Probabilistic Solar Energetic Particle Models

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To plan and design safe and reliable space missions, it is necessary to take into account the effects of the space radiation environment. This is done by setting the goal of achieving safety and reliability with some desired level of confidence. To achieve this goal, a worst-case space radiation environment at the required confidence level must be obtained. Planning and designing then proceeds, taking into account the effects of this worst-case environment. The result will be a mission that is reliable against the effects of the space radiation environment at the desired confidence level.

In this paper we will describe progress toward developing a model that provides worst-case space radiation environments at user-specified confidence levels. We will present a model for worst-case event-integrated solar proton environments that provide the worst-case differential proton spectrum. This model is based on data from IMP-8 and GOES spacecraft that provide a data base extending from 1974 to the present. We will discuss extending this work to create worst-case models for peak flux and mission-integrated fluence for protons. We will also describe plans for similar models for helium and heavier ions.

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Abstract: To plan and design safe and reliable space missions, it is necessary to take into account the effects of the space radiation environment. This is done by setting the goal of achieving safety and reliability with some desired level of confidence. Achieving this goal will require finding a reference worst-case space radiation environment for the mission. Finally, mission design solutions must be found that take into account the effects of this worst-case environment. The result will be a mission that is reliable against the effects of the space radiation environment at the desired confidence level. This paper describes progress toward developing a model that provides worst-case space radiation environments at user-specified confidence levels.

Keywords: Solar energetic particles; Space radiation environment

1 Introduction

1.1 Introduction

Solar energetic particles (SEPs) and galactic cosmic rays pose major radiation hazards for space hardware and astronauts. Penetrating particle radiation adversely affects aircraft avionics and potentially the health of airline crews and passengers on polar flights. Our goal is to deliver the understanding and modeling of the solar particle environment that will be needed for planning missions in Earth orbit and to other destinations in the solar system. The objective is to update and extend the existing probabilistic SEP models for worst-case peak flux and event-integrated fluence energy spectra at user-specified confidence levels.

1.2 Previous Work

The first model of solar proton fluences was based on King's analysis of 10 – 100 MeV protons during solar cycle 20 [1]. One "anomalously large" event, the well-known August 1972 event, dominated the fluence of this cycle so the King Model uses the expected number of such events during a given mission length at a given confidence level to obtain the cumulative proton fluences. Using additional data, a model from JPL [2] emerged for 1 – 60 MeV protons in which Feynman et al. showed that the event magnitude distribution is actually a continuous distribution between small events and much larger events like that of August 1972 [2].

The Emission of Solar Protons (ESP) model developed by Xapsos et al. [3] provides worst-case peak flux and event-integrated fluence spectral models at user-specified confidence levels based on data from solar cycles 20 through 22. The methods of Xapsos et al. are used in this paper and are described in section 2.

Due to the stochastic nature of solar particle events (SPEs), confidence level approaches are often used so that risk-cost-performance tradeoffs can be evaluated by spacecraft designers. In addition to showing that SPEs could be described by a single distribution, the JPL Model introduced some other important points:

- The 11-year solar cycle was divided into 7 active years surrounding solar maximum and 4 inactive years [2]. This division will be used in the model developments proposed here.
- The database used to define the probability distribution must have a constant observational efficiency.
- SPE events must be defined in such a way that they are statistically independent (not coming from the same active region).

The JPL model fits the distribution of event-integrated fluences from statistically independent events to a log-normal distribution. The model then uses Monte Carlo

simulations to calculate the probability distribution for the cumulative flux from n events. This result is combined with a Poisson distribution describing the probability that n events will occur in a time period, T years, to obtain the cumulative fluence during a mission of T years at a given confidence level [2]. The JPL Proton Fluence Model was recently reassessed [4] based on the data that has accumulated between 1991 and 1998. It was found that adding the new data did not require a revision of the model.

Another Monte Carlo-based model has been proposed as an International Standard [5] by Nymmik. This model uses an ad-hoc probability distribution in the form of a power law with an exponential cutoff [5] to represent the size distribution of event-integrated proton fluences and peak integral proton fluxes above 30 MeV. The model then makes the ad-hoc assumption that all SPE proton differential flux and fluence spectra can be represented by power laws in magnetic rigidity with an amplitude, C, and a spectral index, γ. The spectral index is assumed to be constant, γ_0, for energies > 30 MeV, but to "droop" according to a power law in energy with a spectral index, α, for energies $\leqslant$ 30 MeV. The parameters γ and α are assumed, ad-hoc, to be log-normally distributed. The procedure of the model is to generate N mission versions of T years each by Monte Carlo simulation.

Given that the occurrence of SPEs is a stochastic phenomenon it is important to accurately model the underlying distribution of event magnitudes. However, in general it can be rather difficult and even arbitrary to select a probability distribution when the data are limited. As discussed above, numbers of empirical assumptions have been made in the past. For example, King [1], and Feynman [2] have used a log-normal distribution, while Nymmik [5] has used a power law function. The log-normal distribution describes the large events well but underestimates the occurrence probability of smaller events. The power law functions describe the smaller events well but overestimate the occurrence probability of larger events.

Scaling the mean number of events in T years from the smoothed mean monthly sunspot number has been justified [5] by an analysis that shows the frequency of SPEs scales with the monthly smoothed sunspot number and that the cumulative integral fluence distributions for different phases of the solar cycle, when scaled by sums of the smoothed monthly sunspot numbers, are in reasonable agreement, especially at high fluences.

