

# Stream Lifetimes Against Planetary Encounters

G. B. Valsecchi • E. Lega • Cl. Froeschlé

**Abstract** We study, both analytically and numerically, the perturbation induced by an encounter with a planet on a meteoroid stream. Our analytical tool is the extension of Öpik’s theory of close encounters, that we apply to streams described by geocentric variables. The resulting formulae are used to compute the rate at which a stream is dispersed by planetary encounters into the sporadic background. We have verified the accuracy of the analytical model using a numerical test.

**Keywords** meteoroid streams · planetary close encounters

## 1 Introduction

Meteoroids stream orbits can intersect, in specific phases of their evolution, the orbit of the Earth, leading to meteor showers. This causes not only the removal of particles from the stream due to collisions, but also potentially large perturbations of the remaining stream members due to planetary encounters.

We here examine the role of planetary encounters on the dispersion of streams using results from the analytical theory of close encounters. The reason for an analytical approach, which is inevitably affected by some approximations, is to be able to generalize the results to most orbits of interest.

To this purpose, we use the extension of Öpik’s theory of planetary close encounters [Öpik 1976] developed in recent years [Valsecchi et al. 2003]. In it, the gravitational model is a restricted, circular, 3-dimensional 3-body problem in which, far from the planet, the small body moves on an unperturbed heliocentric keplerian orbit. The encounter with the planet is modeled as an instantaneous transition from the incoming asymptote of the planetocentric hyperbola to the outgoing one, taking place when the small body crosses the  $b$ -plane, the plane centered on the Earth and normal to the incoming asymptote of the planetocentric hyperbola (i.e., normal to the unperturbed geocentric velocity  $U$  of the small body). The direction of the latter is defined by two angles,  $\theta(U, a)$  and  $\phi(a, e, i)$  (see Figure 1), such that

$$U = \sqrt{3 - \frac{1}{a} - 2\sqrt{a(1 - e^2)} \cos i}$$

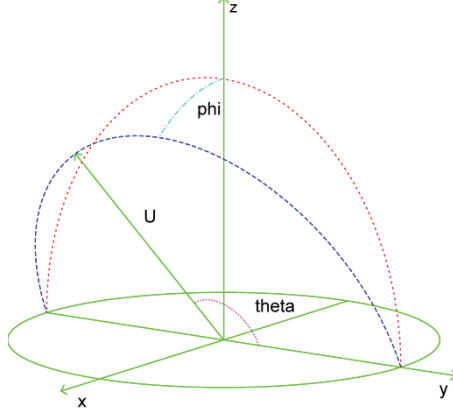
and

$$\begin{aligned} U_x &= U \sin \theta \sin \phi \\ &= \pm \sqrt{2 - \frac{1}{a} - a(1 - e^2)} \end{aligned}$$

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**Figure 1.** The geometric set up of Öpik's theory: the Earth is at the origin of axes and moves in the direction of the  $y$ -axis, while the Sun is on the negative  $x$ -axis; the geocentric velocity vector of the small body is  $U$ ,  $\theta$  is the angle between  $U$  and the  $y$ -axis, and  $\phi$  is the angle between the plane containing  $U$  and the  $y$ -axis, and the  $y$ - $z$  plane.

$$\begin{aligned}
 U_y &= U \cos \theta \\
 &= \sqrt{a(1-e^2)} \cos i - 1 \\
 U_z &= U \sin \theta \cos \phi \\
 &= \pm \sqrt{a(1-e^2)} \sin i
 \end{aligned}$$

and

$$\begin{aligned}
 \cos \theta &= \frac{1-U^2-\frac{1}{a}}{2U} \\
 \sin \phi &= \pm \frac{\sqrt{2-\frac{1}{a}-a(1-e^2)}}{\sqrt{2-\frac{1}{a}-a(1-e^2)\cos^2 i}} \\
 \cos \phi &= \pm \frac{\sqrt{a(1-e^2)} \sin i}{\sqrt{2-\frac{1}{a}-a(1-e^2)\cos^2 i}}
 \end{aligned}$$

where the upper sign in the expressions for  $U_z$  and  $\cos \phi$  apply to encounters at the ascending node, and in the expressions for  $U_x$  and  $\sin \phi$  apply to post-perihelion encounters, while  $a$  is in AU and  $U$  is in units of the orbital velocity of the Earth.

