Fast Approximate Broadband Phase Retrieval for Segmented Systems

Alden S. Jurling and James R. Fienup

University of Rochester
Institute of Optics

Research funded by NASA Goddard Space Flight Center

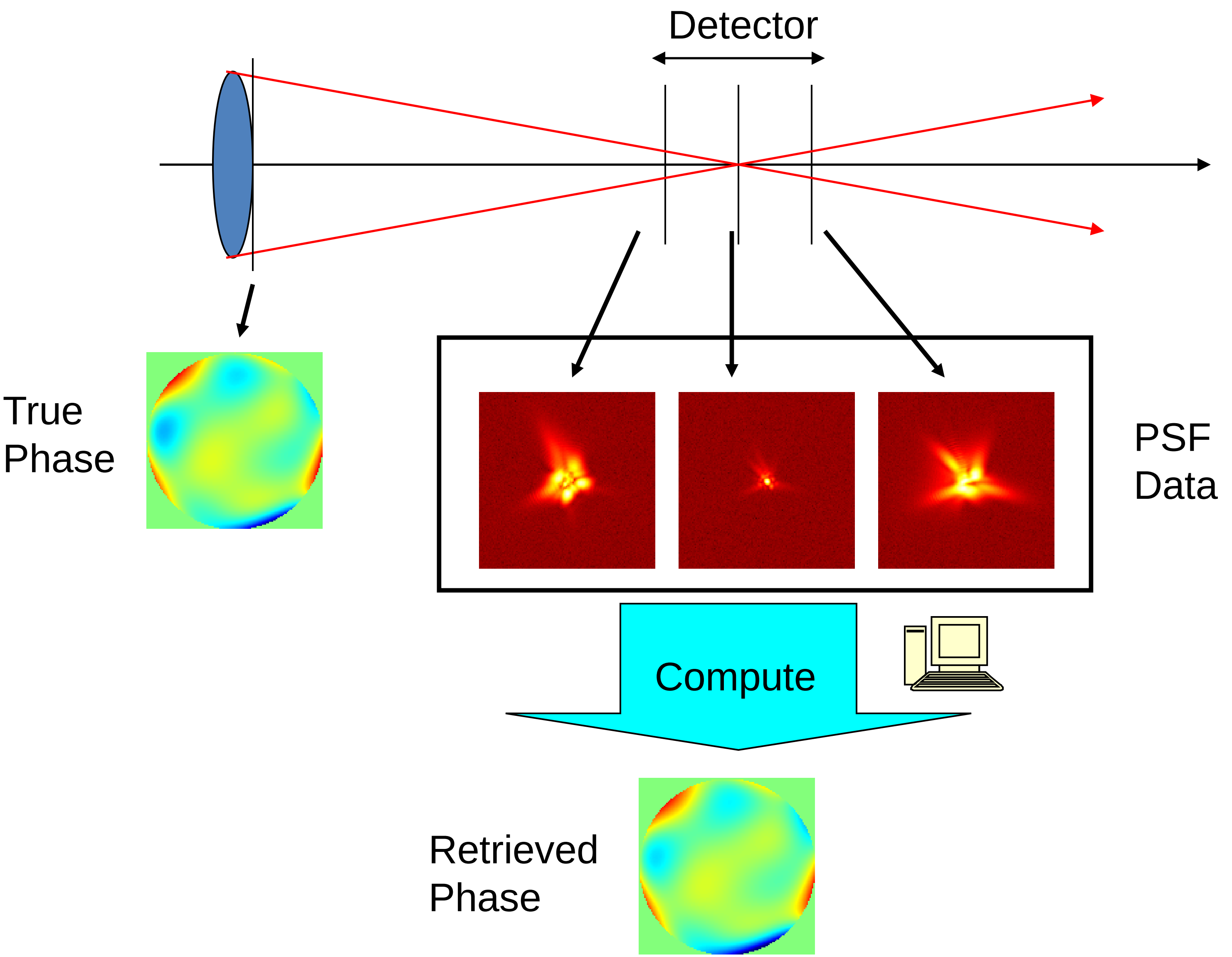
Paper FThD1
Frontiers in Optics 2011, San Jose, CA

Outline

- Application: Wavefront sensing for large telescopes
  - Space telescopes (James Webb Space Telescope)
  - Adaptive Optics for ground based telescopes
- Wavefront sensing method: Focus diverse phase retrieval
- Case of interest: Extremely broadband sources
- Problem
  - Monochromatic phase retrieval algorithms fail above ~10% fractional bandwidth
  - Broadband (polychromatic) algorithms are computationally expensive
- Solution
  - Employ monochromatic algorithm with approximation method
- Result
  - 270x speed up in computational performance for 133% fractional bandwidth
  - Acceptable accuracy
  - Accuracy better for monolithic systems, worse for segmented
- Investigate difference in accuracy between segmented and monolithic systems

The Institute of OPTICS

Focus Diverse Phase Retrieval



Phase Retrieval Overview

- Given
  - System parameters (F/#, detector pixel pitch, etc.)
  - Measured point spread function images (e.g. images of stars) through focus
  - Clear aperture of pupil
- Determine
  - Wavefront error in the exit pupil of the system (phase)
- Issues
  - Optical fields are complex-valued quantities
  - Measured data (intensity) has only magnitude, no phase
  - Measured data corrupted with noise
- Solution strategies
  - Gerchberg-Saxton / iterative transform type algorithms
  - Non-linear optimization type algorithms

The Institute of OPTICS

Nonlinear Optimization Phase Retrieval

- Parameterize problem in terms of a set of variables (e.g. Zernike coefficients)
- Generate wavefront (*W*) and/or field from parameters
- Propagate field to PSF plane
- Use an error metric to evaluate agreement between model and data[1]
- Use standard non-linear optimization algorithm to reduce error metric value
- Analytic expressions for error metric gradients allow use of gradient search algorithms

Gain Bias

$$E(W) = \frac{1}{K}\sum_{k}^{K} \frac{\sum_{u}\left[\alpha_k I_k(u) + \beta_k - D_k(u)\right]^2}{\sum_{u}\left[D_k(u)\right]^2}$$

Model Data

$$I_k(u) = \left| F\left[A(x)\exp\left(\frac{2\pi}{\lambda}W(x)\right)\right]\right|^2$$

(simplified)

[1] S. T. Thurman and J. R. Fienup, "Phase retrieval with signal bias," J. Opt. Soc. Am. A 26, 1008–1014 (2009).

Broadband Phase Retrieval

- Use cases for broadband phase retrieval
  - Narrow spectral filters unavailable
  - Dim sources
  - Low throughput due to misalignment
  - Short exposures times
    - Pointing instability (space)
    - Atmospheric instability (ground based AO)
  - Segment piston determination

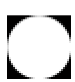
- Traditional approach[1,2]
  - Simulate multiple individual wavelengths
  - Rule of thumb: 1 wavelength per 5% fractional bandwidth
  - Add incoherently to simulate polychromatic PSF
  - Do normal nonlinear optimization phase retrieval

[1] J. R. Fienup, "Phase retrieval for undersampled broadband images," J. Opt. Soc. Am. A **16**, 1831–1837 (1999).
[2] G. R. Brady and J. R. Fienup, "Effect of broadband illumination on reconstruction error of phase retrieval in optical metrology," Proc. SPIE 6617, 66170I (2007).

