Characterization of Deficiencies in the Frequency Domain Forced Response Analysis Technique for Supersonic Turbine Bladed Disks

Andrew M. Brown, Preston Schmauch

Turbine blades in rocket and jet engine turbomachinery experience enormous harmonic loading conditions. These loads result from the integer number of upstream and downstream stator vanes as well as the other turbine stages. Assessing the blade structural integrity is a complex task requiring an initial characterization of whether resonance is possible and then performing a forced response analysis if that condition is met. The standard technique for forced response analysis in rocket engines is to decompose a CFD –generated flow field into its harmonic components, and to then perform a frequency response analysis at the problematic natural frequencies. Recent CFD analysis and water-flow testing at NASA/MSFC, though, indicates that this technique may miss substantial harmonic and non-harmonic excitation sources that become present in complex flows. A substantial effort has been made to account for this denser spatial Fourier content in frequency response analysis (described in another paper by the author), but the question still remains whether the frequency response analysis itself is capable of capturing the excitation content sufficiently. Two studies comparing frequency response analysis with transient response analysis, therefore, of bladed-disks undergoing this complex flow environment have been performed. The first is of a bladed disk with each blade modeled by simple beam elements. Six loading cases were generated by varying a baseline harmonic excitation in different ways based upon cold-flow testing from Heritage Fuel Air Turbine Test. It was hypothesized that the randomness and other variation from the standard harmonic excitation would reduce the blade structural response, but the results showed little reduction. The second study was of a realistic model of a bladed-disk excited by the same CFD used in the J2X engine program. It was hypothesized that enforcing periodicity in the CFD (inherent in the frequency response technique) would overestimate the response. The results instead showed that the transient analysis results were up to 10% higher for "clean" nodal diameter excitations and six times larger for "messy" excitations, where substantial Fourier content around the main harmonic exists. Because the bulk of resonance problems are due to the "clean" excitations, a 10% underprediction is not necessarily a problem, especially since the average response in the transient is similar to the frequency response result, and so in a realistic finite life calculation, the life would be same. However, in the rare cases when the "messy" excitations harmonics are identified as the source of potential resonance concerns, this research does indicate that frequency response analysis is inadequate for accurate characterization of blade structural capability.

ACHARACTERIZATION OF DEFICIENCIES IN THE FREQUENCY DOMAIN FORCED RESPONSE ANALYSIS TECHNIQUE FOR SUPERSONIC TURBINE BLADED DISKS

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ABSTRACT

Turbine blades in rocket and jet engine turbomachinery experience enormous harmonic loading conditions. These loads result from the integer number of upstream and downstream stator vanes as well as the other turbine stages. The standard technique for forced response analysis to assess structural integrity is to decompose a CFD –generated flow field into its harmonic components, and to then perform a frequency response analysis at the problematic natural frequencies. Recent CFD analysis and water-flow testing at NASA/MSFC, though, indicates that this technique may miss substantial harmonic and non-harmonic excitation sources that become present in complex flows. These complications introduce the question of whether frequency domain analysis is capable of capturing the excitation content sufficiently. Two studies comparing frequency response analysis with transient response analysis, therefore, have been performed. The first is of a bladed disk with each blade modeled by simple beam elements. It was hypothesized that the randomness and other variation from the standard harmonic excitation would reduce the blade structural response, but the results showed little reduction. The second study was of a realistic model of a bladed-disk excited by the same CFD used in the J2X engine program. The results showed that the transient analysis results were up to 10% higher for "clean" nodal diameter excitations and six times larger for "messy" excitations, where substantial Fourier content around the main harmonic exists.

NOMENCLATURE

CFD	Computational Fluid Dynamics
ND	Nodal Diameter
N	Running Speed

INTRODUCTION

Turbine blades in rocket and jet engine turbomachinery experience enormous harmonic loading conditions. These loads result from the immediately upstream nozzles or vanes and downstream stator vanes as well as from non-adjacent stages. Assessing the blade structural integrity is a complex task requiring an initial characterization of whether resonance is possible and then performing a forced response analysis if that condition is met. The standard technique for forced response analysis in rocket engine turbines is to decompose a flow field that was generated using computational fluid dynamics (CFD) into its harmonic components, and to then perform a frequency response analysis at the problematic natural frequencies using cyclically symmetric structural dynamic models. Recent CFD analysis and water-flow testing at NASA/MSFC, though, indicates that this technique may miss substantial harmonic and non-harmonic excitation sources that become present in complex supersonic flows. This complex content can only be captured by a CFD flow field encompassing at least an entire revolution of the rotor. A substantial development effort to create a series of software programs to enable application of the 360° forcing function in a frequency response analysis on cyclic symmetric

models has been completed,[1] but the question still remains whether the frequency response analysis itself is capable of capturing the excitation content sufficiently.

Therefore, two studies comparing frequency response analysis with transient response analysis of bladed-disks undergoing this complex flow environment have been performed. In this context, a transient analysis calculates the response to the complete excitation function in the temporal domain. These models are intended to be representative of the response of realistic bladed disks, so the dimensions are roughly equivalent to the J2X rocket engine 1st stage fuel pump turbine. A hypothesis going into the analysis was that perhaps the frequency response was enforcing a temporal periodicity that did not really exist, and so therefore it would overestimate the response. As high dynamic response is a considerable source of stress in the J2X, examining this concept could potentially be beneficial for the program.

The first set of analyses examine a bladed disk with each blade modeled by simple beam elements and the disk modeled with plates (using the finite element code MSC/NASTRAN) in order to focus on the disk-dominated modes, and to enable analyses that can be quickly performed and adjusted. This model is a first step in identifying response differences between transient and frequency forced response analysis techniques. The second study assesses this difference using a much more realistic solid model of a bladed-disk in order to evaluate the effect of the spatial variation in loading on blade-dominated modes.

The literature in this subject is essentially limited to analyses using either the frequency domain or transient techniques, and does not compare the two. Misek, et.al, describe the forced response analysis of a bladed-disk in detail, but only mention that a frequency response technique is adequate because the forcing function is substantially harmonic[2]. The frequency domain technique is also used in a number of other studies, with no mention of its accuracy compared to a transient analysis[3],[4]. This research is therefore a new contribution to practical structural dynamic analysis of gas turbines.

RESULTS AND DISCUSSION

<u>BLADE BEAM MODEL ANALYSES</u>

The blade-beam model consists of plate elements representing a 20 in diameter, 1 in thick disk, and 69 4 in X 1 in X 0.1 in blades each represented by 4 beam elements. The first harmonic of the upstream nozzle count on the fuel pump of the J2X causes the 5th nodal diameter (ND) modes to be excited, and the downstream vanes excite the 12th ND modes, as explained by Tyler and Sofrin[5] and identified using a spatial aliasing chart as shown in table 1. Only nodal diameter modes equal to or less than half the number of blades are possible in a cyclically symmetric system, so combinations exceeding that value are marked N/A in the chart.

Table 1a, b) Blade Nodal Diameter Spatial Aliasing Chart for Upstream and Downstream Excitation.

Upstream Nozzle Multiples	37	74	111	148
Blade multiples				
69	32	-5	N/A	N/A
138	N/A	N/A	27	-10
207	N/A	N/A	N/A	N/A

Downstream Stator Multiples	57	114	171	228
Blade multiples				
69	12	N/A	N/A	N/A
138	N/A	24	-33	N/A
207	N/A	N/A	N/A	-21

A modal analysis was therefore performed on the beam/plate model to identify the 5ND bladed-disk mode to be excited at resonance, which is found to be at 40,167 hz (see figure 1). To perform the frequency response analysis, a CFD analysis encompassing four complete rotor revolutions was performed for the real turbine and scaled in time such that the applicable primary excitation temporal Fourier component $F_o e^{i\Omega t}$ has a frequency equal to the mode of interest, and the complex loading (real and imaginary parts) of the component is applied in the frequency response NASTRAN solution. This analysis is very quick because it uses the modal transformation

$$\{u\} = [\Phi]\{q\} \tag{1}$$

to decompose the very large model into modal components {q}. The response can then be calculated by superimposing the modal single-frequency responses each calculated using the standard complex single degree of freedom frequency response equation

$$q(t) = \bar{H}(\Omega)\bar{U}_{static} = \left[\frac{1 - \Omega/\omega_i^2}{\left(1 - \Omega/\omega_i^2\right)^2} + i\frac{-2\,\zeta\Omega/\omega_i}{\left(1 - \Omega/\omega_i^2\right)^2} \right] \frac{F_o e^{i\Omega t}}{\lambda_i^2} \tag{2}$$

The fourier components of the loading on each blade at the modal frequencies of interest were isolated and applied to the model at not only the resonant frequency but a bandwidth about that frequency to see if potential frequency content in the pressure signal not at 74N would excite other modes. The peak values of the complex magnitude of the axial response for the highest responding nodes are shown in figure 2. Comparisons between the transient and frequency response results are only possible for a translational component of the displacement because of the requirement to make negative peak to positive peak calculations in the transient response, as discussed below.
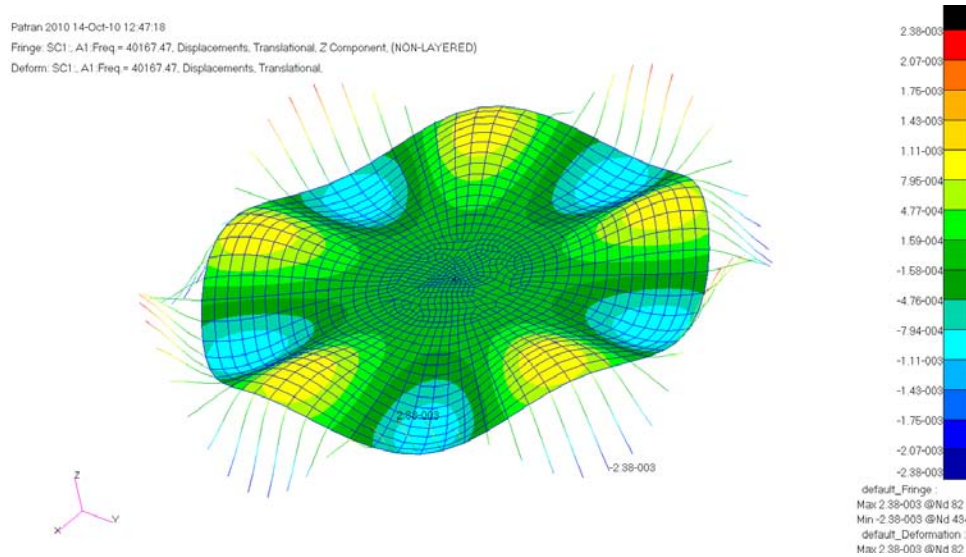

