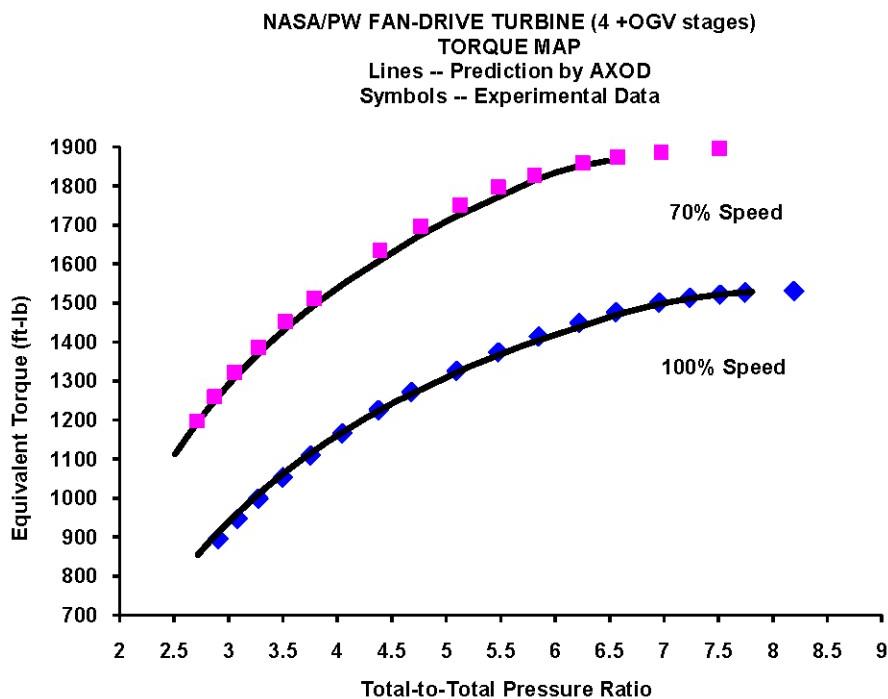
Figure 2.—Comparison of the equivalent torque (TM-X-3498).
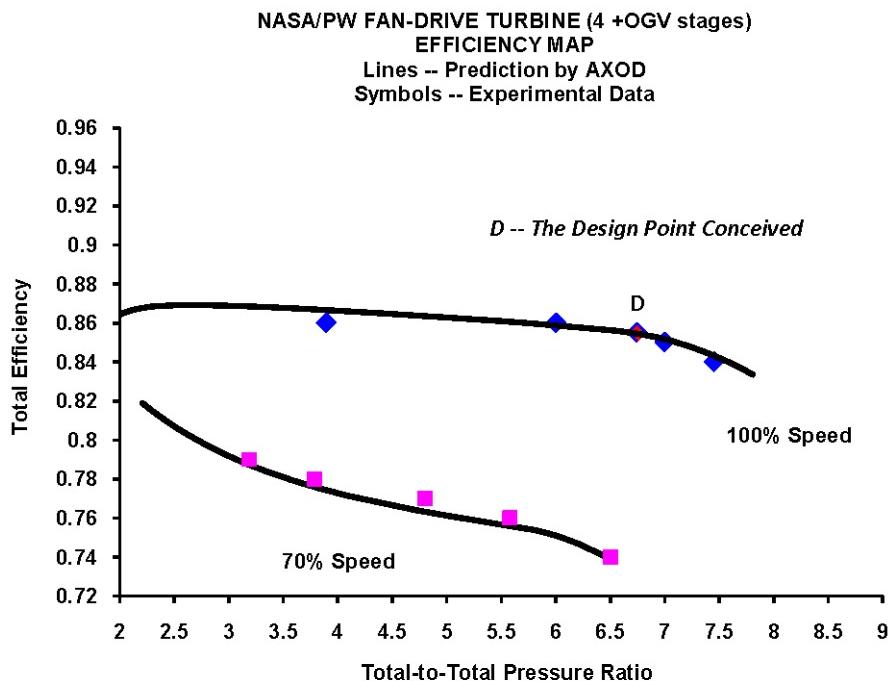
Figure 3.—Comparison of the total efficiency (TM-X-3498).
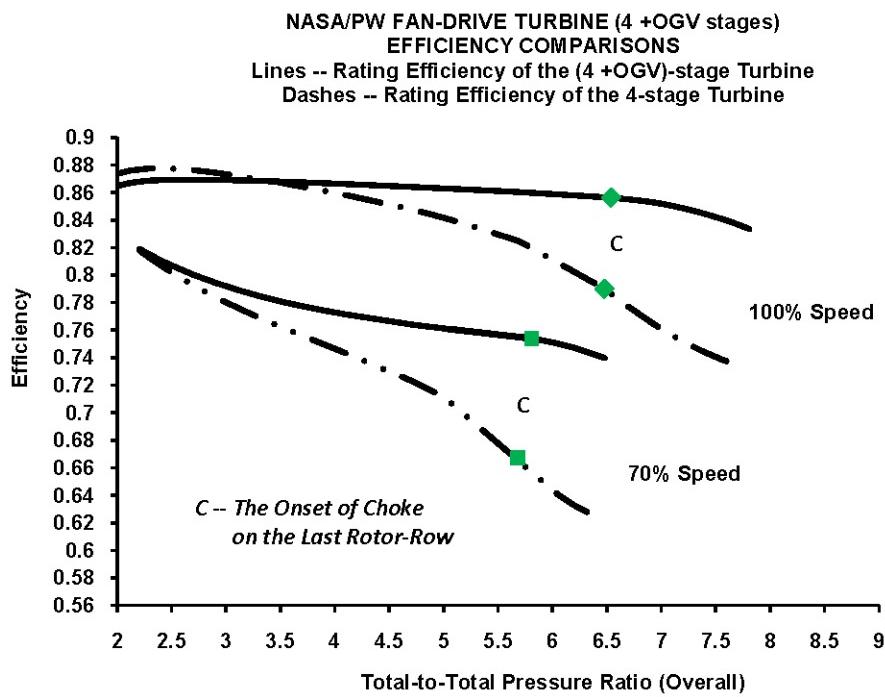
Figure 4.—The rating efficiency comparisons between the 4 1/2-stage and the 4-stage turbines.

