An Analysis of the Internal Truth Files for a CAST 4000 GPS Simulator for Two Rocket Launches and One Weather Balloon Flight

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The CAST GPS 4000 simulator can create scenarios using external trajectories. The following information must be provided: time from a given epoch (a constant sampling rate must be used); position, velocity and acceleration in Earth Centered Earth Fixed (ECEF) coordinates; elements of the matrix that transforms from the body-axis coordinate system to the ECEF frame; and the angular velocity of the body-axis system relative to ECEF coordinates. The initial latitude, longitude, altitude, and UTC time must also be provided during the scenario setup.

The simulator recomputes the positions and velocities using the given accelerations and a constant jerk model. The results are the internal "truth file". The basic algorithm provided by CAST for each ECEF component is

$$\dddot{x} = jerk = j = \text{constant}$$
$$\ddot{x} = a_0 + jt$$
$$\dot{x} = v_0 + a_0 t + \frac{1}{2} jt^2$$
$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2 + \frac{1}{6} jt^3 .$$

The jerk over each interval is computed by simple differencing,

$$j_n = \frac{a_{n+1} - a_n}{\Delta t} .$$

Substituting this into the basic algorithm gives

$$\begin{aligned} v_{n+1} &= v_n + a_n \Delta t + \frac{1}{2}\left(\frac{a_{n+1} + a_n}{\Delta t}\right)\Delta t^2 \\ &= v_n + \frac{\Delta t}{2}\left(a_n + a_{n+1}\right) , \end{aligned}$$

and

$$\begin{aligned} x_{n+1} &= x_n + v_n \Delta t + \frac{1}{2} a_n \Delta t^2 + \frac{1}{6}\left(\frac{a_{n+1} - a_n}{\Delta t}\right)\Delta t^3 \\ &= x_n + v_n \Delta t + \frac{\Delta t^2}{6}\left(2a_n + a_{n+1}\right) . \end{aligned}$$

The external trajectory files require the linear accelerations, but this data is not available from radar or GPS systems and must be derived from the position or velocity data. The technique used at Kennedy Space Center to obtain the accelerations is very simple and is based on the philosophy that the measured data should not be changed unless absolutely necessary, for example, if it is very noisy or has gaps. Noisy data is smoothed with a simple moving average filter whose width is chosen based on the rapidity of the dynamic changes and the data rate, but usually about ± 10 data points is sufficient. Any gaps are filled in with quadratic interpolation. If only position data is available, the velocity is obtained from the position by midpoint differencing,

$$v_n = \frac{x_{n+1} - x_{n-1}}{2\Delta t}.$$

The acceleration is obtained from the velocity in a similar manner,

$$a_n = \frac{v_{n+1} - v_{n-1}}{2\Delta t}.$$

Although this technique has worked for us, it must be used carefully because the derivatives can be very noisy unless the original data is smooth and well-behaved. There are many other ways of smoothing the data and obtaining the derivatives that may be more desirable or appropriate for a well-informed user.