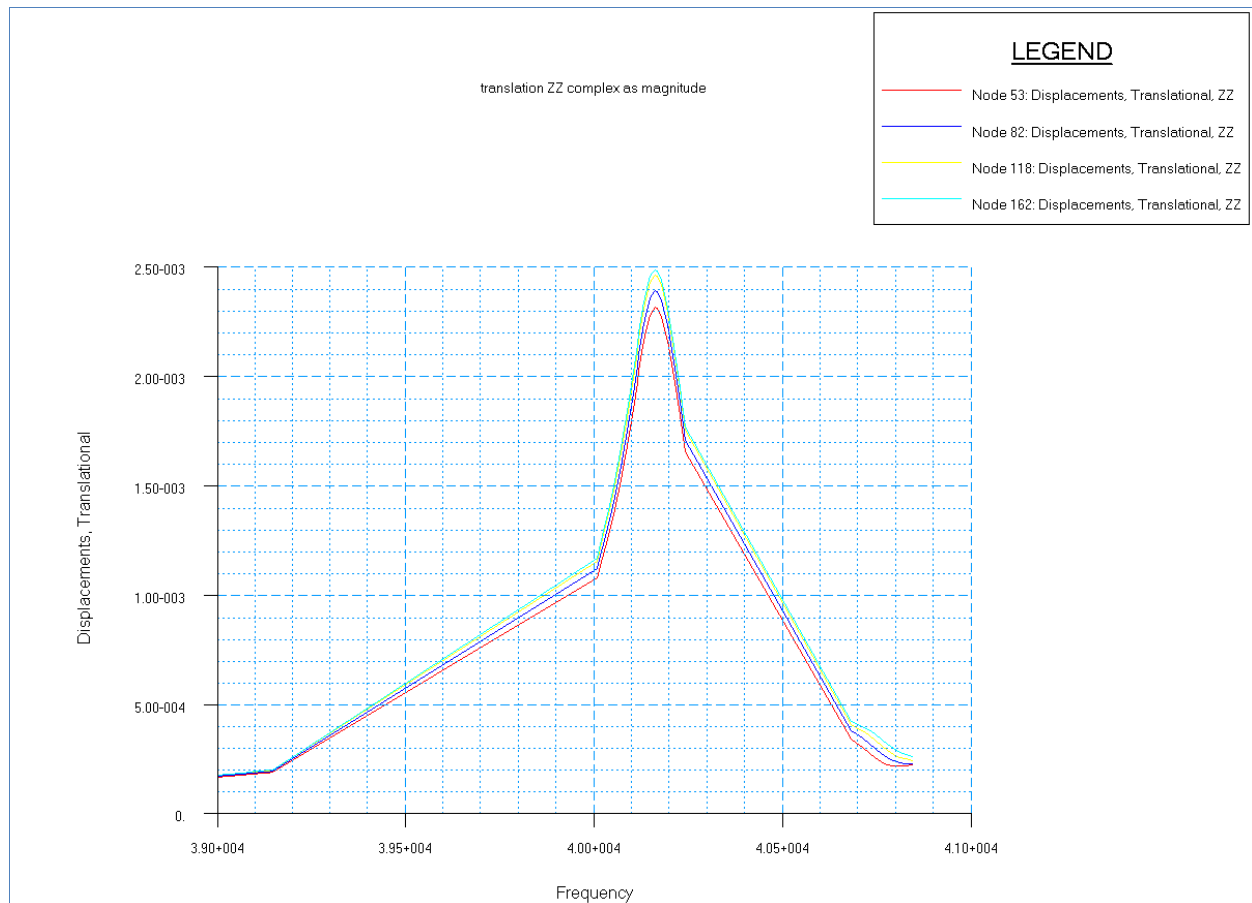Figure 1. Beam/Plate bladed disk model, 5ND mode at 40167hz

Figure 2. Peak Frequency Response of Highest Responding Nodes in Beam/Plate model

The transient analysis was then performed. As this type of analysis typically uses a time marching numerical integration scheme, it is significantly more computer-CPU intensive than frequency response analyses, and so in general is not tractable for realistic high-fidelity solid finite element models of bladed-disks. The main difficulty for this model was creating the NASTRAN format for different high-resolution time histories on each of the blades.

The process for interpreting the results is as follows: 1) plot entire time history of the blade tip node responses for a 1/5 sector of the bladed-disk (corresponding to the 5ND mode shape); 2) Identify which node has the highest response; 3) Subtract out the mean response that represents the static load component, which is included in the transient loading but not the frequency response; 4) Since the response is somewhat random in nature, calculate the statistics of the peaks of the response; 5) Apply the 2σ value as the design value for fatigue calculations (and comparison with other methods), since it has been suggested that a sinusoid of this amplitude will produce the equivalent damage as a full Rayleigh distribution of a narrow-band random signal (envelope of Rayleigh peaks shown in figure 3).

For this case, the node with the highest transient axial response was node 82, which has a mean-to-peak value of 0.00345" and a 2σ value of displacement is .0032".   The frequency response peak is

.0024", a difference of -25%. These results are opposite of what was hypothesized. A promising explanation for the difference is the contributions from the response to non-integer multiples of the frequency components of the excitation close to the fundamental excitation of 74*running speed N (called sidebands), which is all that is used in the frequency response analysis. Since four complete revolutions were processed, a discrete fourier decomposition will have a bin bandwidth of N/4; the 74N energy is therefore in the 4*74=296'th bin. These effects are investigated by taking the top three non-trivial nearby components, as shown in figure 4, bins 294, 295, and 297, corresponding to 73.5N, 73.75N, and 74.25N and performing frequency response analyses using those excitation amplitudes at the slightly off-resonant frequencies. If random phasings of the off-resonant results are assumed, the equation showing the sum of the components is

$$x(t) = 0.000361 \, Cos\left[\frac{\pi}{6} - 238\,200.6\,t\right] + 0.000369 \, Cos[236\,596.6\,t] + 0.0024 \, Sin[237\,398.66\,t] +$$

$$0.0000458 \, Sin\left[\frac{\pi}{4} + 240\,606.7\,t\right]$$

(3)

which has a maximum value of 0.0026605 over the interval studied, still 18.6% below the transient results, so this does not appear to explain the discrepancy. The resolution of this question will require future study. However, this analysis does identify at least one clear source of non-trivial error (accounting for 6.4%) when performing standard frequency response analysis.
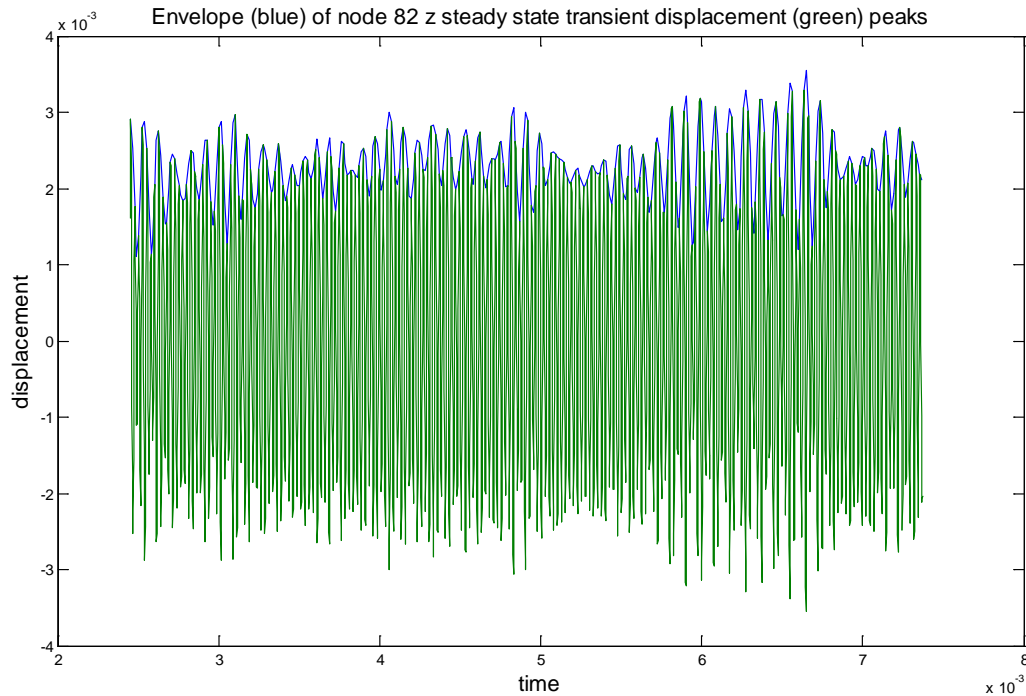

Figure 3. Peak Transient Analysis Response of Highest Responding Node in Beam/Plate Model
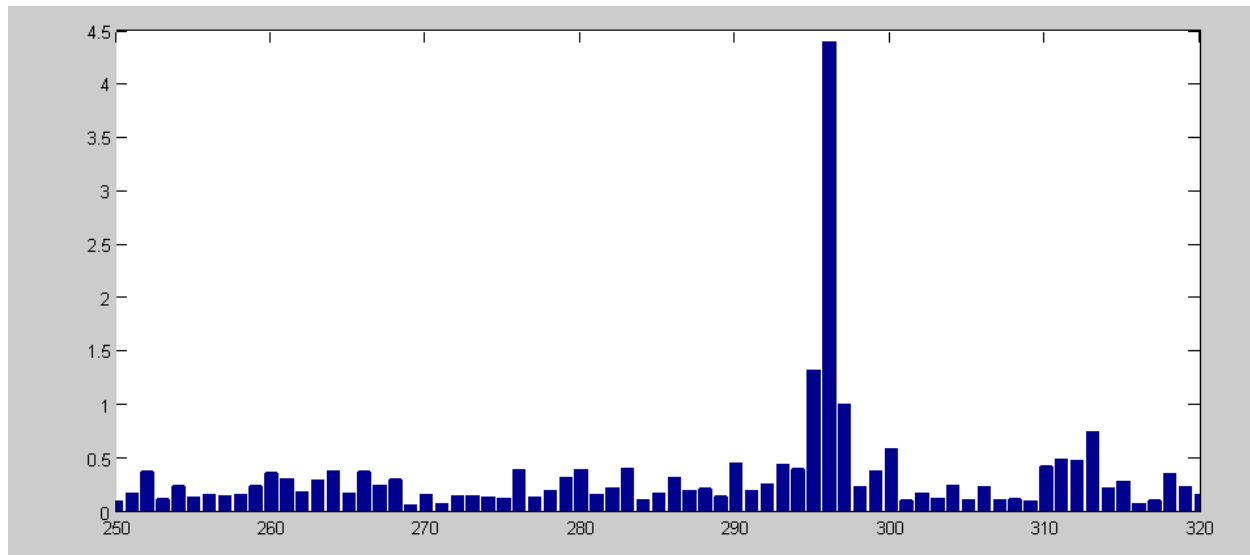
Figure 4 – Magnitude of loading from sidebands of fundamental excitation in bin 296.