These figures are mostly self-explanatory, except the "Rating Efficiency" plotted in Figure 4, which is clarified here. The definition of the Rating Efficiency applied here, is the efficiency in which the kinetic energy of the exit-swirl, contributed from the circumferential velocity component, is considered lost. This efficiency is higher than the static efficiency where all the kinetic energy of the exit flow is considered lost; and it is lower than the total efficiency where none of the kinetic energy of the exit flow is considered lost.

The AXOD predictions in these figures ended at the limit-loading condition. The limit-loading condition was defined earlier in Section 4.0 of this paper, but is repeated here for clarity. The definition of the limit-loading condition applied, and is somewhat different from the original coding of AXOD, is when the supersonic flow over-expansion after the choked last-rotor-row has reached the level, that the lowest sector flow Mach number of the total velocity in the ensuring annulus region reached the sonic condition of Mach 1. In this situation, one declares that the downstream disturbances of the annulus flow will not be able to propagate upstream to further influence the exit flow of the choked rotor, thus the condition of the limit-loading is reached.

Excellent agreement between the AXOD predictions and the experimental data were observed in Figures 1 to 3. The predicted points of limit-loading by AXOD were somewhat lower than the experimentally conceived values, however, the maximum torques achieved between the two were practically at the same level.

The plots in Figure 4 showed that the rating efficiency of the un-guided turbine exit flow was noticeably lower than the turbine with an outlet guide vane (OGV), by as much as 10 percentage points lower. This is a stunning contrast for the justification of employing the outlet guide vane into the turbine configuration, particularly on the highly loaded turbines.

5.2   NASA Designed Fan-Drive Turbine

5.2.1   The Matching Process for the Loss Coefficients in AXOD on TM-X-3482

In the case of NASA in-house designed fan-drive turbine (TM-X-3482), the situation for the matching process is normal, in that the design-point mass flow rate reported by the experiment did not choke the turbine at the design-point pressure ratio. The mass flow rate, without having to go through the choke-iteration process, is a quantity that can be assigned and imposed directly from input. The matching process was conducted in the single-point mode, operating under the same turbine inlet total pressure, total temperature, and the design-point mass flow rate reported by the experiment. This process of matching is the same as those described in Reference 2, except that the index of total-to-static pressure is unavailable here, thus only two design-point performance indices were used for the matching. This process is outlined here:

1. Systematically adjust the set of (YB, YC) values, again, with λ assigned to the fixed value of 3°. Execute AXOD in the single-point operating mode; print the total efficiency and the total-to-total pressure ratio under the imposed inlet mass flow rate.
2. Calculate and minimize the outcome of the RMSE (root-mean-square-error) of

$$\text{RMSE} = \sqrt{\left[\left(\frac{\eta_t - \eta_t^d}{\eta_t^d}\right)^2 + \left(\frac{PR_t - PR_t^d}{PR_t^d}\right)^2\right] \Big/ 2} = \text{Min.} \tag{11}$$

In here, η_t is the total efficiency and PR_t is the total-to-total pressure ratio. The superscript (d) refers to the reported experimental data at the design point.

