Composite Stress Rupture: A New Reliability Model Based on Strength Decay

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Abstract

A model is proposed to estimate reliability for stress rupture of composite overwrap pressure vessels (COPVs) and similar composite structures. This new reliability model is generated by assuming a strength degradation (or decay) over time. The strength decay model will be shown to predict a response similar to that predicted by a traditional reliability model for stress rupture based on tests at a single stress level. In addition, the model predicts that even though there is strength decay due to proof loading, a significant overall increase in reliability is gained by eliminating any weak vessels, which would fail early. The model suggests that strength decays very slowly over most of a component's life. Late in life, as the strength decreases to near the applied stress level, the rate of strength decay increases significantly. This helps explain why strength measurements following prolonged stress rupture testing have not demonstrated a discernable reduction in strength. Any reduction in strength that may have occurred is masked by the scatter in the data. The strength decay model is also consistent with increased levels of acoustic activity late in the life of a COPV. The model predicts that there should be significant periods of safe life following proof loading, because time is required for the strength to decay from the proof stress level to the subsequent loading level.

Validating reliability models for carbon composite stress rupture is difficult because of the high degree of scatter exhibited in observed times to failure. Suggestions for testing the strength decay reliability model have been made which allows observations with a number of test specimens and in a time period, which are reasonable for a test program that could actually be performed. If the strength decay reliability model predictions are shown through testing to be accurate, COPVs may be designed to carry a higher level of stress than is currently allowed, which will enable the production of lighter structures.

Nomenclature

9's   reliability scale where three 9's is 1/1000 chance of failure (R=0.999)

A scaling parameter

b nonlinearity parameter in strength decay model

c time scale parameter in strength decay model

i count of similar specimens that have failed at a given point in stress rupture

N number of similar specimens tested

P probability of occurrence

Pr proof ratio (proof stress/normal operating stress)

R reliability

R_c conditional reliability

$s(t)$ instantaneous strength (strength of a given test specimen at some point in time)

s_o initial strength in a given test specimen

$\hat{s}_o$ scale parameter for Weibull distribution of initial strengths

S strength distribution of a collection of specimens

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SR	Stress ratio ($\sigma_{service} / \hat{s}_o$)
SR_P	Stress ratio during proof loading ($\sigma_{proof} / \hat{s}_o$)
t	time
t_1	initial loading time for proof load
t_f	time to failure
t_{ref}	classic model reference time to failure (scale parameter at SR=1)
t_{safe}	period of safe operation following proof loading
α	Weibull shape parameter for strength
β	Weibull shape parameter for time to failure
λ	Weibull scale parameter
σ	Stress in composite
σ_{proof}	Stress in composite at proof load
$\sigma_{service}$	Stress in composite during normal operations
ρ	Classic model parameter for sensitivity to stress ratio

Introduction

Stress rupture has been observed in composite materials where the material can fail after a period of time when no increase in load is applied. One application where stress rupture may be critical is in Composite Overwrap Pressure Vessels (COPV). This application may be worse than others because the composite material tends to be uniformly loaded to a high stress level for prolonged periods of time. As seen in the Figure 1, COPVs come in various sizes and are either cylindrical or spherical in shape. Several varieties of composite overwrap have been used including Kevlar®/Epoxy, but most COPVs in current use in the aerospace industry are made with carbon fibers. In this introduction, a typical data set for creep rupture of polymer epoxy reinforced carbon composites will be presented. An existing reliability model will then be described to show how reliability predictions are currently established. The concern that proof loading could damage a COPV will be described leading to the need for a new model such as the strength decay model that will be introduced in this paper.