2 Probabilistic Modeling

2.1 Extreme Value Theory

The fundamental approach for developing probabilistic models makes use of the Maximum Entropy Principle [6] which has been applied to the problem of peak integral fluxes [3] and event-integrated integral fluences [3] by Xapsos and his colleagues. It has been argued that this is the best choice that can be made given the limited data [6]. This approach is explained below as it applies to differential fluxes and fluences at some energy, E.

According to the Maximum Entropy Principle, one must choose that probability distribution which maximizes the uncertainty in the predicted outcome, consistent with the constraints that can be placed on the problem. These constraints come from the partial knowledge we have about the nature of the governing probability distribution.

For the cases of peak differential fluxes and event-integrated differential fluencies, the first task is to determine the initial probability distribution, P, which gives the probability that an SPE will not have a flux (or fluence) that exceeds some value, ϕ.

Following Kupur [6], the entropy, S, is defined as

$$S = -\int p(M)\ln[p(M)]dM \qquad (1)$$

where p is the probability density corresponding to P and M is a continuous random variable defined as

$$M = \log_{10}\phi \qquad (2)$$

We can recognize that p has the following constraints:

- There is a known lower limit on the measured flux (or fluence) imposed by instrument sensitivity. Here we take this limit to be ϕ_{min} which gives $M_{min} = log_{10}(\phi_{min})$. Since we seek probabilities for large values of ϕ, these results are insensitive to ϕ_{min}, provided it is small.
- There is a finite but undetermined upper limit on ϕ, giving a finite value M_{max}.
- $p(M)$ can be normalized to unity and it has a well-defined mean.

These constraints give us the equations:

$$\int_{M_{\min}}^{M_{\max}} p(m)dM = 1 \text{ and} \qquad (3)$$

$$\int_{M_{\min}}^{M_{\max}} Mp(M)dM = \overline{M} \qquad (4)$$

Eq. (1) can now be maximized, subject to these constraints, by using the Lagrange Multipliers [7], λ_1 and λ_2 to construct $Q(M)$

$$Q(M) = -\int p(M)\ln(p(M))dM \qquad (5)$$

$$-\lambda_1\left[\int_{M_{\min}}^{M_{\max}} p(M)pM - 1\right] - \lambda_2\left[\int_{M_{\min}}^{M_{\max}} Mp(M)dM - M\right]$$

This gives the truncated exponential distribution,

$$P(\phi) = \left(\phi_{\min}^{-b} - \phi^{-b}\right)/\left(\phi_{\min}^{-b} - \phi_{\max}^{-b}\right) \qquad (6)$$

where $b = \lambda/\ln(10)$

Now if *n* events occur during the specified mission duration containing, *T*, active years then the probability that none of these *n* events will have a flux $\geq \phi$ is $[P(\phi)]^n$. If the average number of SPEs per year that produce a flux (or fluence) $\geq \phi_{min}$ is μ, then the probability that n events will be produced in *T* years is

$$\left[e^{(-\mu T)}(\mu T)^n\right]/n! \qquad (7)$$

So the probability, $F_T(\phi)$, that no event will occur during a mission with a duration of *T* active years having a flux (or fluence) $\phi(E)$ is

$$F_T(\phi) = \exp\left\{-\mu T\left[1 - P(\phi)\right]\right\} \qquad (8)$$

$F_T(\phi)$ is therefore the confidence level which, along with the mission start date and duration is to be specified by the user.

With $F_T(\phi)$ and *T* supplied by the user, the next task is to construct differential energy spectra for the peak flux (or event-integrated fluence), $\phi(E)$, for every element such that a spectrum more intense than the one we construct will not occur during T years at a confidence level, $F_T(\phi)$. We will call this the worst-case spectrum at confidence level $F_T(\phi)$. To do this, we must determine the parameters of the initial probability distribution, $P[\phi(E)]$, and the event rate, $\mu(E)$, for the energy range of interest. These parameters must be determined using the available data on SPEs as discussed in the next section. Once these parameters are determined, eq. (8) will be solved for $\phi(E)$.

2.2 The Experimental Data

The parameters *b* and ϕ_{max} in eq. (6), μ in eq. (8) and σ must be determined from the experimental data. We propose to do this by fitting the data on the peak differential flux and the event-integrated differential fluence for all the measured SPEs in the 20th, 21st, 22nd and 23rd solar cycles.

Reliable space-based measurements of SPE proton spectra are available for solar cycles 20 - 23. The sources of data for solar cycle 20 are IMP-3, -4, -5, -7, and -8. Data for solar cycle 21 are available from IMP-8. GOES-5, -6 and -7 provide data for solar cycle 22. IMP-8, GOES-7 and -8 provide data for solar cycle 23. The proton data from solar cycle 20, 21 and 22 have been analyzed. They available to us from earlier work [3]. We have extended this data set using data from the GOES spacecraft and the Goddard Medium Energy (GME) instrument on IMP-8. These measurements have been cross-checked using data from the PET instrument on SAMPEX and the EPACT instrument on WIND.

Some data have begun to appear from the SOHO spacecraft [8]. While these SOHO data suffer from saturation in large events, they will serve as a cross check on the ACE data after the end of the IMP-8 mission. We have analyzed the IMP-8 proton data from solar cycles 21 and 22 with a time resolution of 30 minutes and the helium and heavy ion data with a time resolution 6 hours. We propose to extend this analysis to the end of the IMP-8 mission. This will provide sufficient time resolution to identify the peak fluxes as well as the event-integrated fluences so that spectra of the more abundant elements for these events can be constructed and fit.

