Fast Approximate Broadband Phase Retrieval for Segmented Systems

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Outline

• Application: Wavefront sensing for large telescopes
  – Space telescopes (James Webb Space Telescope)
  – Adaptive Optics for ground based telescopes
• Wavefront sensing method: Focus diverse phase retrieval
• Case of interest: Extremely broadband sources
• Problem
  – Monochromatic phase retrieval algorithms fail above ~10% fractional bandwidth
  – Broadband (polychromatic) algorithms are computationally expensive
• Solution
  – Employ monochromatic algorithm with approximation method
• Result
  – 270x speed up in computational performance for 133% fractional bandwidth
  – Acceptable accuracy
  – Accuracy better for monolithic systems, worse for segmented
• Investigate difference in accuracy between segmented and monolithic systems
Focus Diverse Phase Retrieval

True Phase

Detector

PSF Data

Compute

Retrieved Phase
Phase Retrieval Overview

• Given
  – System parameters (F/#, detector pixel pitch, etc.)
  – Measured point spread function images (e.g. images of stars) through focus
  – Clear aperture of pupil
• Determine
  – Wavefront error in the exit pupil of the system (phase)
• Issues
  – Optical fields are complex-valued quantities
  – Measured data (intensity) has only magnitude, no phase
  – Measured data corrupted with noise
• Solution strategies
  – Gerchberg-Saxton / iterative transform type algorithms
  – Non-linear optimization type algorithms
Nonlinear Optimization Phase Retrieval

- Parameterize problem in terms of a set of variables (e.g. Zernike coefficients)
- Generate wavefront ($W$) and/or field from parameters
- Propagate field to PSF plane
- Use an error metric to evaluate agreement between model and data\(^1\)
- Use standard non-linear optimization algorithm to reduce error metric value
- Analytic expressions for error metric gradients allow use of gradient search algorithms

\[ E(W) = \frac{1}{K} \sum_{k} \left[ \alpha_k I_k(u) + \beta_k - D_k(u) \right]^2 \]

\[ I_k(u) = \left| F \left[ A(x) \exp \left( \frac{2\pi}{\lambda} W(x) \right) \right] \right|^2 \]  

(simplified)

Broadband Phase Retrieval

• Use cases for broadband phase retrieval
  – Narrow spectral filters unavailable
  – Dim sources
  – Low throughput due to misalignment
  – Short exposures times
    • Pointing instability (space)
    • Atmospheric instability (ground based AO)
  – Segment piston determination

• Traditional approach\textsuperscript{1,2}
  – Simulate multiple individual wavelengths
  – Rule of thumb: 1 wavelength per 5\% fractional bandwidth
  – Add incoherently to simulate polychromatic PSF
  – Do normal nonlinear optimization phase retrieval

Array Sizes

- Fine for modest bandwidth
- Very computationally intensive for large bandwidths
  - More wavelengths $\rightarrow$ more computation
    - Linear in number of wavelengths
  - Shorter wavelengths $\rightarrow$ larger arrays $\rightarrow$ more computation
    - Pupil sampling requirement
      - Avoid $\pi$ phase jumps
    - Phase for given OPD increases with shorter wavelength
    - Array size inversely proportional to wavelength
    - Cost approximately quadratic in array size

Monochromatic Pupil
3 $\mu$m, 128x128

Broadband Pupil
1-5 $\mu$m, 384x384
Array Sizes

- Shorter / longer wavelengths $\rightarrow$ under / over sampling $\rightarrow$ more computation
  - PSF initially computed on large array
  - Requires padding in pupil domain

Monochromatic Pupil
342x342

Broadband Pupil (largest) 1518x1518
Solution Motivation

- Achromatic system
  - Reflecting telescopes
  - Color corrected instruments
- OPD / rays the same at all wavelengths
- Geometrical optics spot diagram the same at all wavelengths
- For poorly corrected systems geometrical optics should predict PSF shape
- PSF shape should be roughly the same at all wavelengths
- Only high frequency diffraction effects depend strongly on wavelength
Approximation Procedure

- Data $D$, blurring kernel $B$, forward model $F$, wavefront $W$, $K$ image planes
- Blur measured PSF data with Gaussian kernel (suppress diffraction effects)
  \[ D_{B,k} = B \otimes D_k \]
- Simulate monochromatic PSF at the center wavelength
  \[ I_k = F_{\lambda,k}[W] \]
- Blur modeled PSF with same Gaussian kernel
  \[ I_{B,k} = B \otimes I_k = B \otimes F_{\lambda,k}[W] \]
- Use non-linear optimization to fit blurred data against blurred model
  \[
  E(W) = \frac{1}{K} \sum_k \frac{\sum_u \left[ \alpha_k I_{B,k}(u) + \beta_k - D_{B,k}(u) \right]^2}{\sum_u \left[ D_{B,k}(u) \right]^2}
  \]
Broadband Test Case (FGS-like)