REALISTIC AIRFOIL SOLID MODEL ANALYSES – 74N EXCITATION OF 5ND MODE

The second phase of the research program was motivated by the question of what the effect of slightly aperiodic excitation would be on blade-dominated modes, rather than the nodal diameter dominated modes of the beam/plate model. This required the construction of a solid element bladed-disk model with a representative airfoil. As before, a 360° model was built to be able to readily load it with the same 360° CFD loading as used in the J2X engine program (five complete revolutions). The model was quite large, as shown in figure 5, but was made manageable by using the new glued-contact formulation in MSC/NASTRAN, which enables a much simpler model of the disk than of the blades by allowing dissimilar meshes at the interface. Once again, for this case, it was hypothesized that the frequency response would overestimate the transient because the requirement that the spatial distribution of the loading on each blade matches the mode shape would be made artificially periodic. A modal analysis was first performed and the 5ND mode at 40,264 hz was identified for resonant excitation (see figure 6).
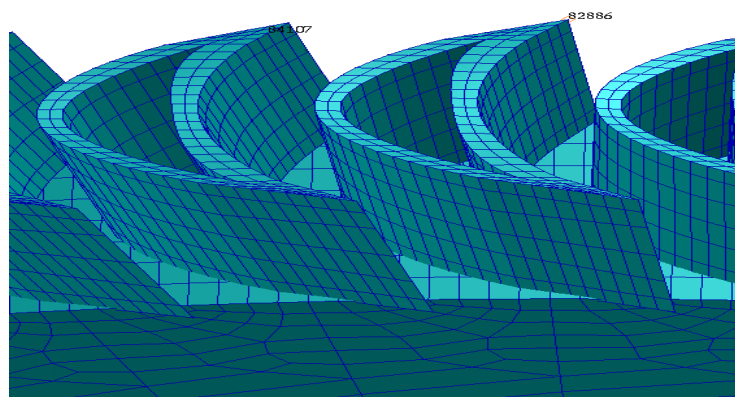

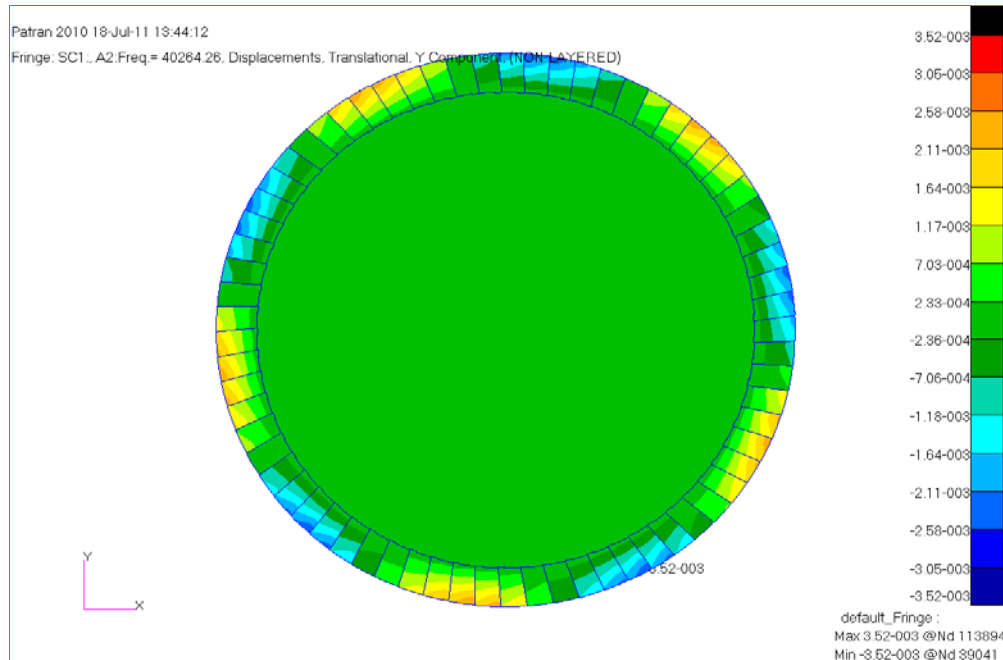Figure 5 – Solid Element Representative Bladed-Disk Model (not to scale)

Figure 6. Realistic Airfoil Solid Blade-Disk Model 5 Nodal Diameter Mode at 40,264 hz

The next step was to perform the frequency response analysis for the 5ND mode. Plots of the entire ring of outermost nodes of the blades at different frequencies within the analysis band were generated and the maximum responding node for each case identified (figure 7). Graphs of the response of those nodes in the entire band were then generated (figure 8), and the peak response of the peak responding node was tabulated.
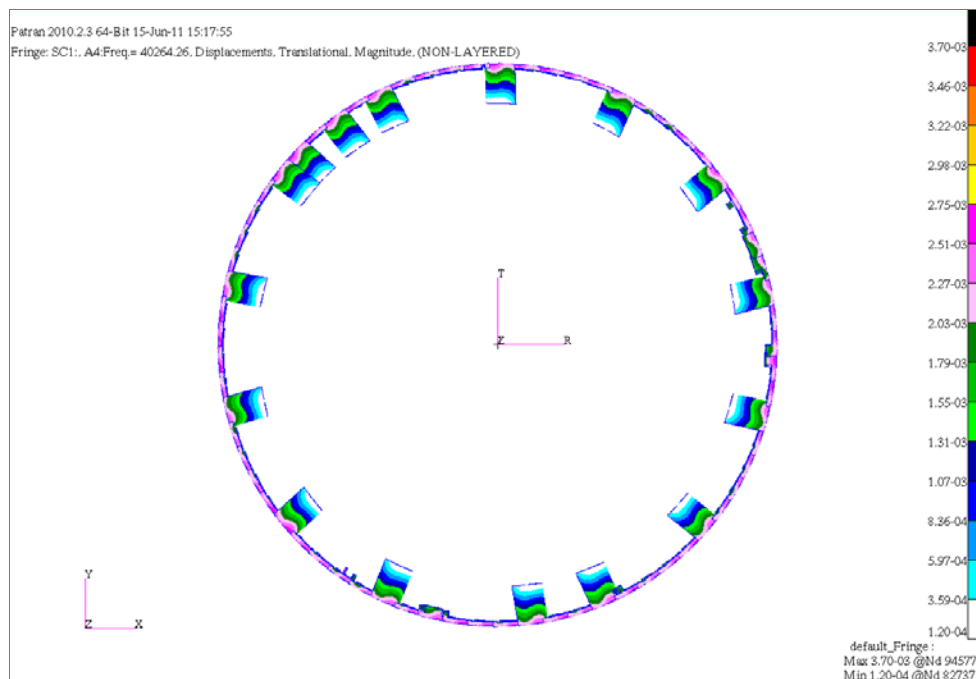

Figure 7. Frequency Response of Outermost Nodes to Harmonic Excitation at 40,264 hz
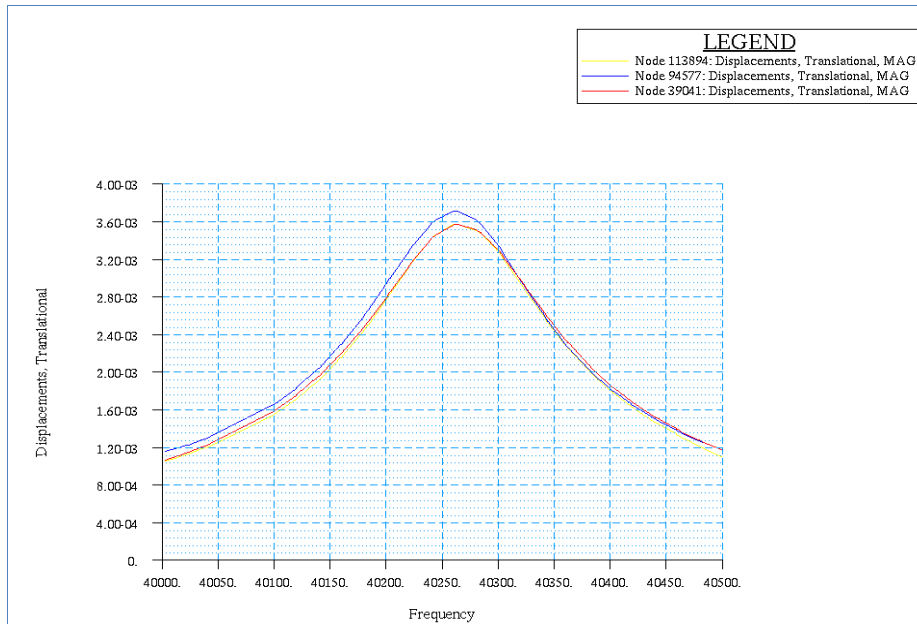
Figure 8. Frequency Response of Peak Responding Nodes in Entire Band of Excitation.

The transient analysis examining the 5ND mode was then performed, and even with the mesh-reduction described previously, it was still tremendously CPU-intensive as a temporal loading for each of 29,000 grids on the blades was applied to the model. Each transient run took approximately 84 clock hours to run on a Linux server. Time histories of the nodes identified from the frequency response analysis were created, and the comparison values were obtained by identifying the time of maximum peak-to-peak oscillations of the response and dividing by two (figure 9).

The final comparison of the peak nodes is shown in Table 2. In addition to the output from the 74N frequency response run, the output amplitudes from the sidebands at 73.8N and 74.2N are also included, as well as the maximum value when the responses are added. For this case, adding the sidebands moves the results to within an acceptable error, with four out of the five nodes examined having an error of less than 5%. The mean+2σ calculation was performed only for the highest responding node, 83711, resulting in .00381", 6.8% less than the summed frequency response value of .00409.