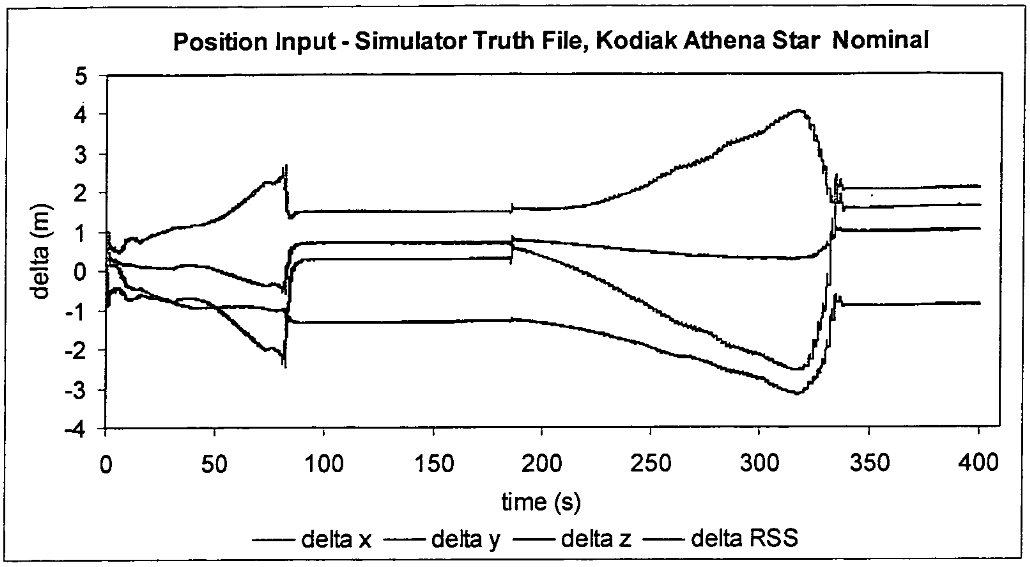
Of course, one wants to know how well the simulator reproduces the position and velocity of the original input file. To answer this question, point-by-point comparisons were made between the positions and velocities from the internal truth and external trajectory files for three operations: the Athena Kodiak Star launch from Kodiak, Alaska; a fictitious three-stage modified Minuteman missile launch from Wallops Island, Virginia called the Wallops Express; and an actual weather balloon flight from Wallops Island, Virginia. The results are shown on the following pages. The velocity and acceleration profiles are also given. All times are seconds from launch.

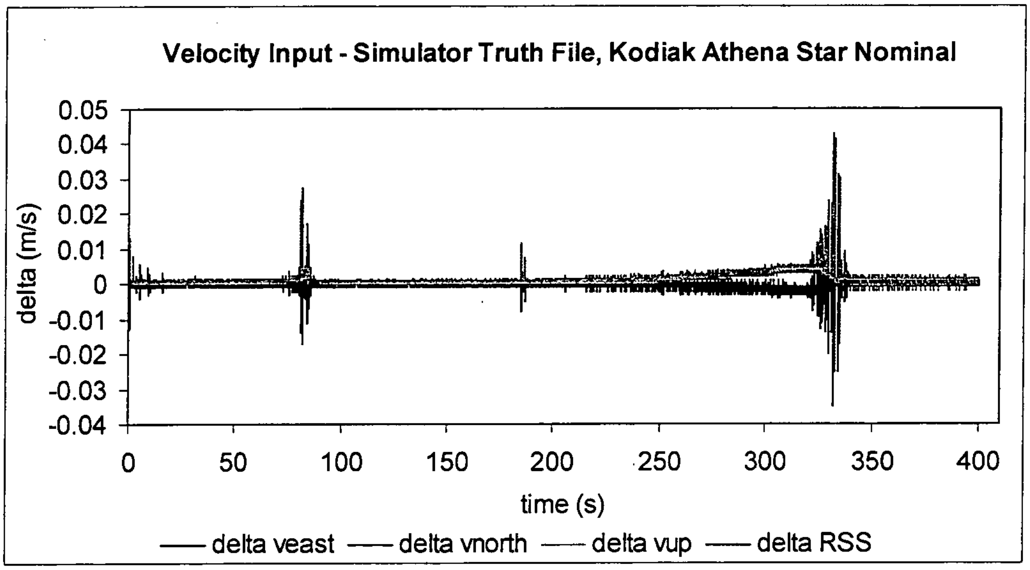
The two launch trajectories contain less than10 minutes of data, and the balloon flight contains about 1 hour and 45 minutes of data. The velocities in the external trajectory files were converted to an East North Up system to agree with the output format of the internal truth file. No attitude data was available and only the center-of-mass coordinates were used.

