# Formation of the Aerosol of Space Origin in Earth's Atmosphere 

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#### Abstract

The problem of formation of the aerosol of space origin in Earth's atmosphere is examined. Meteoroids of the mass range of $10^{-18}-10^{-8} \mathrm{~g}$ are considered as a source of its origin. The lower bound of the mass range is chosen according to the data presented in literature, the upper bound is determined in accordance with the theory of Whipple's micrometeorites. Basing on the classical equations of deceleration and heating for small meteor bodies we have determined the maximal temperatures of the particles, and altitudes at which they reach critically low velocities, which can be called as "velocities of stopping". As a condition for the transformation of a space particle into an aerosol one we have used the condition of non-reaching melting temperature of the meteoroid. The simplified equation of deceleration without earth gravity and barometric formula for the atmosphere density are used. In the equation of heat balance the energy loss for heating is neglected. The analytical solution of the simplified equations is used for the analysis.

As an input parameter we have used the cumulative distribution of space matter influx onto earth on masses in large mass range. Basing on this distribution we have plotted three-dimensional probability density distribution of influx of particles as a function of parameters, which determine the heating and stop altitude of a meteoroid: initial mass $m_{0}$, velocity of entry into the atmosphere $v_{0}$ and radiant zenith angles $z_{\mathrm{R} 0}$. The obtained three-dimensional distribution had been presented first as a product of three independent distributions on the mentioned parameters, then it was transformed using the equation of deceleration into the distribution on the following parameters: $m_{0}, v_{0}$ and "altitude of stopping" $H_{S}$. The final 2-dimensional distribution on parameters $v_{0}$ and $H_{S}$ of the aerosols of space origin in the atmosphere was obtained by means of integration of the previous distribution over $v_{0}$.


Keywords meteoroids • meteors • atmosphere aerosol • aerosol formation • space origin

## 1 Introduction

There are aerosols of both ground and space origin in Earth's atmosphere. Aerosols of the ground origin are presented basically in the lower atmosphere: in the troposphere. The most powerful aerosol layer of the ground-based origin, known also as Junge Layer, is placed at altitudes of $10-25 \mathrm{~km}$. It originated from the condensation of some components of the atmosphere appearing from the photo-chemical transformations of some products of volcano eruptions, for example sulphuric acid vapors. The second confidently established aerosol layer in the atmosphere is placed at altitudes of $80-85 \mathrm{~km}$, corresponding to the minimal atmospheric temperature, in the mesopause. The origin of this aerosol layer in not finally established. Most of scientists, and the authors as well, hold an opinion that all the particles there to be of space origin. Under some special conditions the condensation of water vapors on these particles becomes possible, and we can see, probably, the high-latitudinal silvery clouds.

According to meteor physics investigations (Whipple 1950; Whipple 1951; Levin 1956; Öpik 1956; Lebedinets 1980; Lebedinets 1981; Voloshchuk et al. 1989) most of low-mass particles coming into the atmosphere with initial velocities $\sim 11.2-72.5 \mathrm{~km} / \mathrm{s}$ lose their energy at altitudes of $140-80 \mathrm{~km}$. Small fragments detaching from already heated bigger particles in the atmosphere cannot be decelerated without almost entire loss of their masses due to evaporation. The deeper penetration of a particle to the atmosphere the lower the probability to save its macro size. This task of motion, deceleration and destruction of a separated particle in "abnormal environment" according to the terminology of Öpik (1956) we were considering in Voloshchuk et al. (1989). Such a conclusion is also given from the experimental investigations of chemical analysis of particles, caught in the atmosphere with the help of high airplanes and balloons. Such particles are similar to coaly chondritics (Nady 1975), having a big amount of helium in their surfaces, which penetrated there from the solar wind. Therefore, these are the primary interplanetary particles, which have come though the atmosphere without intensive heating and are not the products of fragmentation of larger bodies (Brownlee and Hodge 1973).

The amount and distribution of the aerosol of space origin in the atmosphere is connected by some authors with planetary global warming. In this work we will try to examine the problem of formation of the aerosol of space origin in Earth's atmosphere basing on the initial meteoroid distributions on the Earth's heliocentric orbit and the equations of classic meteor physics.