Table 2.   Summary of Results for Frequency and Transient Analysis for Full Airfoil Solid Model
          Excitation of 5ND Mode

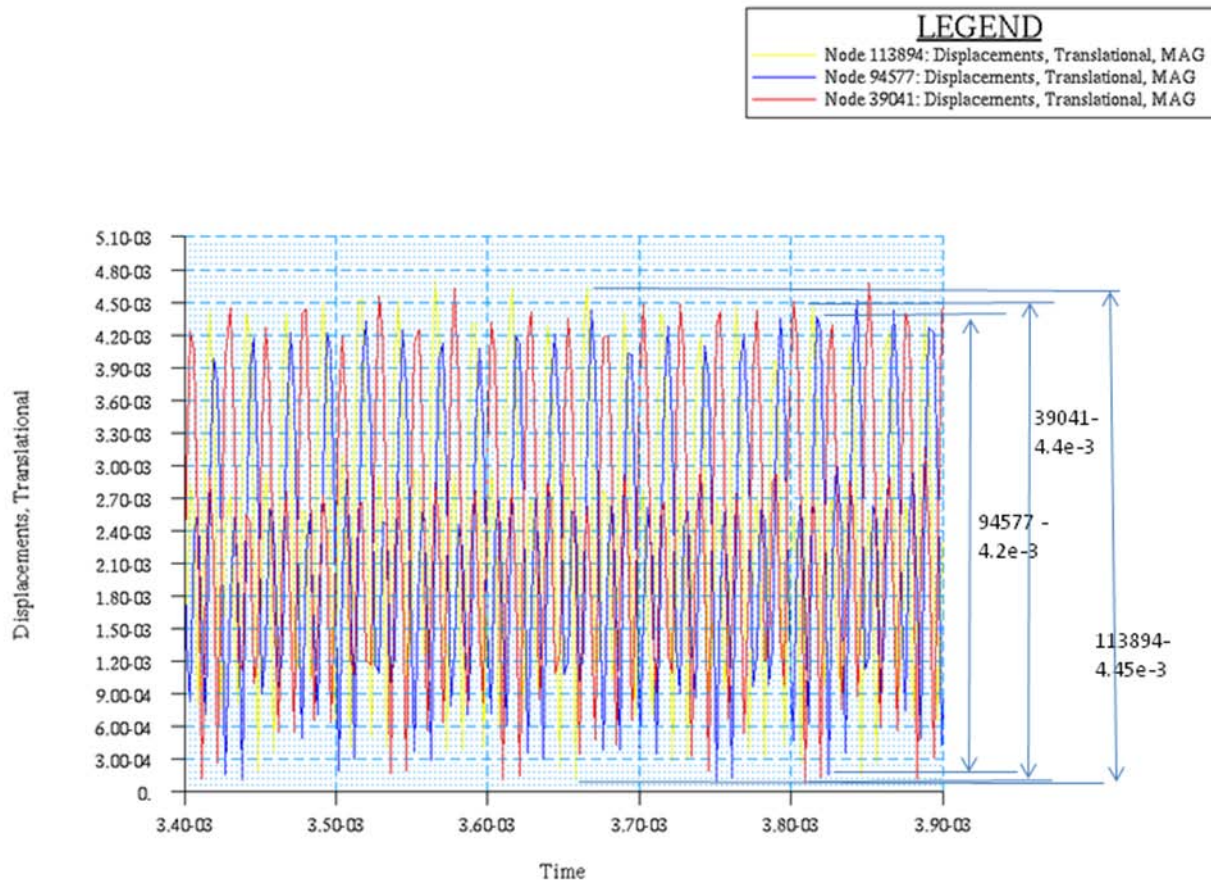| Node	Transient Response Theta Displace-ment	74N Frequency Response Magnitude Displace-ment	74N Frequency Response Theta Displace-ment	Error 74N Freq Resp from Transient Theta	73.8N Frequency Response Theta Dispace-ment	74.2N Frequency Response Theta Dispace-ment	Peak of sum of 74N, 73.8N, and 74.2N	Error sum Freq Resp from Trans-ient Theta
113894	.00380	.00358	.00358	6%	.000658	.000732	.00415	9%
94577	.00403	.0037	.00366	9%	.000606	.000668	.00421	5%
39041	.00398	.00358	.00353	11%	.000694	.000772	.00414	4%
52320	.004	.00345	.00357	11%	.000666	.000740	.00415	4%
83711	.0042	.00355	.00352	16%	.000695	.000664	.00409	-3%

Figure 9. Full Airfoil Solid Model Transient Analysis Displacement Response, 74N=40264hz excitation

57N EXCITATION OF 12ND MODE

Since the 12ND family will be excited by the 1st multiple of the downstream stators as identified in Table 1, a 12ND mode at 40,883 hz (see figure 10) was also identified from the modal run for response analysis. The results, shown in Table 3, are substantially different from the 5ND case. Running the analysis for five revolutions with the amplitude of excitation equal to the 285th bin (five times the 57N excitation), the mean-to-peak transient results are up to six times larger than the frequency response results (figure 11). If the mean + 2σ value obtained from the Rayleigh distribution of the peaks is used, this value was calculated to be $6.52*10^{-4}$ for the peak responding node, number 82462; this is still well over twice the peak value of the frequency response for that node. An investigation was initiated to attempt to explain this huge and potentially significant discrepancy.

1.86+02
1.73+02
1.61+02
1.49+02
1.36+02
1.24+02
1.11+02
9.91+01
8.67+01
7.43+01
6.19+01
4.95+01
3.71+01
2.48+01
1.24+01
0.

1.86+02

Z +

Y
Z   X

default_Fringe :
Max 1.86+02 @Nd 55902
Min 0. @Nd 16835
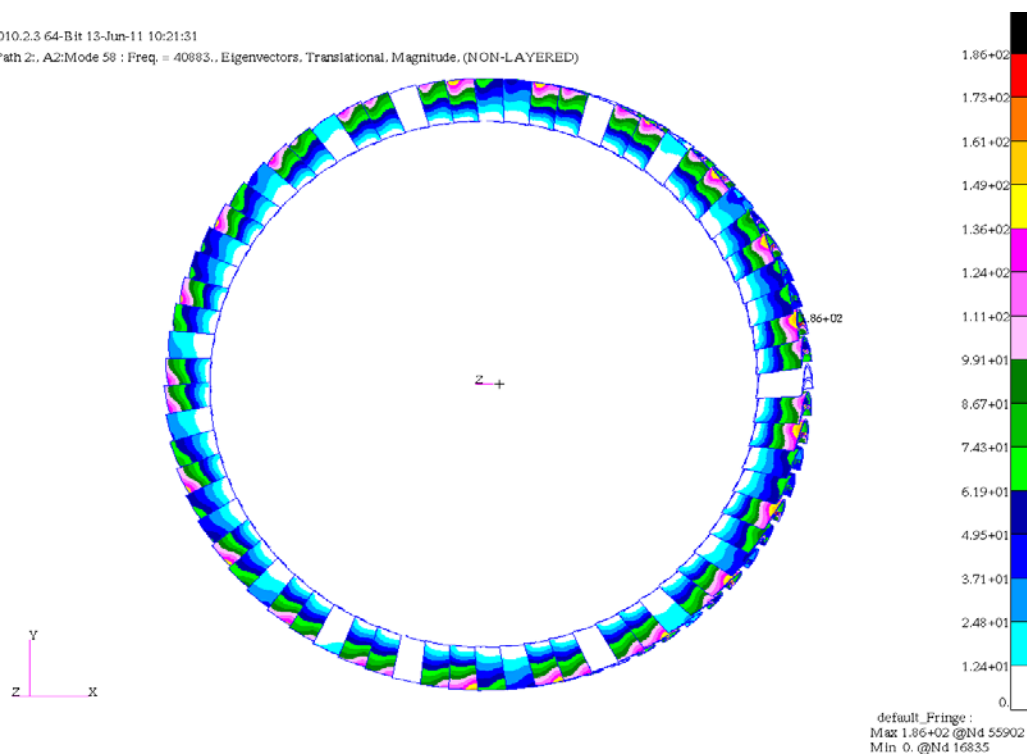
Figure 10.   12 Nodal Diameter shape at 40,883 hz

LEGEND
Node 82462: Displacements, Translational, YY
Node 36585: Displacements, Translational, YY
Node 35378: Displacements, Translational, YY

Displacements, Translational

1.70-03
1.60-03
1.50-03
1.40-03
1.30-03
1.20-03
1.10-03
1.00-03
9.00-04
8.00-04
7.00-04
6.00-04
5.00-04
4.00-04
3.00-04
2.00-04
1.00-04
0.
-1.00-04
-2.00-04

1.45e-3

1.47e-3

1.55e-3

0.   .00007 .00014 .00021 .00028 .00035 .00042 .00049 .00056 .00063 .00070

Time

Figure 11. Transient Response for 12ND excitation

Table 3. Summary of Results for Frequency and Transient Analysis for Full Airfoil
Solid Model Excitation of 12ND Mode

node	transient, theta, 5 revs, excitation at 40882.5hz	57N freqresp, 5 revs, 285th bin (5*57N)	error from transient	57N freqresp, 1 rev, 57th bin	error from transient
35378	1.45E-03	2.02E-04	618%	5.10E-04	184%
36585	1.47E-03	2.00E-04	635%	5.48E-04	168%
82462	1.55E-03	3.20E-04	384%	2.65E-04	485%
110229	1.47E-03	2.80E-04	425%	2.73E-04	438%

Initially, as with the 5ND case, the effect of the non-integer N-multiple sidebands of the 57N present in the 5 revolution pressure excitation is examined. Rather than calculate the effects individually, though, a conservative analysis is to just perform the frequency analysis for the 1 revolution case, where all the sideband energy is inherently lumped into a single, 57th bin. The results from this analysis are also shown in Table 3, and do show some improvement, but the results are still significantly different than the transient results.