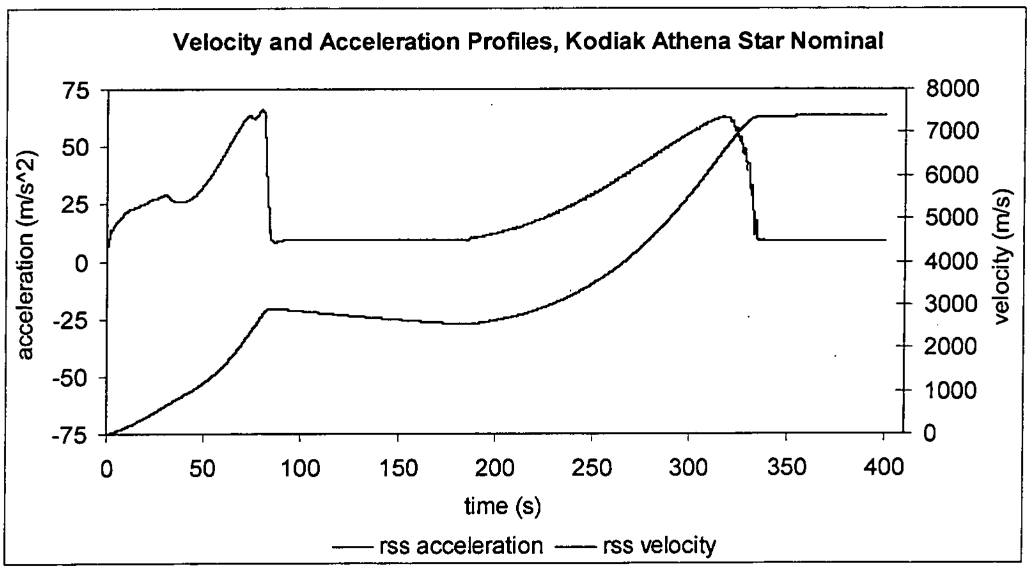
The Kodiak and Wallops Express external trajectory files were based on theoretical nominals with clean, smooth position and velocity data that was quadratically interpolated from 10 Hz to 50 Hz before numerically differentiating the velocities as described above. Because the data was very smooth, this higher rate resulted in smaller jerks. The Wallops balloon external trajectory file was based on position-only 1 Hz data collected from an on-board GPS receiver during the actual flight. This data contained noise and random movements of the balloon and was quadratically interpolated to 10 Hz before numerically differentiating the position and velocities: interpolating to a higher data rate resulted in unacceptably large jerks. No smoothing was done on any of the data.

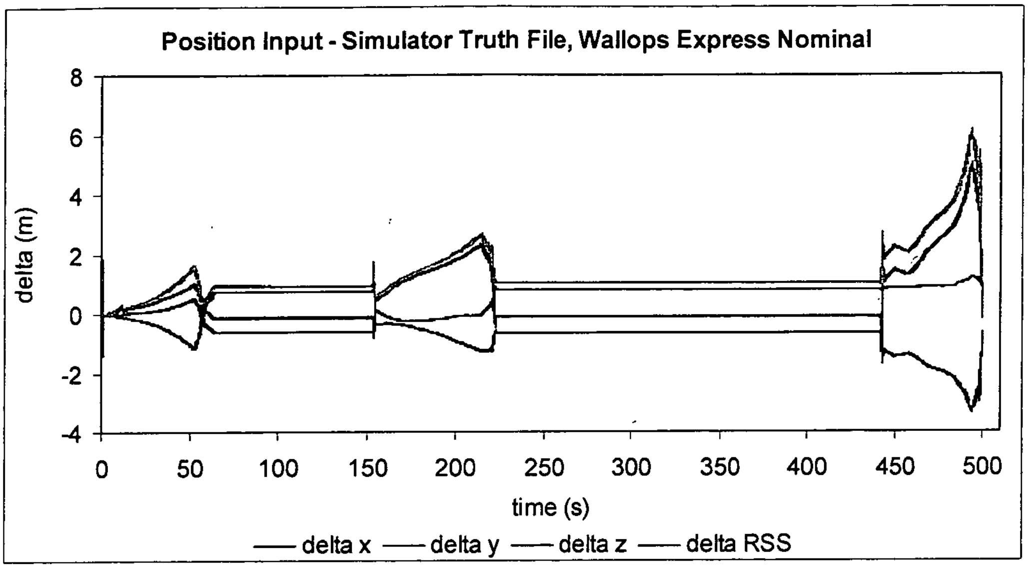
The rocket scenarios required no ramp-up of the velocities, but the balloon scenario required a two-second ramp of the initial velocities because there were large jerks when the balloon was released.

