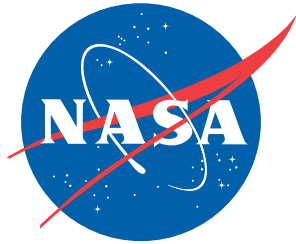


NASA/TM-2012-217335/Volume II  
NESC-RP-08-00448



# Probabilistic Requirements (Partial) Verification Methods Best Practices Improvement

*Variables Acceptance Sampling Calculators: Empirical  
Testing*

*Kenneth L. Johnson/NESC  
Langley Research Center, Hampton, Virginia*

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University of Virginia, Charlottesville, Virginia*

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February 2012

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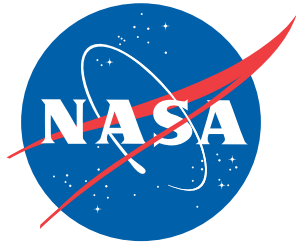
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### Probabilistic Requirements (Partial) Verification Methods Best Practices Improvement

#### Variables Acceptance Sampling Calculators: Empirical Testing

**October 27, 2011**


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## Approval and Document Revision History

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Version	Description of Revision	Office of Primary Responsibility	Effective Date
1.0	Initial Release	Kenneth L. Johnson, Statistics and Trending, Marshall Space Flight Center	10/27/11

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### **1.0 Notification and Authorization**

The NASA Engineering and Safety Center (NESC) was requested to improve on the Best Practices document produced for NESC assessment 07-001-E: Verification of Probabilistic Requirements for the Constellation Program (CxP) by giving a recommended procedure for using acceptance sampling by variables techniques as an alternative to the potentially resource-intensive acceptance sampling by attributes method given in the document.

An NESC out-of-board activity was approved on January 9, 2008. Mr. Kenneth Johnson at Marshall Space Flight Center (MSFC) was selected to lead this assessment. The assessment plan was approved at the NESC Review Board (NRB) on March 20, 2008.

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## 2.0 Signature Page

Submitted by:

*Team Signature Page on File – 2-7-12*

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Kenneth L. Johnson

Date


Significant Contributors:

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K. Preston White, Jr.


Date

Signatories declare the findings and observations compiled in the report are factually based from data extracted from Program/Project documents, contractor reports, and open literature, and/or generated from independently conducted tests, analyses, and inspections.

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## 4.0 Abstract

Acceptance sampling is a method for verifying lot quality or performance requirements using sample data. In this paper, the results of empirical tests intended to assess the accuracy of acceptance sampling plan calculators implemented for six variable distributions—binomial, exponential, normal, gamma, Weibull, inverse Gaussian (IG), and Poisson are presented. In general, these results support the accepted wisdom that variables acceptance plans are superior to attributes (binomial) acceptance plans, in the sense that these provide comparable protection against both producer's and consumer's risk at reduced sampling cost. For the Gaussian and Weibull plans, however, there are ranges of the shape parameters for which the required sample sizes are in fact *larger* than the corresponding attributes plans, dramatically so for instances of large skew. Tests also confirm that the published IG plan is flawed (White and Johnson, 2011). Appendix A provides the protocol for selecting a sampling plan calculator. Appendix B provides a note on hypothesizing an output distribution.

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## 5.0 Introduction

Acceptance sampling by attributes (ASA) assesses the quality of a lot based on the number of nonconforming items discovered in a random sample drawn for inspection. Inspection requires only a pass/fail determination for each item. Because it is conceptually straightforward, easily to implement, and can be applied to qualitative as well as quantitative performance measures, ASA is the first choice for sampling inspection.


For quantitative performance measures, however, a pass/fail determination typically is accomplished by comparing the measured value to a limiting value, without regard to magnitude of conformance or nonconformance for each item tested. It seems reasonable that this additional information might be exploited to decrease the number of items that need to be inspected. This is the rationale behind acceptance sampling by variables (ASV). If there is adequate information to posit a distribution for the measure, then in many instances the ASV alternative translates into significantly smaller samples to achieve the same operating characteristic. While far more restrictive in its assumptions, ASV should be considered only when larger samples required by ASA is an issue.

The objective of this paper is to provide an independent assessment of the accuracy of variables plans reported in the literature. Note that, with the exception of normal plans, the search for off-the-shelf ASV plan calculators was essentially fruitless. This scarcity strongly suggests that non-normal ASV largely has been limited to an academic audience and not fully vetted in practice. The need to implement and test plans reported in the literature is especially important for those plans based on approximations. This paper discusses the results of empirical tests conducted using spreadsheet implementation of calculators developed for this purpose.

## 6.0 Test Protocol

The plans reviewed by White and Johnson (2011) were implemented as spreadsheet calculators and tested empirically using Monte Carlo simulation. The test protocol enforced a limit standard with (1) a specification limit on the measured variable  $X$ , either  $x_{\min}$  or  $x_{\max}$ , as the performance indicator  $I$ , (2) minimum reliability  $\rho = 0.005$ , and (3) maximum consumer's risk  $\beta = 0.100$ . Additionally, maximum producer's risk of  $\alpha = 0.200$  was enforced. These particular test conditions were chosen as representative of certain high-level requirements in the design of spacecraft (White, *et al.*, 2009).

For each test, values were specified for the limit and for any distribution parameters assumed to be known or estimated. The null and alternative means  $\mu_0$  and  $\mu_1$  were determined such that, for a lower limit,  $F(x_{\min}; \mu_0) = 0.001$  and  $F(x_{\min}; \mu_1) = 0.005$ . For an upper limit,  $1 - F(x_{\max}; \mu_0) = 0.999$  and  $1 - F(x_{\max}; \mu_1) = 0.995$ . The corresponding  $(n, \kappa)$  sampling plan was then determined from the appropriate calculator.

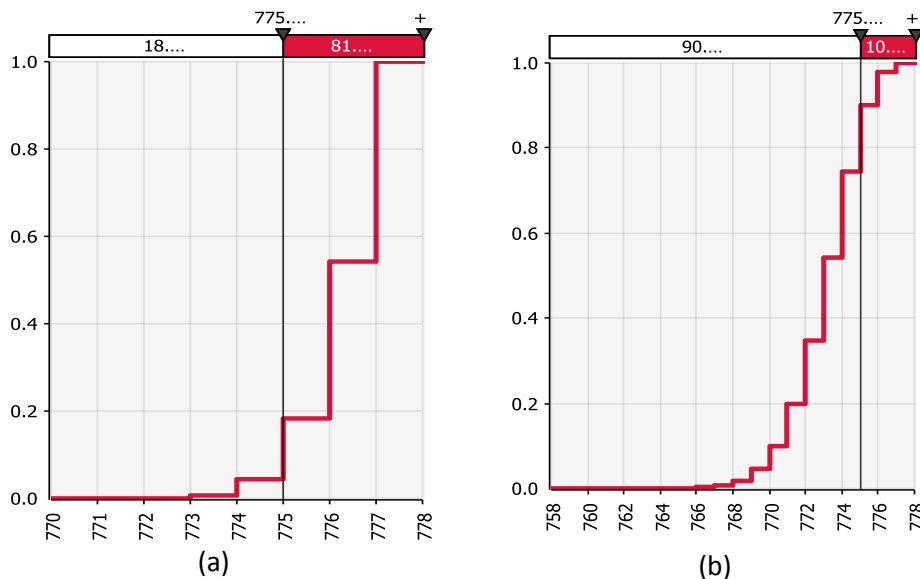
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One-hundred thousand Monte Carlo trials were run for both the null and alternative distributions, each run comprising  $n$  observations as determined by the sampling plan. The proportions  $\hat{\alpha}$  and  $\hat{\beta}$  were estimated from the sampling distribution of the acceptance limit  $A(n,k)$ . These estimates were compared to the specified operating characteristic to assess the accuracy of the plan. The efficiency of the variable sampling plan was determined by comparing the required sample size to that for the closest attributes sampling plan.

## 7.0 Results


### 7.1 Attributes (Binomial) Plan

For the test operating characteristic (OC), the closest attributes sampling plan from the binomial calculator is  $(n,c) = (777,1)$ , with null and alternative test points  $(p_0, 1 - \alpha) = (0.001, 0.8280)$  and  $(p_1, \beta) = (0.005, 0.0998)$ . As shown in Figure 7.1-1, the requirement for integer sample size  $n$  implies that the specified limits on the risks are enforced, but not necessarily as equalities. This plan is otherwise exact, in the sense that (1) the pass/fail determination is necessarily distributed binomial  $(n,p)$  and requires no assumptions regarding the distribution of  $X$  and (2) there is a single unknown parameter  $p$  and additional parameters need not be estimated approximately.



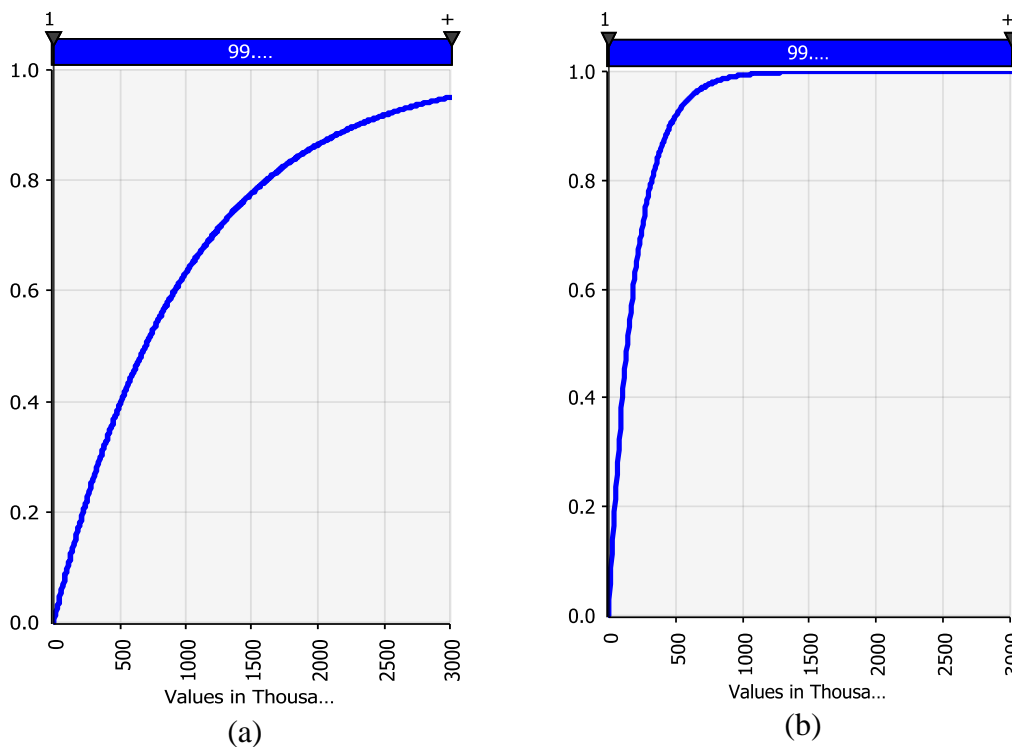
**Figure 7.1-1. Attributes Simulation Results: Distribution Functions and Critical Values for  $X$  Distributed Binomial  $(777,p)$  Under (a) Null Hypothesis  $p_0 = 0.005$  and (b) Alternate Hypotheses and  $p_1 = 0.001$  for an Upper Limit on the Number of Nonconforming Observations**



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## 7.2 Exponential Plan (Lower Limit)

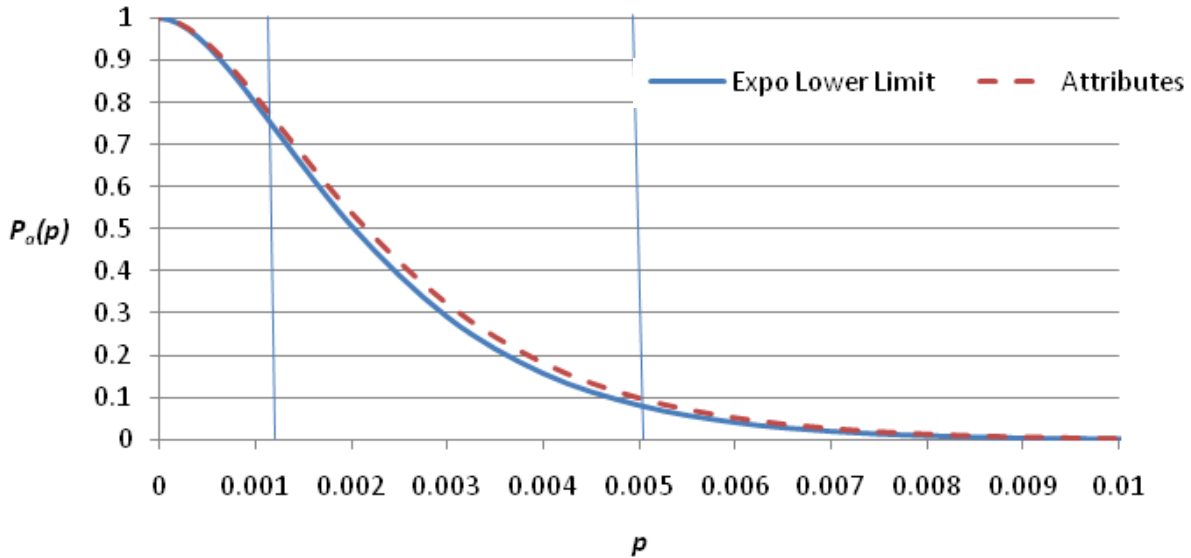
Consider a lower limit of  $x_{\min} = 1000$  for an exponentially distributed random variable  $X$  with unknown mean  $\mu$ . For the test OC, the associated the null and alternative means are  $\mu_0 = 999,500$  and  $\mu_1 = 199,500$ , respectively, as shown in Figure 7.2-1. The variables plan from the exponential calculator is  $(n,k) = (2, 2.42722 \times 10^{-3})$ .