The data we have used is either publically available, available to us from our previous studies or (in the case of IMP-8 CRNC/CRT data) available, but only in a form that is not corrected for dead-time and other instrumental effects. We have corrected much of the IMP-8 CRNC/CRT data for our earlier studies.

2.3 Spectral Fitting

Each of the instruments we propose to use in this study has a natural energy binning determined by the design of the experimental hardware. To determine the measured spectra at a common set of energy points, we propose to fit spectral models to the measured peak elemental flux spectra and the event-integrated fluence spectra. Spectral fitting has been used in many previous investigations. Cohen et al. and Mewaldt et al. were successful in fitting the events of October and November 2003 with either the Ellsion-Ramaty Model or the Band Function. Tylka et al. reports that the Ellsion-Ramaty Model successfully fits the spectra from the event-averaged spectra of the April 21, 2002 event while the Band Function fit the spectra of the August 24, 2002 event. Xapsos et al. [3] have proposed a model based on the Weibel distribution of the smallest values and used it successfully to fit many SPE spectra over a broad energy range.
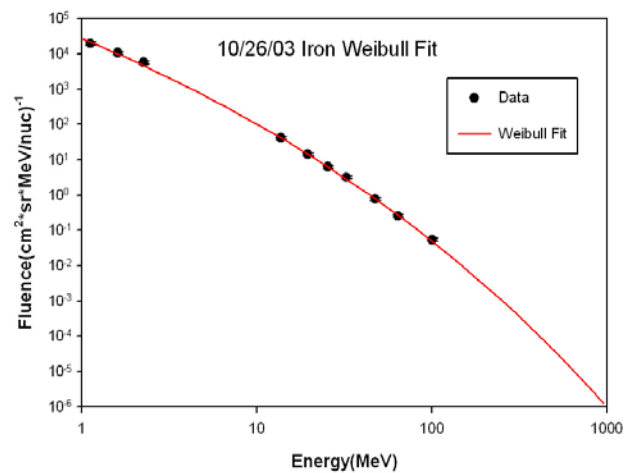

Figure 1: Weibel function fit to the event-integrated fluence spectrum of iron in the Oct 26 2003 SPE.

To investigate our ability to fit the large number of spectra in this investigation, we have tested eight spectral models. These are the Ellsion-Ramaty Model, the Band

function, the Lee-Ramaty Model, an exponential in rigidity, an exponential in energy, a power law in energy, the Weibel Distribution and Mazur's Model. These models were tested by fitting event-integrated or event-averaged spectra obtained from published papers. The reduced χ^2 was computed for each fit and used as a test of the goodness of fit for each model. Figure 1 shows an example of such a fit. In this case the Weibel function was used to fit the event-integrated Fe fluence spectrum from the SPE of October 26, 2003. The reduced χ^2 for this fit is 1.09.

From these tests, we concluded that most of the spectra could be successfully fit, usually with one of three distributions, the Ellsion-Ramaty Model, the Band function, or the Weibel Distribution. Based on our experience with these tests, we believe that the peak flux and event-integrated fluence spectra can be fitted with one of the available models.

2.4 Fitting the Parameters of the Probability Distributions

To fit the parameters of the probability distributions we will follow the procedures laid out by Xapsos et al. [3].

The procedure for fitting the initial distribution of peak fluxes is to use the distribution function in eq. (9) to calculate the number of events per year, N, that have fluxes, $< \phi$. This is

$$N = N_{tot}\left(1 - P(\phi)\right) \text{ or}$$

$$N = N_{tot}\left(\phi_{min}^{-b} - \phi^{-b}\right)/\left(\phi_{min}^{-b} - \phi_{max}^{-b}\right) \quad (9)$$

Here N_{tot} is the average number of measured peak fluxes per active year at energy E and N is the number of peak fluxes with values $<\phi$.

Using the entire data set of fitted spectra for each abundant element from all the active years, we will find the peak fluxes for each event at an energy, E. We will use these data to find μ and ϕ_{min}. From these data we will construct a table of N versus ϕ and fit eq. (9) to these data to find N_{tot}, b and ϕ_{max}.

As an example we show a fit of eq. (9) to the peak integral fluxes > 10 MeV for all events since May 25, 1967 in Fig. 2. These data were assembled from the SPE summary published by Shea and Smart [9] and the NOAA data from the NOAA Space Weather Prediction Center. The fit parameters were determined to be $N_{tot} = 27$; $b = 4.1$; $\phi_{max} = 1.3 \times 10^5$ cm^{-1}s^{-1}sr^{-1} .
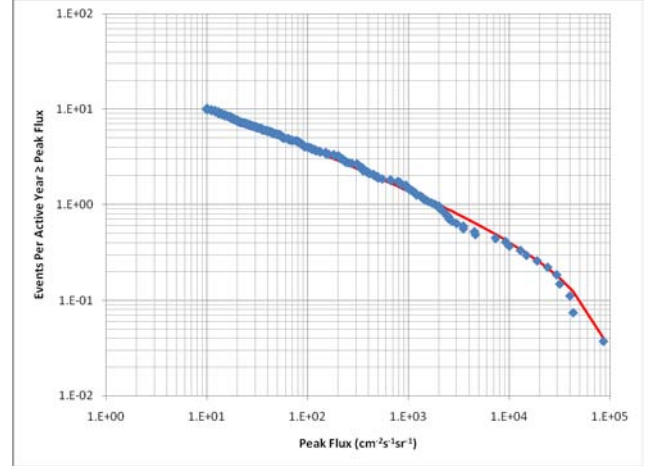

Figure 2: Cumulative distribution of peak SPE proton fluxes (E > 10 MeV) for SPEs since May 25, 1967 and the fit of the truncated exponential distribution, eq. (9).