## 2 Earth Cross-section

As already noted, collisions with the Earth remove meteoroids from a stream. For a given stream, the collisional cross-section of the Earth on the  $b$ -plane is  $\pi b_{\oplus}^2$  with

$$b_{\oplus} = \sqrt{r_{\oplus}^2 + 2cr_{\oplus}},$$

where  $r_{\oplus}$  is the radius of the Earth in AU,  $c = m/U^2$  and  $m$  is the mass of the Earth in solar masses. The values of  $c$  and  $b_{\oplus}$  are tabulated for various streams in Table 1; the values of  $U$  of the stream orbits are taken from [Jopek et al. 1999].

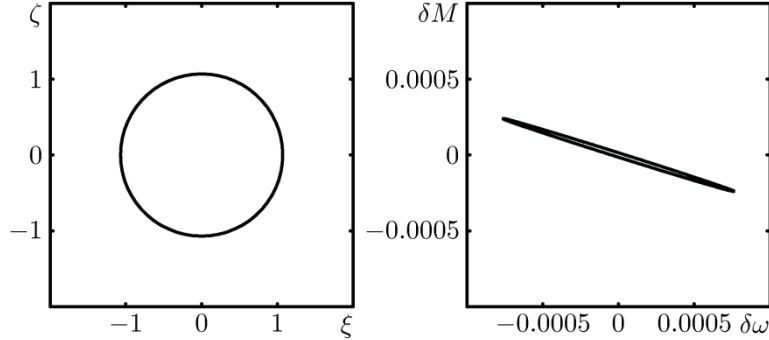
**Table 1.** Values of  $c$  and  $b_{\oplus}$ , in Earth radii, for various streams of interest.

Stream	$c$	$b_{\oplus}$
Leonids	0.013	1.01
Perseids	0.018	1.02
Lyrids	0.029	1.03
Quadrantids	0.038	1.04
Southern $\delta$ -Aquariids	0.038	1.04
Geminids	0.052	1.05
Northern Taurids	0.070	1.07
Northern $\alpha$ -Capricornids	0.12	1.11

Starting from [Valsecchi 2006], [Valsecchi et al. 2005] derived an algorithm to pass from  $b$ -plane coordinates, close to a collision with the planet, to pairs of orbital elements (assuming that all the other elements are kept constant), in the framework of the extended Öpik’s theory. The algorithm neglects second and higher order terms in the distance from the origin.

We here apply it to meteoroid streams encountering the Earth, keeping fixed  $a, e, i, \Omega$  (and thus  $U, \theta, \phi, \lambda$ ), and computing in the  $\omega$ - $M$  plane the area of the ellipse corresponding to a circle centered in the origin of the  $b$ -plane.

Figure 2 shows the collisional cross-section of the Earth on the  $b$ -plane for the Northern Taurids and the corresponding ellipse, computed analytically, in the  $\delta\omega$ - $\delta M$  plane, where  $\delta\omega$  and  $\delta M$  are the displacements in the respective angles relative to a central collision with the Earth.



**Figure 2.** Left: the collisional cross-section of the Earth on the  $b$ -plane for the Northern Taurids; right: the same cross-section in the  $\delta\omega$ - $\delta M$  plane.

An explicit computation, along the lines of [Valsecchi et al. 2005], shows that the area of the ellipse in the  $\delta\omega$ - $\delta M$  plane is

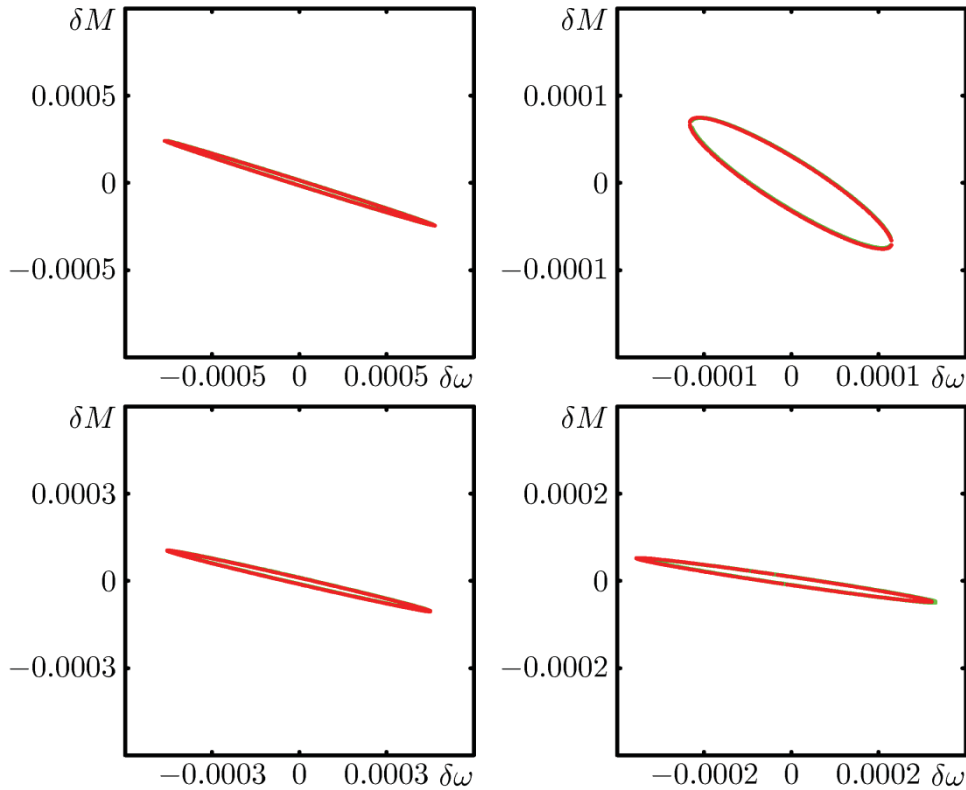
$$A(b_{\oplus}) = \frac{\pi b_{\oplus}^2}{(a^{3/2} \sin i \sin \theta |\sin \phi|)},$$