**Figure 7.2-1. Exponential Lower Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Lower Limit  $x_{\min} = 1000$  Under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 999,500$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 199,500$ )**

Figure 7.2-2 compares the OC curves for the exponential and attribute plans. These curves are similar, with the exponential sampling plan modestly more conservative with respect to risks. Note that here and throughout, the OC curves are theoretical and derived from the plans, and may or may not agree with empirical results.

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*Figure 7.2-2. Comparison of the Exponential (Lower Limit) and Attributes Sampling OC Curves for the Test Case*

Simulations were run to estimate the distribution of the test statistic, the allowable limit  $\hat{A} = k\hat{\mu}$ , where the estimated mean  $\hat{\mu}$  is the sample mean  $\bar{X}(n)$  and  $n$  and  $k$  are given by the sampling plan. Figure 7.2-3(a) and (b) shows the sample distributions of  $\hat{A}$  for the null and alternate hypotheses, respectively. From these distributions, it was observed that there was no practical difference between the estimated risks of  $\hat{\alpha} = 0.200$  and  $\hat{\beta} = 0.082$  and those predicted by the OC curve.

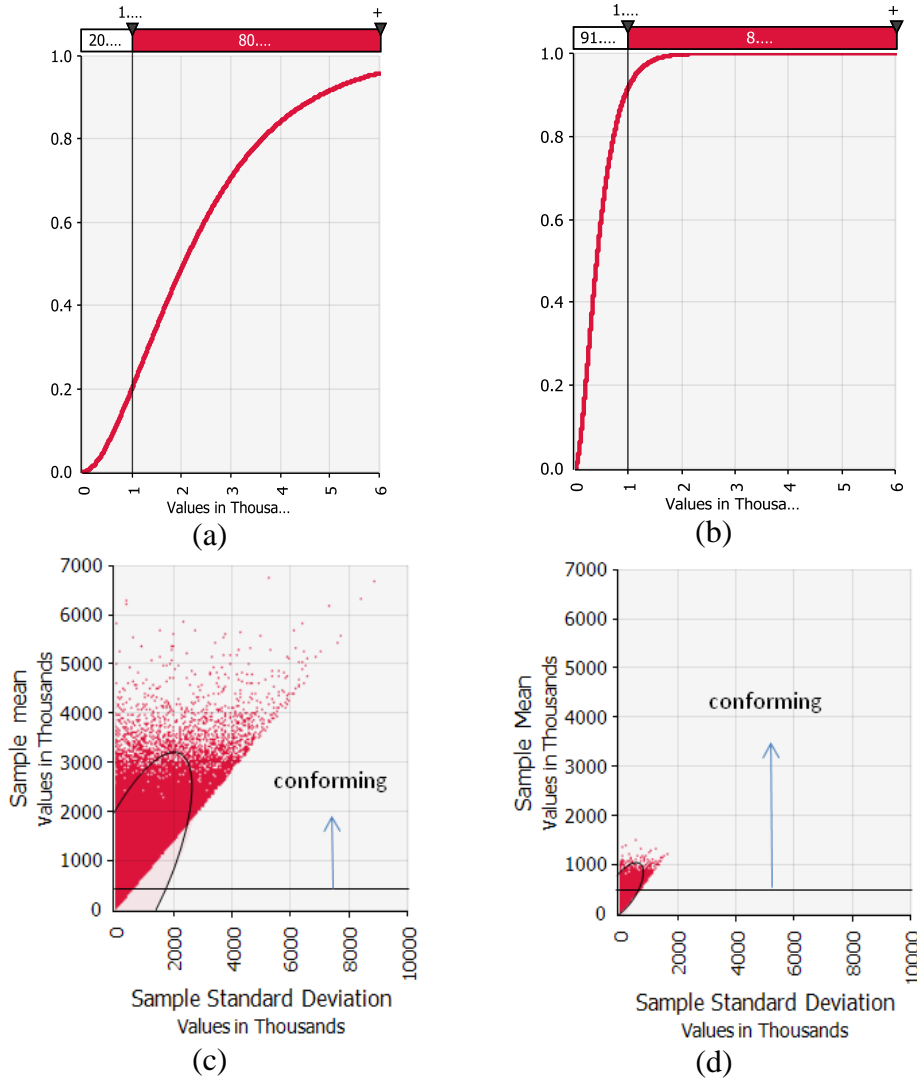
Figure 7.2-3(c) and (d) are scatterplots of the sample mean and standard deviation for each of the 100,000 simulation trials for the null and alternate hypotheses, respectively. Also shown on each scatterplot is the threshold value of  $A$ , the constant  $\mu = x_{\min}/k = 411,993$ . This line divides each sample into two sets, with the conforming observations above the line and nonconforming below.



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**Figure 7.2-3. Exponential Lower-Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

Each figure shows that the proportion of nonconforming observations subtended by the line is approximately 20 percent for the null distribution and the proportion of conforming observations subtended by the line is approximately 8.20 percent for the alternate distribution. Increasing the limiting value  $x_{\min}$  will decrease the proportion conforming (increasing  $\alpha$  and decreasing  $\beta$ ). Increasing the value of  $k$  will have a similar effect.

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The respective 99-percent confidence bounds for each data set are shown as ovals on the scatterplots. The area subtended by these bounds depends on the number runs  $n$ . Increasing the number of runs will decrease the radius of the confidence bound, decreasing  $\alpha$  and for fixed  $\beta$ .

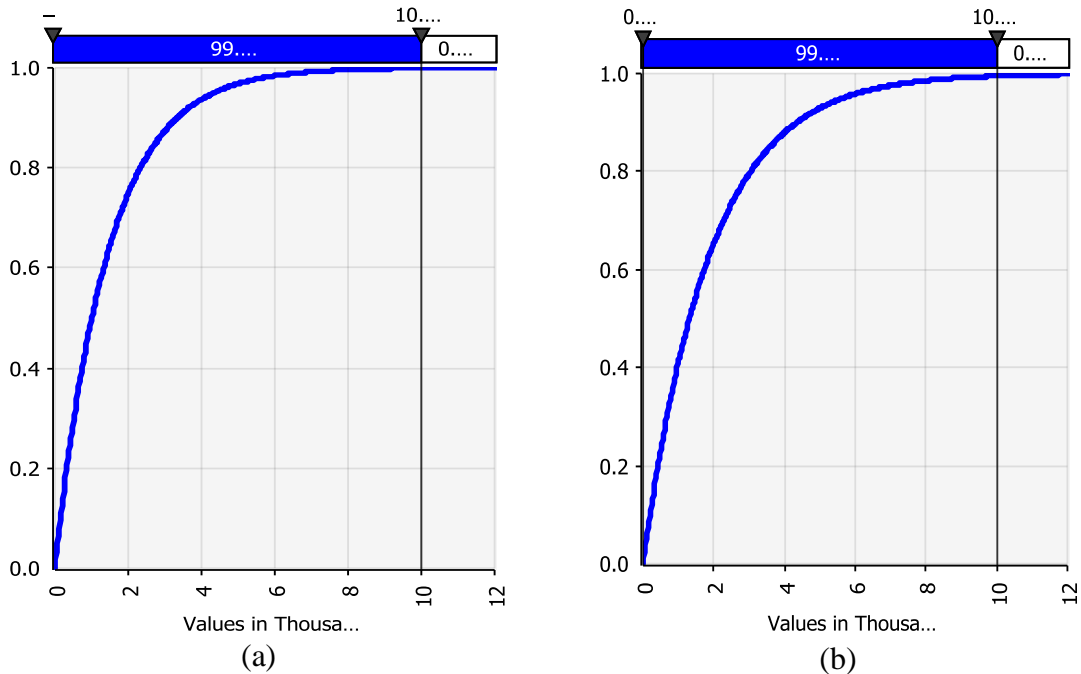
The lower-limit exponential plan clearly outperforms the attributes plan, by providing equivalent risk protection at a dramatically reduced sampling cost. The number of simulations indicated by the binomial is  $n = 777$ , whereas the variables plans require only  $n = 2$ , which is 0.26 percent of the computational effort of the attributes plan. From Figure 7.2-1, it should be clear that this economy was achieved because the null and alternate distributions are distinct—observations that are typical of one distribution are rare for the other. The economy was further enhanced by the need to estimate only a single parameter sample from the data.

It seems obvious, however, that two observations were inadequate to determine the exponential distributional form if it is otherwise unknown. Thus, in the case of exponentially distributed measurement, it appears that the number of trials required in many cases will be dictated by the need to determine the form of the output distribution, rather than the need to satisfy a limit standard.

### 7.3 Exponential Plan (Upper Limit)

Consider an upper limit of  $x_{\max} = 10,000$  for an exponentially distributed random variable  $X$  with unknown mean  $\mu$ . For the test OC, the associated the null and alternative means are  $\mu_0 = 1447.65$  and  $\mu_1 = 1887.4$ , respectively, as shown in Figure 7.3-1. The variables plan from the exponential calculator is  $(n, k) = (66, 6.26922)$ .

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**Figure 7.3-1. Exponential Upper Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Upper Limit  $x_{max} = 10,000$  and Under (a) Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 1447.65$ ) (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 1887.4$ )**

Figure 7.3-2 compares the OC curves for the exponential upper limit and attributes plans. These curves again are similar and almost identical at the OC points tested. Simulations were run to generate the distribution of the test statistic  $A = k\hat{\mu}$  for the exponential tests. Results are shown in Figure 7.3-3. There is no practical difference between the estimated risks of  $\hat{\alpha} = 0.200$  and  $\hat{\beta} = 0.082$  and those shown on the OC curve.

The exponential upper-limit plan also outperforms the attributes plan, by providing equivalent or superior risk protection at a substantially reduced sampling cost. The number of simulations indicated by the binomial is  $n = 777$ , whereas the variables plans require  $n = 66$ , which is 8.49 percent of the computational effort of the attributes plan. From Figures 7.3-1 and 7.3-3, it is clear that the null and alternate distributions are not nearly as distinct as in the case of a lower bound, owing to the skew. Thus, the economy is not as great as that for a lower bound, but is still impressive. Again, it may be that 66 observations are inadequate to determine the exponential distributional form and that the number of trials required in many cases will be dictated by this need, rather than the need to satisfy a limit standard.



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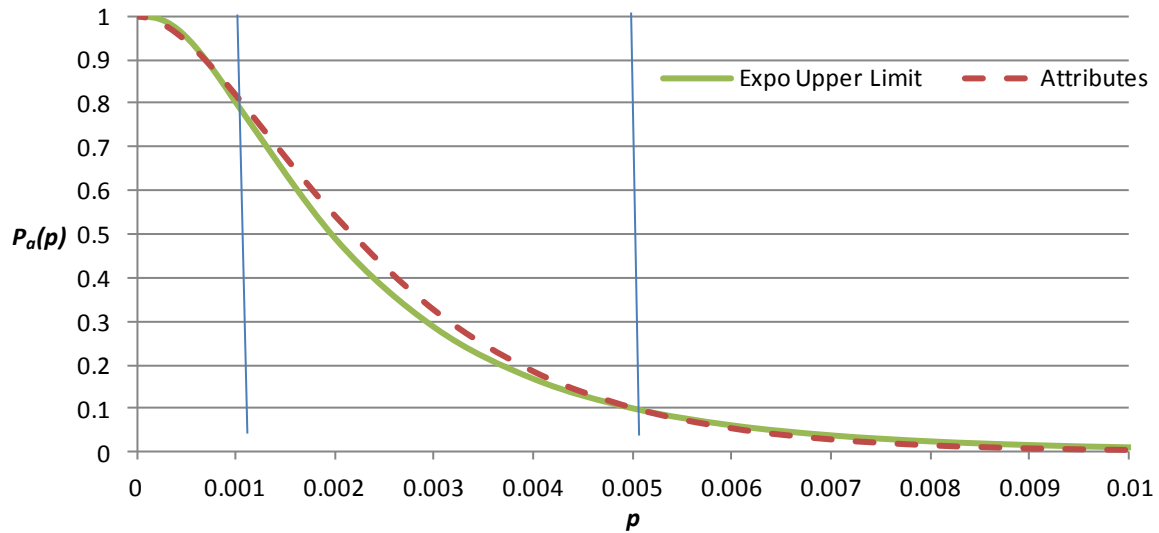
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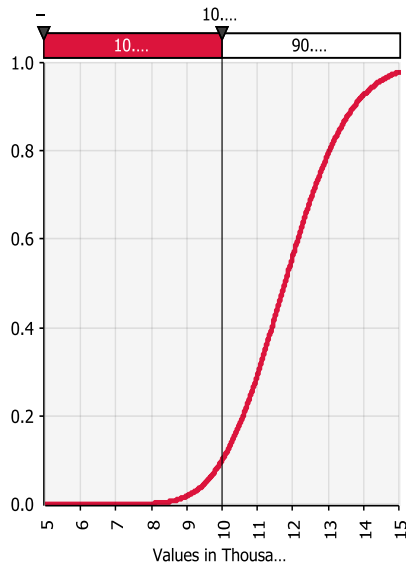
*Figure 7.3-2. Comparison of the Exponential (upper limit) and Attributes Sampling OC Curves for the Test Case*



