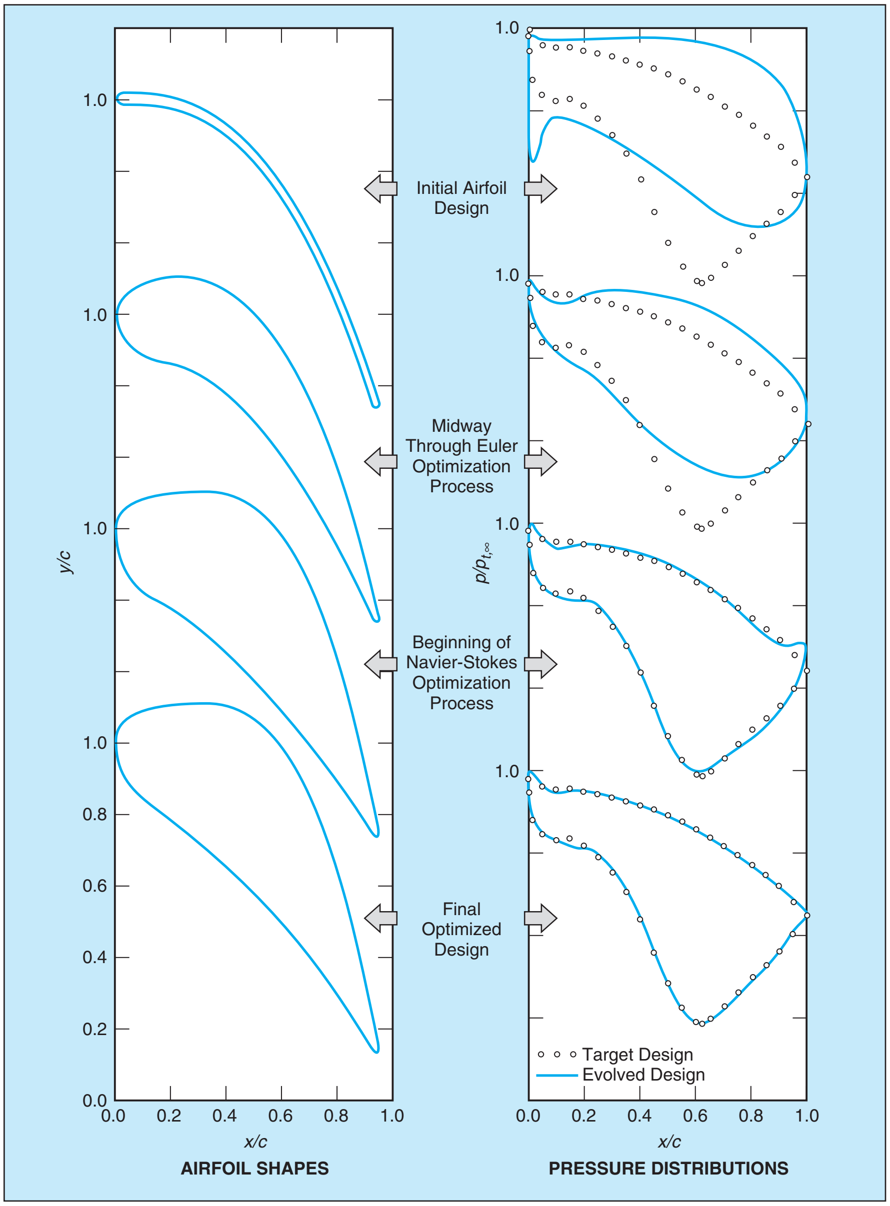


The **Design Procedure** can evolve a generic shape into an optimized airfoil that matches a target pressure distribution. Position coordinates (*x* and *y*) are normalized to the chord length (*c*); local pressure (*p*) is normalized to the turbine inlet total pressure (p_t, ∞).

some of its capabilities include the inverse design of an optimal turbine airfoil starting from a generic shape and the redesign of transonic turbines to improve their unsteady aerodynamic characteristics.