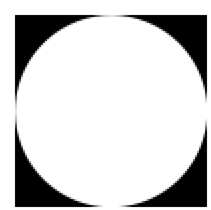
Array Sizes

- Fine for modest bandwidth
- Very computationally intensive for large bandwidths
  - More wavelengths → more computation
    - Linear in number of wavelengths
  - Shorter wavelengths → larger arrays → more computation
    - Pupil sampling requirement
      - Avoid π phase jumps
    - Phase for given OPD increases with shorter wavelength
    - Array size inversely proportional to wavelength
    - Cost approximately quadratic in array size

Monochromatic Pupil
3 μm, 128x128



Broadband Pupil
1-5 μm, 384x384



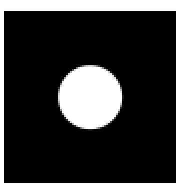
Array Sizes

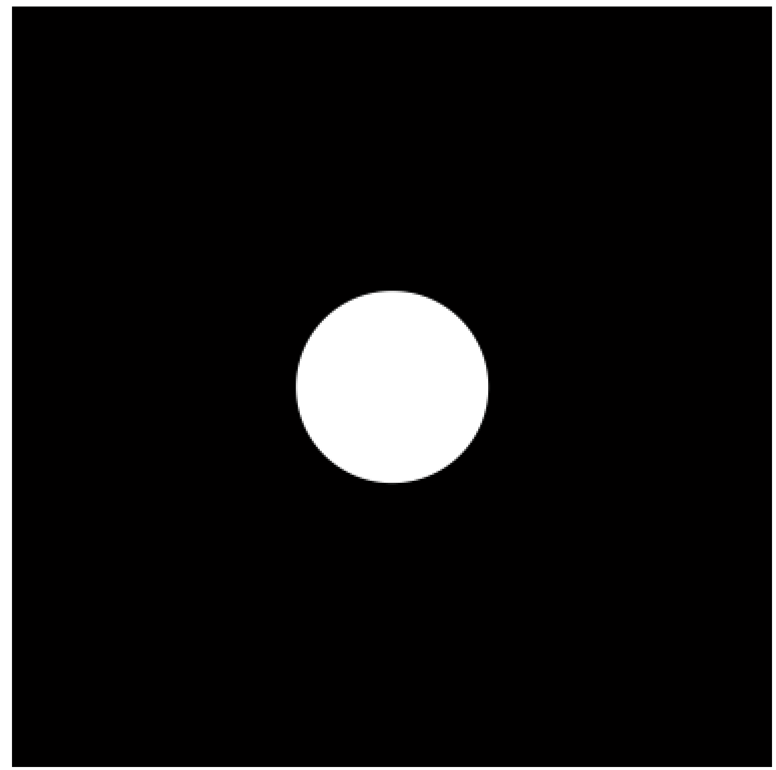
– Shorter / longer wavelengths → under / over sampling
→ more computation

- PSF initially computed on large array
- Requires padding in pupil domain

Broadband Pupil
(largest) 1518x1518

Monochromatic Pupil
342x342





Solution Motivation

- Achromatic system
  - Reflecting telescopes
  - Color corrected instruments
- OPD / rays the same at all wavelengths
- Geometrical optics spot diagram the same at all wavelengths
- For poorly corrected systems geometrical optics should predict PSF shape
- PSF shape should be roughly the same at all wavelengths
- Only high frequency diffraction effects depend strongly on wavelength

Approximation Procedure

- Data *D,* blurring kernel *B*, forward model *F,* wavefront *W, K* image planes
- Blur measured PSF data with Gaussian kernel (suppress diffraction effects)

$$D_{B,k} = B \otimes D_k$$

- Simulate monochromatic PSF at the center wavelength

$$I_k = F_{\lambda,k}[W]$$

- Blur modeled PSF with same Gaussian kernel

$$I_{B,k} = B \otimes I_k = B \otimes F_{\lambda,k}[W]$$

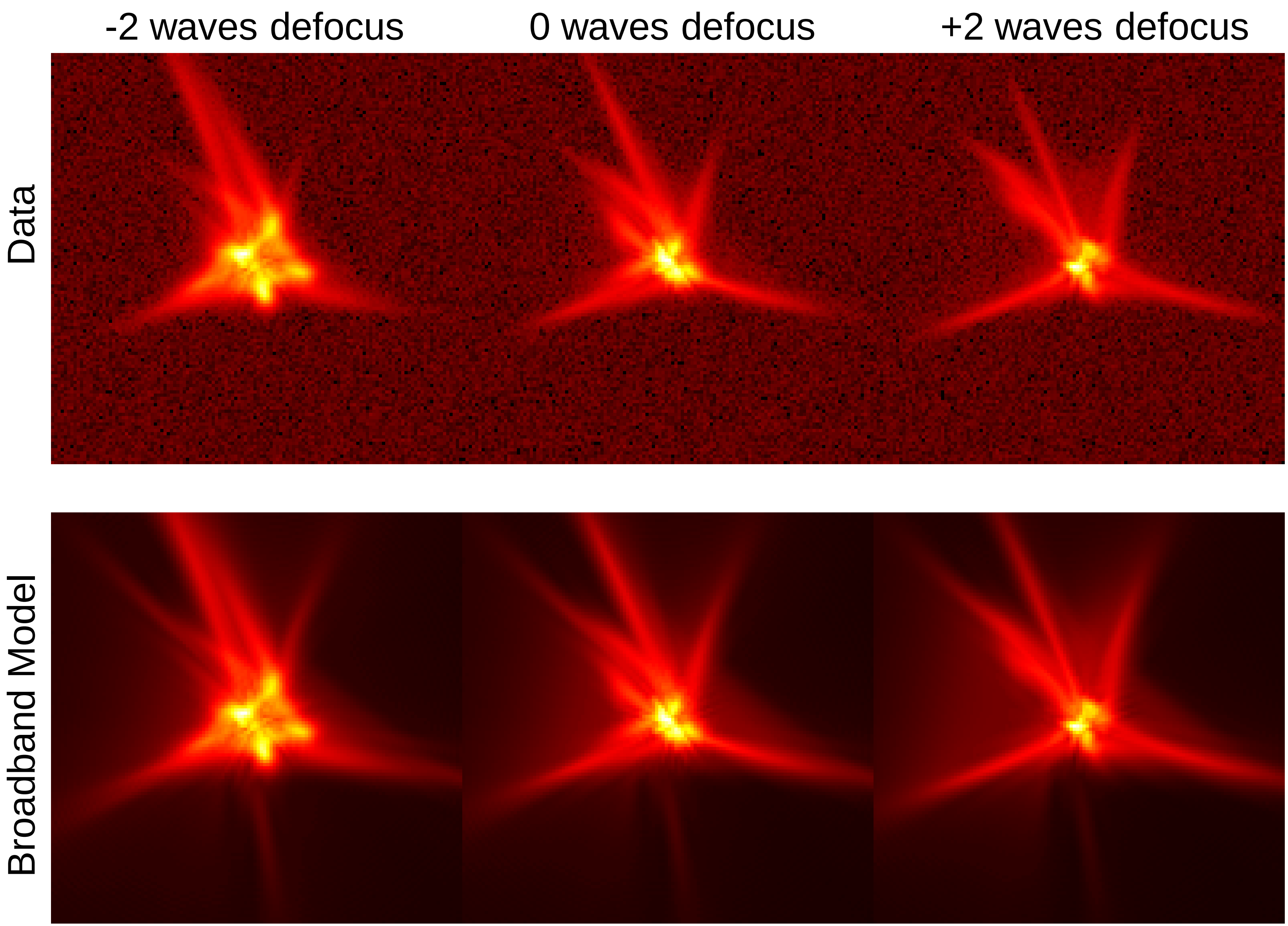
- Use non-linear optimization to fit blurred data against blurred model

$$E(W) = \frac{1}{K} \sum_{k}^{K} \frac{\sum_u \left[\alpha_k I_{B,k}(u) + \beta_k - D_{B,k}(u) \right]^2}{\sum_u \left[D_{B,k}(u) \right]^2}$$

Broadband Test Case (FGS-like)