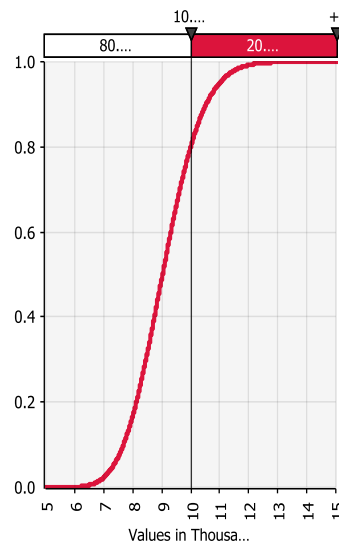
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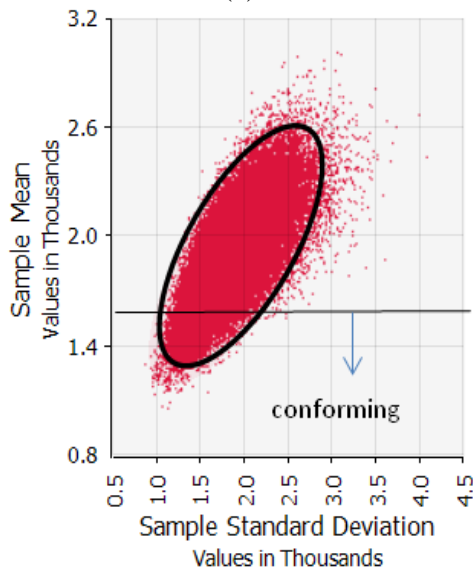
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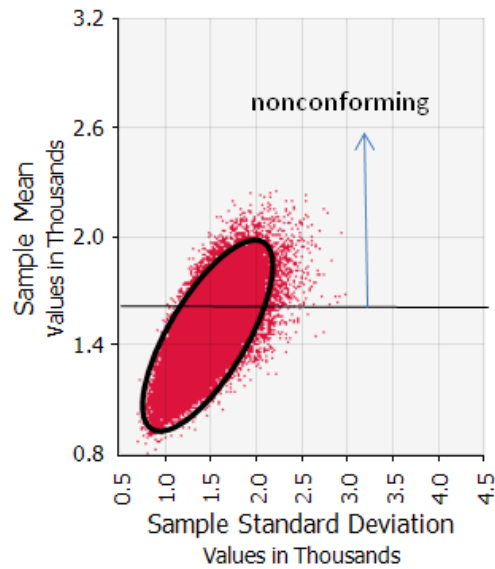
(a)



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(c)



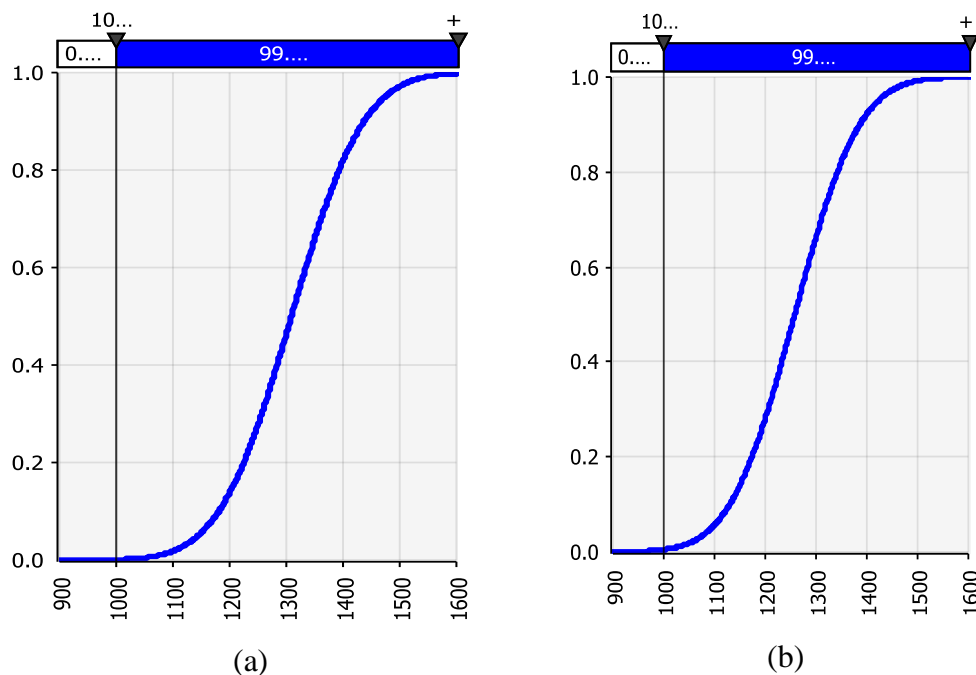
(d)

**Figure 7.3-3. Exponential Upper Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

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## 7.4 Normal Plan ( $\sigma$ -known)

Consider a lower limit of  $x_{\min} = 1000$  for a normally distributed random variable  $X$  with unknown mean  $\mu$  and known standard deviation  $\sigma = 100$ . For the test OC, the associated the null and alternative means are  $\mu_0 = 1309$  and  $\mu_1 = 1258$ , respectively, as shown in Figure 7.4-1. The variables plan from the norm calculator is  $(n, k) = (18, 2.88632)$  is also shown.



**Figure 7.4-1. Normal  $\sigma$ -known, Lower-Limit, Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Lower Limit  $x_{\min} = 1000$  under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 1258$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 1309$ )**

Figure 7.4-2 compares the OC curves for the normal and attributes plans. These curves again are similar, with the normal plan modestly more conservative in the region tested. Simulations were run to generate the distribution of the test statistic  $A_0 = \hat{\mu} - 100k$  for the normal tests. Results are shown in Figure 7.4-3. There was no practical difference between estimated risks of  $\hat{\alpha} = 0.193$  and  $\hat{\beta} = 0.096$  and those predicted by the OC curve.

The normal plan outperformed the attributes plan, by providing equivalent or superior risk protection at a dramatically reduced sampling cost. The number of simulations indicated by the binomial was  $n = 777$ , whereas the variables plans require only  $n = 18$ , which is 2.32 percent of the computational effort of the attributes plan. Given the symmetry of the normal distribution, the same results for an upper bound were expected.

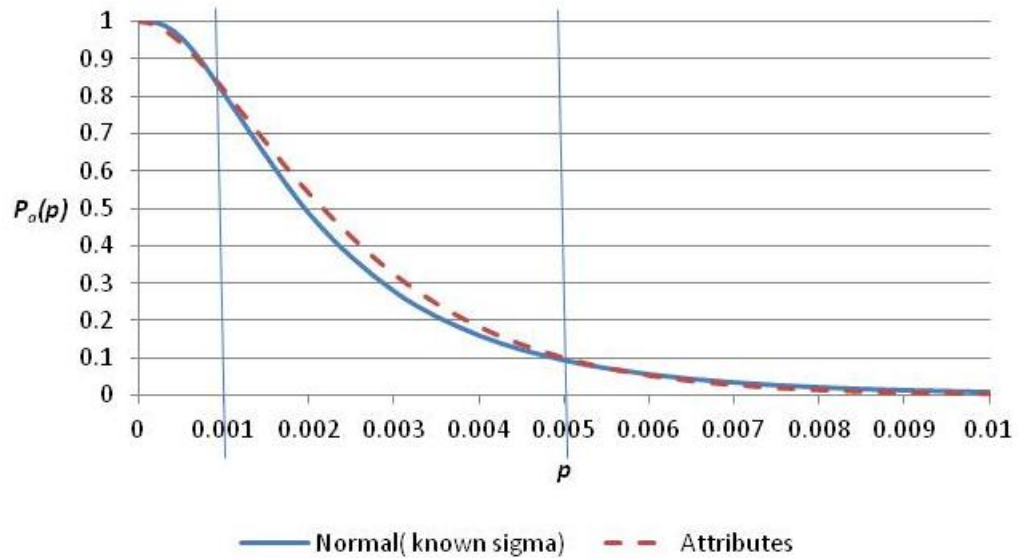




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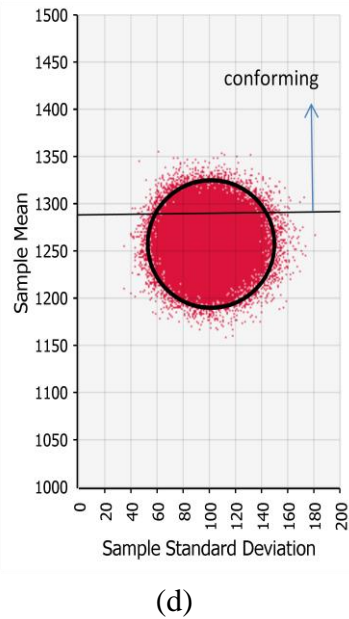
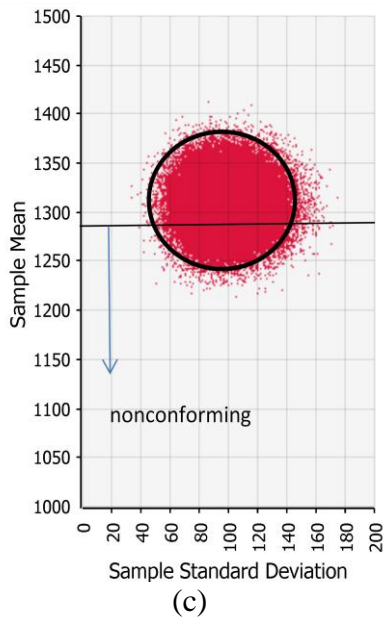
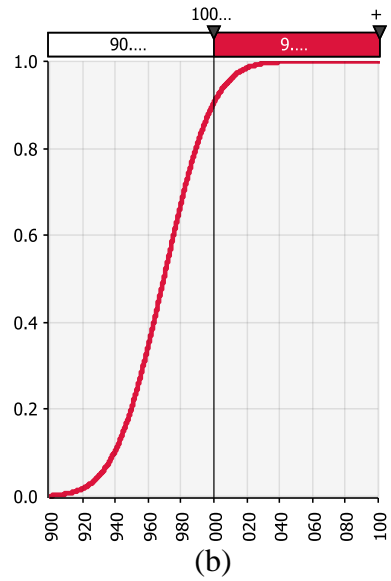
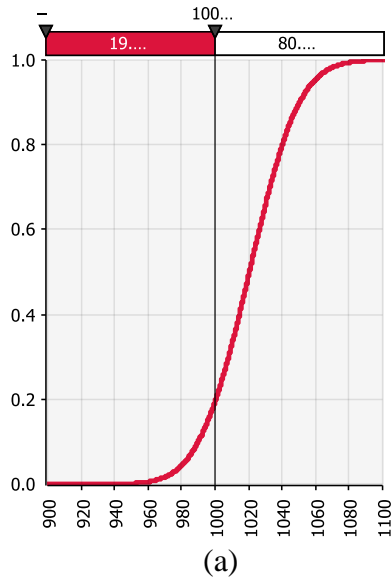
*Figure 7.4-2. Comparison of the Normal Plan ( $\sigma$ -known) and Attributes Sampling OC Curves for the Test Case*




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**Figure 7.4-3. Normal ( $\sigma$ -known), Lower-Limit, Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

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## 7.5 Normal Plan ( $\sigma$ -unknown)

Test conditions are identical to the  $\sigma$ -known case, except that standard deviation is now  $\hat{\sigma}$ , and estimated from the data for each trial. The variables plan from the normal calculator is  $(n,k) = (88,2.88632)$  with OC curve identical to that for the  $\sigma$ -known case.

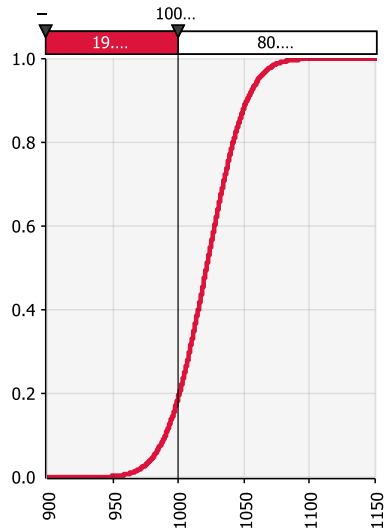
Simulations were run to generate the distribution of the test statistic  $A = \hat{\mu} - 100k$  for the normal tests. Results are shown in Figure 7.5-1. The estimated risks of  $\hat{\alpha} = 0.191$  and  $\hat{\beta} = 0.109$  are not quite those and predicted by the OC curve, with modestly lower producer's risk and modestly higher consumer's risk. The normal plan outperforms the attributes plan, by providing equivalent or protection at a dramatically reduced sampling cost. The number of simulations indicated by the binomial was  $n = 777$ , whereas the variables plans require only  $n = 88$ , which is 11.3 percent of the computational effort of the attributes plan. Given the symmetry of the normal distribution, the same results for an upper bound was expected.