3   Conclusions

Single column figures

4   References

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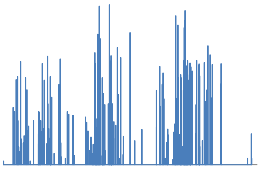
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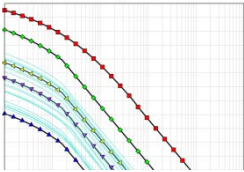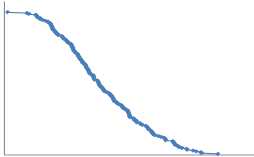[3]NASA Goddard Space Flight Center

Outline
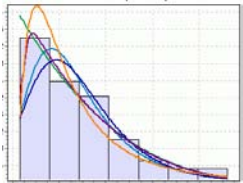
Determining a reference worst-case environment.

Selecting the SPEs.

Determining the energy spectra for each event.

Constructing the cumulative spectra.

Finding the extreme-value distribution.

Reference worst-case environment

- A reference worst-case environment in the local interplanetary medium is needed for:
  - Spacecraft design
  - Mission planning
- Objective: Determine an environment that:
  - Won't be exceeded at a user-specified confidence level
  - For a user-specified conditions, e.g.
    - Launch date, mission duration, heliospheric location
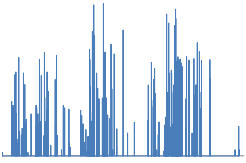
The NOAA Event Selection Criterion

Solar Proton Events (>10 MeV)
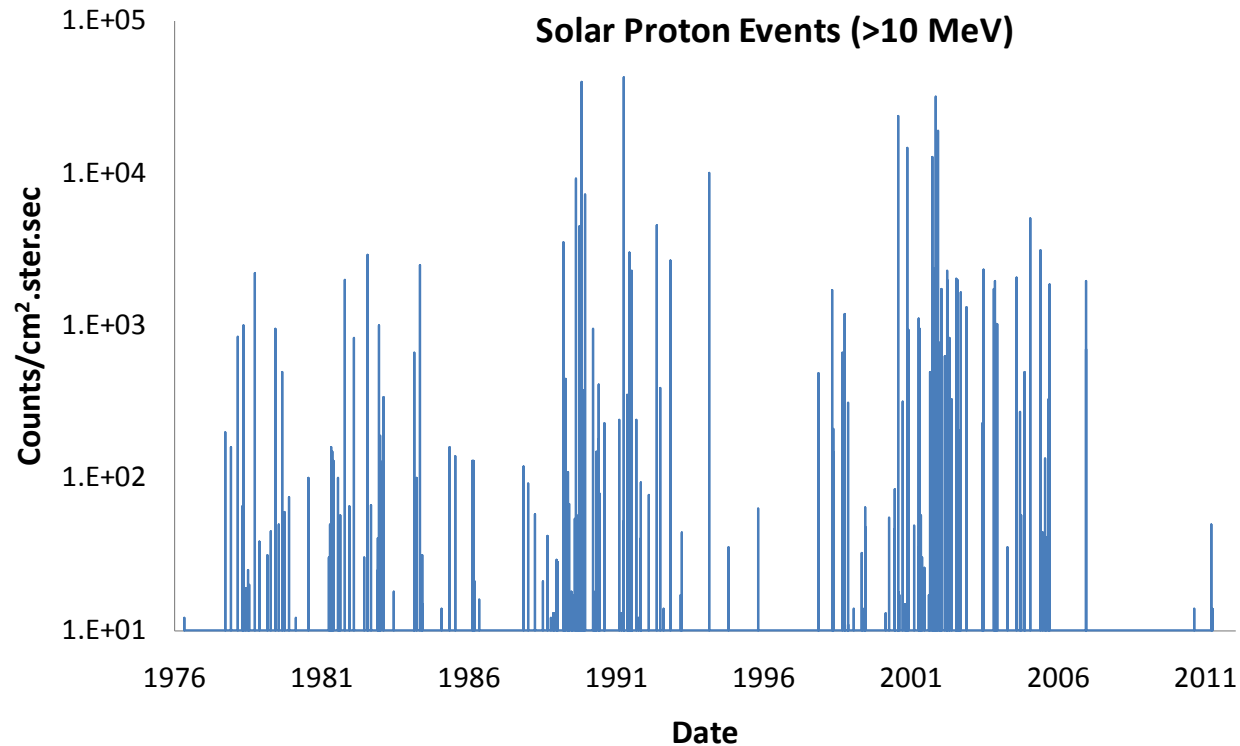
- **Onset:** *The first of 3 consecutive data points with >10 MeV proton fluxes ≥ 10 PFU*.*