The second pertinent piece of data evaluated is that the energy on the blade leading edge is at least an order of magnitude larger than on the trailing edge, as seen on plots of the temporally fourier-decomposed pressures (figure 12). Even at the 57N, which is a direct multiple of the downstream stator count, the leading edge 57N content is much higher. Although the effect of any downstream obstruction will always be much less than the effect of upstream flow distortions, the asymmetries and supersonic conditions of this turbine introduce substantially denser fourier content than previously assumed. As a result, the 12ND shape dominates the trailing edge excitation spatial field at 57N , but the noise and asymmetric effects on the leading edge are of same magnitude as the TE 12ND at other ND's, and in particular the -32ND shape at the LE, as shown in a spatial fourier decomposition of the 57N temporal fourier component (figure 13).
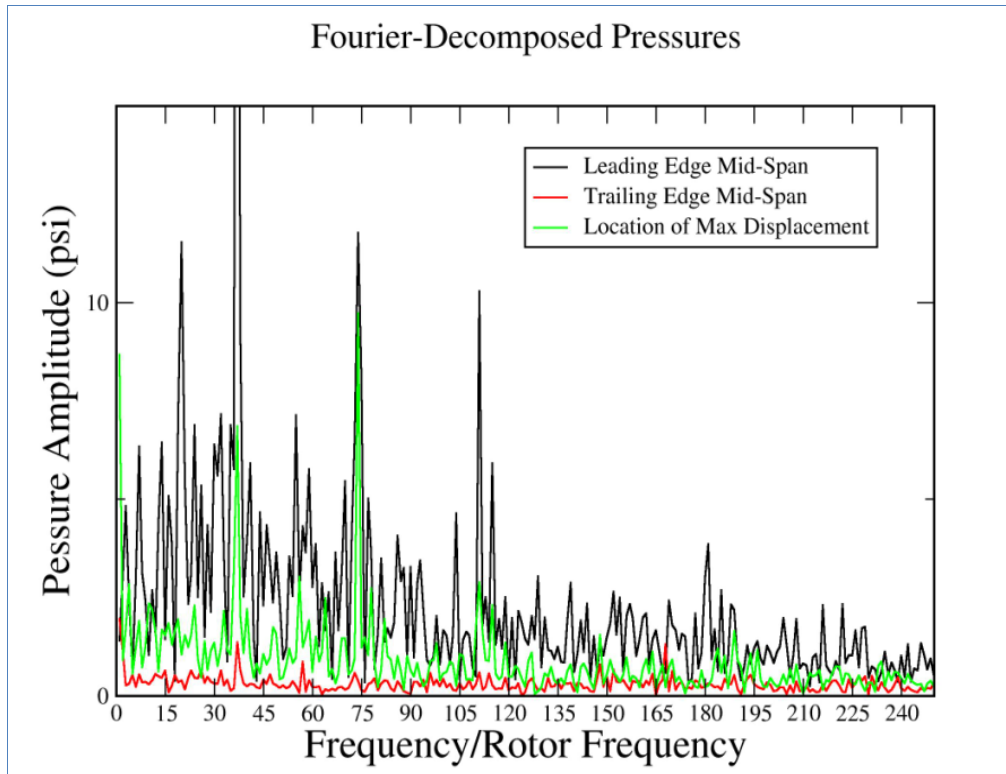
Figure 12. Temporal Fourier Decomposition of Pressures on selected Leading Edge, Trailing Edge, and Maximum Displacement Nodes
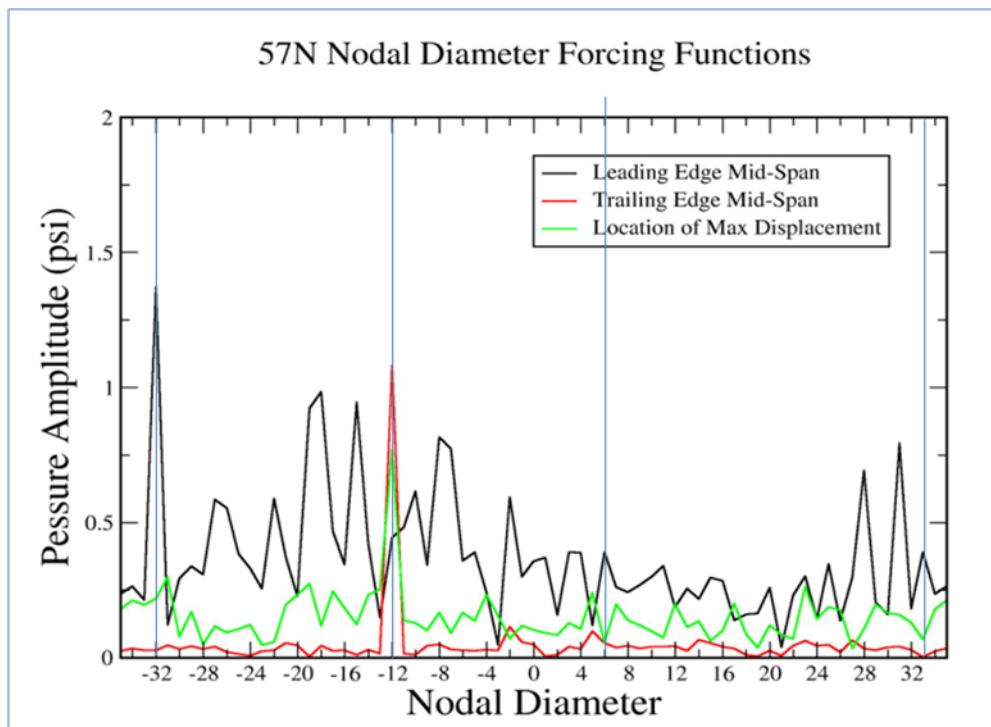

Figure 13. Spatial Fourier Decomposition of the 57N Temporal Fourier Component of the Flow-Field

Next, a wideband frequency response analysis indicates that the peak response for many of the nodes is not at 40882 hz (figure 14). This is in stark contrast to the results for the 5ND analysis, where the peaks for all the nodes occur at 40264 hz (figure 2). The implication of this result is that the dense array of spatial shapes included in the 57N would excite similarly-shaped modes away from 40882 hz. However, since the transient is performed at the 57N frequency, the relevance of this conclusion is unknown.

In addition, the transient response peaks are not nearly as narrow-banded as the 5ND results, as shown in the wide variability of response in figure 11. A fourier transform of the transient response shows significant energy at frequencies other than the 57N, as shown in figure 15. In effect, the typical SDOF oscillator filtering mechanism that converts narrow-to-wide band excitation into single-band response is quite weak in this case. A probable cause of the discrepancy, therefore, can be hypothesized by combining this information with the fact that the flow field has significant spatial density at nodal diameter modes other than 12ND. If the temporal field has frequencies at modes other than the 12ND at 40882hz, and the spatial field also has content that matches these shapes as well, then the response at these frequencies could be significant and would not be seen in the response to the 57N by itself.

A method to examine this hypothesis is to first examine the spatial content of these non-57N peaks from figure 15 and potentially corresponding modes. Looking at the peak at 29N at a frequency of 21,146 hz, there is a mode within 50 hz, but the shape is in the concentric circle family, so a nodal diameter type excitation could not excite it (the existence of concentric shapes in the flow field has not been examined). For the peak at 63N at 45208hz, there is a higher-order nodal diameter mode shape close to that value that is difficult to classify. The contribution of this mode can be determined by applying the fourier content at 63N in a frequency response analysis. Unfortunately, the resulting peak response is only 8.35*10^-5, an order of magnitude smaller than the 12ND frequency response. At this point, therefore, the discrepancy between the transient and frequency response results for this case is not fully understood and will require continued investigation.
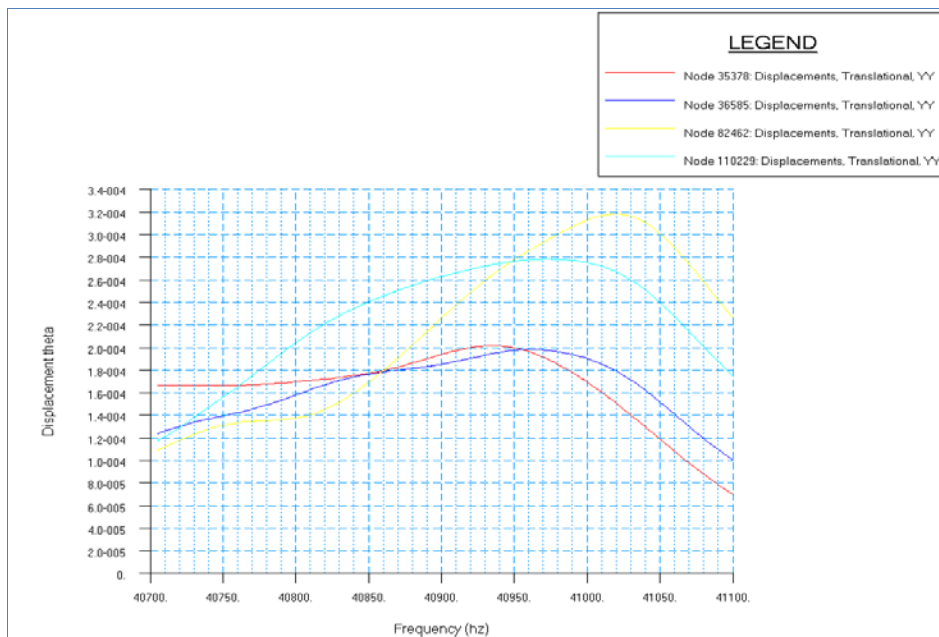

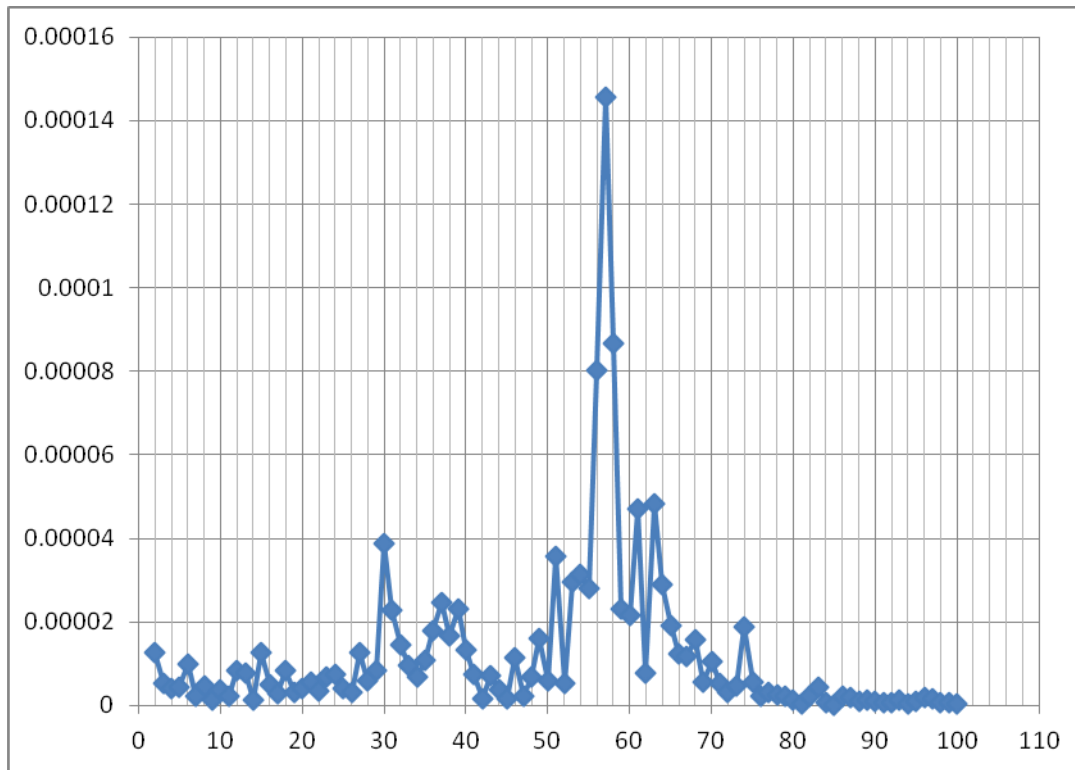Figure 14. Frequency Response Analysis for 57N Excitation