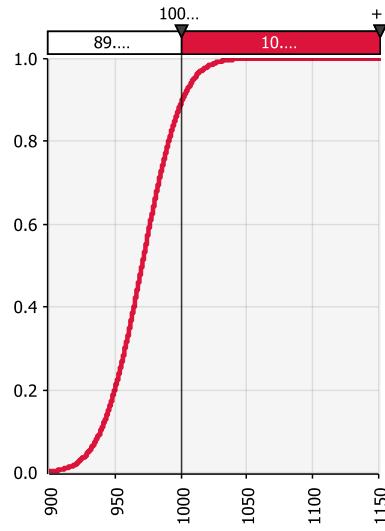
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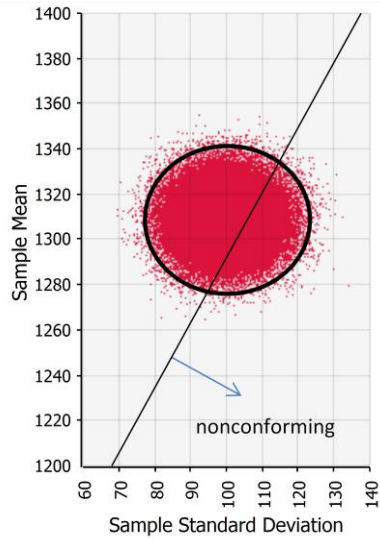
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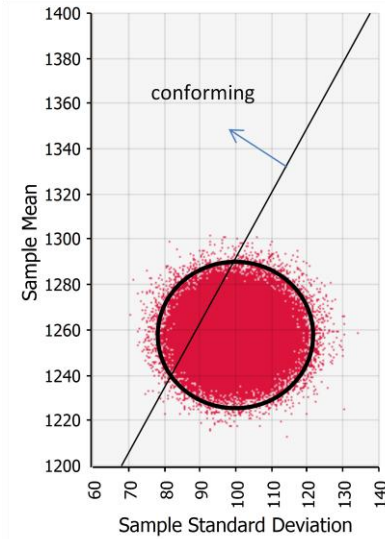
(a)



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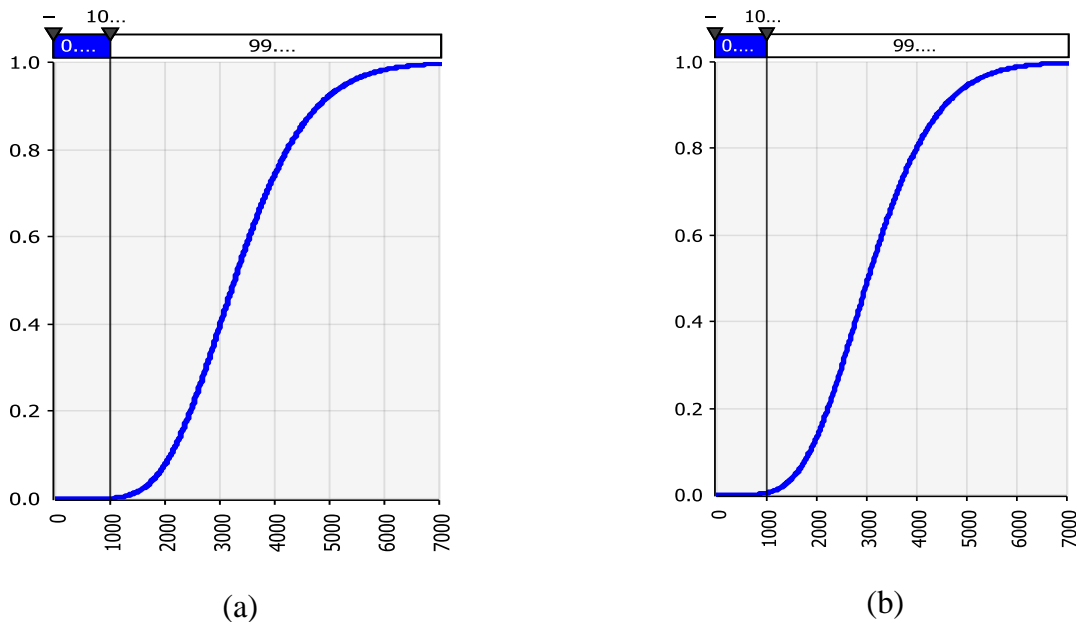
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Figure 7.5-1. Normal ( $\sigma$ -Unknown), Lower-Limit, Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Lower Limit  $x_{min} = 1000$  under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 1258$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 1309$ ). Ovals are 99-percent Confidence Bounds on the Scatter

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## 7.6 Gamma Plan (Lower Limit)

Consider a lower limit of  $x_{\min} = 1000$  for a random variable  $X$  distributed gamma with unknown shift parameter  $\delta$  and estimated shape and scale parameters,  $\hat{\nu} = 10$  and  $\hat{\lambda} = 337.779$ . For the test OC, the associated null and alternative means were  $\mu_0 = 3377.79$  and  $\mu_1 = 3140.79$ , as shown in Figure 7.6-1. The variables plan from the gamma calculator was  $(n, k) = (206, 2.13126)$ . Simulations were run to generate the distribution of the test statistic  $A_0 = \hat{\mu} - 100k$ . Results are shown in Figure 7.6-2. There was no practical difference between estimated risks of  $\hat{\alpha} = 0.191$  and  $\hat{\beta} = 0.096$  and those prescribed. The gamma plan outperforms the attributes plan, by providing equivalent or superior risk protection at a reduced sampling cost. The number of simulations indicated by the binomial was  $n = 777$ , whereas the variables plans require  $n = 206$ , which is 25.5 percent of the computational effort of the attributes plan.



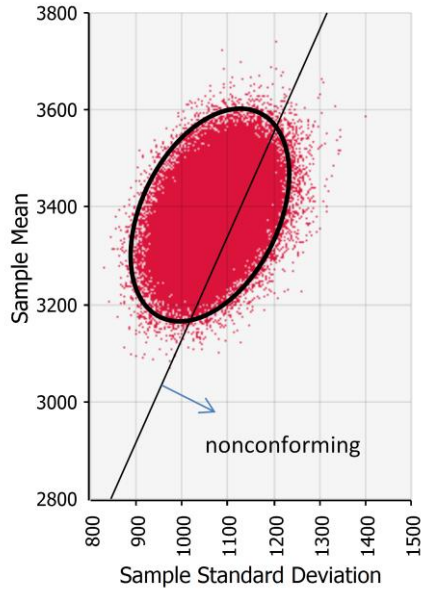
**Figure 7.6-1. Gamma Lower-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Lower Limit  $x_{\min} = 1000$  under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 3377.79$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 3117.79$ )**



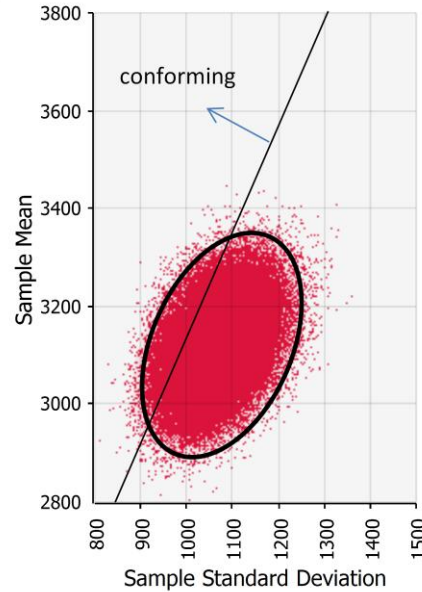
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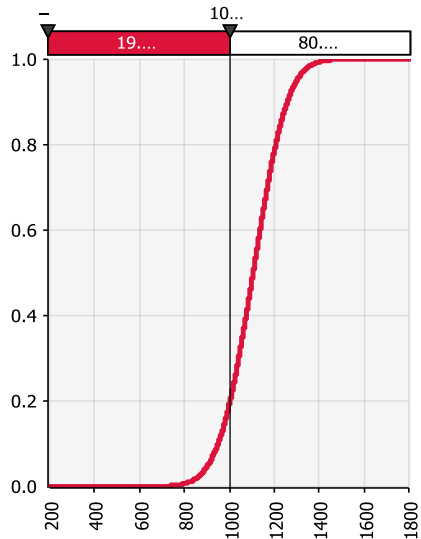
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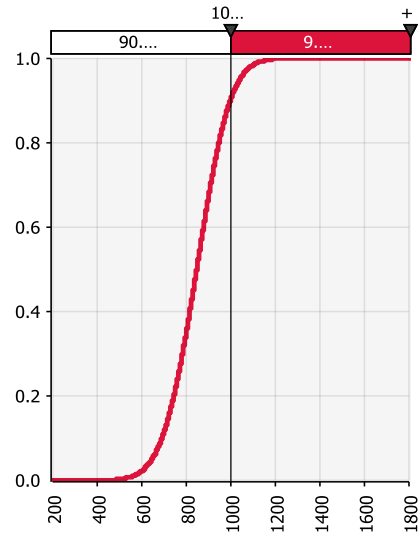
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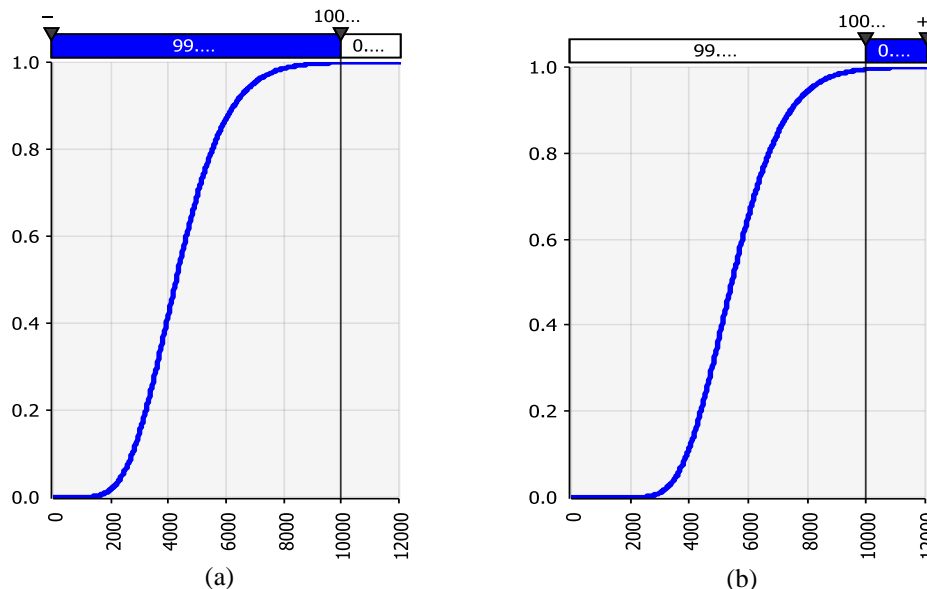
**Figure 7.6-2. Gamma Lower-Limit Simulation Results: Sampling Distributions for the Allowable Limit A under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

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Note, however, for the test OC and  $\hat{\nu} < 3.431$ , the variables plan required samples larger than the attributes plan. For example,  $\hat{\nu} = 0.200$  sampling plan from the gamma calculator was  $(n, k) = (1960, 1.46100)$ —nearly three times larger than the attributes plan. Given the other advantages of attributes sampling, gamma plans would never be used under this condition. This contradicts the accepted wisdom that ASV will always be more efficient than ASA. The operative point was that the sample size for any variables plan should be compared to the corresponding attributes plan and the benefit of the potential computational savings achieved (if any) assessed in the light of the restrictive assumptions imposed by variables sampling.


### 7.7 Gamma Plan (Upper Limit)

Consider an upper limit of  $x_{\max} = 10,000$  for a random variable  $X$  distributed gamma with unknown shift parameter  $\delta$  and estimated shape and scale parameters,  $\hat{\nu} = 10$  and  $\hat{\lambda} = 441.358$ . For the test OC, the associated the null and alternative means are  $\mu_0 = 4413.58$  and  $\mu_1 = 5587.57$ , respectively, as shown in Figure 7.7-1. The variables plan from the gamma calculator is  $(n, k) = (77, 3.66693)$ .



**Figure 7.7-1. Gamma Upper-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Upper Limit  $x_{\max} = 10,000$  Under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 4413.58$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 5587.57$ )**

Simulations were run to generate the distribution of the test statistic  $A_0 = \hat{\mu} + 100k$  for the Gamma test. Results are shown in Figure 7.7-2. The estimated risks of  $\hat{\alpha} = 0.186$  and

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$\hat{\beta} = 0.104$  are not quite those and predicted by the OC curve, with modestly lower producer's risk and modestly higher consumer's risk.

The gamma plan outperforms the attributes plan, by providing equivalent or superior risk protection at a reduced sampling cost. The number of simulations indicated by the binomial is  $n = 777$ , whereas the variables plans require  $n = 77$ , which is 9.91 percent of the computational effort of the attributes plan.