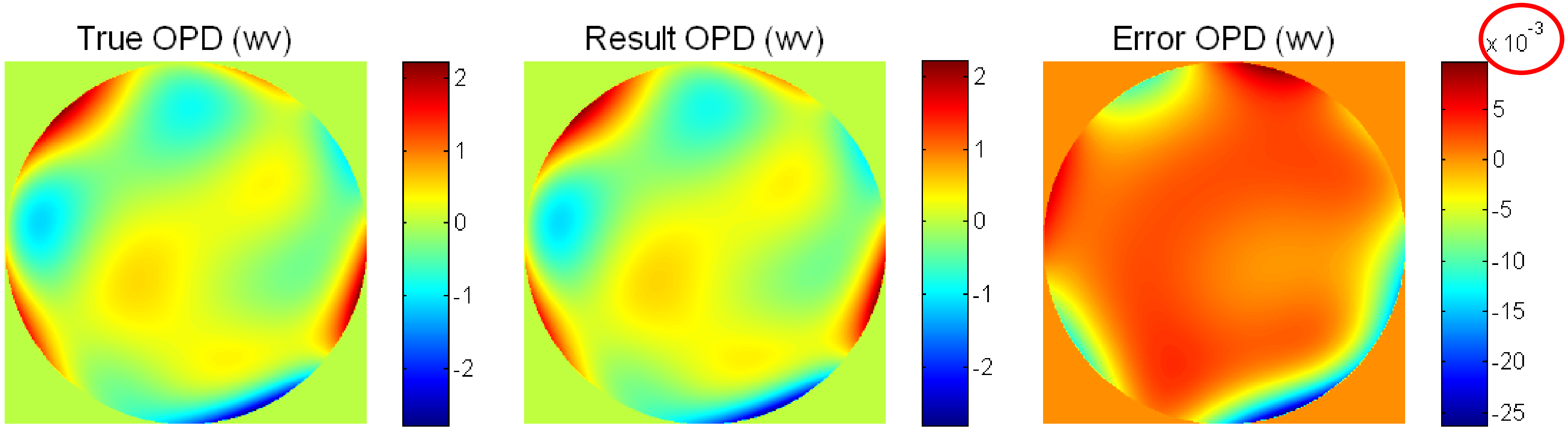
- Similar to JWST Fine Guidance Sensor (FGS)
- F/8 system
- 18 μm pixels
- 1-5 μm bandwidth (133% fractional bandwidth)
  - Flat spectrum
- Modeled detector area: 128x128 pixels (artificially small)
- Noise model: 50,000 photons in peak pixel, 25 photons of read noise, 100 photon noise-free bias
- Measured point spread functions at –2, 0, 2 waves center to edge defocus
- Modeled with monolithic aperture
- Monte Carlo simulation
  - 64 trials
  - Minumum wavefront error: 0.025 waves RMS
  - Maximum wavefront error: 0.500 waves RMS

Broadband Phase Retrieval Results



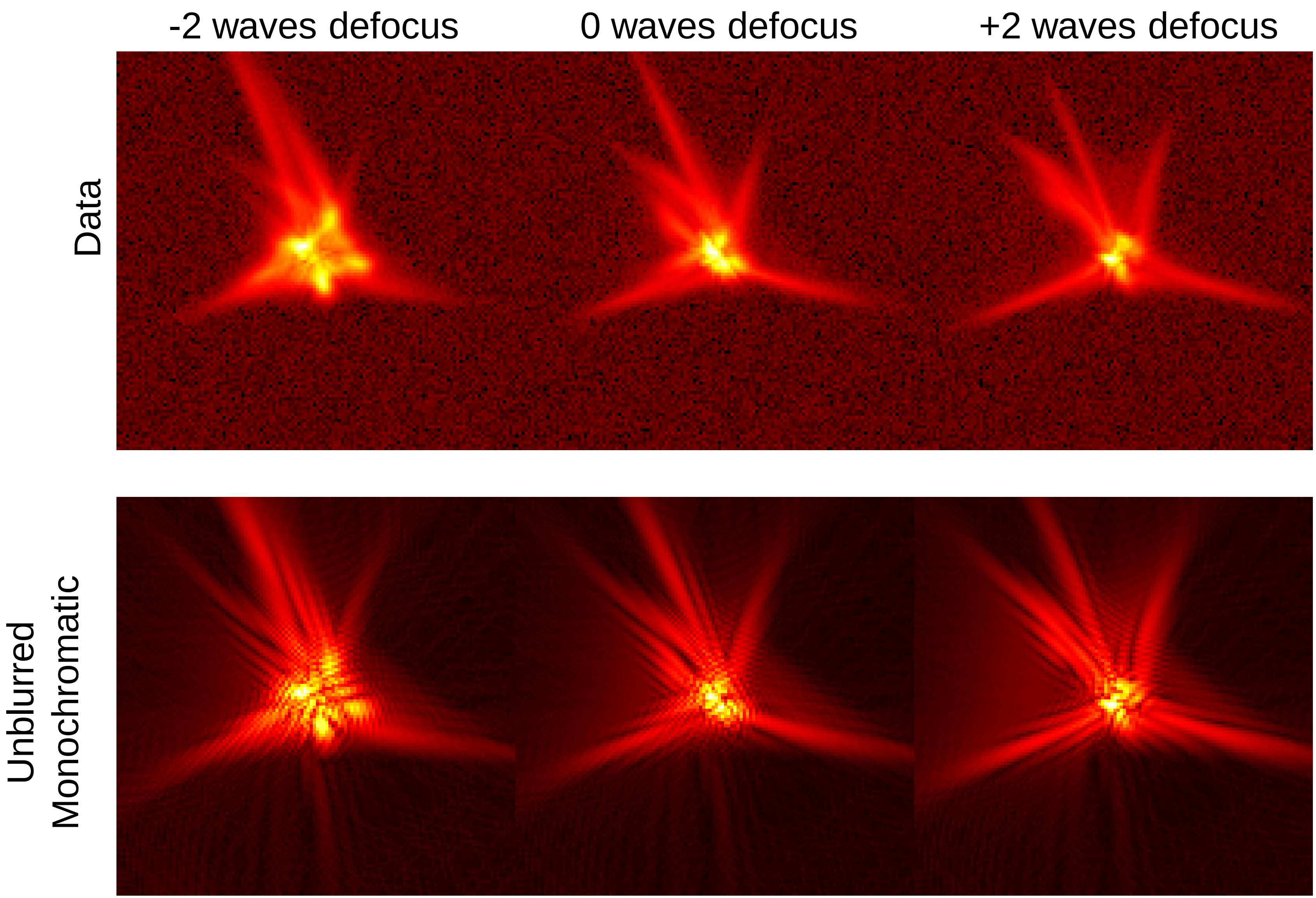
Quarter power stretch

Broadband Phase Retrieval Results



OPD in waves

Unblurred PSFs (Monolithic)