Figure 15. Temporal Fourier Transform of Transient Response for excitation such that 57N=40883hz

CONCLUSIONS AND FUTURE WORK

An exhaustive study has been performed to characterize any possible deficiencies in the standard frequency response method of forced response analysis of bladed-disks. The time-marching transient analysis method has been chosen as a baseline for comparison. Two different models were analyzed; a very simple bladed-disk where the blades are modeled as simple beams, which was built to examine the differences due to the excitation of the disk type modes, and a more detailed solid model using the shape of the J2-X upper stage rocket engine fuel turbopump airfoil, which was intended on focusing on potential differences due to the blade mode shape excitation. The initial hypothesis that the frequency response would enforce a periodicity that would incorrectly magnify the response was usually incorrect. For cases where a strong exciting mechanism, such as an upstream flow distortion, excites the mode of concern, the difference between the techniques is less than 10% so is generally acceptable for design. In cases when the primary excitation mechanism is weaker though, the frequency response method indicates an underprediction error which can be enormous. For these situations, the existence of supersonic flow in asymmetric turbines causes significant variance in the cyclic symmetry of the excitation, which introduces spatial shapes in the flow other than those that normally exist due to blade-vane interaction. It appears that these shapes, combined with non-trivial temporal fourier content at frequencies other than at the primary driver, excite structural modes that would be ignored by the single frequency excitation used in frequency response analysis. Unfortunately, the authors were unable to verify this assertion. The situation where a secondary, weak excitation source will be the primary driver of a design is rare but not unheard of, so the validation of the reasons for the underprediction as well as the development of guidelines for the appropriate use of the frequency response method will be continued.

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*Huntsville, AL, Dec.5-9, 2011*

- Introduction, Motivation, Previous Work
- Blade Beam Model Analysis
  - Tyler-Sofrin Blade-Vane Interaction Charts
  - Contribution of Sidebands
- Realistic Airfoil Solid Model Analysis
  - 74N Excitation of 5ND Mode
  - 57N Excitation of 12 ND Mode
- Conclusions and Future Work

- Turbine Blades experience large harmonic excitations.
  - Upstream, Downstream nozzles, vanes, and harmonics.
- Analysis in frequency domain therefore thought to be appropriate.
- Recent testing, analysis at MSFC has shown substantial asymmetric and non-periodic content.
  - Inlet asymmetry
  - Influence of Non-adjacent stages
  - Turbulence and other flow distortions due to supersonic flow.
- This content can only be captured by CFD over 360° of revolution.
  - New technique applying 360° on cyclic symmetric structural model described in another paper.
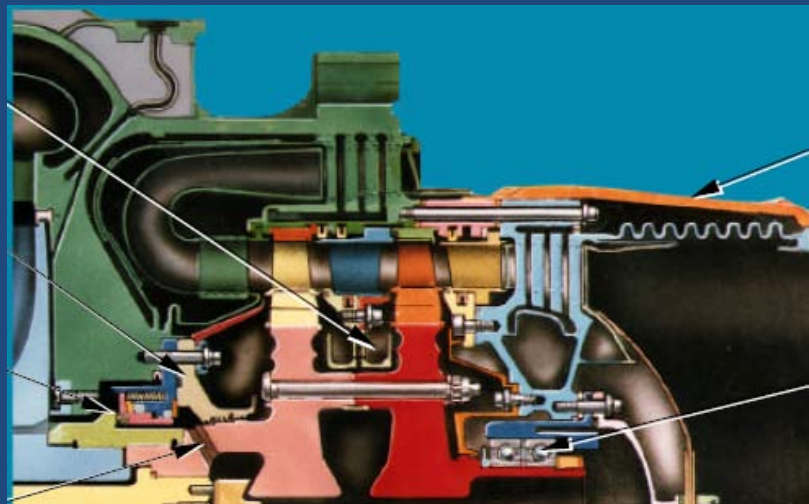
- Non-periodic content raises question: is Frequency Response Analysis itself accurate?
  - Might "enforced periodization" increase response?
- Two Studies comparing Frequency Response Analysis with "Transient" (time history) analysis performed.
  - Bladed-disk with disk modeled as plates, blades as beams, to focus on disk-mode excitation and to enable rapid turnaround time.
  - Bladed-disk with disk and blades modeled as solids, accurate model of J2-X airfoil to focus on blade-mode excitation.
- Literature survey shows question has not been addressed.
  - Misek, et.al, recent paper detailing forced response analysis of bladed disk, states frequency response technique is adequate because forcing function harmonic.

Tyler-Sofrin Blade-Vane Interaction Charts

Upstream Nozzle Multiples	37	74	111	148
Blade multiples				
69	32	-5	N/A	N/A
138	N/A	N/A	27	-10
207	N/A	N/A	N/A	N/A

Downstream Stator Multiples	57	114	171	228
Blade multiples				
69	12	N/A	N/A	N/A
138	N/A	24	-33	N/A
207	N/A	N/A	N/A	-21

- 20" x 1" disk, 69 blades 4" x 1" x .1"
- 5ND mode at 40,167 hz
- 4 revolution CFD analysis
- Scaled such that primary temporal Fourier component $F_o e^{i\Omega t}$ has frequency of 40,167 hz.
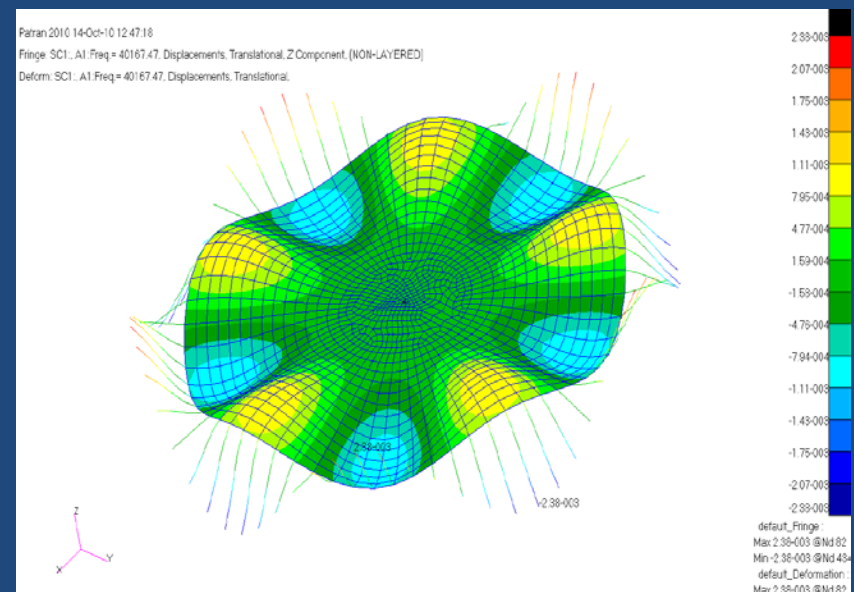
- Frequency response analysis:

$$\{u\} = [\Phi]\{q\}$$

$$q(t) = \bar{H}(\Omega)\bar{U}_{static} = \left[\frac{1 - \Omega/\omega_i^2}{\left(1 - \Omega/\omega_i^2\right)^2} + i\frac{-2\zeta\Omega/\omega_i}{\left(1 - \Omega/\omega_i^2\right)^2}\right]\frac{F_o e^{i\Omega t}}{\lambda_i^2}$$

translation ZZ complex as magnitude

LEGEND

—— Node 53: Displacements, Translational, ZZ
—— Node 82: Displacements, Translational, ZZ
—— Node 118: Displacements, Translational, ZZ
—— Node 162: Displacements, Translational, ZZ
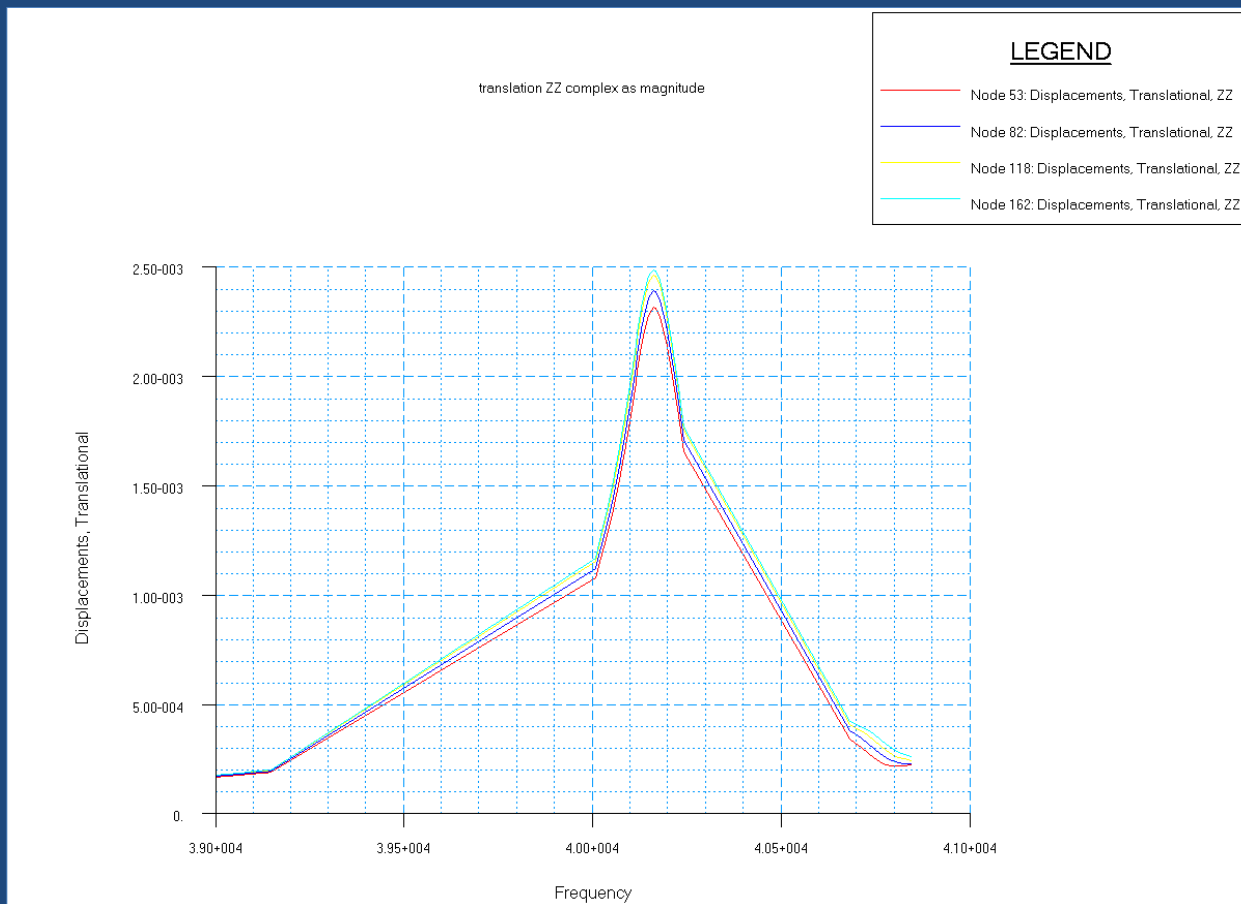
- Time Marching numerical integration, CPU intensive, not usually tractable.
- Time histories for all nodes on every blade assembled.
- Process:
  - 1) plot time history of blade tip node responses for a 1/5 sector of the bladed-disk
  - 2) Identify which node has the highest response.
  - 3) Subtract out mean response
  - 4) Calculate the statistics of the peaks of the response
  - 5) Apply 2σ value as value for comparison since sinusoid of this amplitude will produce equivalent damage as full Rayleigh distribution of narrow-band random signal.