## 2 Meteor Physics Equations to be Used

In this chapter we consider the basic equations of meteor physics to be used in the work, namely the equation of heating of the meteoroid, and the equation of its deceleration. In addition, the simplification of the equations in order to realize the final investigation analytically is substantiated.

### 2.1 Complete Equations of Meteoroid Deceleration and Heating

The base assumption for the transformation of a small meteoroid into an aerosol particle, not into a meteor, consists in non-reaching by the meteoroid its melting temperature. Therefore, we have to determine the mass interval, and other parameters of meteoroids, which coming into the Earth's atmosphere, do not reach the melting temperature because of their deceleration and heat radiation.

### 2.1.1 Heating Balance Equation

The theory of heating of low-mass meteoroids with their deceleration, which plays an important role in this case, were developed by Whipple (1950), Whipple (1951) and later by Fecenkov (1955). They have obtained the name of Whipple's micro-meteorites. It is known (Levin 1956) that the particles having the size less than $x_{0}$ warm up to the same temperature ( $x_{0}$ is the warming up depth at which the temperature of the body is less to $e$ times relatively the surface). According to Öpik (1937) and Levin (1956) such particles have radius $r_{0} \leq 10^{-3} \mathrm{~cm}$. The change of temperature of such a particle with taking into account the energy loss for heating and radiation can be written as:

$$
\begin{equation*}
S_{M 0} E d t=m_{0} c d T+\beta \sigma\left(T^{4}-T_{0}^{4}\right) S_{F 0} d t \tag{1}
\end{equation*}
$$

where $S_{M 0}=$ const and $S_{F 0}=$ const are the middle section and entire surface area of the particle accordingly, $m_{0}=$ const is its initial mass, $c$ is the specific heat capacity and $\sigma$ is Stefan's constant, $T$ and
$T_{0}$ are the current temperature for time $t$ and initial temperature of the particle in the field of solar radiation at the distance of 1 a.u., $\beta \leq 1$ is a coefficient of thermal radiation of the meteoroid characterizing the digression from black body radiation, $E=\Lambda \rho_{A} v^{3} / 2$ is the energy incoming to unity of the meteoroid surface due to its collision with atmosphere molecules, $\Lambda$ is the dimensionless coefficient of heat conductivity, $\rho_{A}$ is the atmosphere density.

### 2.1.2 Deceleration Equation

If the space particle is not warmed up to the melting temperature it becomes the aerosol particle. So, the next question we should answer: at which altitude will it stop? In order to solve this problem we consider the equation of deceleration, which can be written in the most common vector view as

$$
\begin{equation*}
m_{0} \frac{d \mathbf{v}}{d t}=-c_{R} S_{M 0} \rho_{A} v \mathbf{v}+m_{0} \mathbf{g}, \tag{2}
\end{equation*}
$$

or separated into constituent parts:

$$
\begin{align*}
& m_{0} \frac{d v}{d t}=-c_{R} S_{M 0} \rho_{A} v^{2}+m_{0} g \cos z_{R}  \tag{3}\\
& v \frac{d z_{R}}{d t}=-g \sin z_{R}, \tag{4}
\end{align*}
$$

where $v$ is the meteoroid velocity, $z_{R}$ is the zenith angle of its radiant, $c_{R}$ is the resistance coefficient, $g$ is the free fall deceleration constant.

### 2.2 Accepted Assumptions

In our calculations we use some assumptions and simplifications. First, we suppose the particles of space origin producing the aerosol are of meteor mass range. Lower bound of meteoroid initial mass is $10^{-18} \mathrm{~g}$ according to meteoroid mass distributions presented in literature (Ceplecha et al. 1992), higher bound corresponds to the $r_{0}$, and is approximately equal to $10^{-8} \mathrm{~g}$ (Öpik 1937). The second assumption is that we consider just warmed up and evaporated particles and neglect the mass loss due to blowing meteoroid molecules away in its "cold" state. The next, the most doubtful assumption consists in the fact we use the barometric formula for the atmosphere density:

$$
\begin{equation*}
\rho_{A}(H)=\rho_{A}(0) \exp \left(-\frac{H}{H^{*}}\right) . \tag{5}
\end{equation*}
$$

Here $\rho_{A}(0), H^{*}$ are the atmosphere density at the sea level and altitude of the homogeneous atmosphere accordingly. For precise calculations one should use the numerical solution of the equations (1) and (2) and take the real atmosphere density distribution from modern models of atmosphere, especially for altitudes over approximately 120 km . We use the formula (5) here just for the purpose of obtaining the analytical solution of (3) and (4) in order to understand the physics of the aerosol layer formation. Then,
we consider the sporadic meteoroids as the main source of aerosol particles, i.e. the particles which are supposed to be of the stone composition. Finally, we calculate the mean aerosol influx during a year.