In one practical application, the procedure was used to reconstruct the shape of a turbine airfoil given a desired pressure distribution and some relevant flow and geometry parameters. The shape of the airfoil was not known beforehand. Instead, it was evolved from a simple curved section of nearly uniform thickness. The evolved optimal airfoil closely matched the shape of the original airfoil that was used to obtain the pressure distribution. The progression of the design is depicted in the figure. The airfoil shape evolution is shown on the left, while the corresponding pressure distributions and the target pressure distribution are shown on the right. The surface pressures approach the target distribution as the design progresses until the optimal airfoil shown at the bottom has a pressure distribution that matches closely the target.

The technology developed and implemented in the neural-network-based design optimization procedure offers a unique capability that can be used in other aerospace applications such as external aerodynamics and multidisciplinary optimization, and has potential applications beyond aerospace design.

This work was done by Man Mohan Rai and Nateri K. Madavan of **Ames Research Center**. *Further information is contained in a TSP (see page 1).*

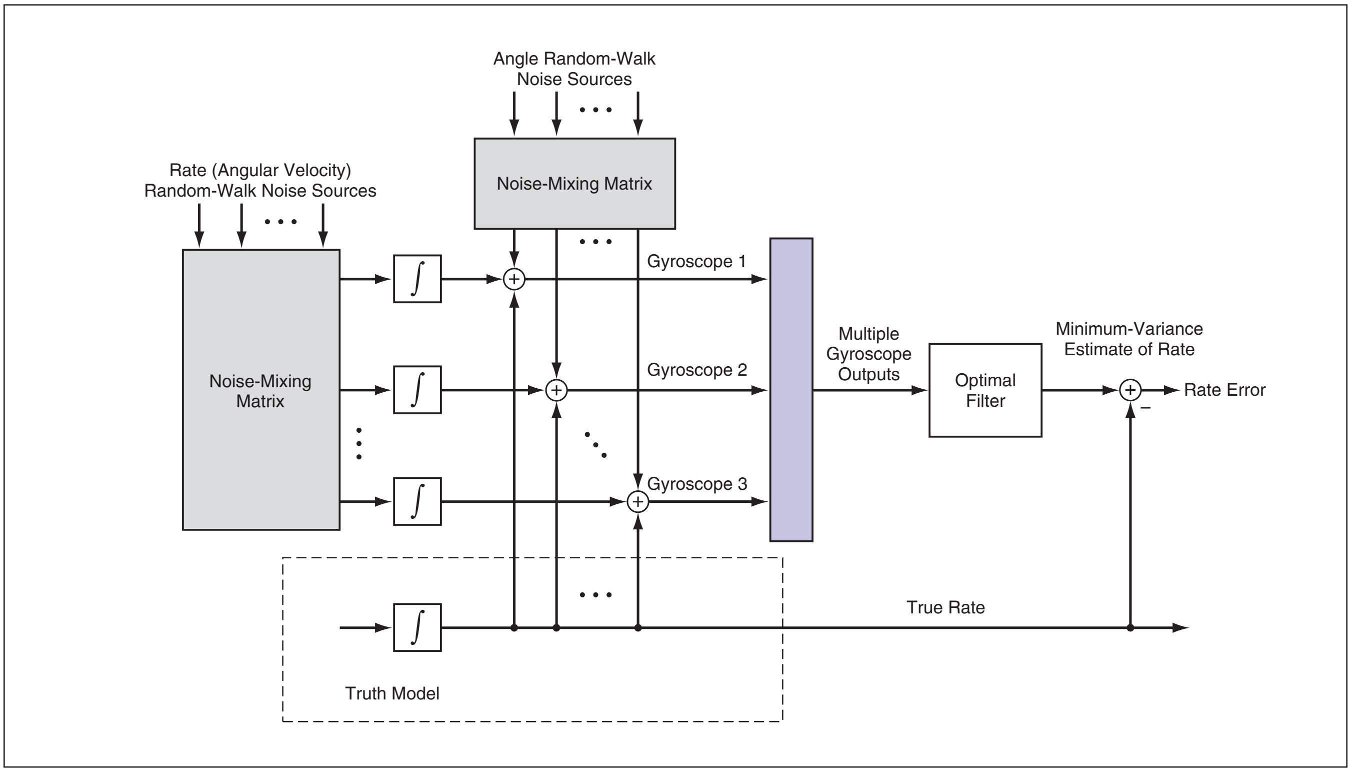
This invention is owned by NASA, and a patent application has been filed. Inquiries concerning nonexclusive or exclusive license for its commercial development should be addressed to the Patent Counsel, Ames Research Center, (650) 604-5104. Refer to ARC-14281.