- **End:** *The last data point ≥ 10 PFU*.*

*protons/cm².ster.sec.
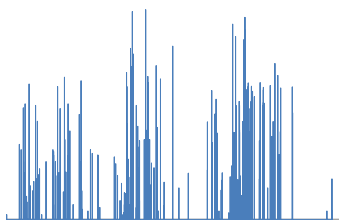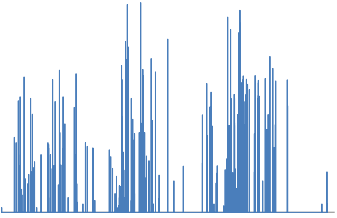
Identifying events and finding onset and end times.
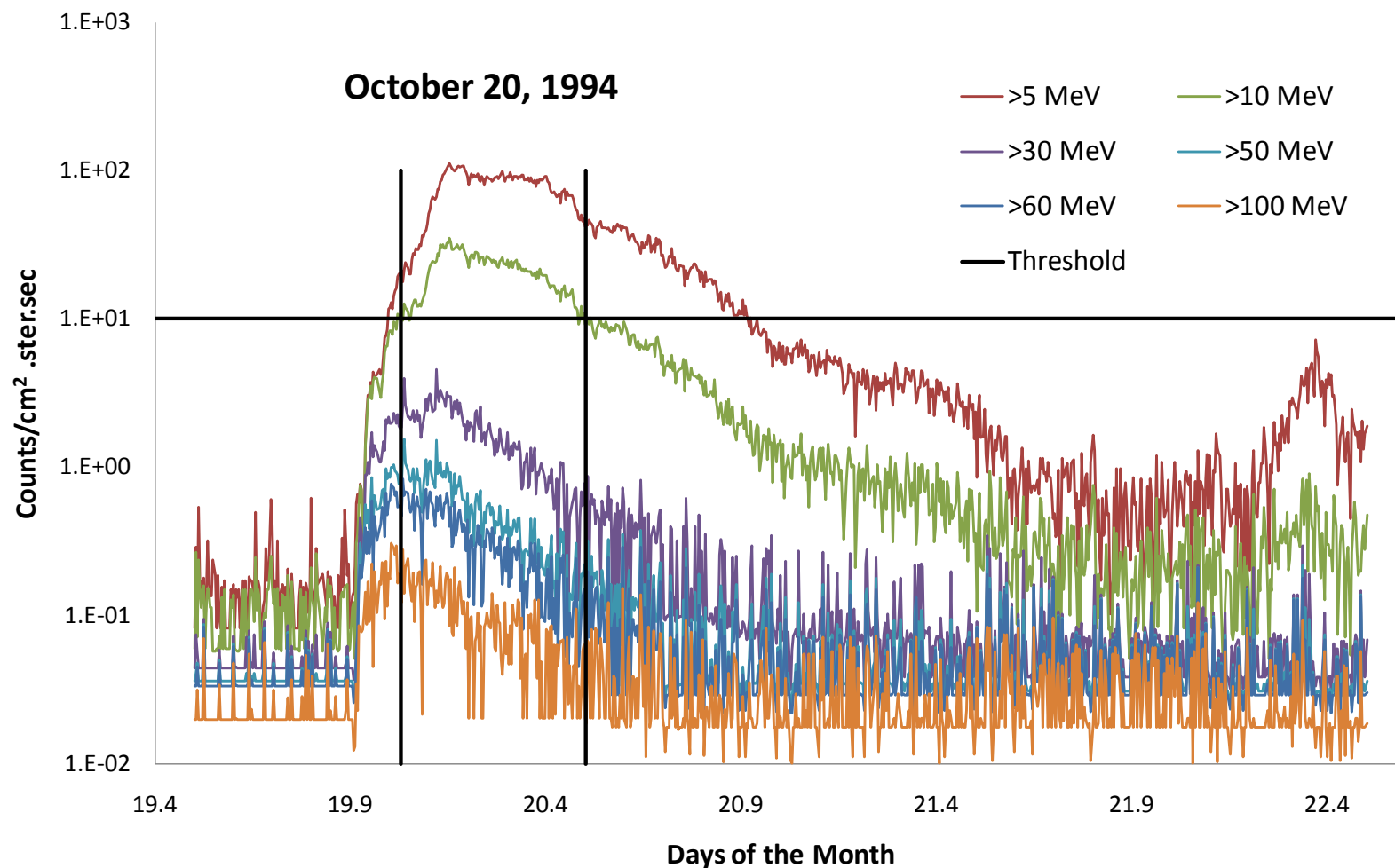