• Similar to JWST Fine Guidance Sensor (FGS)
• F/8 system
• 18 μm pixels
• 1-5 μm bandwidth (133% fractional bandwidth)
  — Flat spectrum
• Modeled detector area: 128x128 pixels (artificially small)
• Noise model: 50,000 photons in peak pixel, 25 photons of read noise, 100 photon noise-free bias
• Measured point spread functions at –2, 0, 2 waves center to edge defocus
• Modeled with monolithic aperture
• Monte Carlo simulation
  — 64 trials
  — Minimum wavefront error: 0.025 waves RMS
  — Maximum wavefront error: 0.500 waves RMS
Broadband Phase Retrieval Results

-2 waves defocus  0 waves defocus  +2 waves defocus

Data

Broadband Model

Quarter power stretch
Broadband Phase Retrieval Results

True OPD (wv)  Result OPD (wv)  Error OPD (wv)

OPD in waves
Unblurred PSFs (Monolithic)

-2 waves defocus  |  0 waves defocus  |  +2 waves defocus

Data

Unblurred Monochromatic

Quarter power stretch
Blurred PSFs (Monolithic)

-2 waves defocus  |  0 waves defocus  |  +2 waves defocus

Data

Blurred Monochromatic

Quarter power stretch
Wavefront Retrieval Results With Blurring

True OPD (wv)  Result OPD (wv)  Error OPD (wv)

OPD in waves
Monte Carlo Results

Accuracy Comparison

- Trials
- Break-even line

Approximate Better
Approximate Worse
Segmented Systems

• Monolithic System Results
  – Good results for monolithic system
  – 270x speed improvement
  – Small loss in accuracy
  – How does approximation perform for a segment system?

• Segmented test case
  – Same 1-5 µm FGS-like system
  – Global aberration model: Third order aberrations
  – Segment aberration model: Piston, tip, tilt
  – Monte Carlo simulation
    • 16 trials
    • 0.1 waves RMS wavefront errors
Unblurred PSFs (Segmented)

-2 waves defocus  0 waves defocus  +2 waves defocus

Data

Unblurred Monochromatic
Blurred PSFs (Segmented)

-2 waves defocus 0 waves defocus +2 waves defocus

Data

Blurred Monochromatic
Results, Segmented System
Accuracy Comparison: Segmented System

Accuracy Comparison for Segmented System

- Blurred Monochromatic
- Unblurred Monochromatic
Segmented System

- Reduced Accuracy with segmented system
- Possible causes
  - More diffraction
  - More variables (higher order phases)
  - Which is responsible?
- To investigate
  - Apply segmented system mask to low order phases
  - Same phase model as monolithic aperture
  - Same aperture mask as segmented system
  - Isolates diffraction effect from higher order phase effect
  - Limit study to 0.25 waves wavefront to avoid issues with large wavefronts
Results: Segmented Mask

- True OPD (waves)
- Result OPD (waves)
- Error OPD (waves)
<table>
<thead>
<tr>
<th>-2 waves defocus</th>
<th>0 waves defocus</th>
<th>+2 waves defocus</th>
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**Data**

**Unblurred Monochromatic**
Blurred PSFs (Segmented Mask)

-2 waves defocus  0 waves defocus  +2 waves defocus

Data

Blurred Monochromatic
Accuracy Comparison: Three Cases

Medians:
- Monolithic: 0.004 waves
- Mask: 0.009 waves
- Segments: 0.0308 waves

Both diffraction and higher order phase reduce accuracy!
Conclusion

- Broadband phase retrieval needed when:
  - Narrow spectral filters unavailable
  - Dim sources
  - Low throughput due to misalignment
  - Short exposures times
    - Pointing instability (space)
    - Atmospheric instability (ground based AO)
- Traditional approach is computationally burdensome for extreme bandwidths
- Approximate approach
  - Substitute monochromatic model
  - Blur model and data
- Test case performance
  - ~270x reduction in computational cost for FGS-like test case
  - Good accuracy for monolithic system
  - Acceptable accuracy for segmented systems
  - Reduced by diffraction
  - Reduced by higher order segment model
Questions?