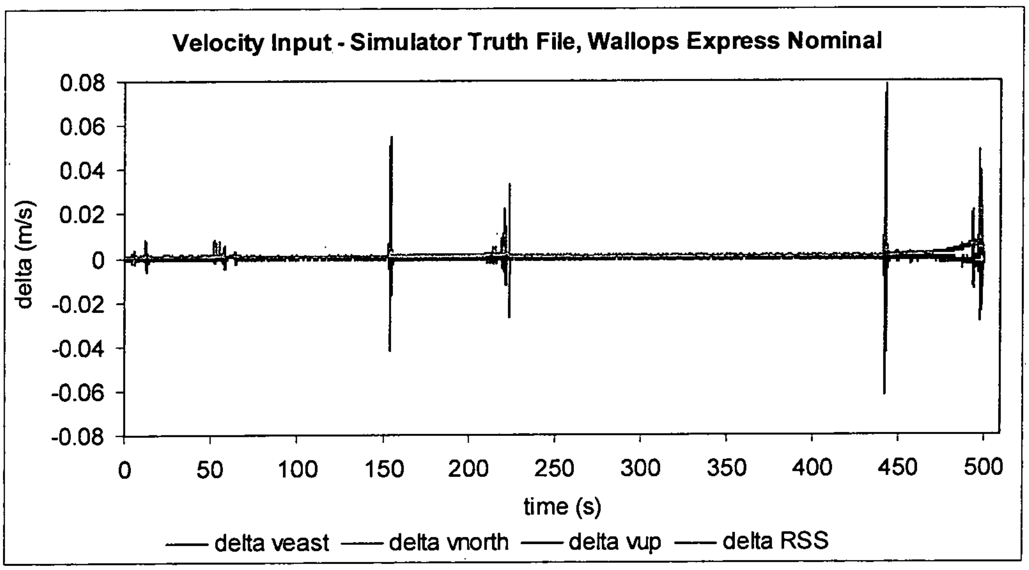
The position differences between the input and internal truth data are less than a few meters and the velocity differences are typically much less than a few centimeters per second. The largest differences occur during times of large or rapidly changing accelerations (i.e., jerks). Because numerical integration is used, the mean errors gradually increase. For all but the most demanding applications, it is reasonable to conclude that that these differences are negligible. Although is not known why the balloon data is noisy at about T+4000 s, atmospheric turbulence is a likely cause.

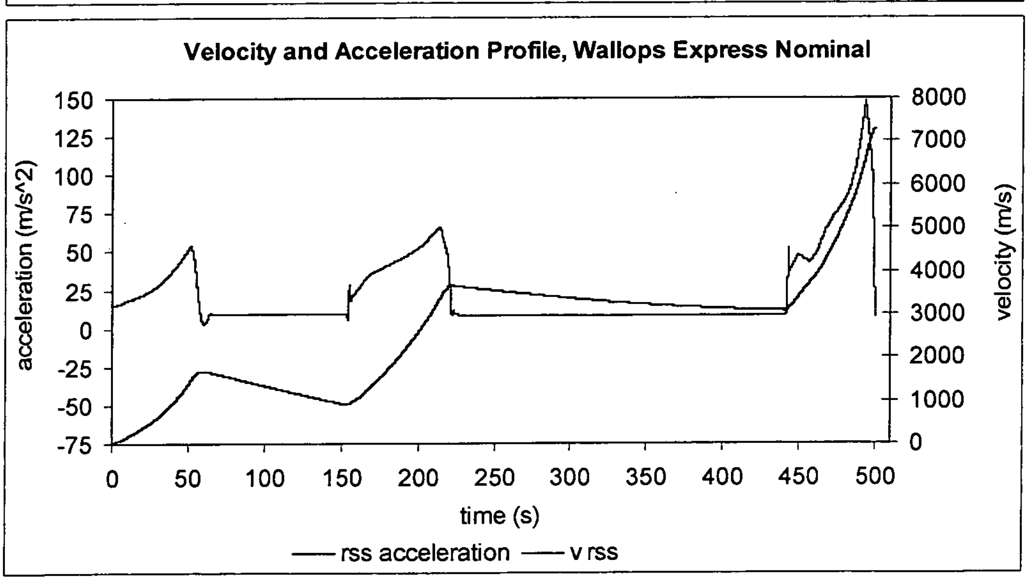

Position Input - Simulator Truth File, Kodiak Athena Star Nominal
delta (m)
time (s)
delta x
delta y
delta z
delta RSS