These critical structures operate under very high pressures, and an unanticipated failure of a vessel would likely lead to loss of life and to loss of mission. The structure is made of a thin metal liner that is then wrapped with composite. To avoid stress rupture in the composite, the pressure in the COPV is normally limited to a relatively low percentage of the vessel burst pressure (typically less than 50%). Reliability models are used to predict the likelihood of stress rupture at various pressure levels. By keeping the stress level in the composite low, the chance of a failure is
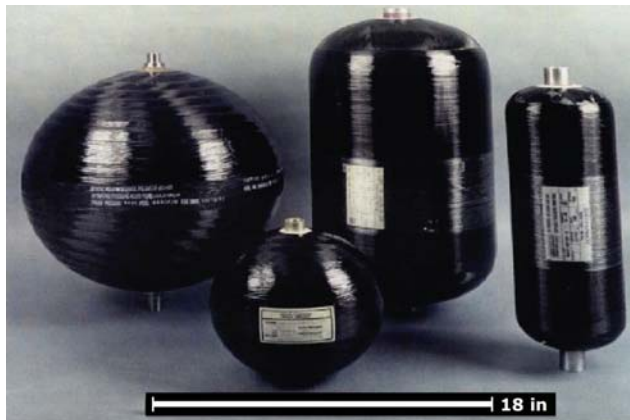

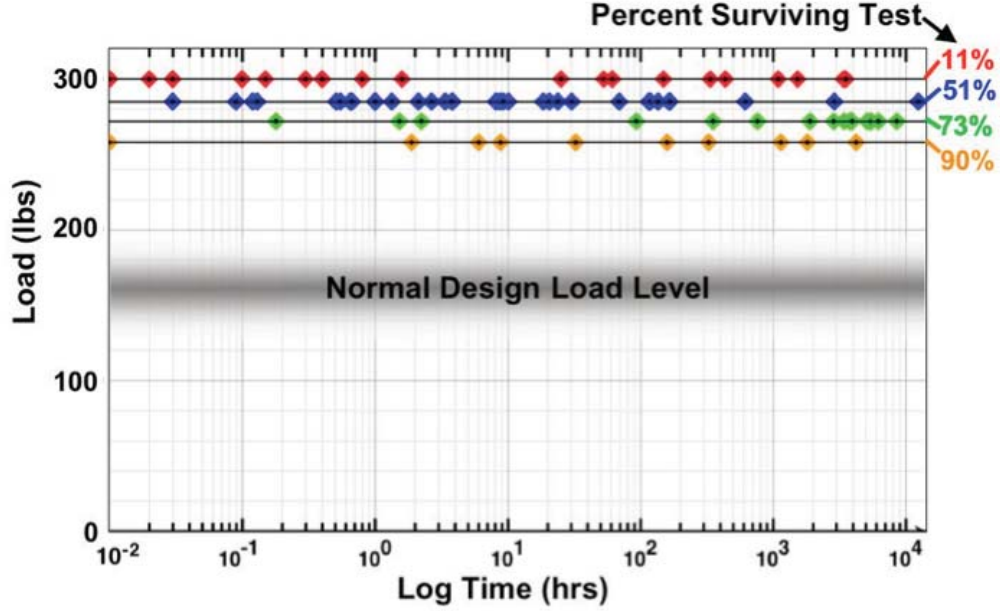**Figure 1. Composite overwrap pressure vessels (COPV).**

Figure 2. Time to failure data from a stress rupture study on IM6 composite test specimens[1].

kept acceptably small (e.g., 1 in a million). The reliability models are based on data, but there is significant extrapolation from the reliability levels where tests can be performed to the predictions that are desired. For example, a test of a million vessels to show a one in a million chance of failure is not feasible. An example of composite stress rupture data that can be collected is presented in Figure 2. The data in Figure 2 are collected from strand test specimens which are a single tow of fiber (perhaps 1000 fiber filaments) impregnated with a polymer. These strand specimens exhibit the creep rupture phenomenon and are much cheaper test specimens than full COPVs. Two things are clear from the stress rupture data presented in Figure 2: as the stress level (i.e. load) is decreased, the percentage of surviving specimens grows dramatically; and the time to failure of apparently identical test specimens under identical loading can be vastly different.