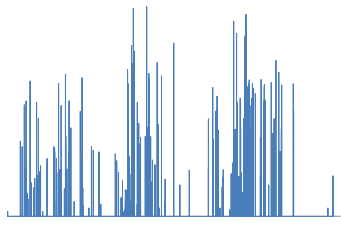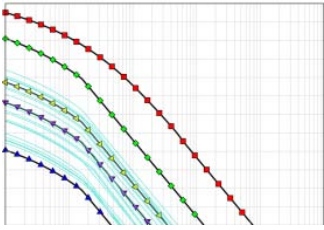
The smallest events are affected most by the NOAA criterion.
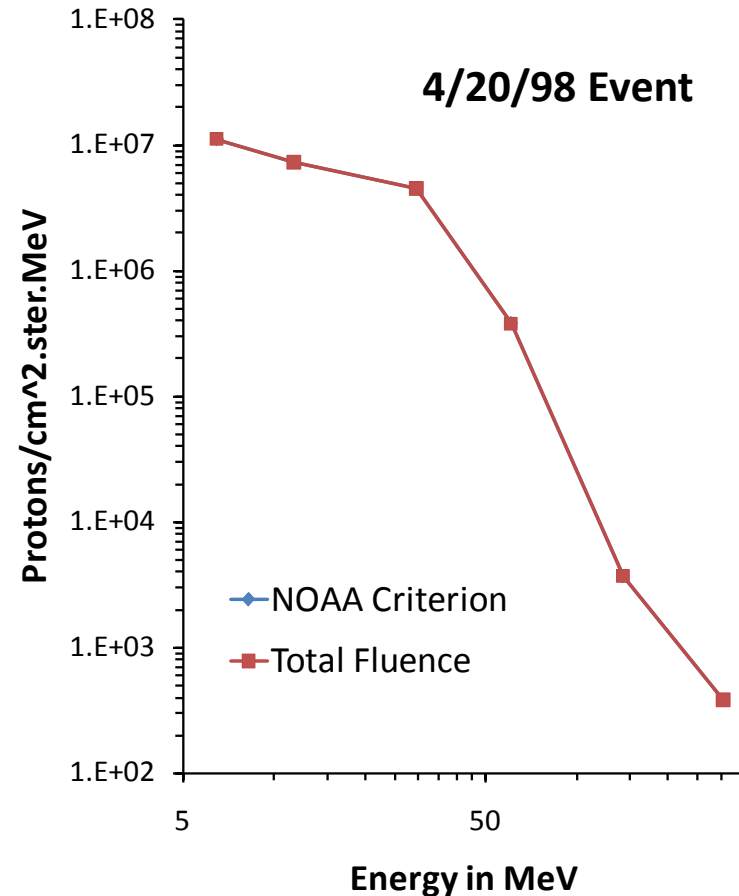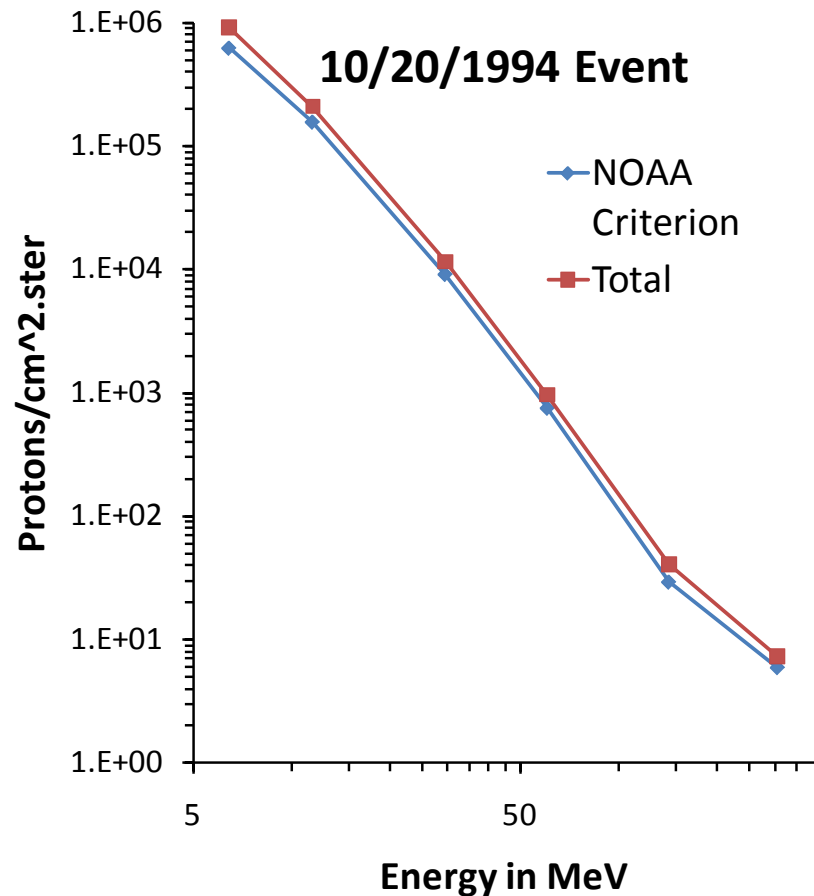
Data sources for the energy spectra for each event.