Quarter power stretch

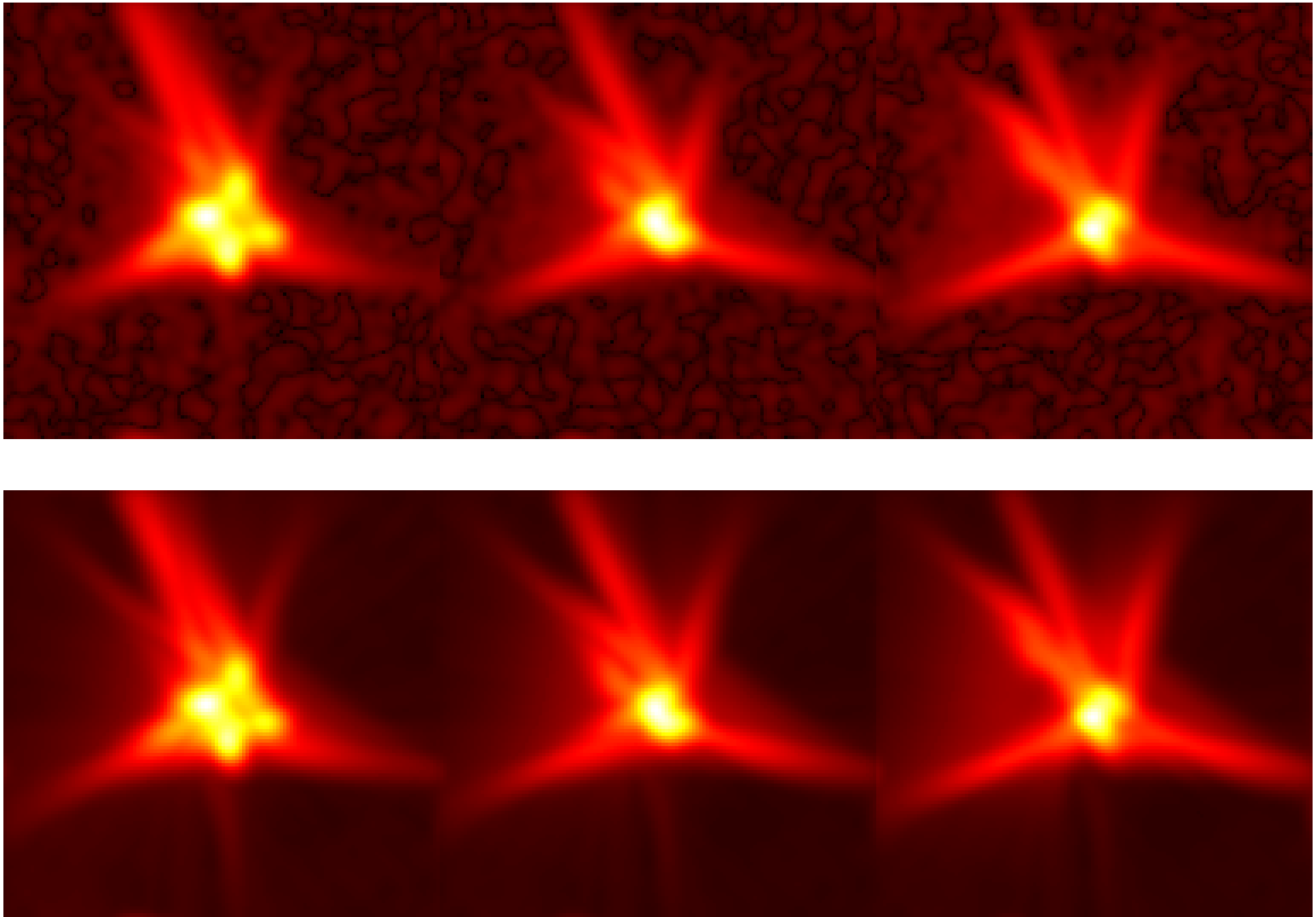
The Institute of OPTICS

Blurred PSFs (Monolithic)