### 2.3 Simplification of the Equations

According to Öpik (1937), the small meteoroid spends almost all its energy for the thermal radiation if its radius $r \leq 10^{-3} \mathrm{~cm}$ (corresponds to $m_{0} \approx 10^{-8} \mathrm{~g}$ for spherical particles), so we can neglect the first term in the equation (1):

$$
\begin{equation*}
T^{4}-T_{0}^{4}=\frac{S_{M 0} \Lambda \rho_{A} v^{3}}{2 \beta \sigma S_{F 0}} \tag{6}
\end{equation*}
$$

Since we deal with low-mass particles we can suppose they are decelerated relatively fast, so we can neglect the gravity term in the equations (2). The equations (3) and (4) in this case transform into the equation

$$
\begin{equation*}
m_{0} \frac{d v}{d t}=-c_{R} S_{M} \rho_{A} v^{2}, \tag{7}
\end{equation*}
$$

and $z_{R}=z_{R 0}=$ const .
Also we use the relation between the time $t$ and altitude $H$ of the particle

$$
\begin{equation*}
d H=-v \cos z_{R} d t \tag{8}
\end{equation*}
$$

and express the middle section and surface area of the particle through the shape parameter $A: A=$ $S_{M} / V^{2 / 3}$, where $V$ is the meteoroid volume. Supposing the particle is spherical $S_{\mathrm{F} 0}=4 S_{\mathrm{M} 0}=4 A\left(m_{0} / \rho_{M}\right)^{2 / 3}$, the shape parameter for spherical particles to be $A=\pi(3 / 4 \pi)^{2 / 3}$.

### 2.4 Variation Parameters, Constants and Final Equations

Using (6), (7), (8), the shape parameter and barometric formula (5) we obtain

$$
\begin{align*}
& T^{4}-T_{0}^{4}=\frac{\Lambda \rho_{A} v_{0}^{3}}{8 \beta \sigma} \exp \left(-\frac{3 c_{R} A H^{*}}{m_{0}^{1 / 3} \rho_{M}^{2 / 3} \cos z_{R 0}} \rho_{A}\right)  \tag{9}\\
& v=v_{0} \exp \left(-\frac{c_{R} A H^{*}}{m_{0}^{1 / 3} \rho_{M}^{2 / 3} \cos z_{R 0}} \rho_{A}\right) . \tag{10}
\end{align*}
$$

Reaching by the particle of maximal temperature along its trajectory can be derived from $d T / d \rho_{A}=0$, and so from (9):

$$
\rho_{A T \max }=\frac{m_{0}^{1 / 3} \rho_{M}^{2 / 3} \cos z_{R 0}}{3 c_{R} A H^{*}}
$$

Putting it back into (9) we obtain

$$
\begin{equation*}
T_{\max }^{4}-T_{0}^{4}=\frac{\Lambda m_{0}^{1 / 3} \rho_{M}^{2 / 3} \cos z_{R 0} 0_{0}^{3}}{24 \beta \sigma c_{R} A H^{*} \exp (1)} . \tag{11}
\end{equation*}
$$

Thus, the condition of transformation of space particle into the aerosol can be expressed now as

$$
\begin{equation*}
T_{\max } \leq T_{\text {melt }}, \tag{12}
\end{equation*}
$$

where $T_{\text {max }}$ has to be expressed from (11), $T_{\text {melt }}$ is the melting temperature of the particle.
Looking at the equations (10) and (11) we can note that there are three parameters of a meteoroid (under the assumptions made above) having an influence onto its belonging to the class of aerosols or meteors, and to the altitude of stopping in the case of the aerosols. These are initial mass of the particle $m_{0}$, velocity $v_{0}$, and zenith radiant angle $z_{R 0}$. The ranges of their variations are: $m_{0}=10^{-18}-10^{-8} \mathrm{~g}$ according to Ceplecha et al. (1992) for the lower limit and Öpik (1937) for the higher limit (see above), $v_{0}=11.2-72.5 \mathrm{~km} / \mathrm{s}$, i.e. the particles belonging to the solar system are considered, $z_{R 0}=0^{\circ}-90^{\circ}$, all possible entrance angles are taken into account.