- **Protons and Helium**
 - 7/1974 - 10/2001: IMP-8 GME
  - 11/2001 – present: GOES

- **Heavy Ions**
 - 7/1974 - 10/2001: IMP-8 CRNE
  - 11/2001 – present: ACE SIS

# Putting all the spectra in a standard format

- The IMP-8 GME format with 29 energy bins was chosen
  - The spectra for the events before Nov. 2001 are taken from GME measurements
- For events after October 2001, GOES data were used
  - The 7-energy-bin GOES spectra were fitted
  - The best spectral fits were used to re-bin the data into the GME format with 29 bins
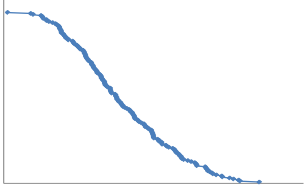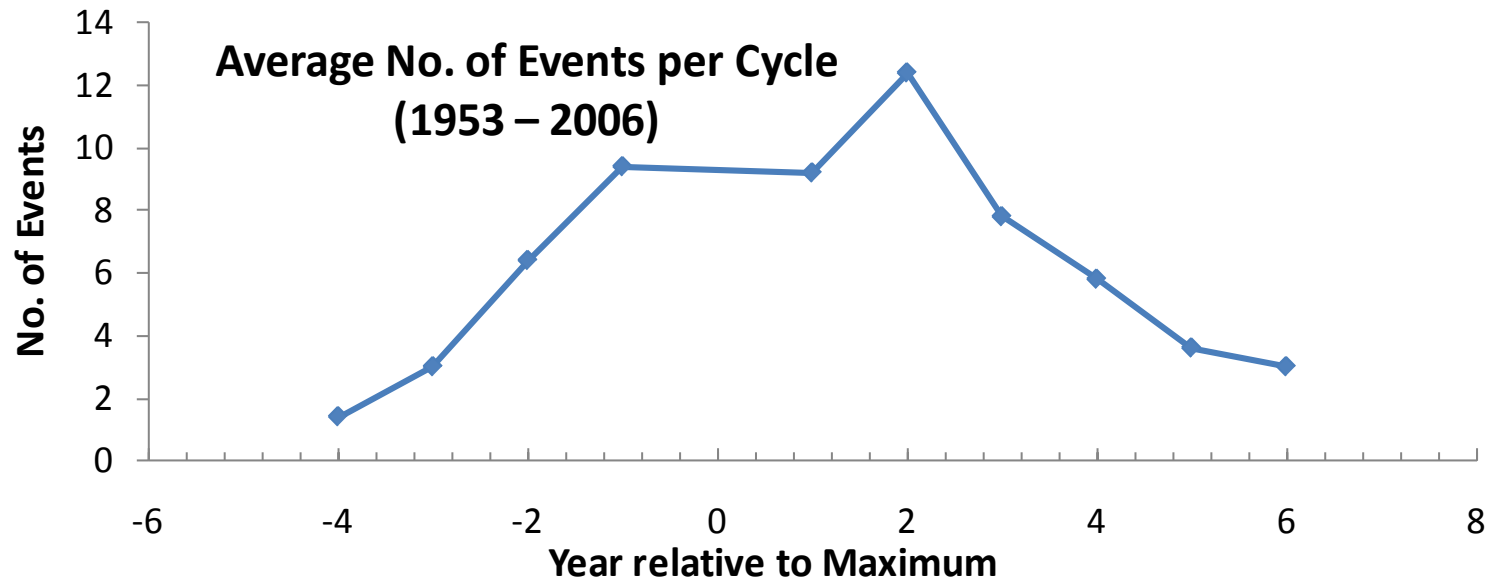
Cumulative spectra

- Cumulative spectra were constructed for each energy bin.
  - An example is shown below
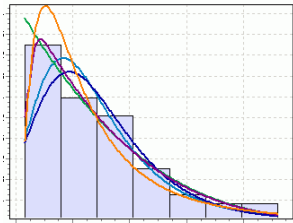

12.9 MeV protons
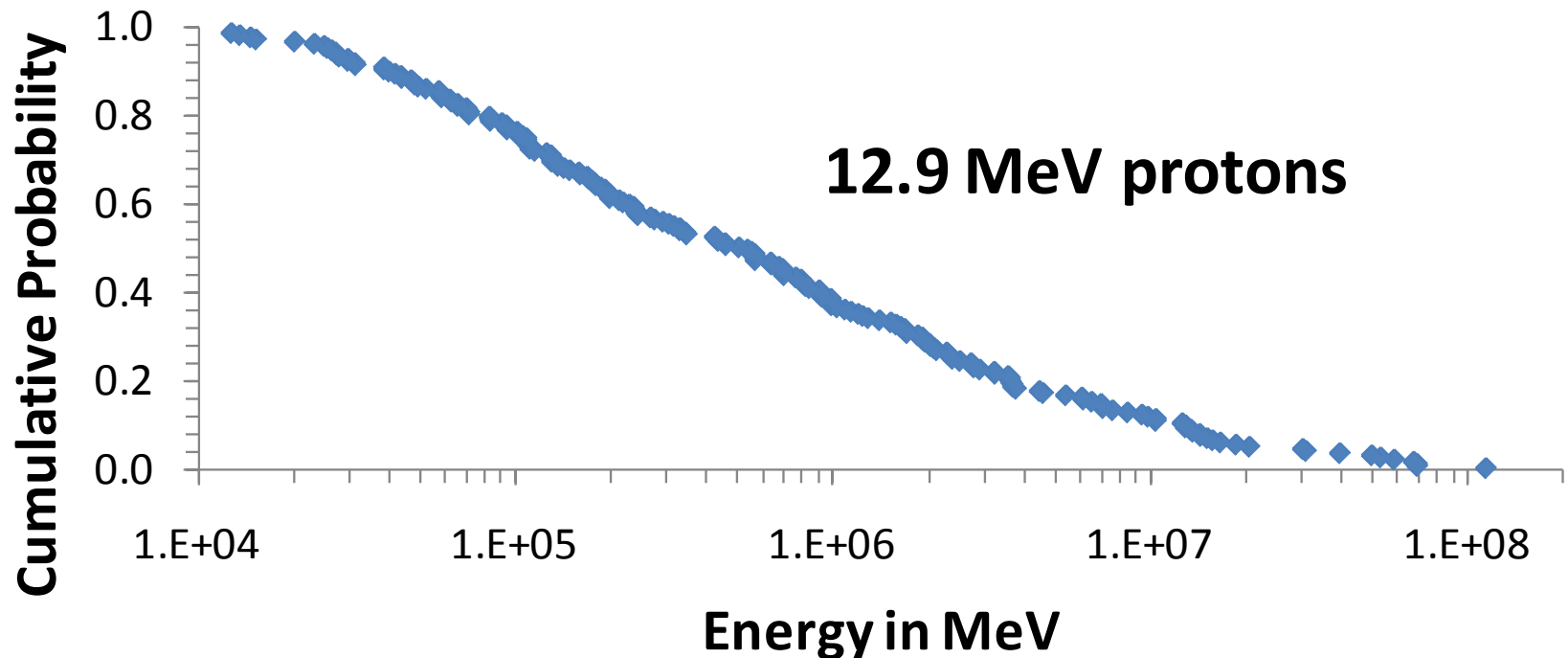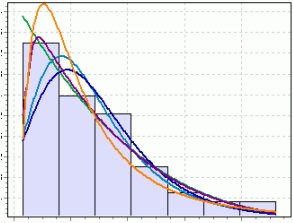
Solar Cycle variation is SPE probability