-2 waves defocus | 0 waves defocus | +2 waves defocus

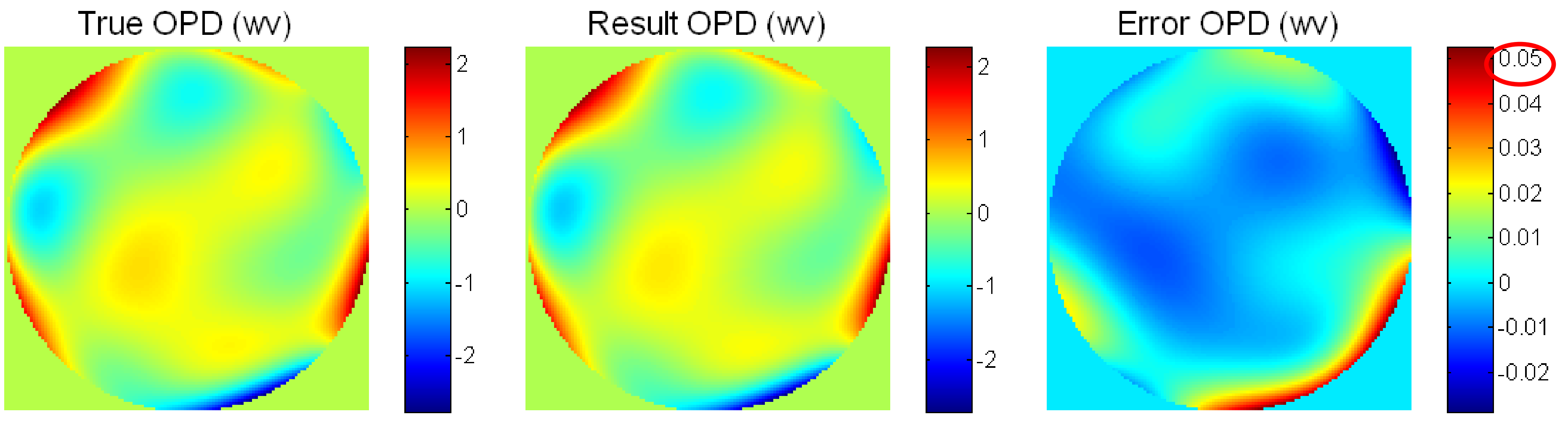
Data

Blurred Monochromatic



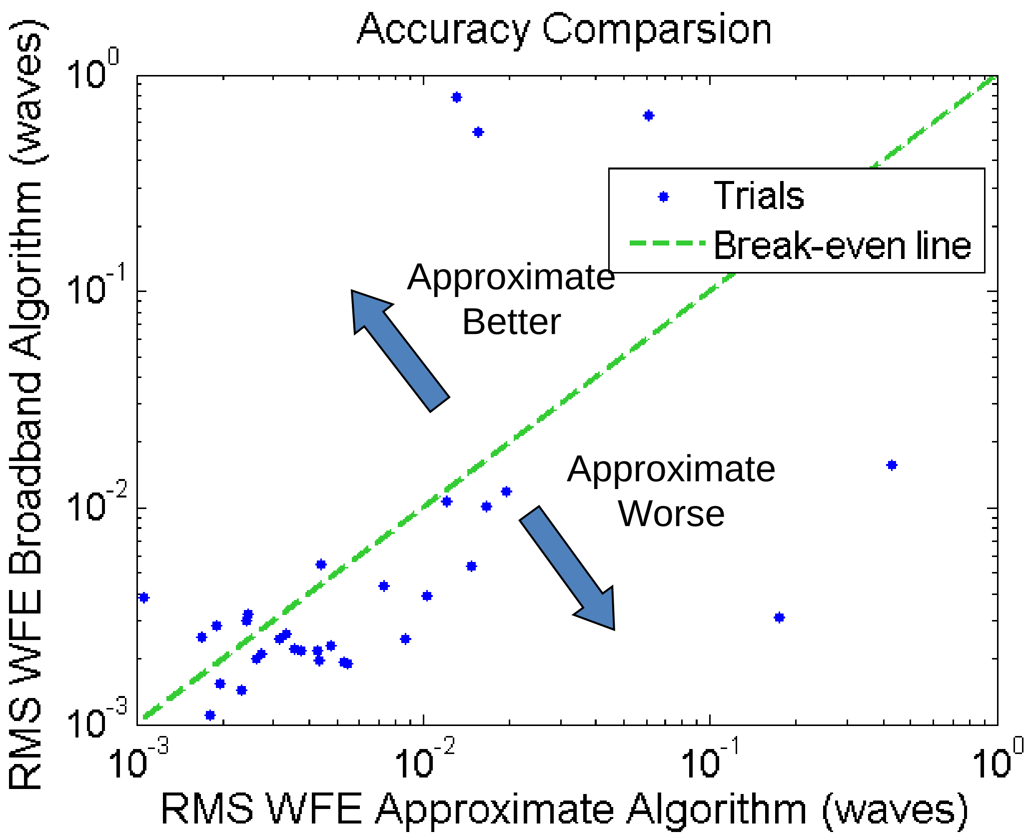
Quarter power stretch

Wavefront Retrieval Results With Blurring

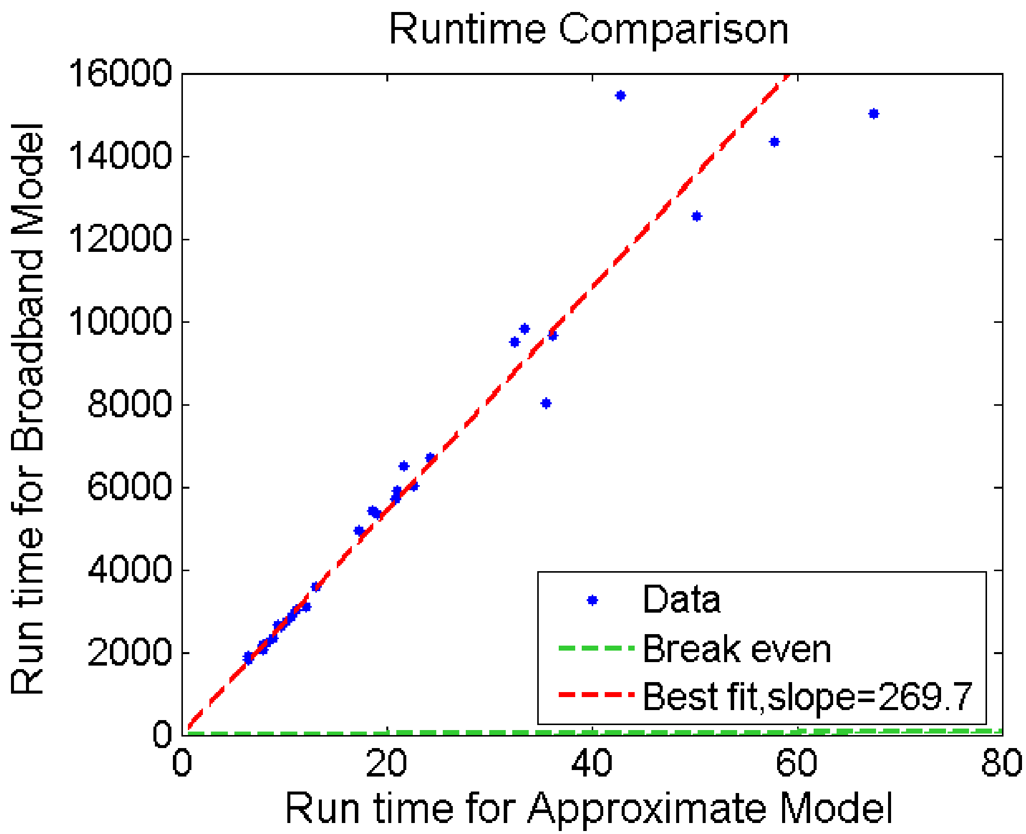


OPD in waves

Monte Carlo Results



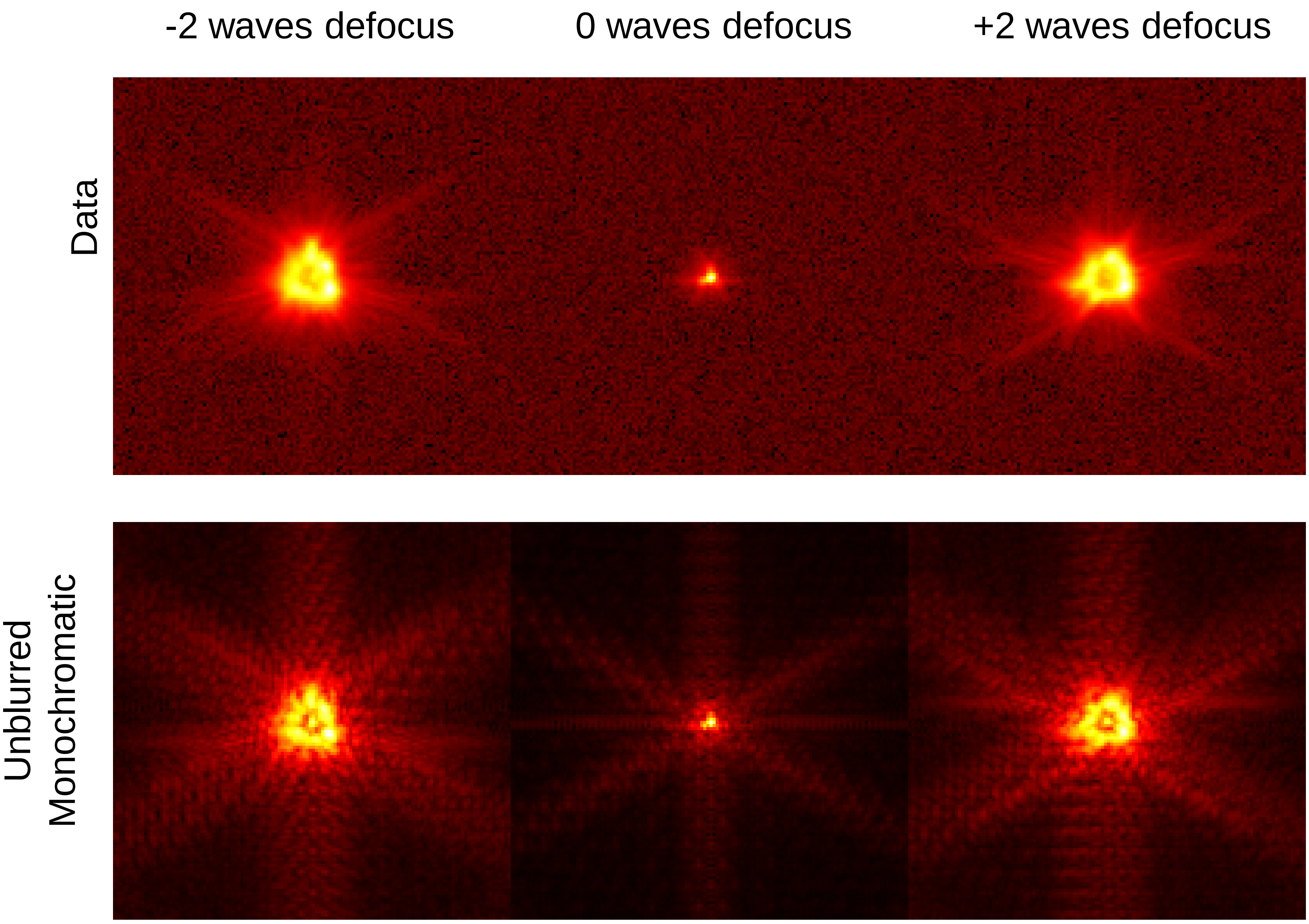
Monte Carlo Results



Segmented Systems

- Monolithic System Results
  - Good results for monolithic system
  - 270x speed improvement
  - Small loss in accuracy
  - How does approximation perform for a segment system?
- Segmented test case
  - Same 1-5 μm FGS-like system
  - Global aberration model: Third order aberrations
  - Segment aberration model: Piston, tip, tilt
  - Monte Carlo simulation
    - 16 trials
    - 0.1 waves RMS wavefront errors

Unblurred PSFs (Segmented)