where we take the values for  $a, i, \theta, \phi$  for the stream from [Jopek et al. 1999].

To test the validity of the analytic approach, we have set up the following numerical experiment: in the restricted, circular, 3-dimensional 3-body problem, we start a suitable number of meteor particles at a large distance from the Earth; all the particle orbits have the same  $a$ ,  $e$ ,  $i$ ,  $\Omega$ , while  $\omega$ ,  $M$  are distributed on a regularly spaced grid. We follow the particles through an encounter with the Earth, and check which of them actually collide with it (i.e., those for which the minimum geocentric distance along the perturbed trajectory is less or equal to  $r_{\oplus}$ ); interpolating in the grid, we can then find the initial values of  $\omega$ ,  $M$  for which the minimum geocentric distance is exactly  $r_{\oplus}$ .

We have used a fourth order Runge-Kutta integrator on the equations of motion regularized through Kustaanheimo-Stiefel regularization [Kustaanheimo and Stiefel 1965]. The reader can find in [Froeschlé 1970] a detailed derivation of the regularized equations of motions using the Lagrangian formalism and in [Celletti 2002] a review of regularization theory. As recently shown in [Celletti et al. 2010] and in [Lega et al. 2010], when integrating orbits undergoing close encounters or even collisions, the existence of the singularity cannot be canceled, neither by changing the integration scheme, nor through a better precision computation; by singularity we mean that the solution does not behave as a power series about the point, while usual integration schemes are based on the development in power series of the solution.

The results of these computations are compared to those of our analytical approach in Figure 3 and, as the plots show, are definitely satisfactory.



**Figure 3.** The collisional cross-section of the Earth in the  $\delta\omega$ - $\delta M$  plane for the Northern Taurids (top left), the Geminids (top right), the Northern  $\alpha$ -Capricornids (bottom left), and the Quadrantids (bottom right); superimposed on the analytical estimates (green lines) are the results of a numerical computation (red dots).

### 3 Stream Dispersion

The agreement between the analytic computation and the numerical check encourages us in the use of the former in order to study the dispersion induced in a stream by its passage close to the Earth. To get a quantitative description of stream dispersion, we start by recalling the orbital similarity criterion based on  $U$ ,  $\cos \theta$ ,  $\phi$  and  $\lambda$ , the longitude of the Earth at the time of the meteor fall, introduced by [Valsecchi et al. 1999] to classify meteoroids in streams; it is based on the quantity  $D_N$  defined by:

$$D_N^2 = [U_2 - U_1]^2 + [\cos \theta_2 - \cos \theta_1]^2 + \Delta \Xi^2$$

where

$$\Delta \Xi^2 = \min [\Delta \phi_I^2 + \Delta \lambda_I^2, \Delta \phi_{II}^2 + \Delta \lambda_{II}^2]$$

and

$$\begin{aligned} \Delta \phi_I &= 2 \sin \frac{\phi_2 - \phi_1}{2} & \Delta \phi_{II} &= 2 \sin \frac{\pi + \phi_2 - \phi_1}{2} \\ \Delta \lambda_I &= 2 \sin \frac{\lambda_2 - \lambda_1}{2} & \Delta \lambda_{II} &= 2 \sin \frac{\pi + \lambda_2 - \lambda_1}{2} \end{aligned}$$

As discussed in [Valsecchi et al. 1999], this criterion basically uses the geocentric speed, the anti-radiant coordinates (in a frame rotating with the Earth about the Sun) and the date of meteor fall, instead of the usual orbital elements. Of these quantities, encounters with the Earth affect only the anti-radiant coordinates, and therefore  $\theta$ ,  $\phi$ , since  $U$  is an invariant; we disregard changes in  $\lambda$ , since they are far smaller than those in  $\theta$  and  $\phi$ .