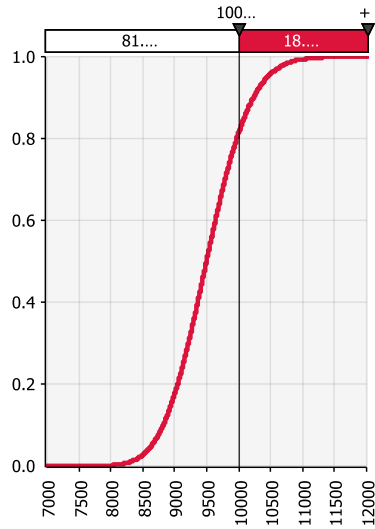




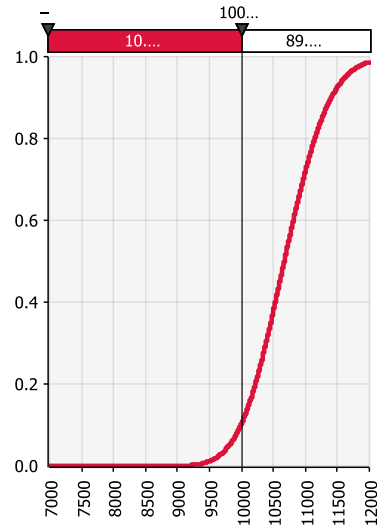
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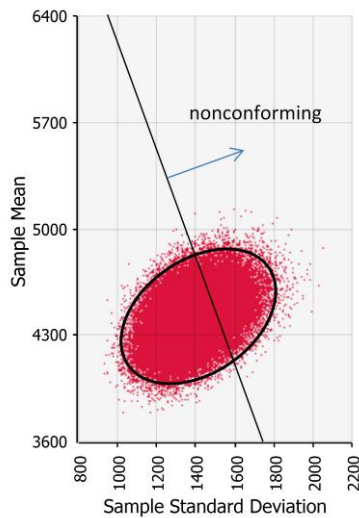
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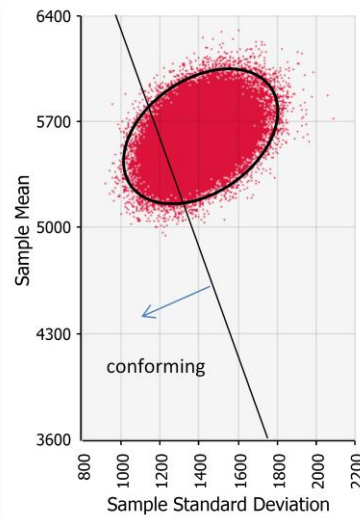
(a)



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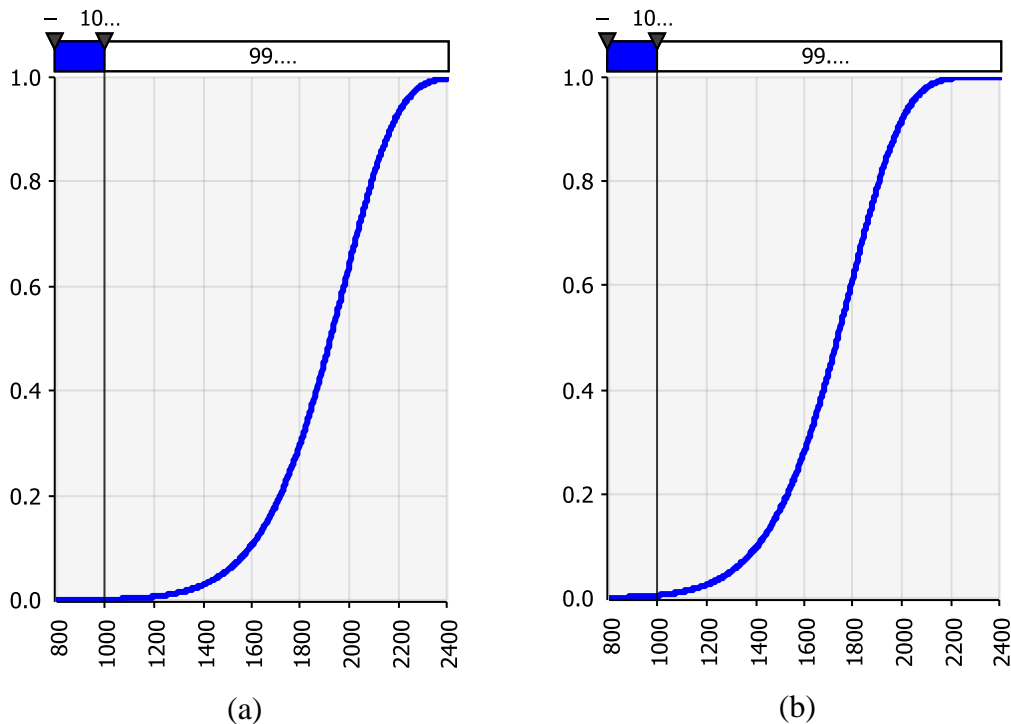
Figure 7.7-2. Gamma Upper-Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.

### 7.8 Weibull Plan (Lower Limit)


Consider a lower limit of  $x_{\min} = 1000$  for a random variable  $X$  distributed Weibull with unknown shift parameter  $\delta$  and estimated shape and scale parameters,  $\hat{\nu} = 10$  and  $\hat{\lambda} = 1995.16$ . For the test

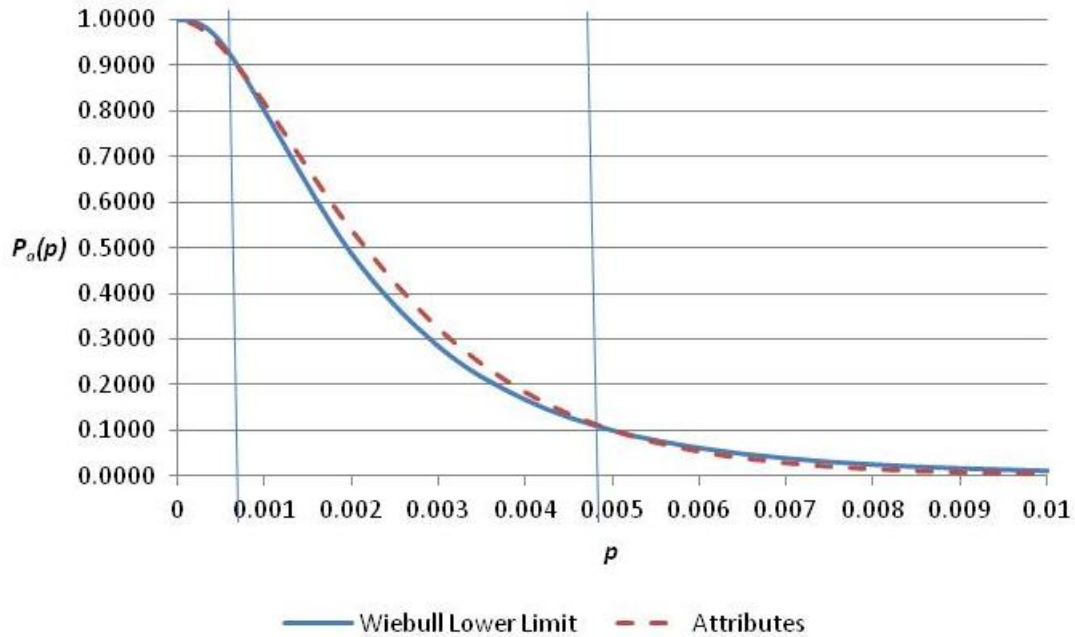
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OC, the associated the null and alternative means are  $\mu_0 = 1898.10$  and  $\mu_1 = 1712.00$ , respectively, as shown in Figure 7.8-1. The variables plan from the Weibull calculator is  $(n, k) = (91, 3.62929)$ . Figure 7.8-2 compares the OC curves for the normal and attributes plans. These curves again are similar.



**Figure 7.8-1. Weibull Lower-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Lower Limit  $x_{max} = 10,000$  Under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 1898.10$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 1712.00$ )**


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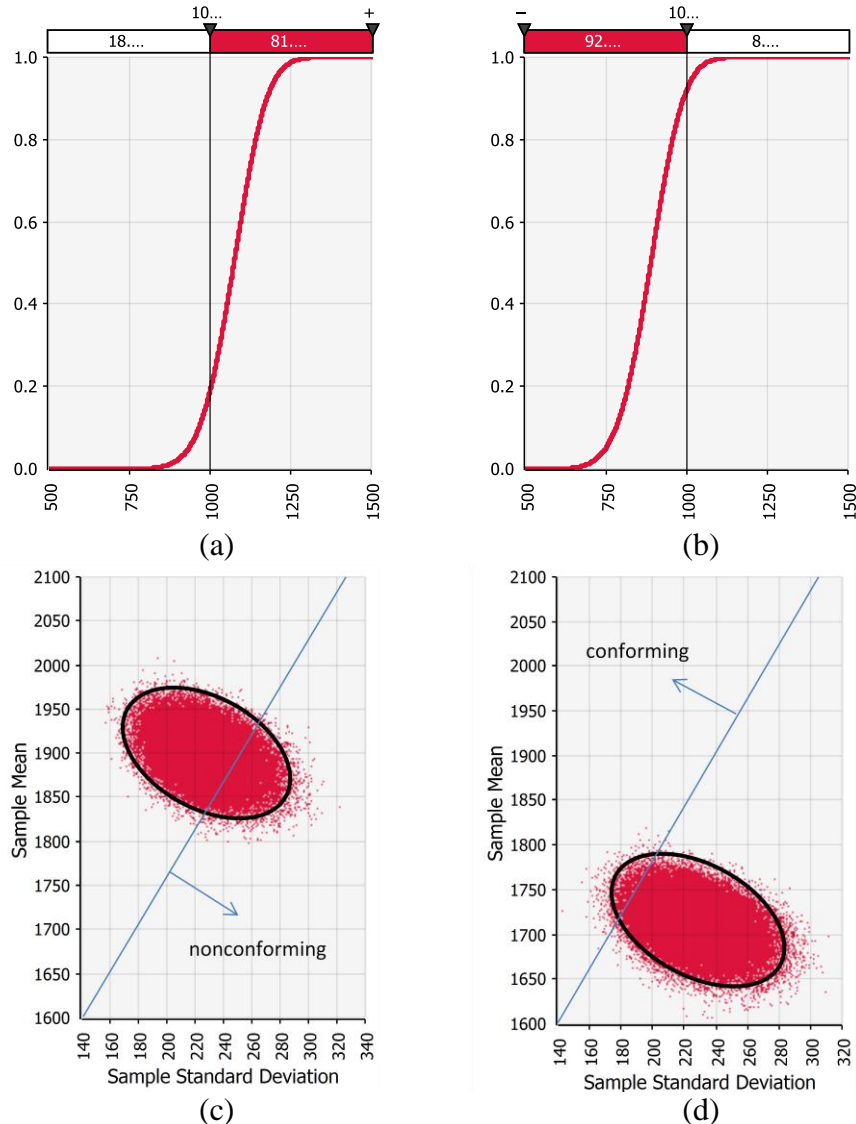


**Figure 7.8-2. Comparison of the Weibull Lower-limit Normal Plan and Attributes Sampling OC Curves for the Test Case**

Simulations were run to generate the distribution of the test statistic  $A_0 = \hat{\mu} - k\hat{\sigma}$  for the Weibull test. Results are shown in Figure 7.8-3. The estimated risks of  $\hat{\alpha} = 0.1889$  and  $\hat{\beta} = 0.0794$  were modestly more conservative than those predicted by the OC curve and there is no practical difference between the estimated risks and those predicted by the OC curve. The Weibull plan outperforms the attributes plan, by providing equivalent or superior risk protection at a reduced sampling cost. The number of simulations indicated by the Binomial  $n = 777$ , whereas the variables plans require  $n = 91$ , which is 11.7 percent of the computational effort of the attributes plan.

Note that for the test OC and  $\hat{\nu} < 2.157$ , the variables plan requires samples larger than the attributes plan and consequently would never be used. For example,  $\hat{\nu} = 2$  and all else being equal, the sampling plan from the Weibull calculator is  $(n, k) = (1071, 1.81126)$ . This contradicts the accepted wisdom that ASV will always be more efficient than ASA.


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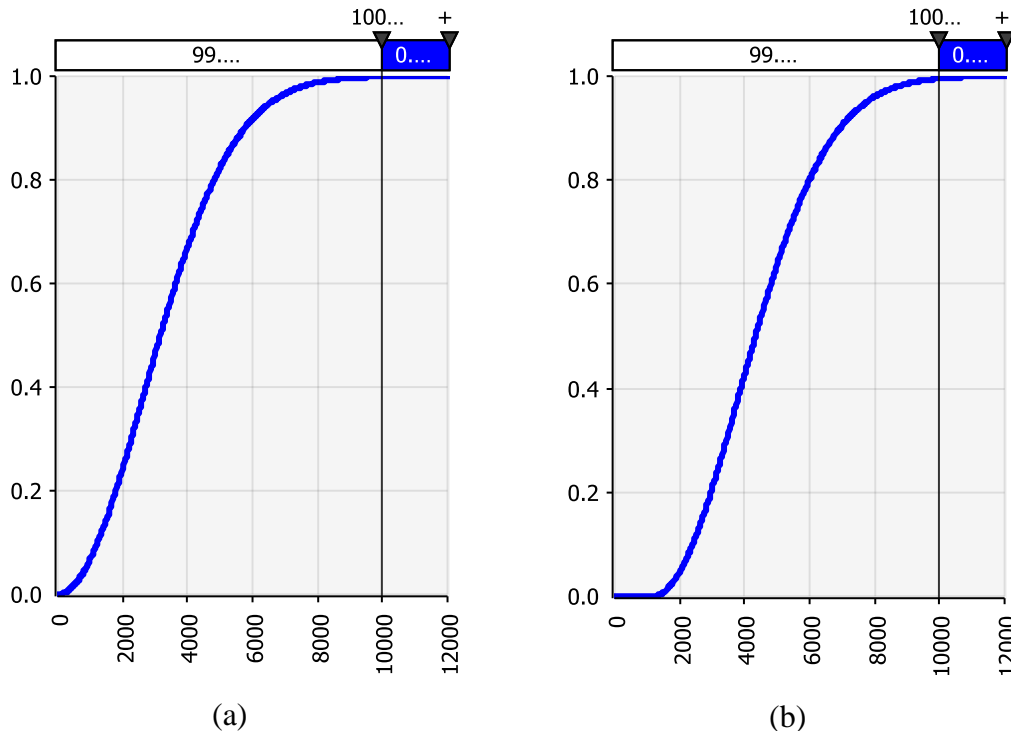
**Figure 7.8-3. Weibull Lower-Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

## 7.9 Weibull Plan (Upper Limit)

Consider an upper limit of  $x_{\max} = 10,000$  for a random variable  $X$  distributed gamma with unknown shift parameter  $\delta$  and estimated shape and scale parameters,  $\hat{\nu} = 10$  and  $\hat{\lambda} = 3800.00$ . For the test OC, the associated the null and alternative means are  $\mu_0 = 7841.64$  and  $\mu_1 = 8121.07$ ,

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respectively, as shown in Figure 7.9-1. The variables plan from the gamma calculator was  $(n,k) = (156,2.17779)$ .