Expressing (12) through the variation parameters we obtain the final inequality of separation of the meteoroids onto aerosols and meteors

$$
\begin{equation*}
m_{0}^{1 / 3} v_{0}^{3} \cos z_{R 0} \leq C_{T} \tag{13}
\end{equation*}
$$

where $C_{T}=24 \beta \sigma c_{R} A H^{*} \exp (1)\left(T_{\text {melt }}^{4}-T_{0}^{4}\right) / \Lambda \rho_{M}^{2 / 3}$.
If the condition (13) is realized we can find the altitude of stopping $H_{S}$ of the aerosol particle from (10), supposing the velocity of stopping $v_{\mathrm{S}}$ is a small enough value. Here we continue to use the equation (10) except the Stokes formula for low velocities, where the deceleration is proportional to the first power of the velocity, so $H_{S}$ can be found from the expression

$$
\begin{equation*}
v_{0}=v_{S} \exp \left(\frac{C_{V} \rho_{A}(0) \exp \left(-\frac{H_{S}}{H^{*}}\right)}{m_{0}^{1 / 3} \cos z_{R 0}}\right), \tag{14}
\end{equation*}
$$

where $C_{V}=c_{R} A H^{*} \rho_{M}^{-2 / 3}$.
During the calculations the following values of constants are taken (Levin, 1956): $\Lambda=1, \sigma=$ $5.67032 \times 10^{-5} \mathrm{erg} \cdot \mathrm{cm}^{-2} \cdot \mathrm{~K}^{-2} \cdot \mathrm{~s}^{-1}, \beta=1, c_{R}=1, \mathrm{H}^{*}=7 \times 10^{5} \mathrm{~cm}, \rho_{A}(0)=1.6 \times 10^{-3} \mathrm{~g} / \mathrm{cm}^{3}, \rho_{M}=3 \mathrm{~g} / \mathrm{cm}^{3}, T_{0}=$ $276 \mathrm{~K}, T_{\text {melt }}=1600 \mathrm{~K}, v_{\mathrm{S}}=0.5 \mathrm{~km} / \mathrm{s}$. For an iron particle $\Lambda=0.75 ; \rho_{M}=7.6 \mathrm{~g} \cdot \mathrm{~cm}^{-3} ; c_{R}=1.25 ; T_{\text {melt }}=$ 1800 K.

## 3 The Statistical Approach to the Process of Space Origin Aerosol Formation in the Atmosphere

Here we propose the statistical model for the description of atmospheric aerosol formation from meteoroids. We will construct the 3-dimensional distribution on variation parameters having an
influence onto the probability of the aerosol formation and onto the altitude of the aerosol layer. Let's represent the distribution as a multiplication of three independent single-parameter probability density distributions:

$$
\begin{equation*}
p_{m v z}\left(m_{0}, v_{0}, z_{R 0}\right)=p_{m}\left(m_{0}\right) p_{v}\left(v_{0}\right) p_{z}\left(z_{R 0}\right) . \tag{15}
\end{equation*}
$$

It is obvious that this distribution should be normalized to unity over all three parameters, i.e. there must be

$$
\iiint p_{m v z}\left(m_{0}, v_{0}, z_{R 0}\right) d m_{0} d v_{0} d z_{R 0}=1
$$

where the integration is carried out inside all possible ranges of parameter values.

### 3.1 Primary Distribution of Meteoroids

Let us find all three 1-dimensional primary probability density distributions, and start from the distribution on mass.

