

method used to develop the initial power spectral density (PSD) environments for deriving the random forces for the rocket engine case is based on the Barrett Criteria developed at Marshall Space Flight Center in 1963. This

invention approach can be applied in the aerospace, automotive, and other industries to obtain reliable dynamic loads and responses from a finite element model for any structure subject to multi-point random vibration excitations.

*This work was done by Lawrence Shen of Pratt & Whitney Rocketdyne for Marshall Space Flight Center. For more information, contact Sammy Nabors, MSFC Commercialization Assistance Lead, at [sammy.a.nabors@nasa.gov](mailto:sammy.a.nabors@nasa.gov). Refer to MFS-32714-1.*

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## ➤ Optimal Control Via Self-Generated Stochasticity

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The problem of global maxima of functionals has been examined. Mathematical roots of local maxima are the same as those for a much simpler problem of finding global maximum of a multi-dimensional function. The second problem is instability — even if an optimal trajectory is found, there is no guarantee that it is stable. As a result, a fundamentally new approach is introduced to optimal control based upon two new ideas.

The first idea is to represent the functional to be maximized as a limit of a probability density governed by the ap-

propriately selected Liouville equation. Then, the corresponding ordinary differential equations (ODEs) become stochastic, and that sample of the solution that has the largest value will have the highest probability to appear in ODE simulation. The main advantages of the stochastic approach are that it is not sensitive to local maxima, the function to be maximized must be only integrable but not necessarily differentiable, and global equality and inequality constraints do not cause any significant obstacles.

The second idea is to remove possible instability of the optimal solution by equipping the control system with a self-stabilizing device.

The applications of the proposed methodology will optimize the performance of NASA spacecraft, as well as robot performance.

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## ➤ Space-Time Localization of Plasma Turbulence Using Multiple Spacecraft Radio Links

**This technology has applications in forecasting adverse effects on satellites.**

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Space weather is described as the variability of solar wind plasma that can disturb satellites and systems and affect human space exploration. Accurate prediction requires information of the heliosphere inside the orbit of the Earth. However, for predictions using remote sensing, one needs not only plane-of-sky position but also range information — the third spatial dimension — to show the distance to the plasma disturbances and thus when they might propagate or co-rotate to create disturbances at the orbit of the Earth. Appropriately processed radio signals from spacecraft having communications lines-of-sight passing through the inner heliosphere can be used for this space-time localization of plasma disturbances.

The solar plasma has an electron density- and radio-wavelength-dependent index of refraction. An approximately monochromatic wave propagating through a thin layer of plasma turbulence causes a geometrical-optics phase shift proportional to the electron density

at the point of passage, the radio wavelength, and the thickness of the layer. This phase shift is the same for a wave propagating either “up” or “down” through the layer at the point of passage. This attribute can be used for space-time localization of plasma irregularities.

The transfer function of plasma irregularities to the observed time series depends on the Doppler tracking “mode.” When spacecraft observations are in the two-way mode (downlink radio signal phase-locked to an uplink radio transmission), plasma fluctuations have a “two-pulse” response in the Doppler. In the two-way mode, the Doppler time series  $y_2(t)$  is the difference between the frequency of the downlink signal received and the frequency of a ground reference oscillator. A plasma blob localized at a distance  $x$  along the line of sight perturbs the phase on both the up and down link, giving rise to two events in the two-way tracking time series separated by a time lag depending the blob's

distance from the Earth:  $T_2 - 2x/c$ , where  $T_2$  is the two-way time-of-flight of radio waves to/from the spacecraft and  $c$  is the speed of light.

In some tracking situations, more information is available. For example, with the 5-link Cassini radio system, the plasma contribution to the up and down links,  $y_{up}(t)$  and  $y_{dn}(t)$ , can be computed separately. The time series  $y_{up}(t)$  and  $y_{dn}(t)$  respond to a localized plasma blob with one event in each time series. These events are also separated in time by  $T_2 - 2x/c$ . By cross-correlating the up and down link Doppler time series, the time separation of the plasma events can be measured and hence the plasma blob's distance from the Earth determined. Since the plane-of-sky position is known, this technique allows localization of plasma events in time and three space dimensions.

*This work was done by John W. Armstrong and Frank B. Estabrook of Caltech for NASA's Jet Propulsion Laboratory. For more information, contact [iaoffice@jpl.nasa.gov](mailto:iaoffice@jpl.nasa.gov). NPO-46952*