- The probability of occurrence of solar particle events varies over the 11-year solar cycle as shown below.
  - This variation is used to construct time-independent Poisson Distributions for each year.

Average No. of Events per Cycle (1953 – 2006)

Extreme Value Model

- By convolving the annual Poisson distributions with the cumulative distributions for each energy bin, extreme value distributions have been constructed

- Below is an example of a worst case spectrum for the 90% confidence level during a three year mission beginning Year 0 of the solar cycle
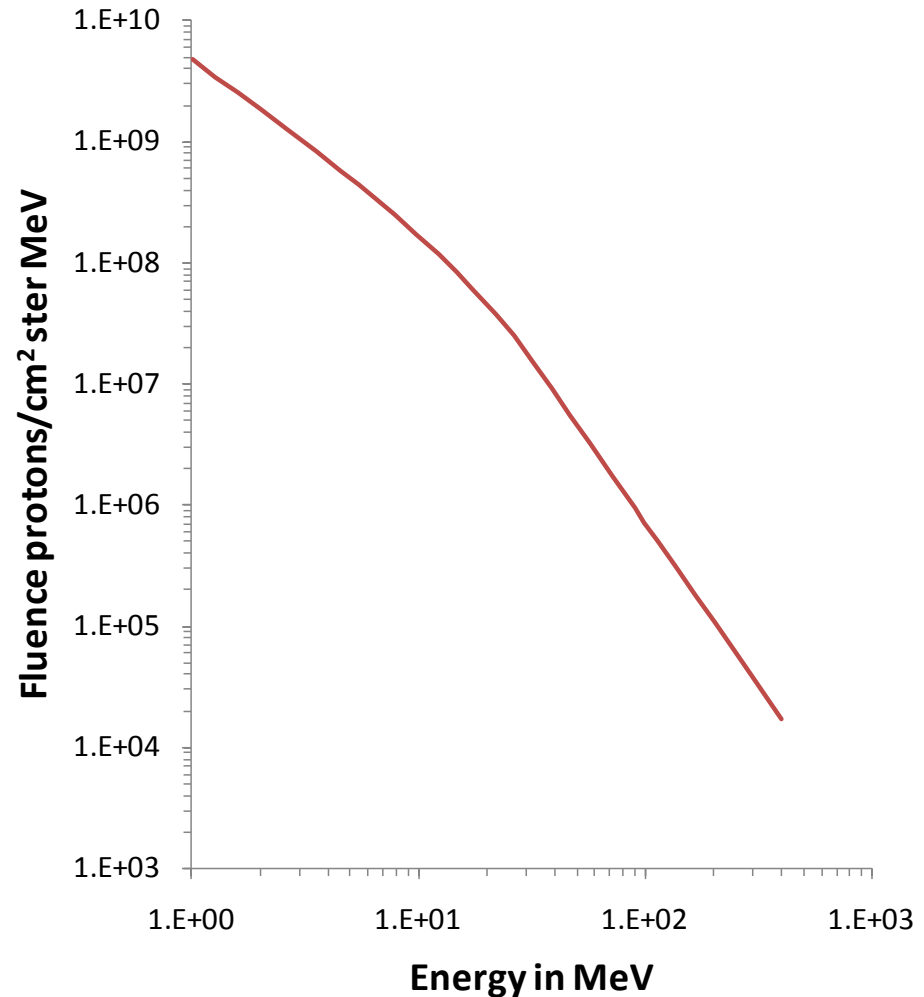
Summary

- We have developed a model for estimating the worst-case episode-integrated proton spectrum that is:
  - Specific to the mission start date and duration
  - At a user-specified confidence level
- We plan to extend this model to:
  - Alpha particles and Heavy ions
- We plan to construct similar models for peak flux and mission-integrated fluences.

Backup charts

How to define the environment

- Identify a large sample of SPEs
- Determine their elemental spectra:
  - Over the energy range of interest
  - In many energy channels
- Form the cumulative distribution in each energy channel and fit it to get the **Initial Distribution.**
- Find the SPE frequency in each phase of the solar cycle to find the time-dependent **Poisson distribution.**

Extreme Value Distribution

- Combine the Initial and Poisson distributions to obtain the extreme value distributions for:
  - Each phase of the solar cycle
  - Each energy bin
- Use the user input to find the flux in each energy bin of each elemental spectrum
- Fit the spectra to obtain analytic representations of the worst-case spectra.