### 3.1.1 Probability Density Distribution on Initial Mass

There can be found in the literature distributions of space matter onto Earth as cumulative distributions of number of particles on their masses, for example Ceplecha (1992), Kruchynenko (2002), Kruchynenko (2004). We will use the linear dependence (Kruchynenko 2002, Kruchynenko 2004) for the further calculations:

$$
\begin{equation*}
\log _{10} N\left(m_{0}^{\prime} \geq m_{0}\right)=C_{0}-k \log _{10} m_{0}, \tag{16}
\end{equation*}
$$

where $N\left(m_{0}{ }^{\prime} \geq m_{0}\right)$ is a number of particles with masses not less than $m_{0}$ coming into all Earth atmosphere during a year, $C_{0}=7.86, k=0.892$.

The probability density distribution on mass $p_{m}\left(m_{0}\right)$ according to cumulative distribution (16) can be described by Pareto distribution:

$$
\begin{align*}
& p_{m}\left(m_{0}<m_{0 l}\right)=0 \\
& p_{m}\left(m_{0} \geq m_{0 l}\right)=\frac{k m_{0 l}^{k}}{m_{0}^{k+1}} \tag{17}
\end{align*}
$$

where $m_{0 l}$ is chosen freely. The probability density function is normalized to unity in the value range $0-$ $+\infty$. There are the following obvious consequences:

$$
\begin{equation*}
F\left(m_{0}\right)=\int_{m_{0 l}}^{m_{0}} p_{m}\left(m_{0}\right) d m_{0}=1-\int_{m_{0}}^{+\infty} p_{m}\left(m_{0}\right) d m_{0}=1-\frac{m_{0 l}^{k}}{m_{0}^{k}}, \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d N\left(m_{0}\right)}{N_{l}\left(m_{0 l} \leq m_{0} \leq+\infty\right)}=d F\left(m_{0}\right)=p_{m}\left(m_{0}\right) d m_{0}, \tag{19}
\end{equation*}
$$

where $\mathrm{F}\left(m_{0}\right)$ is the cumulative probability, $N_{l}\left(m_{0} \geq m_{0 l}\right)$ is a sample of all particles in the chosen range, which can be found from (16) as $N_{l}\left(m_{0 l}\right)=10^{C_{0}} / m_{0 l}^{k}$. We suppose $m_{0 l}=10^{-18} \mathrm{~g}$, so $N_{l}=8.24 \times 10^{23}$.

### 3.1.2 Probability Density Distribution on Initial Velocity

The probability density distribution on velocity $\rho_{v}\left(v_{0}\right)$ will be chosen according to radar meteor observations, for example (Voloshchuk et al. 1989):

$$
\begin{equation*}
p_{\nu}\left(v_{0}\right)=P G\left(\overline{v_{1}}, \sigma_{\nu 1}\right)+(1-P) G\left(\overline{v_{2}}, \sigma_{\nu 2}\right), \tag{20}
\end{equation*}
$$

where

$$
G(v)=\frac{1}{\sigma_{v} \sqrt{2 \pi}} \exp \left(-\frac{(v-\bar{v})^{2}}{2 \sigma_{v}^{2}}\right)
$$

are Gaussians with the following parameters: $\overline{v_{1}}=32.32 \mathrm{~km} / \mathrm{s}, \sigma_{v 1}=6.51 \mathrm{~km} / \mathrm{s}, \overline{v_{2}}=54.26 \mathrm{~km} / \mathrm{s}, \sigma_{v 2}=$ $5.15 \mathrm{~km} / \mathrm{s}$. The value $P$ is changing during a year. For the mean value we choose $P \approx 0.33$ (Voloshchuk et al. 1989). It is obvious that the probability density function is normalized to unity in the range $0-+\infty$.

### 3.1.3 Probability Density Distribution on Initial Radiant Zenith Angle

The probability density distribution on radiant zenith angle $\rho_{z}\left(z_{R 0}\right)$ will be derived from the following thoughts: let suppose that the number of particles $d N(r, r+d r)$ entering into earth atmosphere from some direction in the range $d r$ in some spatial angle $d \Omega$ (see Figure 1) per time unity can be expressed as $d N(r$, $r+d r) \sim 2 n_{0} \pi r d r d \Omega d t$, where $n_{0}$ is a spatial concentration of meteoroids. Since $r=R_{\oplus} \sin z_{R}$, we have $d N\left(z_{R}, z_{R}+d z_{R}\right) \sim 2 n_{0} \pi R_{\oplus}^{2} \sin z_{R} \cos z_{R} d z_{R} d \Omega d t$. So we have to use the sine-cosine distribution $\sin z_{R 0} \cos$ $z_{R 0}$.