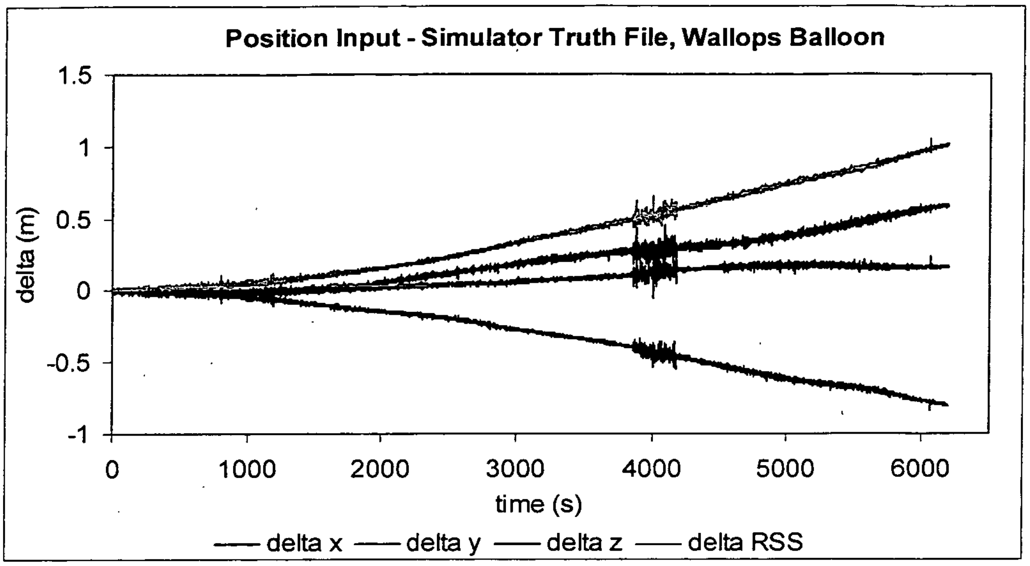

Velocity Input - Simulator Truth File, Kodiak Athena Star Nominal
delta (m/s)
time (s)
delta veast
delta vnorth
delta vup
delta RSS

