



## ➤ Finding Every Root of a Broad Class of Real, Continuous Functions in a Given Interval

**This robust and reliable algorithm is capable of locating the zeros of a continuous, nonlinear function.**

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One of the most pervasive needs within the Deep Space Network (DSN) Metric Prediction Generator (MPG) view period event generation is that of finding solutions to given occurrence conditions. While the general form of an equation expresses equivalence between its left-hand and right-hand expressions, the traditional treatment of the subject subtracts the two sides, leaving an expression of the form  $f(x) = 0$ . Values of the independent variable  $x$  satisfying this condition are roots, or solutions. Generally speaking, there may be no solutions, a unique solution, multiple solutions, or a continuum of solutions to a given equation.

In particular, all view period events are modeled as zero crossings of various metrics; for example, the time at which the elevation of a spacecraft reaches its maximum value, as viewed from a Deep Space Station (DSS), is found by locating that point at which the derivative of the eleva-

tion function becomes zero. Moreover, each event type may have several occurrences within a given time interval of interest. For example, a spacecraft in a low Moon orbit will experience several possible occultations per day, each of which must be located in time. The MPG is charged with finding all specified event occurrences that take place within a given time interval (or "pass"), without any special clues from operators as to when they may occur, for the entire spectrum of missions undertaken by the DSN. For each event type, the event metric function is a known form that can be computed for any instant within the interval.

A method has been created for a mathematical root finder to be capable of finding all roots of an arbitrary continuous function, within a given interval, to be subject to very lenient, parameterized assumptions. One assumption is that adjacent roots are separated at least by a given amount,  $xGuard$ . Any point

whose function value is less than  $\epsilon f$  in magnitude is considered to be a root, and the function values at distances  $xGuard$  away from a root are larger than  $\epsilon f$ , unless there is another root located in this vicinity. A root is considered found if, during iteration, two root candidates differ by less than a pre-specified  $\epsilon x$ , and the optimum cubic polynomial matching the function at the end and at two interval points (that is within a relative error fraction  $\epsilon L$  at its midpoint) is reliable in indicating whether the function has extrema within the interval. The robustness of this method depends solely on choosing these four parameters that control the search. The roots of discontinuous functions were also found, but at degraded performance.

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## ➤ Kalman Orbit Optimized Loop Tracking

**This method has application in military GNSS receivers.**

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Under certain conditions of low signal power and/or high noise, there is insufficient signal to noise ratio (SNR) to close tracking loops with individual signals on orbiting Global Navigation Satellite System (GNSS) receivers. In addition, the processing power available from flight computers is not great enough to implement a conventional ultra-tight coupling tracking loop. This work provides a method to track GNSS signals at very low SNR without the penalty of requiring very high processor throughput to calculate the loop parameters.

The Kalman Orbit-Optimized Loop (KOOL) tracking approach constitutes a filter with a dynamic model and using the

aggregate of information from all tracked GNSS signals to close the tracking loop for each signal. For applications where there is not a good dynamic model, such as very low orbits where atmospheric drag models may not be adequate to achieve the required accuracy, aiding from an IMU (inertial measurement unit) or other sensor will be added. The KOOL approach is based on research JPL has done to allow signal recovery from weak and scintillating signals observed during the use of GPS signals for limb sounding of the Earth's atmosphere. That approach uses the onboard PVT (position, velocity, time) solution to generate predictions for the range, range rate, and ac-

celeration of the low-SNR signal. The low-SNR signal data are captured by a directed open loop. KOOL builds on the previous open loop tracking by including feedback and observable generation from the weak-signal channels so that the MSR receiver will continue to track and provide PVT, range, and Doppler data, even when all channels have low SNR.

The KOOL algorithm will also reduce the processor throughput requirements. This is enabled because the dynamic model of the receiver motion is very smooth, so that the full physical orbit model can be run at a low rate; for example, every 10 seconds. This contrasts with the signal tracking loop requirement for