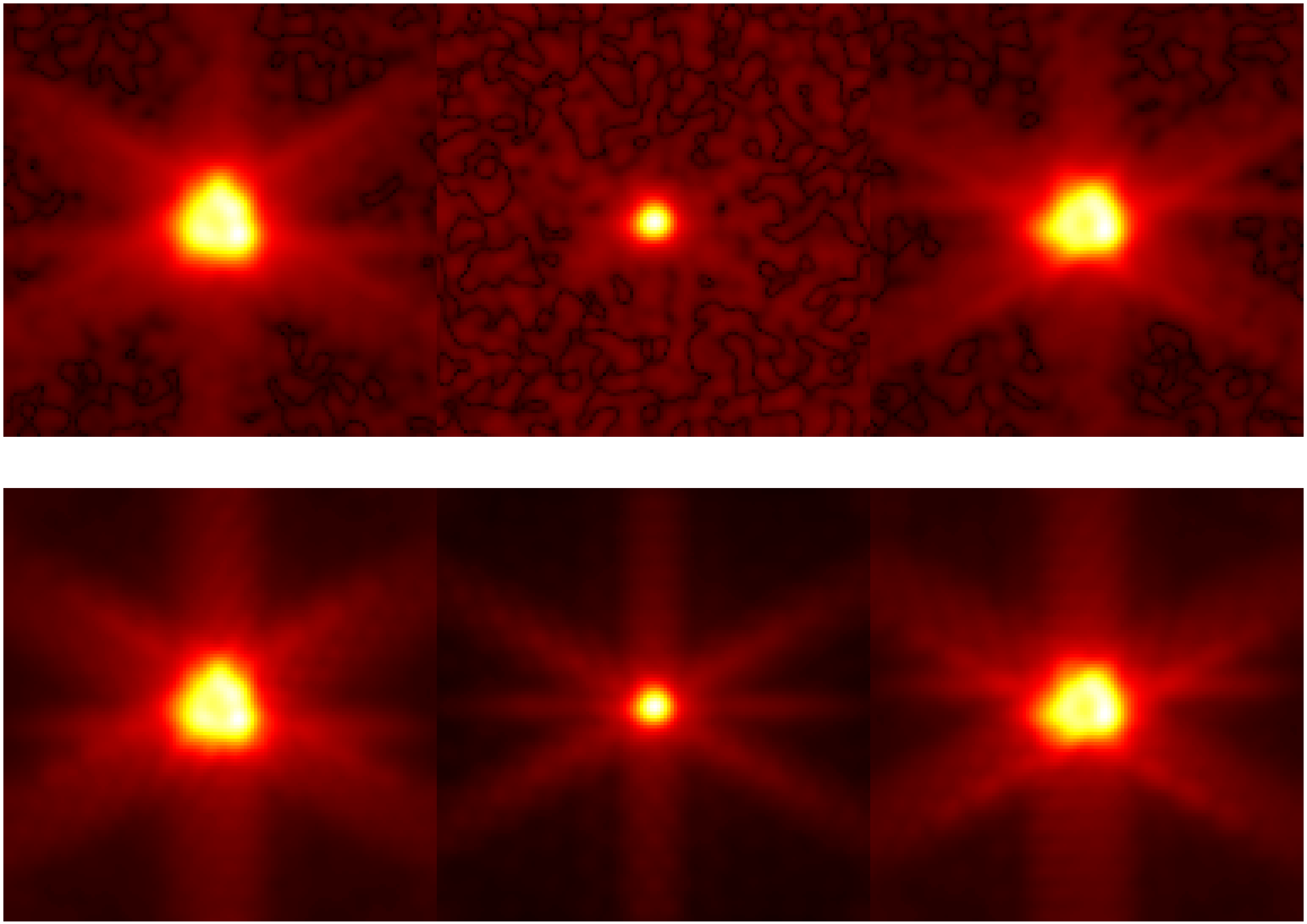


Blurred PSFs (Segmented)

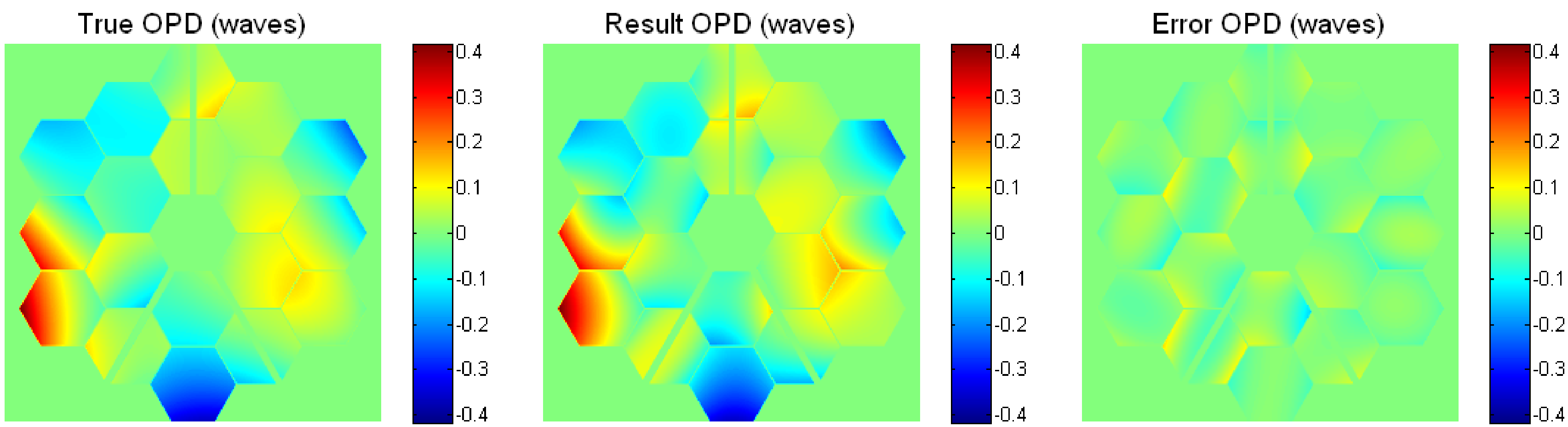
-2 waves defocus | 0 waves defocus | +2 waves defocus