Envelope (blue) of node 82 z steady state transient displacement (green) peaks

- Peak nodal transient = .00345", 2σ = .032", freq response=.0024 underpredicting by 25% (opposite of hypothesis).

- Potential source of error – contribution of non-integer multiples of running speed excitation close to 74N ("sidebands").

- Since 4 revs of CFD performed, $\Delta F=N/4$, so 74N will be 296'th bin.
  - Non-trivial amplitudes in bins 294=73.5N, 295=73.75N, 297=74.25N
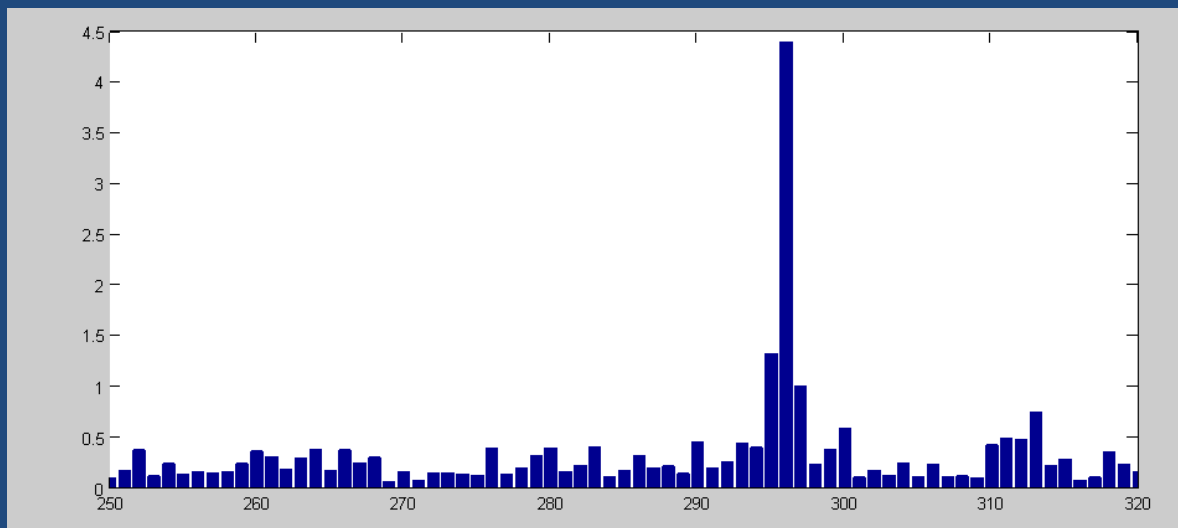
- Perform Freq. Response Analysis at sideband frequencies using sideband amplitude, sum total:

$$x(t) = 0.000361 \, Cos\left[\frac{\pi}{6} - 238200.6 \, t\right] + 0.000369 \, Cos[236596.6 \, t] + 0.0024 \, Sin[237398.66 \, t] +$$

$$0.0000458 \, Sin\left[\frac{\pi}{4} + 240606.7 \, t\right]$$

- Peak=0.00266", Є=18.6%

- 2nd phase of Study – what would effect of non-periodic components be on blade-dominated modes?

- Used more realistic solid model of disk, same diameter as J2-X, and solid model of blades using airfoil shapes from J2-X.

- 5ND mode at 40264hz identified.

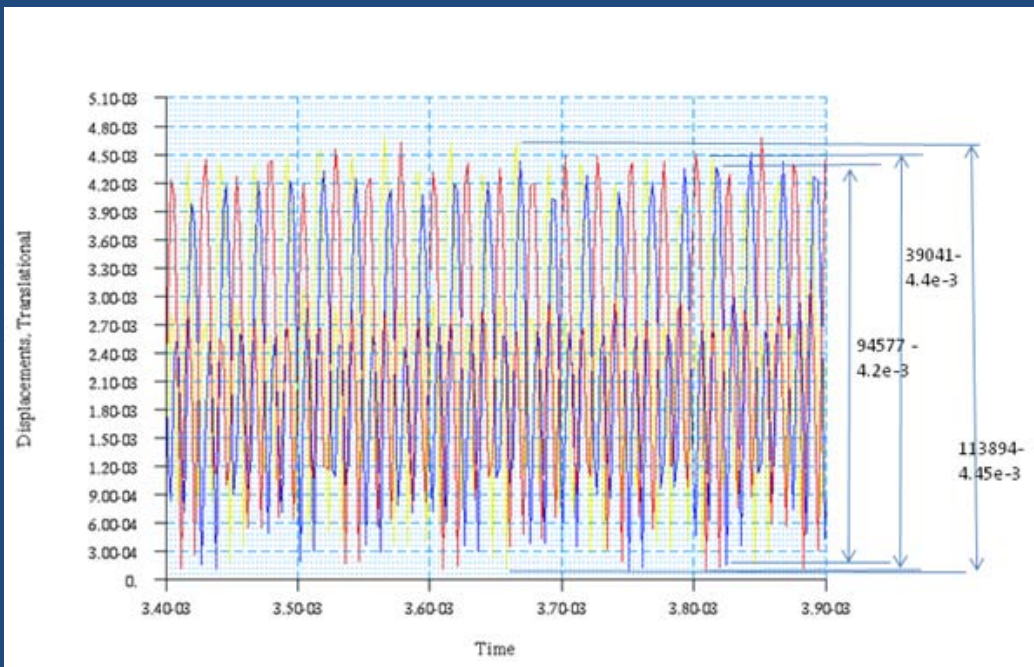- Frequency response for 74N excitation performed for peak responding nodes.

Solid Element Representative Bladed-Disk Model (not to scale)
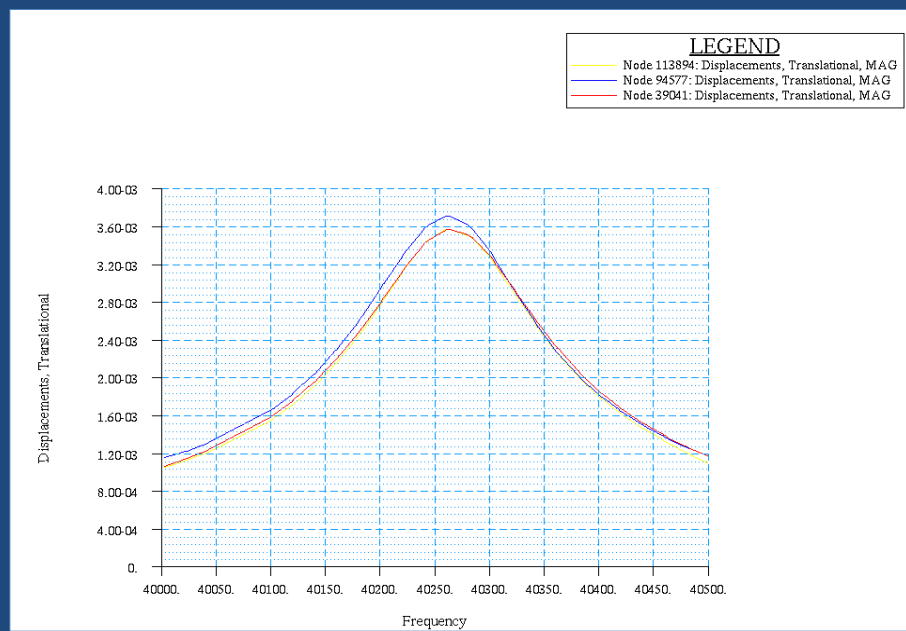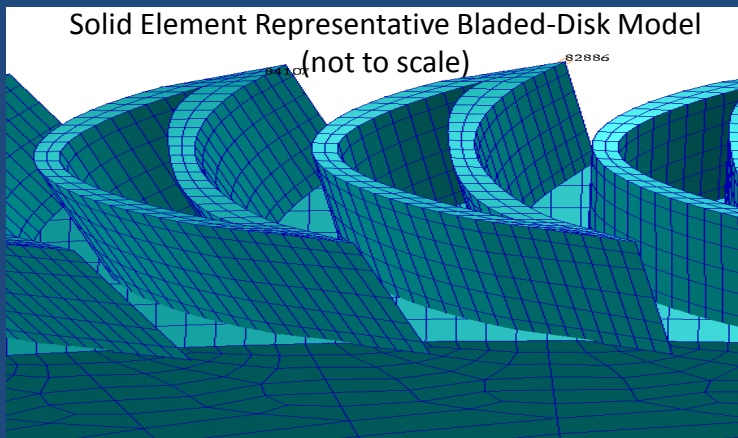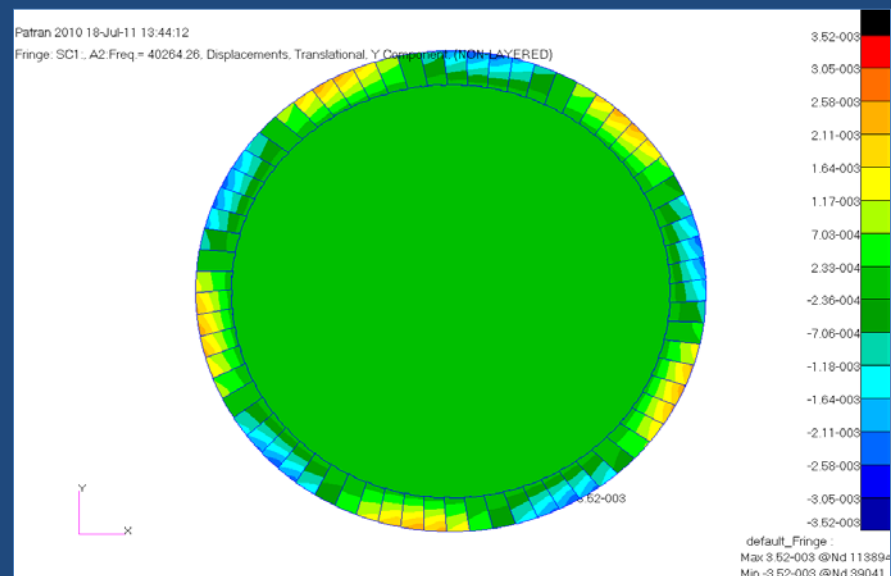
- Transient analysis required 52,000 point time history on each of 29,000 nodes, took 84 clock hours.