**Figure 7.9-1. Weibull Upper-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Upper Limit  $x_{max} = 10,000$  Under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 7841.64$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 8121.07$ )**

Figure 7.9-2 compares the OC curves for the normal and attributes plans. These curves again are very similar, with the normal plan modestly more conservative in the region tested. Simulations were run to generate the distribution of the test statistic  $A_0 = \hat{\mu} - 100k$  for the Weibull test.

Results are shown in Figure 7.9-3. The estimated risks of  $\hat{\alpha} = 0.1883$  and  $\hat{\beta} = 0.0806$  were modestly more conservative than those predicted by the OC curve.

The Weibull plan outperformed the attributes plan, by providing equivalent or superior risk protection at a reduced sampling cost. The number of simulations indicated by the Binomial  $n = 777$ , whereas the variables plans require  $n = 156$ , which is 20.1 percent of the computational effort of the attributes plan. Note that for the test OC and  $\hat{\nu} < 0.4069$ , the variables plan requires samples larger than the attributes plan and consequently would never be used. For example,  $\hat{\nu} = 0.200$  sampling plan from the gamma calculator is  $(n,k) = (4368,9.43402)$ —more than five times larger than the attributes plan.



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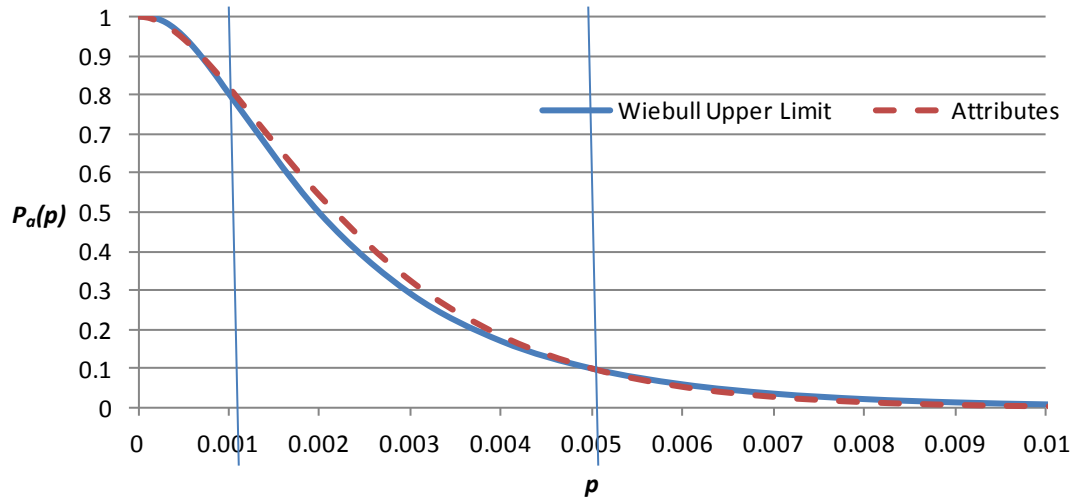
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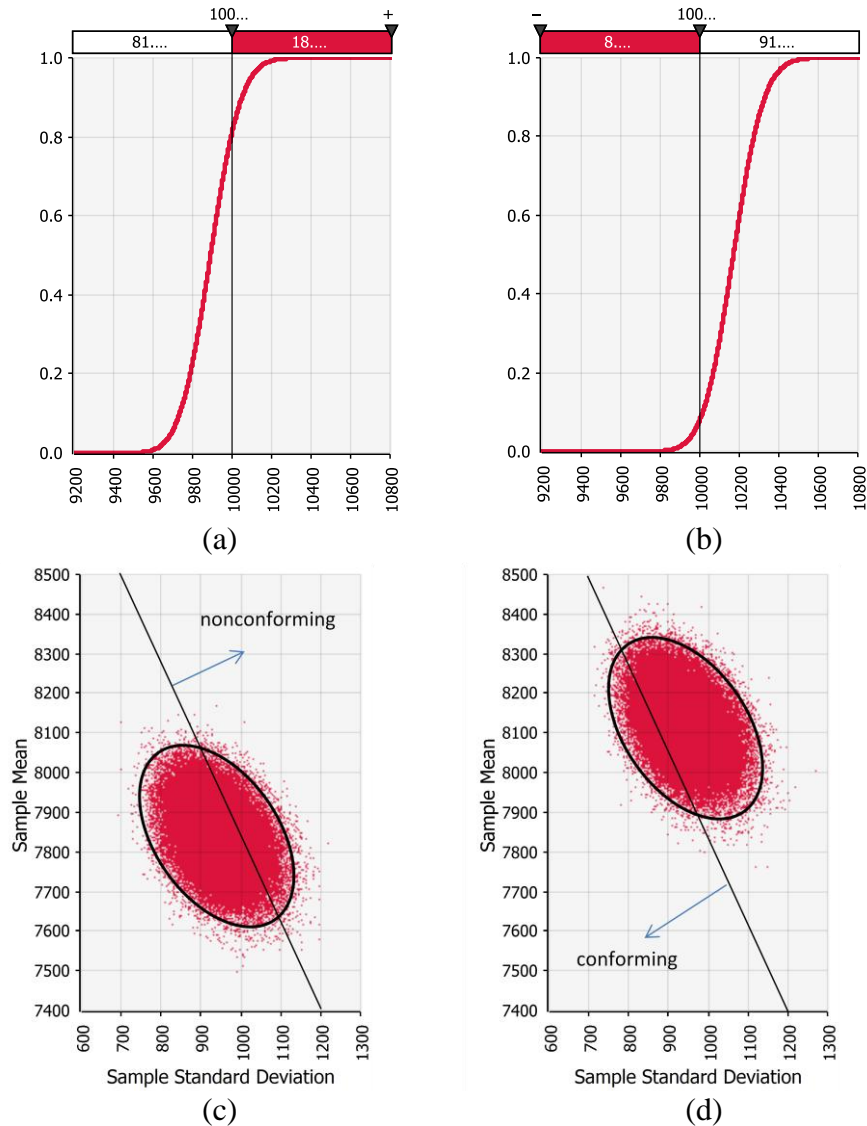
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*Figure 7.9-2. Comparison of the Weibull Upper-Limit Normal Plan and Attributes Sampling OC Curves for the Test Case*


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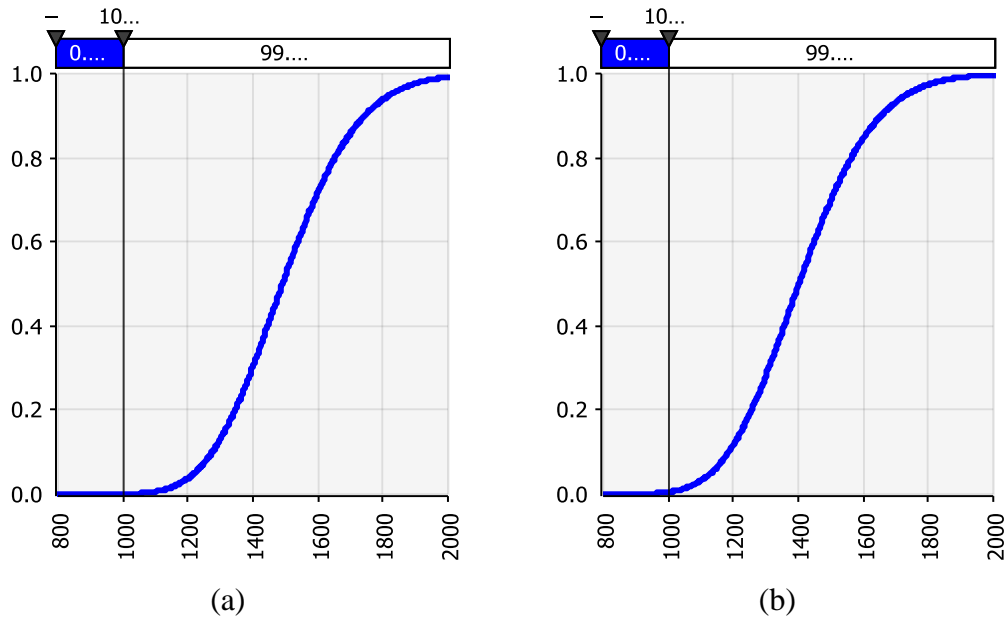
**Figure 7.9-3. Weibull Upper-Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

## 7.10 Inverse Gaussian

Consider a lower limit of  $x_{\max} = 1000$  for a random variable  $X$  distributed Poisson with unknown shift parameter  $\delta$  and estimated mean  $\hat{\mu} = 1502$  and shape parameter  $\hat{\lambda} = 100,000$ . For the test

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OC, the associated null and alternative means were  $\mu_0 = 1502.00$  and  $\mu_1 = 1413.00$ , respectively, as shown in Figure 7.10-1. The variables plan from the IG calculator was  $(n,k) = (18,2.88632)$ .



**Figure 7.10-1. IG Lower-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Upper Limit  $x_{min} = 1000$  Under (a) the Null Hypothesis  $p_0 = 0.001$  ( $\mu_0 = 1502.00$ ) and (b) Alternate Hypotheses and  $p_1 = 0.005$  ( $\mu_1 = 1413.00$ )**

Figure 7.10-2 compares the OC curves for the normal and attributes plans. These curves were similar, with the normal plan modestly more conservative in the region tested. Note, however, that the theoretical OC curve for the published plan did not match the empirical data, as shown below.

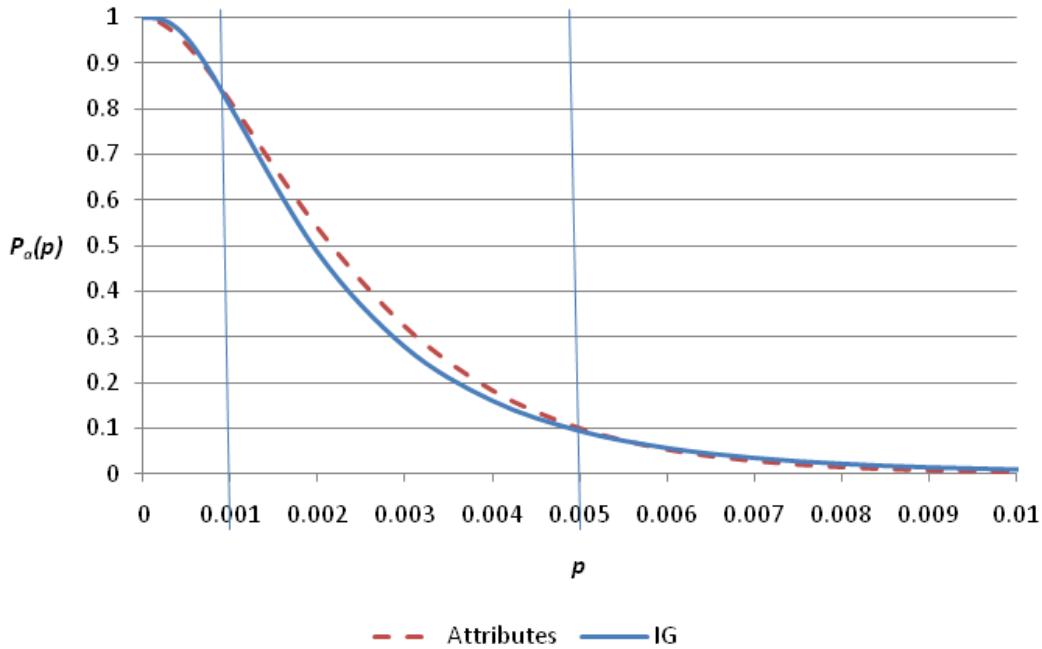




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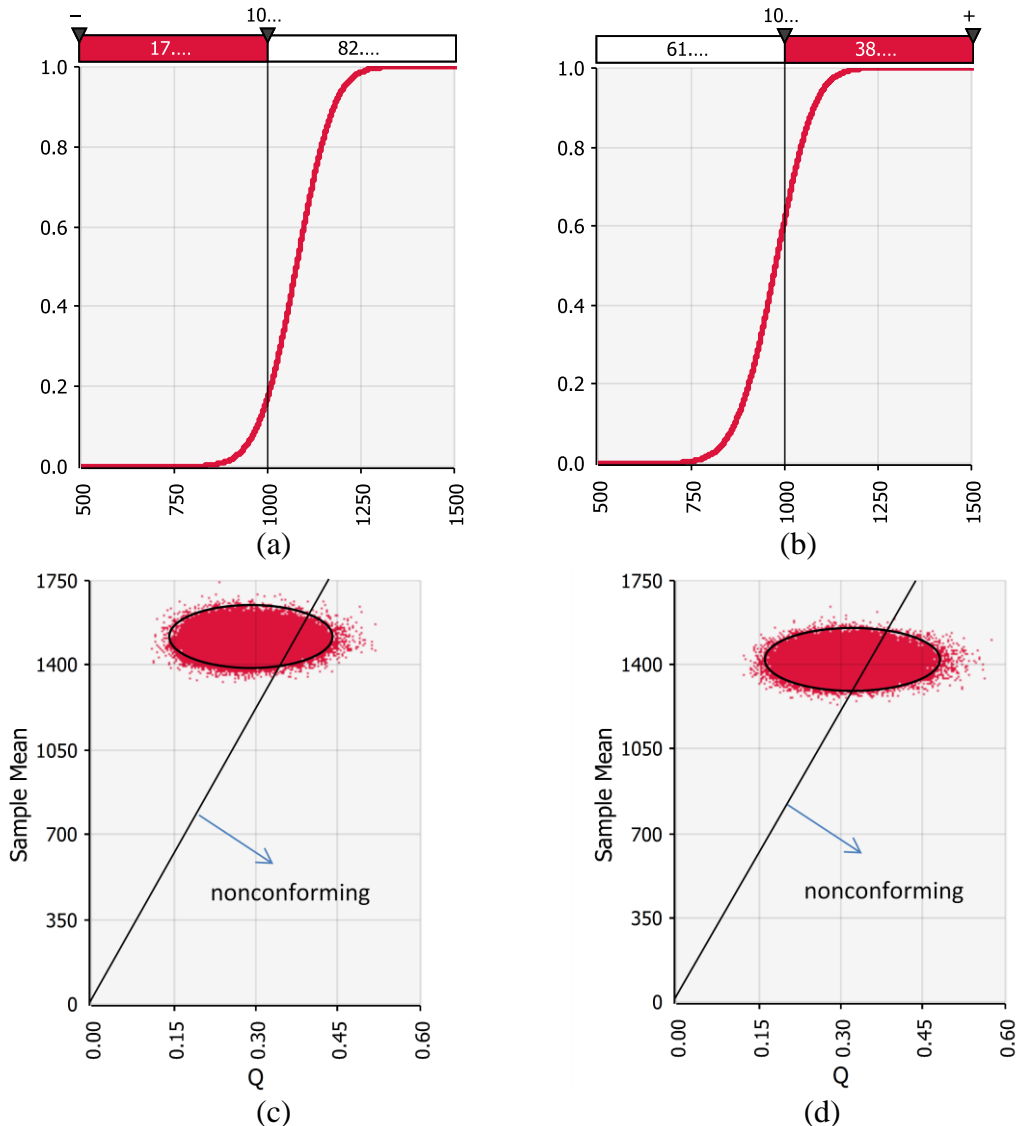
**Figure 7.10-2. Comparison of the Weibull Upper-Limit Normal Plan and Attributes Sampling OC Curves for the Test Case**