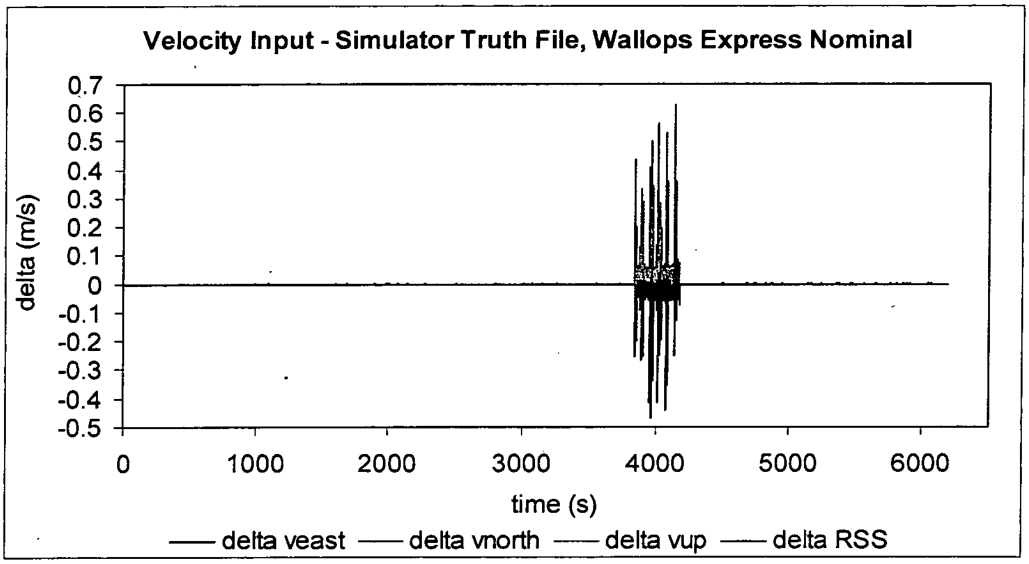

Velocity and Acceleration Profiles, Kodiak Athena Star Nominal
acceleration (m/s^2)
velocity (m/s)
time (s)
rss acceleration
rss velocity

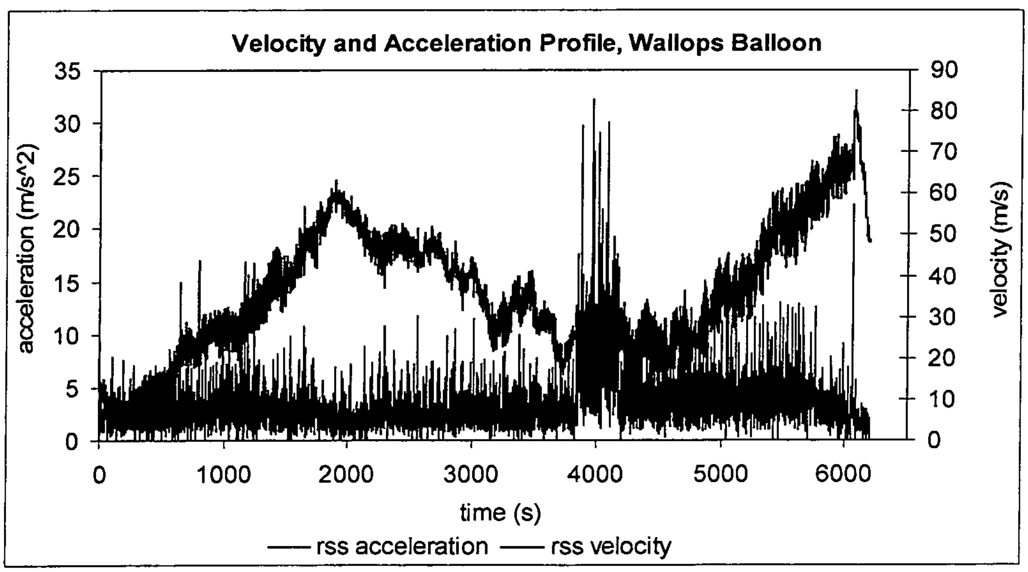

Position Input - Simulator Truth File, Wallops Express Nominal
delta (m)
8
6
4
2
0
-2
-4
0 50 100 150 200 250 300 350 400 450 500
time (s)
delta x
delta y
delta z
delta RSS


Velocity Input - Simulator Truth File, Wallops Express Nominal
delta (m/s)
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0 50 100 150 200 250 300 350 400 450 500
time (s)
delta veast
delta vnorth
delta vup
delta RSS


Velocity and Acceleration Profile, Wallops Express Nominal
acceleration (m/s^2)
150
125
100
75
50
25
0
-25
-50
-75
velocity (m/s)
8000
7000
6000
5000
4000
3000
2000
1000
0
0 50 100 150 200 250 300 350 400 450 500
time (s)
rss acceleration
v rss


Position Input - Simulator Truth File, Wallops Balloon
delta (m)
1.5
1
0.5
0
-0.5
-1
0
1000
2000
3000
4000
5000
6000
time (s)
delta x
delta y
delta z
delta RSS


Velocity Input - Simulator Truth File, Wallops Express Nominal
delta (m/s)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1000
2000
3000
4000
5000
6000
time (s)
delta veast
delta vnorth
delta vup
delta RSS


Velocity and Acceleration Profile, Wallops Balloon
acceleration (m/s^2)
35
30
25
20
15
10
5
0
velocity (m/s)
90
80
70
60
50
40
30
20
10
0
0
1000
2000
3000
4000
5000
6000
time (s)
rss acceleration
rss velocity