Data

Blurred Monochromatic

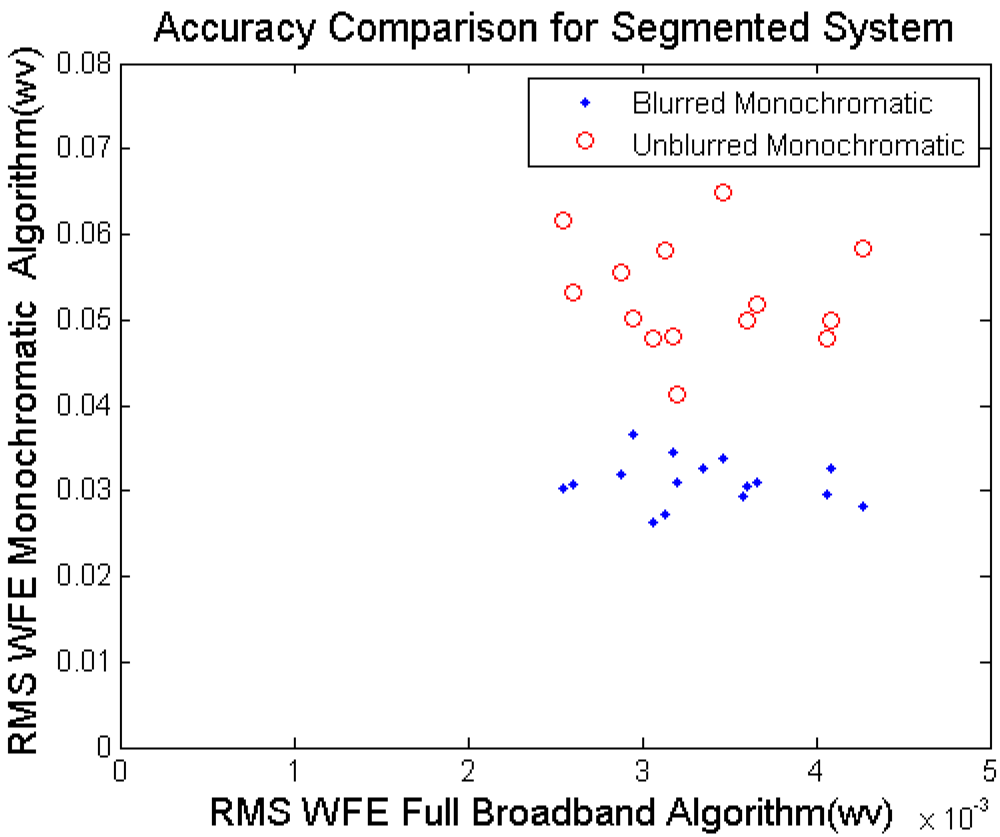


Results, Segmented System



The Institute of OPTICS

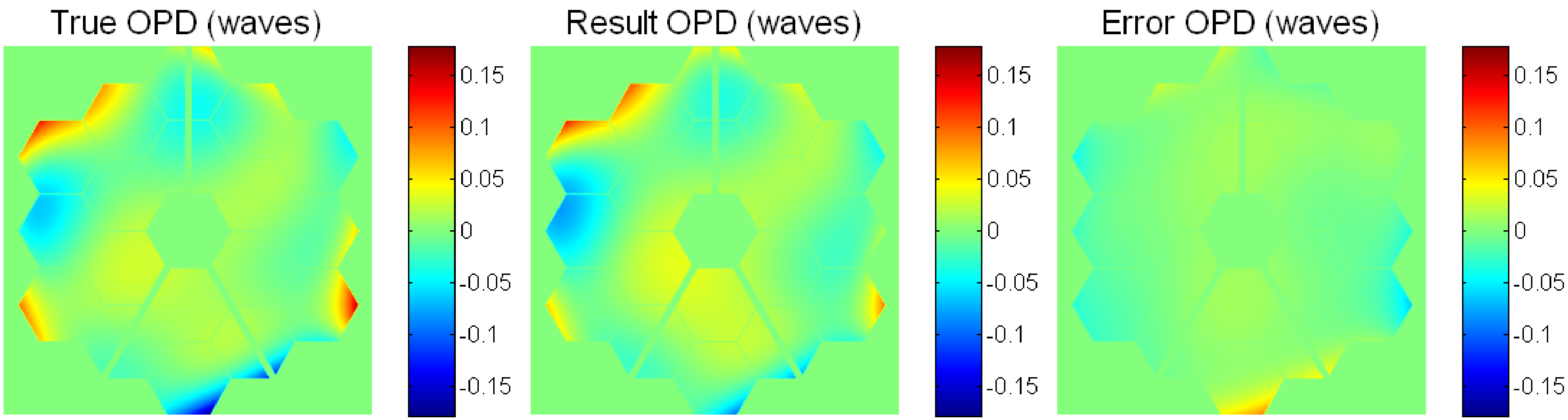
Accuracy Comparison: Segmented System



Segmented System

- Reduced Accuracy with segmented system
- Possible causes
  - More diffraction
  - More variables (higher order phases)
  - Which is responsible?
- To investigate
  - Apply segmented system mask to low order phases
  - Same phase model as monolithic aperture
  - Same aperture mask as segmented system
  - Isolates diffraction effect from higher order phase effect
  - Limit study to 0.25 waves wavefront to avoid issues with large wavefronts

Results: Segmented Mask

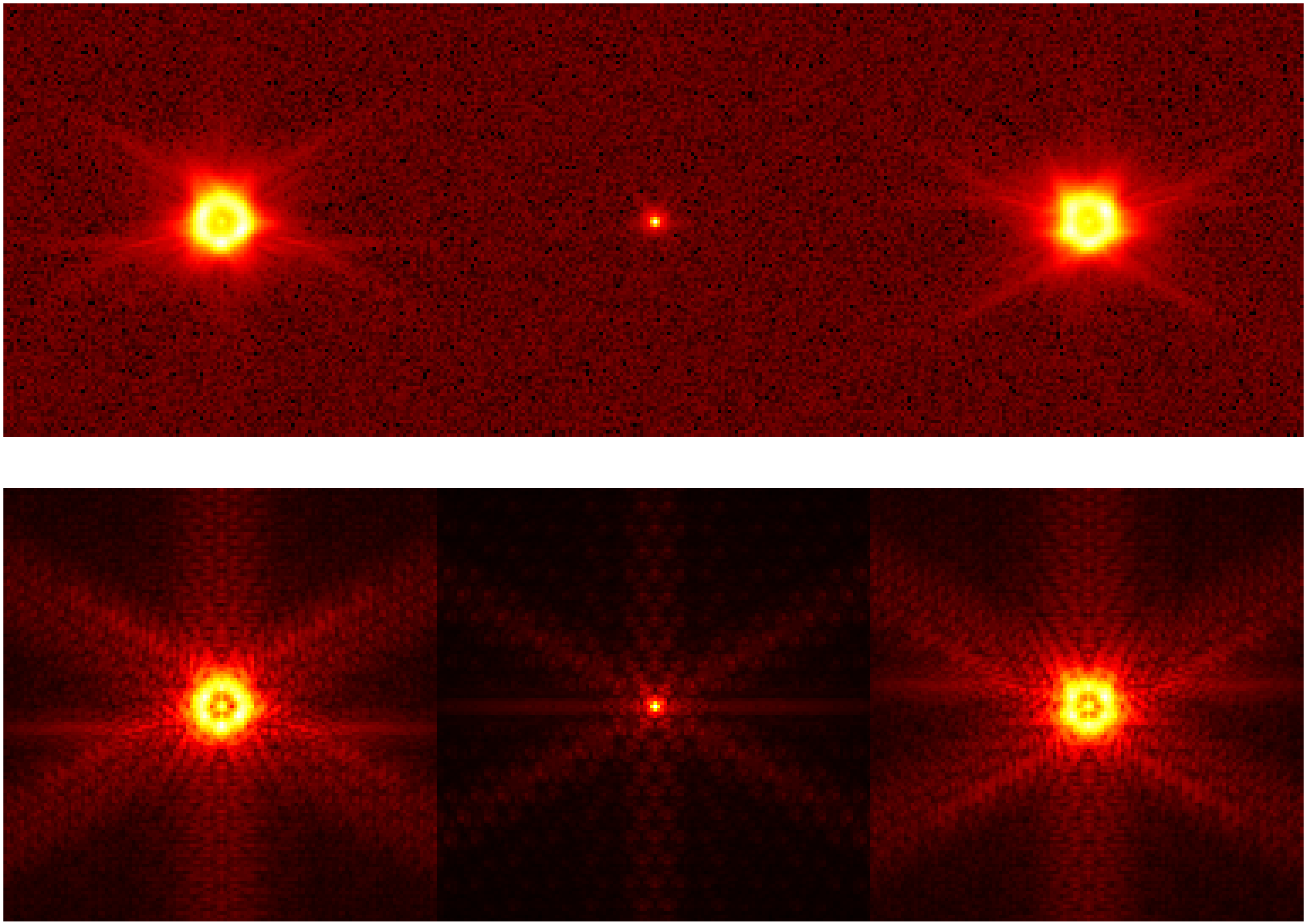


Unblurred PSFs (Segmented Mask)