The test statistic for the IG plan (Aminzadeh, 1996) is:

$$A = \hat{\mu} \left( 1 + k \sqrt{x_{\min} / \hat{\lambda}} \right) = \hat{\mu} \left( 1 + Q(k, x_{\min}, \hat{\lambda}) \right) \geq x_{\min}.$$


Simulations were run to generate the distribution of the statistic and the results are shown in Figure 7.10-3. The estimated producer’s risk of  $\hat{\alpha} = 0.173$  was modestly conservative. However, the estimated consumer’s risk of  $\hat{\beta} = 0.382$  was unacceptably large—almost four times that specified—and far from that predicted for the theoretical OC. This confirms the error in derivation reported by White and Johnson (2011). The scatter diagrams plot the value of the sample mean  $\hat{\mu}$  against the parameter group  $Q$  for all 100,000 simulation trials. It was apparent that the 99-percent confidence bounds on the simulation data are too large and that the  $(n, k) = (18, 2.88632)$  sampling plan is too small.

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**Figure 7.10-3. IG Lower-Limit Simulation Results: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**

As a check, second simulation experiment was run using the same IG distribution, but this time following the normal sampling plan  $(n,k) = (88,2.88632)$  and corresponding normal test statistic. Results are shown in Figure 7.10-4. The larger sample size has the desired effect of compacting the confidence bounds. However, the estimated producer's risk of  $\hat{\alpha} = 0.743$  was exceedingly

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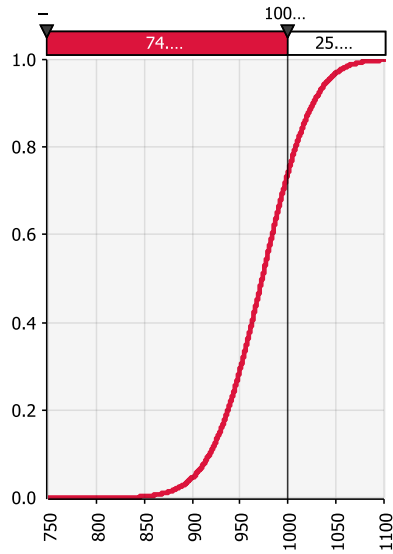
large—almost four times that specified— while the estimated consumer’s risk of  $\hat{\beta} = 0.020$  was highly conservative. The reason for this is clear from Figure 7.10-5, which compares the IG and normal density functions with the same mean and standard deviation. Even though the skew in the IG distribution is modest, the lower tail of the normal distribution is considerably fatter than that of the IG. Therefore, the normal plan is conservative with respect to consumer’s risk for IG data unless these data are suitably transformed.



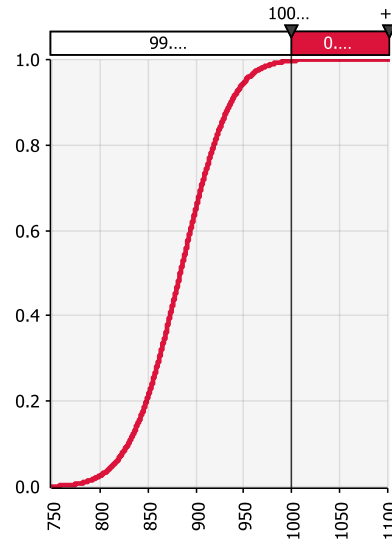
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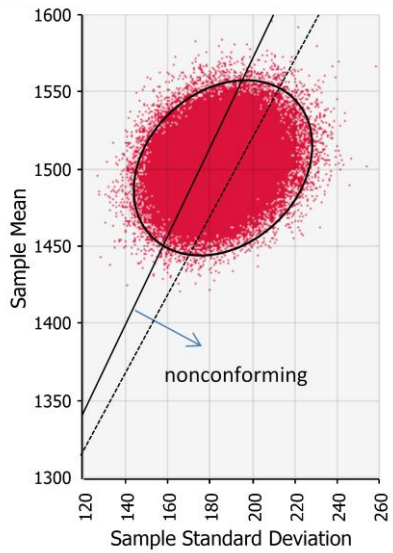
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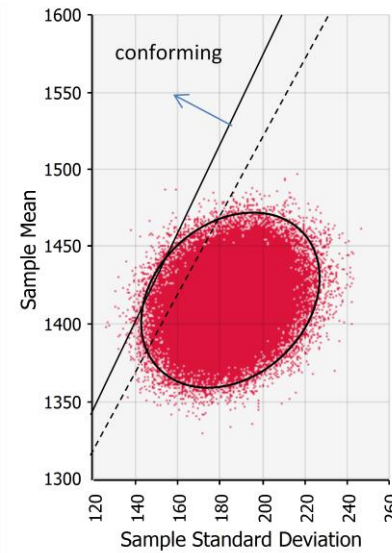
(a)



(b)

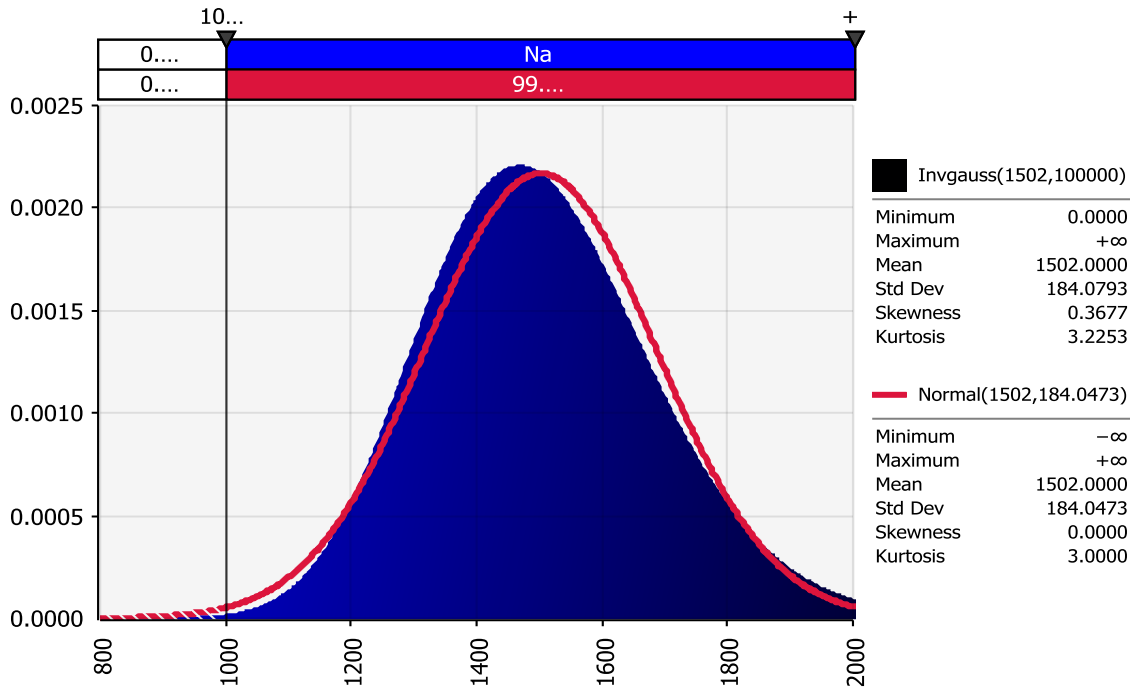


(c)



(d)

**Figure 7.10-4. IG Lower-Limit Simulation Results Applying a Normal Sampling Plan: Sampling Distributions for the Allowable Limit A Under the (a) Null and (b) Alternate Hypotheses; Scatterplots of the Sample Statistics Under the (c) Null and (d) Alternate Hypotheses. Ovals are 99-percent Confidence Bounds on the Scatter.**



**Figure 7.10-5. Comparison of the IG and Normal Distributions with the Same Mean and Standard Deviation**

Figure 7.10-3 shows the critical value of the test statistic for the normal plan,  $k = 2.88632$ , as a solid line. The effect of reducing this value to  $k = 2.60000$  as a dashed line is also shown. The  $(n, k) = (88, 2.60000)$ , while approximate, was much closer to the desired plan for the test OC than either the normal or published IG plans.

### 7.11 Poisson (Upper Limit)

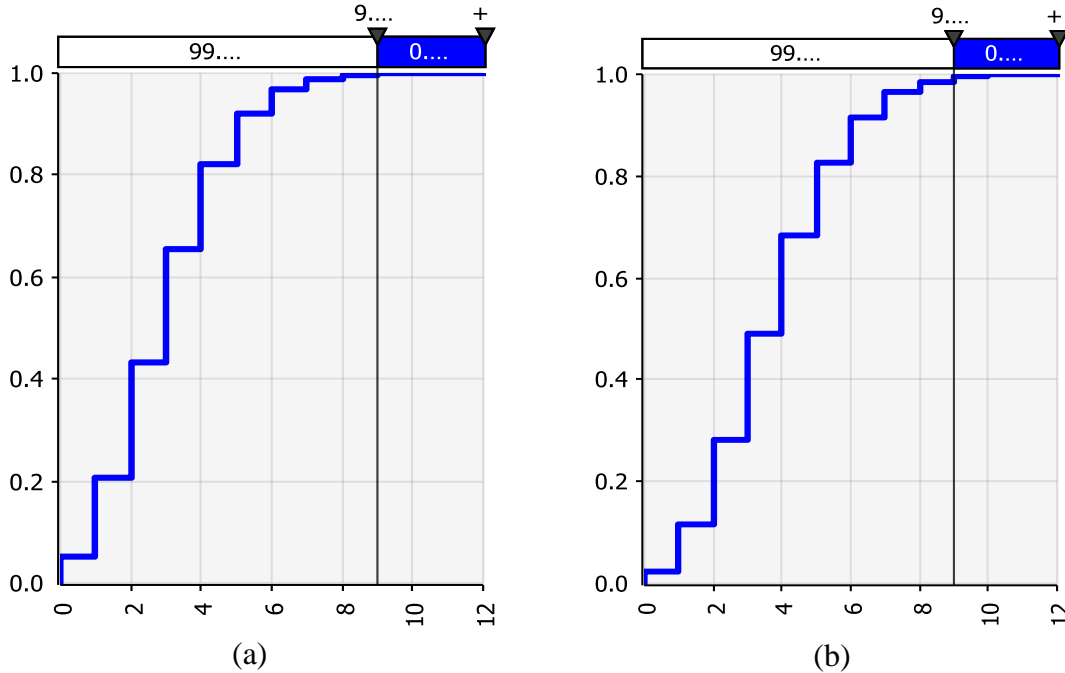
Consider an upper limit of  $x_{\max} = 10$  for a random variable  $X$  distributed Poisson with unknown mean  $\mu$ . For the test OC, the associated null and alternative means were  $\mu_0 = 2.96051$  and  $\mu_1 = 3.79612$ , respectively, as shown in Figure 7.11-1. The variables plan from the gamma calculator was  $(n, c) = (27, 88)$ .