The outcome of this best-match practice is:

TABLE 5.—COMPARISON OF THE DESIGN POINT PERFORMANCE INDICES BETWEEN AXOD AND TM-X-3482

	Total efficiency	Total-to-total pressure ratio	Equivalent flow (lbm/s)
TM-X-3482	0.855	2.290	44.12
AXOD	0.8551	2.2895	44.12

and the optimum loss coefficients obtained are:

TABLE 6.—THE OPTIMUM LOSS COEFFICIENTS OBTAINED BY AXOD ON TM-X-3482

	λ on the 1st stator (degrees)	Blade row efficiency (1st stator) $(1\text{-}YB)$	Blockage factor (1st stator) $(1\text{-}YC)$
AXOD	3.0	0.9753	0.9801

5.2.2 The Comparison of the Results Between AXOD and the Experiment of TM-X-3482

The same as the first case studied, now that the loss model is closed and the loss coefficients found, AXOD was then executed by sweeping over two different speed-lines, and the results in comparison with the experimental data reported in TM-X-3482 were plotted in Figures 5 to 8.
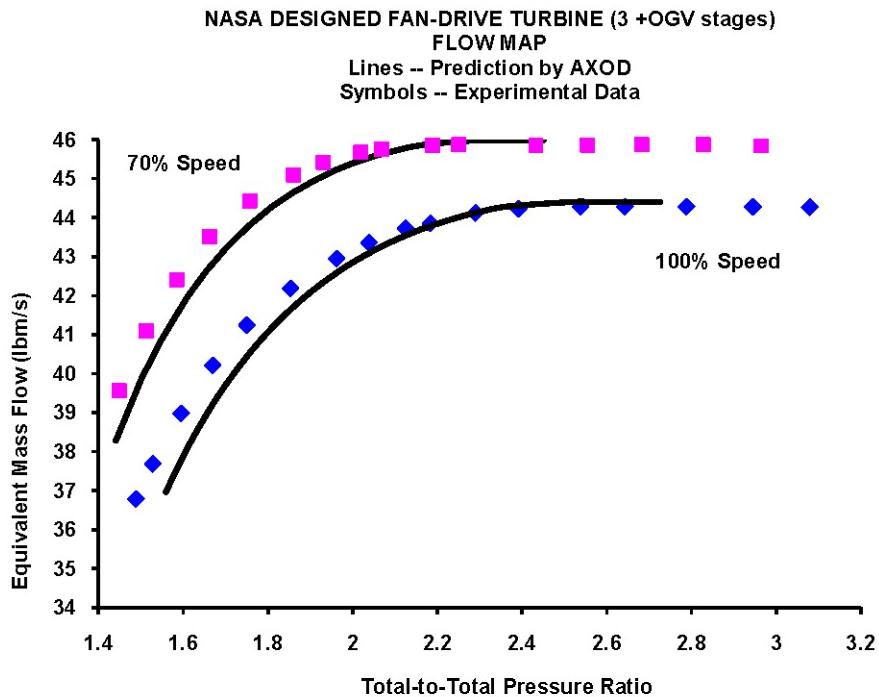

Figure 5.—Comparison of the equivalent mass flow rate (TM-X-3482).