-2 waves defocus | 0 waves defocus | +2 waves defocus

Data

Unblurred Monochromatic

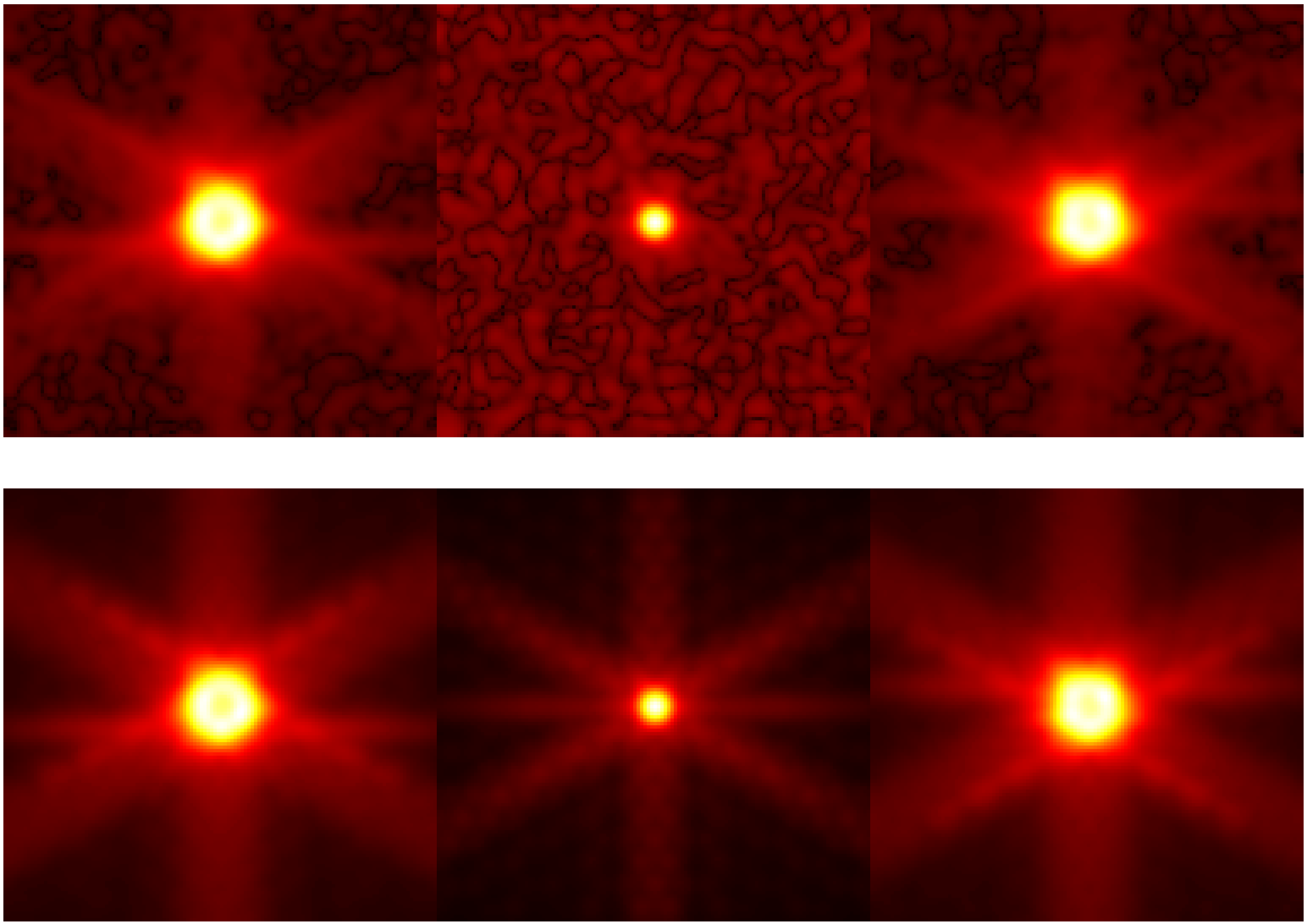


Blurred PSFs (Segmented Mask)

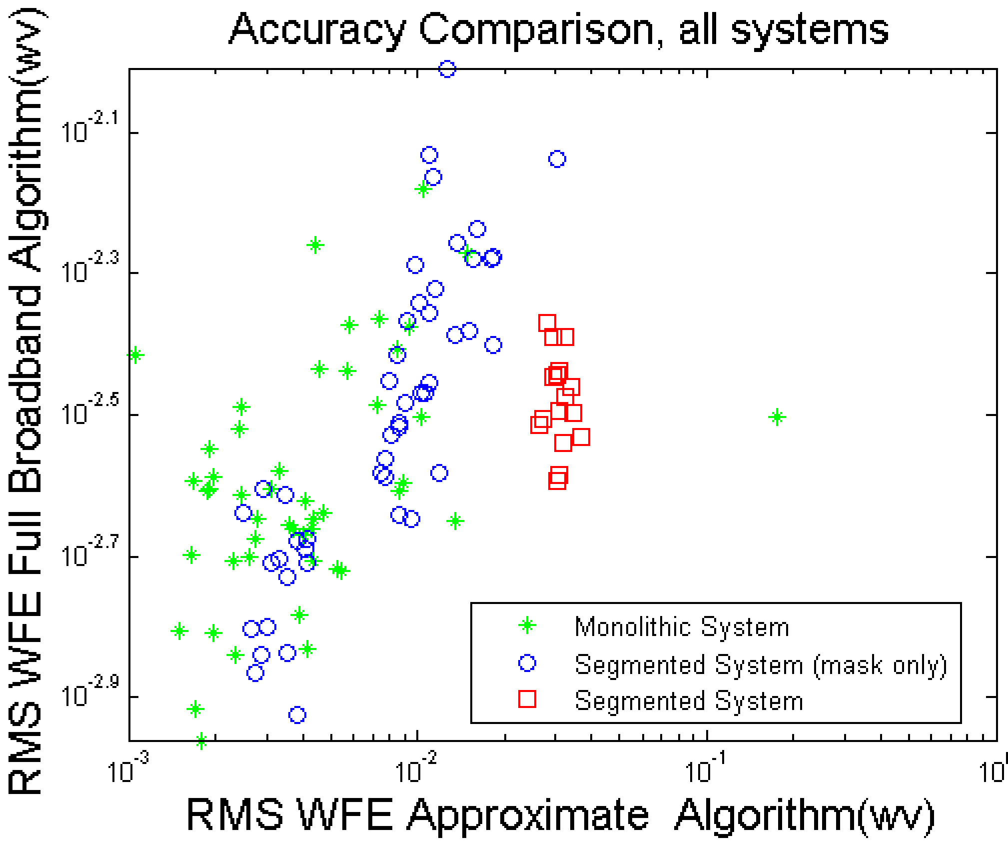
-2 waves defocus | 0 waves defocus | +2 waves defocus

Data

Blurred Monochromatic



Accuracy Comparison: Three Cases



Medians:
Monolithic: 0.004 waves
Mask: 0.009 waves
Segments: 0.0308 waves

Both diffraction and higher order phase reduce accuracy!

Conclusion

- Broadband phase retrieval needed when:
  - Narrow spectral filters unavailable
  - Dim sources
  - Low throughput due to misalignment
  - Short exposures times
    - Pointing instability (space)
    - Atmospheric instability (ground based AO)
- Traditional approach is computationally burdensome for extreme bandwidths
- Approximate approach
  - Substitute monochromatic model
  - Blur model and data
- Test case performance
  - ~270x reduction in computational cost for FGS-like test case
  - Good accuracy for monolithic system
  - Acceptable accuracy for segmented systems
    - Reduced by diffraction
    - Reduced by higher order segment model

Questions?