After normalization to unity we obtain the final distribution on zenith radiant angle

$$
\begin{equation*}
p_{Z}\left(z_{R 0}\right)=2 \sin z_{R 0} \cos z_{R 0} . \tag{21}
\end{equation*}
$$

Strictly saying, this distribution will be distorted by the Earth gravity, but we use it due to its simplicity.


Figure 1. To the derivation of the probability density function on radiant zenith angle. $\mathrm{R}_{\oplus}$ is Earth's radius.

### 3.2 Separation of the Distribution into Aerosols and Meteors

The primary distribution of meteoroids can be conceived as a geometrical 3-dimensional cube, where the three considered parameters $m_{0}, v_{0}$ and $z_{R 0}$ determine the dimensions along its three ribs-axes, limited by the permissible parameter values. The "intensity" in each point inside such a cube is expressed by the value of $p_{m v z}\left(m_{0}, v_{0}, z_{R 0}\right)$. The real number of particles $d N$ in the range of $d m_{0}, d v_{0}$, and $d z_{R 0}$ can be found from (19). If we make a few sections perpendicularly to the cube rib describing the mass $m_{0}$ we will obtain the 2-dimensional pictures in coordinates $v_{0} \leftrightarrow z_{R 0}$ for the fixed mass values, where the relative value of $p_{m o z}$ can be expressed with the help of lines of the similar values, for instance. In the Figure 2 we show only two maximums of the $p_{m v z}$ corresponding to modal values of bimodal distribution of velocity and the maximum of zenith radiant angle value $z_{R 0}=45^{\circ}$. Figure 2 a corresponds to $m_{0}=10^{-12} \mathrm{~g}$, Figure 2 b to $m_{0}=10^{-9} \mathrm{~g}$. The regions of aerosols and meteors are separated by solid line according to inequality (13).


Figure 2. The separation of meteoroids onto aerosols and meteors. Letter $\boldsymbol{A}$ and dark gray color corresponds to aerosols, otherwise to meteors (letter $\boldsymbol{M}$ and light gray color). The picture $\boldsymbol{a}$ corresponds to mass $m_{0}=10^{-12} \mathrm{~g}, \boldsymbol{b}$ to $m_{0}=10^{-9} \mathrm{~g}$. Dashed lines describe the equal altitudes of stopping. Signs " + " show positions of modes of the distribution.

We can see in Figure 2 that the region of aerosols (denoted in the picture with the letter $A$ and dark gray color) is increasing while the mass is decreasing (the region of meteors denoted as $M$ with lighter gray color is decreasing accordingly). Therefore, there must be a mass value lower of which all particles remain aerosols. The probability for a meteoroid to become a meteor is proportional to its velocity and cosine of zenith radiant angle. Setting according parameters to their maximal values $v_{0}=$ $72.5 \mathrm{~km} / \mathrm{s}$ and $\cos z_{R 0}=1$ we get the critical mass value $m_{0 c r} \approx 1.7 \times 10^{-14} \mathrm{~g}$. Finally, all space particles entering into the Earth atmosphere remain aerosols if their masses are lower than the critical value, then the rate of aerosols is decreasing almost down to zero while the mass is increasing up to the value of approximately $m_{0}=10^{-8} \mathrm{~g}$. This rate $g_{A}\left(m_{0}\right)$ can be easily calculated with the help of the formula

$$
g_{A}\left(m_{0}\right)=\int_{0}^{\pi / 2} p_{z}\left(z_{R 0}\right) d z_{R 0} \int_{v_{0 M N}}^{v_{0}\left(z_{R 0}\right)} p_{v}\left(v_{0}\right) d v_{0} .
$$

The power coefficient in cumulative mass distribution $k$ from (16) is running a range of values: $k=$ 0.892 for $m_{0} \leq 1.7 \times 10^{-14} \mathrm{~g}$, then $k=1.087$ for $m_{0} \approx 10^{-13} / 10^{-12} \mathrm{~g}, k=1.189$ for $m_{0} \approx 10^{-11} / 10^{-10} \mathrm{~g}, k=$ 1.438 for $m_{0} \approx 10^{-9} / 10^{-8} \mathrm{~g}$ (the average value for all the mass range $m_{0} \approx 10^{-13} / 10^{-8} \mathrm{~g}$ is $k=1.232$ ).