NASA DESIGNED FAN-DRIVE TURBINE (3 +OGV stages)
TORQUE MAP
Lines -- Prediction by AXOD
Symbols -- Experimental Data
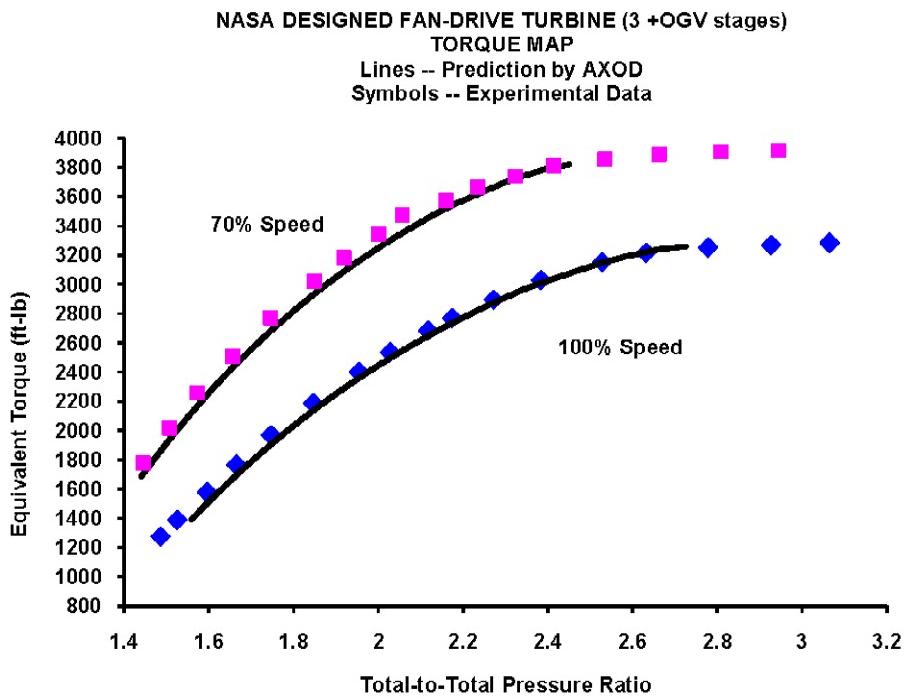
Figure 6.—Comparison of the equivalent torque (TM-X-3482).

NASA DESIGNED FAN-DRIVE TURBINE (3 +OGV stages)
EFFICIENCY MAP
**Lines -- Prediction by AXOD**
Symbols -- Experimental Data
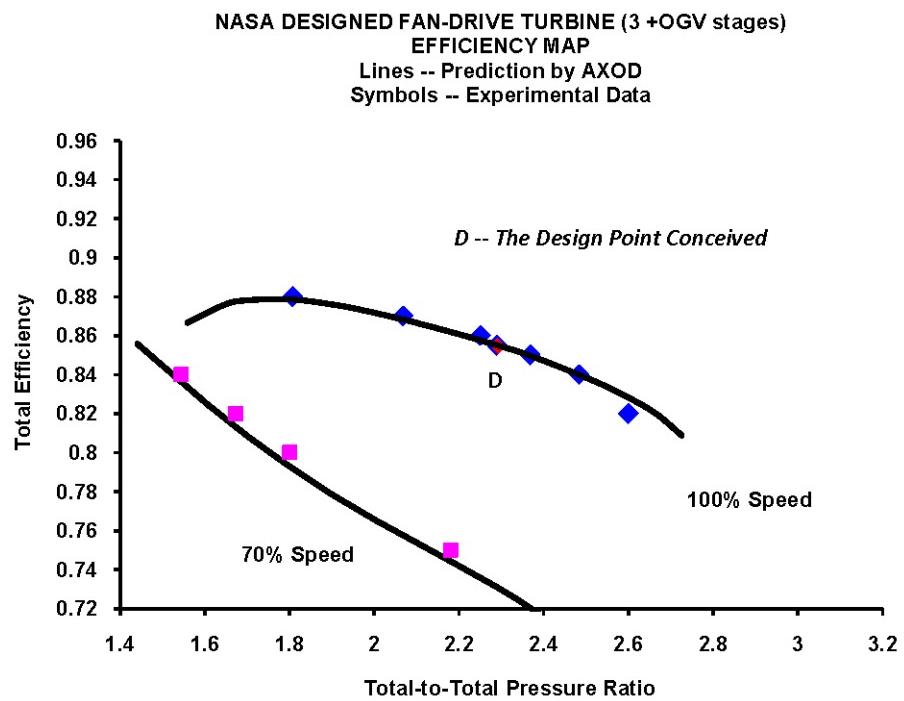
Figure 7.—Comparison of the total efficiency (TM-X-3482).
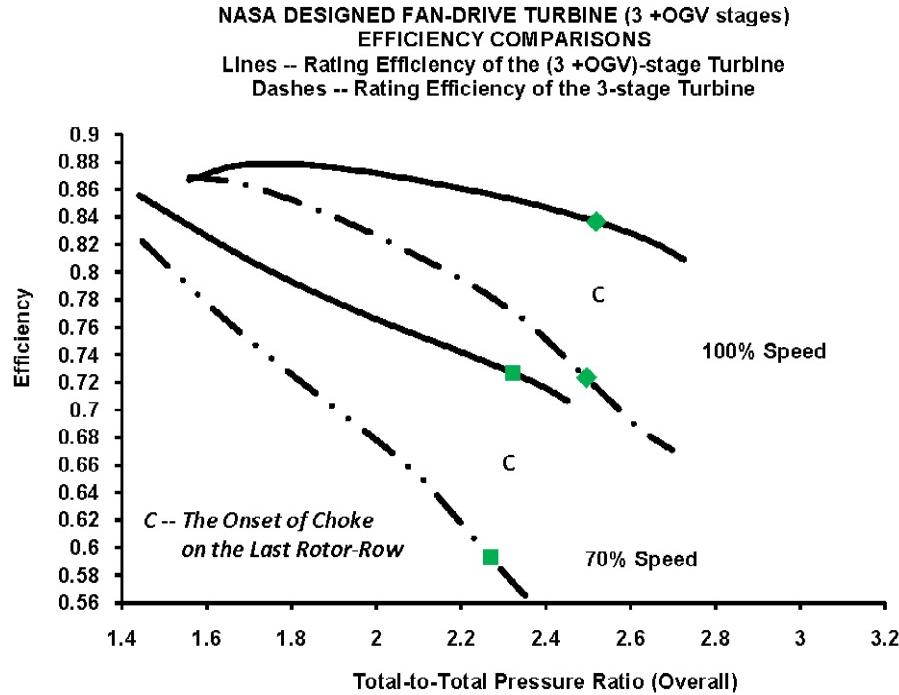
Figure 8.—The rating efficiency comparisons between the 3 1/2-stage and the 3-stage turbines.