Combining Multiple Gyroscope Outputs for Increased Accuracy

A lightweight, low-power, compact MEMS gyroscope array could perform comparably to a larger more-conventional gyroscope.

NASA's Jet Propulsion Laboratory, Pasadena, California

A proposed method of processing the outputs of multiple gyroscopes to increase the accuracy of rate (that is, angular-velocity) readings has been developed theoretically and demonstrated by computer simulation. Although the method is applicable, in principle, to any gyroscopes, it is intended especially for application to gyroscopes that are parts of microelectromechanical systems (MEMS). The method is based on the concept that the collective performance of multiple, relatively inexpensive, nominally identical devices can be better than that of one of the devices considered by itself. The method would make it possible to synthesize the readings of a single, more accurate gyroscope (a "virtual gyroscope") from the outputs of a large number of microscopic gyroscopes fabricated together on a single MEMS chip. The big advantage would be that the combination of the MEMS gyroscope array and the processing circuitry needed to implement the method would be smaller, lighter in



Readings From Multiple Gyroscopes would be combined by use of an optimal (minimum-variance) filter, thereby synthesizing the readings of what amounts to a more accurate virtual gyroscope.

weight, and less power-hungry, relative to a conventional gyroscope of equal accuracy.

The method (see figure) is one of combining and filtering the digitized outputs of multiple gyroscopes to obtain minimum-variance estimates of rate. In the combining-and-filtering operations, measurement data from the gyroscopes would be weighted and smoothed with respect to each other according to the gain matrix of a minimum-variance filter. According to Kalman-filter theory, the gain matrix of the minimum-variance filter is uniquely specified by the filter covariance, which propagates according to a matrix Riccati equation. The present method incorporates an exact analytical solution of this equation.

The analytical solution reveals a wealth of theoretical properties and enables the consideration of several practical implementations. Among the most notable theoretical properties are the following:

- Even though the terms of the Riccati equation grow in an unbounded fashion, the Kalman gain can be shown to approach a steady-state matrix. This result is fortuitous because it simplifies implementation and can serve as a basis of practical schemes for realizing the optimal filter by use of a constant- gain matrix.
- The analytical solution enables the development of a complete statistical theory that characterizes the drift of the virtual gyroscope and provides theoretical limits of improvement obtainable by use of an ensemble of correlated sensors as a single virtual sensor.
- The minimum-variance gain matrix has been analyzed in detail. The structure of the gain matrix indicates the presence of a marginally unstable pole, which would make implementation impossible if it were not properly understood and compensated. In addition, a simple algebraic method for computing the optimal gain matrix has been developed, making it possible to avoid the Riccati equation completely.
- The notion of statistical common-mode rejection (CMR) has been characterized mathematically. For statistically uncorrelated gyroscopes, it is shown that the component drift variances add like parallel resistors (e.g., in units of rad^2/sec^3). For identical devices, this means that the combined drift is reduced by a factor of $1/\sqrt{N}$ compared to the individual gyroscope drift, where N is the number of gyroscopes being combined.
- Potential improvement in drift is much more impressive when the devices are correlated. For correlated gyroscopes, an exact mathematical expression is developed for the combined drift. The expression indicates that the internal noise sources count to the extent that they infect multiple devices coherently (e.g., with common sign), and can be removed to the extent that they infect multiple devices incoherently (e.g., with randomized signs). Noise eliminated by correlations between devices can potentially reduce the virtual gyroscope drift far beyond the $1/\sqrt{N}$ factor attainable using uncorrelated devices.

This work was done by David S. Bayard of Caltech for **NASA's Jet Propulsion Laboratory**. *Further information is contained in a TSP (see page 1).*

In accordance with Public Law 96-517, the contractor has elected to retain title to this invention. Inquiries concerning rights for its commercial use should be addressed to

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