In the approximation  $c^2 \ll b^2$ , valid for all of the streams we examined, we have that an encounter with the Earth at unperturbed distance  $b$  rotates the geocentric velocity vector by an angle  $\gamma$  given by:

$$\sin \gamma \approx \frac{2c}{b}.$$

Thus, in order to turn the direction of  $U$  (i.e., to change the radiant) by a significant quantity, say by  $\gamma \geq 0.1$  rad, we need an encounter with the Earth taking place at:

$$b_{\gamma \geq 0.1} \leq \frac{2c}{0.1}.$$

Table 2 gives the values of  $b_{\gamma \geq 0.1}$  for the same streams of Table 1; note that for all the tabulated streams, with the exception of the Northern Taurids and the Northern  $\alpha$ -Capricornids, the collisional cross-section is larger than the deflection cross-section.

**Table 2.** Values of  $b_{\gamma \geq 0.1}$  and of  $b_{\oplus}$ , in Earth radii, for various streams of interest.

Stream	$c$	$b_{\gamma \geq 0.1}$	$b_{\oplus}$
Leonids	0.013	0.25	1.01
Perseids	0.018	0.36	1.02
Lyrids	0.029	0.57	1.03
Quadrantids	0.038	0.75	1.04
Southern $\delta$ -Aquariids	0.038	0.75	1.04
Geminids	0.052	1.03	1.05
Northern Taurids	0.070	1.4	1.07
Northern $\alpha$ -Capricornids	0.12	2.3	1.11

For a generic, not too large value of  $b$ , the area  $A(b)$  on the  $\delta\omega$ - $\delta M$  plane covered by the ellipse corresponding to a circle of radius  $b$  centered in the origin of the  $b$ -plane is given, as seen before, by:

$$A(b) = \frac{\pi b^2}{a^{3/2} \sin i \sin \theta |\sin \phi|}$$

Every year, when the Earth crosses a stream, a fraction of the latter will be removed, either by collision or by deflection by an angle larger than a suitable threshold. On the other hand, at the Earth crossing the meteoroids have, in general,  $0 \leq \delta M \leq 2\pi$ , while the values of  $\delta\omega$  are characterized by  $(\delta\omega)_{min} \leq \delta\omega \leq (\delta\omega)_{max}$ , with values of  $(\delta\omega)_{min}$  and  $(\delta\omega)_{max}$  different for each stream, as function of, among other things, the age of the stream itself.

Thus, the fraction  $f(b)$  removed each year from a stream, either by collision or by

$$f(b) = \frac{A(b)}{2\pi\Delta\omega},$$

where  $\Delta\omega = (\delta\omega)_{max} - (\delta\omega)_{min}$ .

Table 3 gives the fraction, to be divided by a suitable value of  $\Delta\omega$  for each stream, eliminated yearly by collisions and/or close encounter with the Earth; as it is readily seen, the Earth seems not to have a major effect in dispersing streams. Note, however, that some of the tabulated streams intersect the orbit of Jupiter, something that would greatly accelerate their dispersion.

**Table 3.** Values of  $f(b_{\gamma \geq 0.1}) \cdot \Delta\omega$  and of  $f(b_{\oplus}) \cdot \Delta\omega$ , in Earth radii, for various streams of interest.

Stream	$f(b_{\gamma \geq 0.1}) \cdot \Delta\omega$	$f(b_{\oplus}) \cdot \Delta\omega$
Leonids	—	$1.9 \cdot 10^{-9}$
Perseids	—	$6.8 \cdot 10^{-11}$
Lyrids	—	$4.1 \cdot 10^{-12}$
Quadrantids	—	$3.1 \cdot 10^{-9}$
Southern $\delta$ -Aquariids	—	$5.7 \cdot 10^{-10}$
Geminids	—	$1.7 \cdot 10^{-9}$
Northern Taurids	$1.1 \cdot 10^{-8}$	$6.6 \cdot 10^{-9}$
Northern $\alpha$ -Capricornids	$1.2 \cdot 10^{-8}$	$2.5 \cdot 10^{-9}$

## 4 Conclusion

We have presented an analytic formula relevant for the rate of dispersion of a meteoroid stream induced by encounters and collisions with the Earth, and have checked its validity by numerical computations in the restricted, circular 3-body problem.

We plan to pursue this work, extending it to encounters with more than one planet, and investigating the coupled effects of planetary close encounters and of secular perturbations.

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