### 3.3 Transformation of the Primary Distribution to New Variables

The relation (14) connects four variation parameters: three ones included into the primary distribution, and the fourth one, the altitude of stopping $H_{S}$. This parameter can be called conditionally the "free" parameter. The dotted curves corresponding to some of its values (minimal altitude of stopping, minimal and maximal velocities altitudes) to be expressed in kilometers are shown in Figure 2.

Since the main aim of our investigations is to plot the two-dimensional distribution $p_{m H}\left(m_{0}, H_{S}\right)$ of the aerosol formation into the atmosphere, we will solve it in two steps. The first one is to change the variable $z_{R 0}$ in the primary distribution $p_{m v H}\left(m_{0}, v_{0}, z_{R 0}\right)$ to the altitude of stopping $H_{S}$ of the aerosol particle. The second step will be consisting in the reducing of 3 -dimensional distribution $p_{m u H}\left(m_{0}, v_{0}, H_{S}\right)$ to $p_{m H}\left(m_{0}, H_{S}\right)$ by means of integration of $p_{m v H}\left(m_{0}, v_{0}, H_{S}\right)$ over all range of $v_{0}$.

According to statistical probability density distribution transformations and taking into account that only one variable is changing $\left(z_{R 0} \rightarrow H_{S}\right)$ we can write

$$
\begin{equation*}
p_{H}\left(H_{S}\right)=p_{z\left(H_{S}\right)}\left(z_{R 0}\left(H_{S}\right)\right)\left|\frac{\partial z_{R 0}\left(H_{S}\right)}{\partial H_{S}}\right|, \tag{22}
\end{equation*}
$$

where $z_{R 0}\left(H_{S}\right)$ and determinant of transition $\frac{\partial z_{R 0}\left(H_{S}\right)}{\partial H_{S}}$ can be found from (14). Let us denote

$$
C_{Z}\left(m_{0}, v_{0}, H_{S}\right) \equiv \cos z_{R 0}=C_{V} \rho_{A}\left(H_{S}\right) / m_{0}^{1 / 3} \ln v_{0} / v_{S} .
$$

Then we can write

$$
p_{Z}\left(z_{R 0}\left(H_{S}\right)\right)=2 C_{Z} \sqrt{\left(1-C_{Z}^{2}\right)}, \frac{\partial z_{R 0}}{\partial H_{S}}=\frac{1}{H^{*}} \frac{C_{Z}}{\sqrt{1-C_{Z}^{2}}}
$$

and put them into (22):

$$
\begin{equation*}
p_{H}\left(m_{0}, v_{0}, H_{S}\right)=\frac{2}{H^{*}} C_{Z}^{2}\left(m_{0}, v_{0}, H_{S}\right) \tag{23}
\end{equation*}
$$

The final view of the obtained distribution $p_{m o H}\left(m_{0}, v_{0}, H_{S}\right)$ while taking into account (23) is shown in Figure 3 for the same masses as in Figure 2. The "free" parameter now is the cosine of the zenith radiant angle, and the dashed curves correspond to different values of $z_{R 0}$ expressed in degrees. The value $z_{R 0}=0$ is placed lower than others in Figure 3 and shown with a solid curve. The region to be placed lower than value $z_{R 0}=0$ is forbidden for both aerosol and meteor particles.

An interesting fact is that the inequality (13) is now transformed in the "stable" state in new coordinates and does not depend on the mass:

$$
H_{S} \geq H^{*} \ln \left(\frac{C_{V} \rho_{A}(0) v_{0}^{3}}{C_{T} \ln \left(v_{0} / v_{S}\right)}\right)
$$

It is shown in Figure 3 with a solid diagonal line.


Figure 3. The same distributions as in Fig. 2 but in new variables. Dashed lines describe the equal radiant zenith angles of meteoroids.