The agreement between the AXOD predictions and the experimental data were excellent, though the equivalent mass flow rates plotted in Figure 5 did show a somewhat lesser degree of agreement in the lower pressure ratio regime, when compared with the experimental data of TM-X-3482. Again, the same as previously noted, the predicted points of limit-loading by AXOD were somewhat lower than the experimentally conceived values, but the maximum torques achieved between the two were practically the same.

The plots of Figure 8 have confirmed the comments made previously on Figure 4, that the rating efficiency of the un-guided turbine exit flow was noticeably lower than the turbine with an outlet guide vane (OGV), and showed in here, by as much as 15 points. A conclusion can be made, then, that an outlet guide vane is a necessary part of the turbine configuration, when the turbine is designed to be highly loaded.

6.0   Concluding Remarks

The axial flow turbine off-design computer program AXOD has been upgraded to include the outlet guide vane into its acceptable turbine configurations. The mathematical bases and the techniques used for the code implementation were described and discussed. The logics for terminating the program execution were updated. This extended capability was verified and validated with two cases of highly loaded fan-drive turbines. The first case is a 4 1/2-stage turbine with an average stage loading factor of 4.66, designed by Pratt & Whitney Aircraft; the second case is a 3 1/2-stage turbine with an average loading factor 4.0, designed in-house by the NASA Lewis Research Center (now Glenn). The processes conducted in these studies were described in detail in this paper, and the results in comparison with the experimental data were presented and discussed. The results between the AXOD predictions and the experimental data of the first case, published in NASA TM-X-3498, were in excellent agreement. The results between the AXOD predictions and the experimental data published in NASA TM-X-3482 were in equally good agreement except the equivalent mass flow rates, which have shown slight discrepancies in the lower pressure ratio regime. These results indicted that the capability extension to AXOD in including the outlet guide vane into the analysis was fully successful. An outlet guide vane can now be a part of the turbine configuration for the off-design computer program AXOD.

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14. ABSTRACT
The axial flow turbine off-design computer program AXOD has been upgraded to include the outlet guide vane (OGV) into its acceptable turbine configurations. The mathematical bases and the techniques used for the code implementation are described and discussed in lengths in this paper. This extended capability is verified and validated with two cases of highly loaded fan-drive turbines, designed and tested in the V/STOL Program of NASA. The first case is a 4 1/2-stage turbine with an average stage loading factor of 4.66, designed by Pratt & Whitney Aircraft. The second case is a 3 1/2-stage turbine with an average loading factor of 4.0, designed in-house by the NASA Lewis Research Center (now the NASA Glenn Research Center). Both cases were experimentally tested in the turbine facility located at the Glenn Research Center. The processes conducted in these studies are described in detail in this paper, and the results in comparison with the experimental data are presented and discussed. The comparisons between the AXOD results and the experimental data are in excellent agreement.

15. SUBJECT TERMS
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