- Transient Mean + 2σ for node 83711 =.00381", 6.8% > summed freq. resp of .00409"

Node	Transient Response Theta Displace-ment	74N Frequency Response Magnitude Displace-ment	74N Frequency Response Theta Displace-ment	Error 74N Freq Resp from Transient Theta	73.8N Frequency Response Theta Dispace-ment	74.2N Frequency Response Theta Dispace-ment	Peak of sum of 74N, 73.8N, and 74.2N	Error sum Freq Resp from Trans-ient Theta
113894	.00380	.00358	.00358	6%	.000658	.000732	.00415	9%
94577	.00403	.0037	.00366	9%	.000606	.000668	.00421	5%
39041	.00398	.00358	.00353	11%	.000694	.000772	.00414	4%
52320	.004	.00345	.00357	11%	.000666	.000740	.00415	4%
83711	.0042	.00355	.00352	16%	.000695	.000664	.00409	-3%

- 12ND mode at 40883 hz identified, freq. resp. & trans response analyses performed.

node	transient, theta, 5 revs, excitation at 40882.5hz	57N freqresp, 5 revs, 285th bin (5*57N)	error from transient	57N freqresp, 1 rev, 57th bin	error from transient
35378	1.45E-03	2.02E-04	618%	5.10E-04	184%
36585	1.47E-03	2.00E-04	635%	5.48E-04	168%
82462	1.55E-03	3.20E-04	384%	2.65E-04	485%
110229	1.47E-03	2.80E-04	425%	2.73E-04	438%

- Energy on LE>> energy on TE, even at 57N
- Fourier 2D (Spatial) Decomposition shows high spatial density.
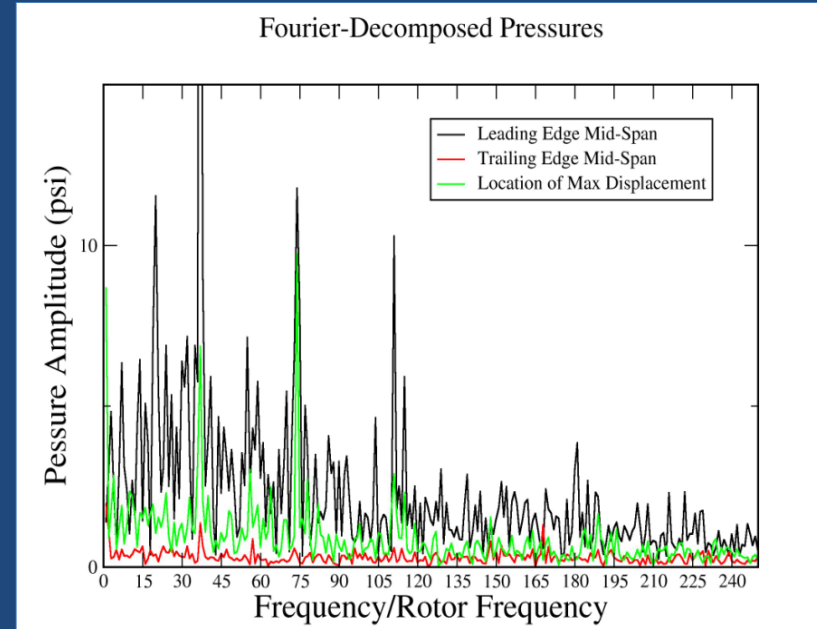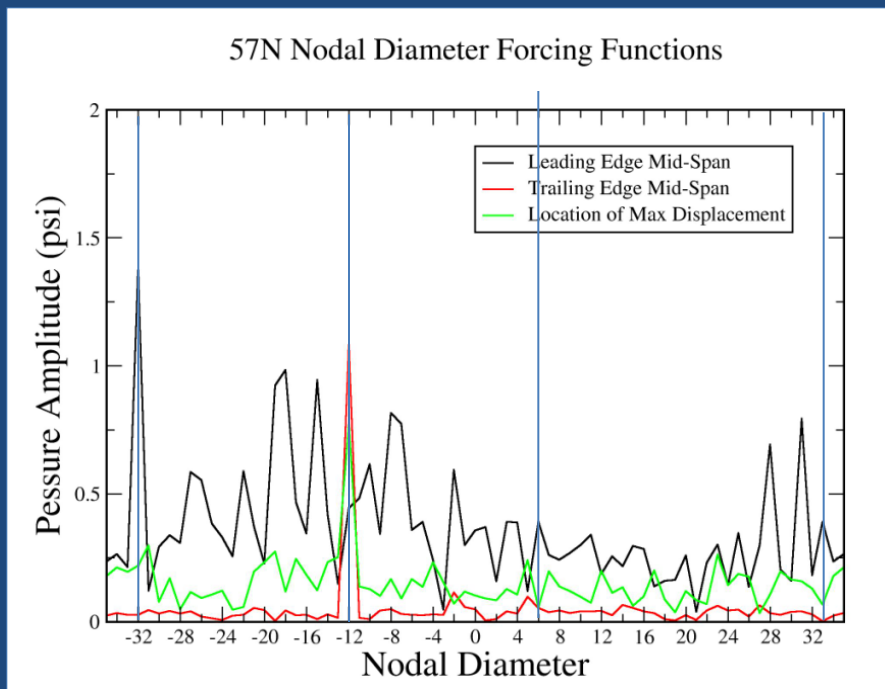
- Peak frequency response not at 40883.
- Transient response not narrow-banded.

- Hypothesis – high non-12ND mode frequencies and spatial content of excitation match a mode shape.

- Examine modal excitation due to high non-57N peaks 29N and 63N shown in temporal Fourier decomposition of transient response.

- Mode close to 29N at 21146hz, but it is concentric circle mode.

- High-order nodal diameter mode shape near 63N at 45208 hz.
  - Frequency response analysis yields response << 57N response though.

- Discrepancy will require further investigation.
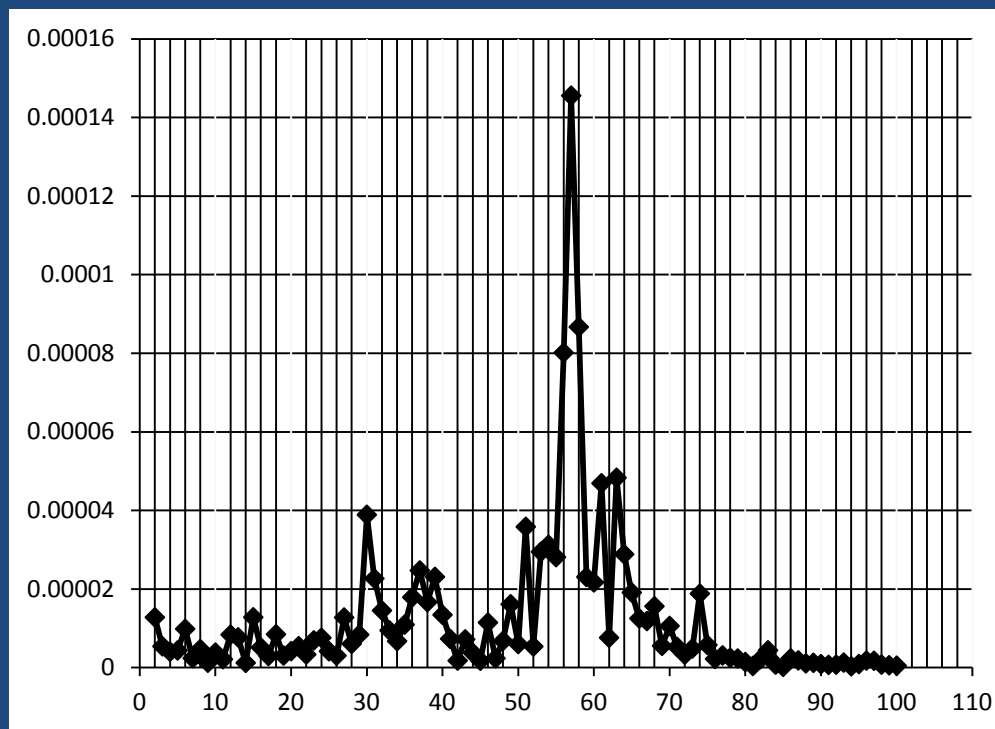
Fourier Transform of Transient Response for excitation such that 57N=40883hz

- Non-periodic content in CFD motivated study to characterize possible deficiencies in standard frequency response analysis for Bladed-Disks.
- Transient, or temporal solution solved by numerical integration used as baseline.
- Two models used: simple beam-blade model and realistic solid-airfoil model.
- For cases where strong excitation mechanism exists, difference between frequency and transient response results < 10%.
- Cases when excitation mechanism weak can exhibit errors > 100%.
- Hypothesis that spatial shapes in flow other than those from vane-blade interaction and which have significant temporal fourier content interact with structural modes unproven.
- Situations where these cases drive design are rare but not unheard of, so search for validation of this or other hypotheses will continue.