### 3.4 Resultant Distribution Reducing

The final transformation of the distribution (23) consists in reducing it to the two-dimensional state by means of integration over $v_{0}$. The limits of the integration can be easily determined from Figure 3 and according formulae. We obtain the following

$$
\begin{equation*}
p_{m H}\left(m_{0}, H_{S}\right)=p_{m}\left(m_{0}\right) \int_{v_{1}\left(m_{0}, H_{S}\right)}^{v_{2}\left(m_{0}, H_{S}\right)} p_{v}\left(v_{0}\right) p_{H}\left(m_{0}, v_{0}, H_{S}\right) d v_{0} \tag{24}
\end{equation*}
$$

If we denote the integral, which has to be taken numerically, as

$$
I_{\nu}\left(m_{0}, H_{S}\right)=\int_{v_{01}\left(m_{0}, H_{S}\right)}^{v_{02}\left(m_{0}, H_{S}\right)} \frac{p_{v}\left(v_{0}\right)}{\ln ^{2}\left(v_{0} / v_{S}\right)} d v_{0}
$$

the final formula for the formation of the aerosol of space origin in the atmosphere can be written as

$$
\begin{equation*}
p_{m H}\left(m_{0}, H_{S}\right)=p_{m}\left(m_{0}\right) \frac{2}{H^{*}}\left(\frac{C_{V} \rho_{0} \exp \left(-H_{S} / H^{*}\right)}{m_{0}^{1 / 3}}\right)^{2} I_{v}\left(m_{0}, H_{S}\right) . \tag{25}
\end{equation*}
$$

The function $p_{m H}\left(m_{0}, H_{S}\right)$ is shown in Figure 4.


Figure 4. Final two-dimensional distribution of influx of aerosol of space origin into Earth's atmosphere. 10logarithm scale

## 4 Conclusion

As it can be seen from the Figure 4 the minimal altitude which can be reachable by the aerosol stone particle of space origin is approximately $\sim 79.6 \mathrm{~km}$. This value corresponds to the meteoroid which is moving vertically with the velocity $\sim 16.6 \mathrm{~km} / \mathrm{s}$. A particle of the same mass and with lower velocity will stop higher, with higher velocity will transform into a meteor.

The meteoroids with the mass less or equal to $\sim 1.7 \times 10^{-14} \mathrm{~g}$ remain the aerosols always. For masses $10^{-14} / 10^{-8} \mathrm{~g}$ cumulative distribution coefficient $k$ increases from 0.892 to 1.438 while the mass increases.

The Figure 4 also demonstrates that aerosols of mass range $10^{-14} / 10^{-8} \mathrm{~g}$ stop in relatively thin altitude range $80-120 \mathrm{~km}$. Evidently, the aerosols do not stay at these altitudes forever but immediately start to move downwards under gravitational force and the resistance force of air, which can be described by Stokes formula. How it occurs is the goal for the future work.

## References

D.E. Brownlee, P.W. Hodge Space Res., 13/2, 1139-1151 (1973).

Ceplecha Z. Astron. Astrophys., 263, 361-366 (1992).
Fecenkov V.G. Meteoritics, 12, 3-14 (1955).
Kruchynenko V.G. Kin. And Phys. Nebesn. Tel, 18, 2, 114 - 127 (2002).
Kruchynenko V.G. Kin. And Phys. Nebesn. Tel, 20, 3, 269 - 282 (2004).
B.Y. Levin Moscow: AN SSSR, 296 pp. (1956).
V.N. Lebedinets Leningrad: Hidrometeoizdat, 250 pp. (1980).
V.N. Lebedinets Leningrad: Hidrometeoizdat, 272 pp. (1981).
B. Nady Amsterdam - New York, 747 pp. (1975).
E.J. Öpik Publ. Obs. Astr. Tartu, 29, 5, 67 (1937).
E.J. Öpik Irish Astron. J., 4, 3/4, 84-135 (1956).
Y.I. Voloshchuk, B.L. Kashcheev, V.G. Kruchynenko Kyiv: Naukova dumka, 294 pp. (1989).
F. L. Whipple Proc. Nat. Acad. Sci. Amer., 36, 12, 686-695 (1950).
F. L. Whipple Proc. Nat. Acad. Sci. Amer., 37, 1, 19-29 (1951).