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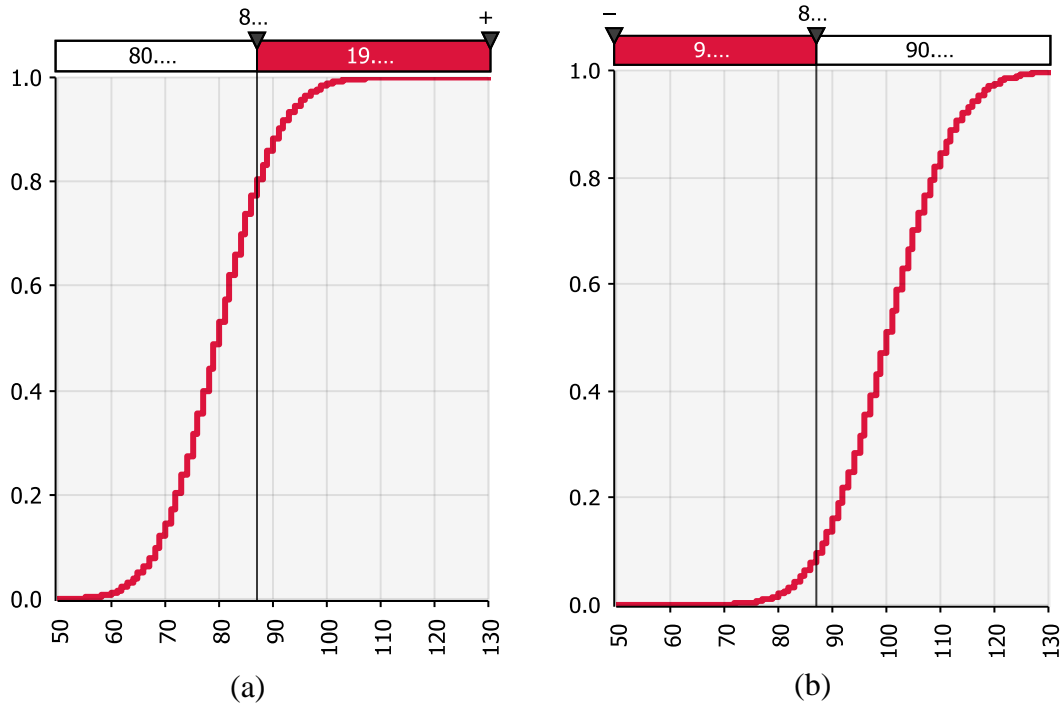
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**Figure 7.11-1. Poisson Upper-Limit Simulation Inputs: Distribution Functions and Critical Values for  $X$  with Upper Limit  $x_{max} = 10$  Under (a) the Null Hypothesis  $p_0 = 0.001$  and (b) Alternate Hypotheses and  $p_1 = 0.005$**

Simulations were run to generate the distribution of the test statistic  $Y = \sum_{j=0}^n X_j$ . Results are shown in Figure 7.11-2. There was no practical difference between the estimated risks of  $\hat{\alpha} = 0.192$  and  $\hat{\beta} = 0.097$  and those specified by the OC.

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**Figure 7.11-2. Poisson Simulation Results: Distribution Functions and Critical Values for  $X$  Distributed Poisson  $(27, p)$  with Limit  $x_{max} = 10$  and Under (a) Null Hypothesis  $p_0 = 0.001$  and (b) Alternate Hypotheses and  $p_1 = 0.005$**

The Poisson plan dramatically outperformed the attributes plan, by providing equivalent or superior risk protection at a reduced sampling cost. The number of simulations indicated by the binomial was  $n = 27$ , whereas the variables plans require  $n = 777$ , which is 0.347 percent of the computational effort of the attributes plan.

## 8.0 Summary and Conclusions

This paper presents the results of empirical tests designed to provide an independent assessment of the validity and accuracy of six published ASV procedures. These results are summarized in Tables 8.0-1 through 8.0-3. Overall, the plans were shown to provide adequate or superior protection against producer's and consumer's risks for samples substantially smaller than those required for the corresponding ASA plans. The exception is the IG plan, which was previously shown to be in error.

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**Table 8.0-1. Summary of Test Results for Continuous Variables Given Lower Limit  $x_{min} = 1000$  and Nominal OC  $(p_0, \alpha) = (0.001, 0.2)$  and  $(p_1, \beta) = (0.001, 0.1)$**

Variable	$n$	$k$	$\hat{\alpha}$	$\hat{\beta}$	$n_v/n_a$
Exponential( $\mu$ )	2	$2.43 \times 10^{-3}$	0.200	0.082	0.003
Normal( $\mu, \sigma = 100$ )	18	2.886	0.191	0.097	0.023
Normal( $\mu, \hat{\sigma}$ )	88	2.886	0.191	0.097	0.099
Gamma( $\hat{\nu} = 10, \hat{\lambda} = 338, \delta$ )	206	2.131	0.193	0.096	0.224
Weibull( $\hat{\nu} = 10, \hat{\lambda} = 1995, \delta$ )	91	3.623	0.189	0.079	0.117
IG( $\hat{\mu} = 1502, \hat{\lambda} = 100,000, \delta$ )	18	2.886	0.173	0.382	unusable

**Table 8.0-2. Summary of Results for Continuous Variables Given Upper Limit  $x_{max} = 10,000$  and Nominal OC  $(p_0, \alpha) = (0.001, 0.2)$  and  $(p_1, \beta) = (0.001, 0.1)$**


Variable	$n$	$k$	$\hat{\alpha}$	$\hat{\beta}$	$n_v/n_a$
Exponential( $\mu$ )	66	6.26922	0.200	0.082	0.085
Gamma( $\hat{\nu} = 10, \hat{\lambda} = 441, \delta$ )	77	3.667	0.189	0.104	0.099
Weibull( $\hat{\nu} = 10, \hat{\lambda} = 3800, \delta$ )	156	3.623	0.188	0.081	0.201

**Table 8.0-3. Summary of Results for Discrete Variables**

Variable	$n$	$c$	$\hat{\alpha}$	$\mathbf{b}$	$n_v/n_a$
Binomial ( $n, p$ )	777	1	0.188	0.100	1
Poisson ( $n, p$ )	21	88	0.191	0.097	0.035

While this overall conclusion supports the assertion in the literature that ASV plans require smaller samples than ASA plans, it was also discovered that this assertion does not hold absolute. In particular, for gamma and Weibull variables with small shape parameters, the ASV



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plans were in fact larger than the corresponding ASV plans. It is believed that this discovery is original and needs to be assimilated into the literature on acceptance sampling.

## 9.0 Acronyms List

ASA	Acceptance Sampling by Attributes
ASV	Acceptance Sampling by Variables
ATK	Alliant Techsystems, Inc.
CxP	Constellation Program
IG	Inverse Gaussian
LaRC	Langley Research Center
MSFC	Marshall Space Flight Center
MTSO	Management Technical Support Office
NESC	NASA Engineering and Safety Center
NRB	NESC Review Board
OC	Operating Characteristic

## 10.0 References

1. Aminzadeh, M. S. (1996), "Inverse-Gaussian Acceptance Sampling Plans by Variables," *Communications in Statistics--Theory and Methods* 25(5)923-935.
2. White, K. P., Jr., Johnson, K. L., and Creasey, R. R., Jr. (2009). "Attribute acceptance sampling as a tool for verifying requirements using Monte Carlo simulation, *Quality Engineering*, 21(2)203-214).
3. White, K. P., Jr., and Johnson, K. L. (2011), "Variables Acceptance Sampling Calculators: Derivations and Verification of Plans," NASA Engineering Statistics Team, working paper available from the authors.

## 11.0 Appendices

- Appendix A. Protocol for Selecting a Sampling Plan Calculator
- Appendix B. Note on Hypothesizing an Output Distribution

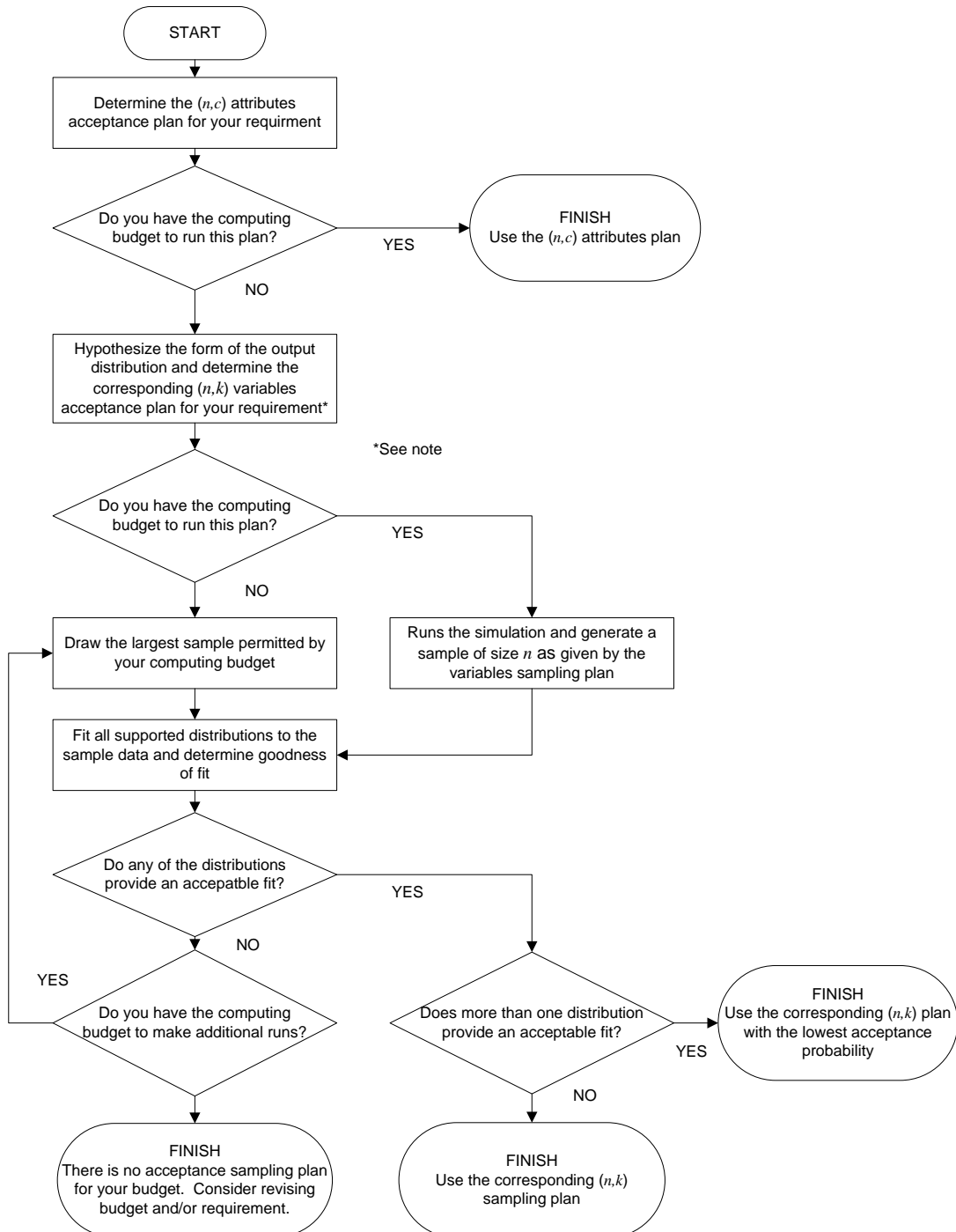



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### Appendix A. Protocol for Selecting a Sampling Plan Calculator



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## Appendix B. Note on Hypothesizing an Output Distribution

The sampling plan calculators can be used to determine the number of simulation replications or runs needed, i.e., the number of Monte Carlo trials yielding the sample size  $n$  specified by the plan for a given requirement. However, the choice of which calculator to use depends on knowing the form of the distribution of the verification output. The size of sampling plans for different distributions can vary by orders of magnitude depending on this distribution.

Pass/fail output always have a *binomial* distribution and therefore the  $n$  can be determined using the binomial (attributes) calculator before any simulation runs are made. For variables output, the choice may not be as obvious.


If historical or test data exist for variables outputs that are thought to be similar in nature to the expected verification output, a verifier may be able to make use of such prior knowledge to guess what form the distribution might assume. S/he needn't be concerned about the values of the distribution parameters at this step, just with the general shape of the distribution. The choice of distribution (hypothesis) and parameter estimates can be checked once data are generated for the verification analysis.

There also are theoretical considerations that might suggest one or more distributions, or at least rule out some of the distributions available. For example, the *Normal* distribution often is a reasonable model for errors of various types, where the magnitude of the error is equally likely to be positive or negative and smaller errors are more common than larger errors. By virtue of the central limit theorem, the Normal distribution also is often a reasonable model for the sum of a large number of other random quantities, no matter how these quantities are distributed. Because the Normal distribution has a negative tail, however, it may not be a reasonable model for a time, or a count, or some other output that must be strictly positive.

Because of the “memoryless property”, the *exponential* distribution is a suitable model for the time between the occurrence of events, if these occurrences have a constant rate and the numbers of occurrence in disjoint time periods are independent (i.e., the time of occurrence of one event does not influence the time of occurrence of subsequent events). Events might be the failure of a component or system, or the arrival of a customer needing service, or the placing of orders against an inventory. By the same property, the number of events occurring during any fixed period of time under these same assumptions is an integer count distributed *Poisson*.

The *gamma* and *Weibull* distributions frequently are used to model the time to some event, where the event might be a failure or the completion of some task, such as a service or repair. The Weibull is quite flexible in application, can have positive or negative skew, and frequently is used as the assumed distribution when modeling in the absence of data.

The *lognormal* distribution (the normal distribution applied to the natural logarithm of the data) has applications similar to the gamma and Weibull. It takes on shapes similar to the gamma and

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Weibull with positive skew, but can have a large “spike” at the mode close to zero. There are other transformations to the Normal that may also be appropriate.

Before constructing a hypothesis and collecting any data, it is a good idea to consult a statistician to help in this activity. It is also important to be aware that the data, once generated, may result in a finding that the hypothesized distribution is not the conservative choice. The analyst, managers and other verifiers should have a contingency plan covering the possibility that the simulation trials already run are not sufficient for verification at the confidence level required.

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<b>14. ABSTRACT</b> The NASA Engineering and Safety Center was requested to improve on the Best Practices document produced for the NESC assessment, Verification of Probabilistic Requirements for the Constellation Program, by giving a recommended procedure for using acceptance sampling by variables techniques as an alternative to the potentially resource-intensive acceptance sampling by attributes method given in the document. In this paper, the results of empirical tests intended to assess the accuracy of acceptance sampling plan calculators implemented for six variable distributions are presented.					
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