To account for the large variation in failure times for essentially identical test specimens, statistical models are used. Many statistical models have been proposed with most assuming a Weibull distribution in time to failure, for specimens tested at a given load level. The equation for Weibull reliability is given by eq. 1, where β is the Weibull shape parameter and is a measure of the scatter, while λ is the scale parameter (the time at which only 36.8% (e^{-1}) would survive).

$$R(t) = e^{-\left(\frac{t}{\lambda}\right)^{\beta}} \tag{1}$$

To account for specimens tested at various load levels, it is often assumed that the scale parameter is a power law function of the stress. In eq. 2, the power law exponent is $-\rho$, and A is a proportionality constant.

$$\lambda = A\, \sigma^{-\rho} = t_{ref}\left(\frac{\sigma}{\hat{s}_o}\right)^{-\rho} = t_{ref}\; SR^{-\rho} \tag{2}$$

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As further shown in eq. 2, it is common to normalize the stress level by the strength. However, strengths of a group of specimens also exhibit scatter. The distribution of strengths, S, is often characterized with a Weibull distribution similar to that expressed in eq. 1, but where time, t, is replaced by strength level s_o as given by eq. 3. Equation 3 gives the probability of a specimen having initial strength greater then s_o.

$$R(s_o) = P(initial\ strength > s_o) = e^{-\left(\frac{s_o}{\hat{s}_o}\right)^\alpha} \tag{3}$$

It is therefore the scale parameter of initial strength, $\hat{s}_o$, that is used to normalize the stress level. The ratio of applied stress to the strength scale parameter is termed the stress ratio, SR. A proportionality constant, t_{ref}, is still needed and is chosen to be the scale parameter when $SR=1$. Substituting eq. 2 into eq. 1, a common model for stress rupture is attained. This model is given by eq. 4 and has been called the "Classic Model" in previous publications [2, 3].

$$R(t) = e^{-\left(\frac{t}{SR^{-\rho}\ t_{ref}}\right)^\beta} \tag{4}$$

where t_{ref}, ρ, β are model parameters, and t and SR are input parameters.

The IM6 data introduced in Figure 2 are replotted in Figure 3 to show how the classic model fits the data. The data are now plotted on a "Reliability in 9's" scale where 1 is a 1 in 10 chance of failure ($R=0.9$) and 3 is a 1/1000 chance ($R=0.999$). The scale below 1 is not so easily interpreted but the time to failure at zero 9's is the time scale parameter at a given stress level. The transformation used to create the scale is given by eq. 5 where $R(t)$ is the reliability given by eq. 4 for the classic model.

$$Reliability\ in\ 9's = -\log\left(-\ln(R(t))\right) \tag{5}$$

Notice that a combination of natural log and log base 10 is used in this transformation. For the test data marked as points joined with solid lines on the right-side plot of Figure 3, the R(t) is determined when a specimen fails based on how many similar specimens failed previously and how many remain unfailed. When the second specimen out of 10 fails, it would normally have a reliability of 0.8 or 1- (2/10). However, it has been shown that this produces a bias in the data when sample sizes are small as is generally the case with creep rupture data, so Bernard's Median Rank approximation (eq. 6) [4] is used as a better estimate.

$$R(t) = 1 - \frac{i - 0.3}{N + 0.4} \tag{6}$$

where N is the total number of similar specimens tested while i is the number of specimens that have failed at a given point in time. So as time progresses more specimens fail, and the reliability of specimens at a given stress level decreases. Note that the IM6 stress rupture tests were run at four stress levels, but when the data were analyzed, it was found that specimens came from two different spools, which had slightly different strengths. When the specimens were grouped according to SR (stress normalized by strength), the 7 different groups plotted in Figure 3 resulted (the lowest SR from 1 spool resulted in no failures and is not shown). The data from the two spools are plotted with different colors.
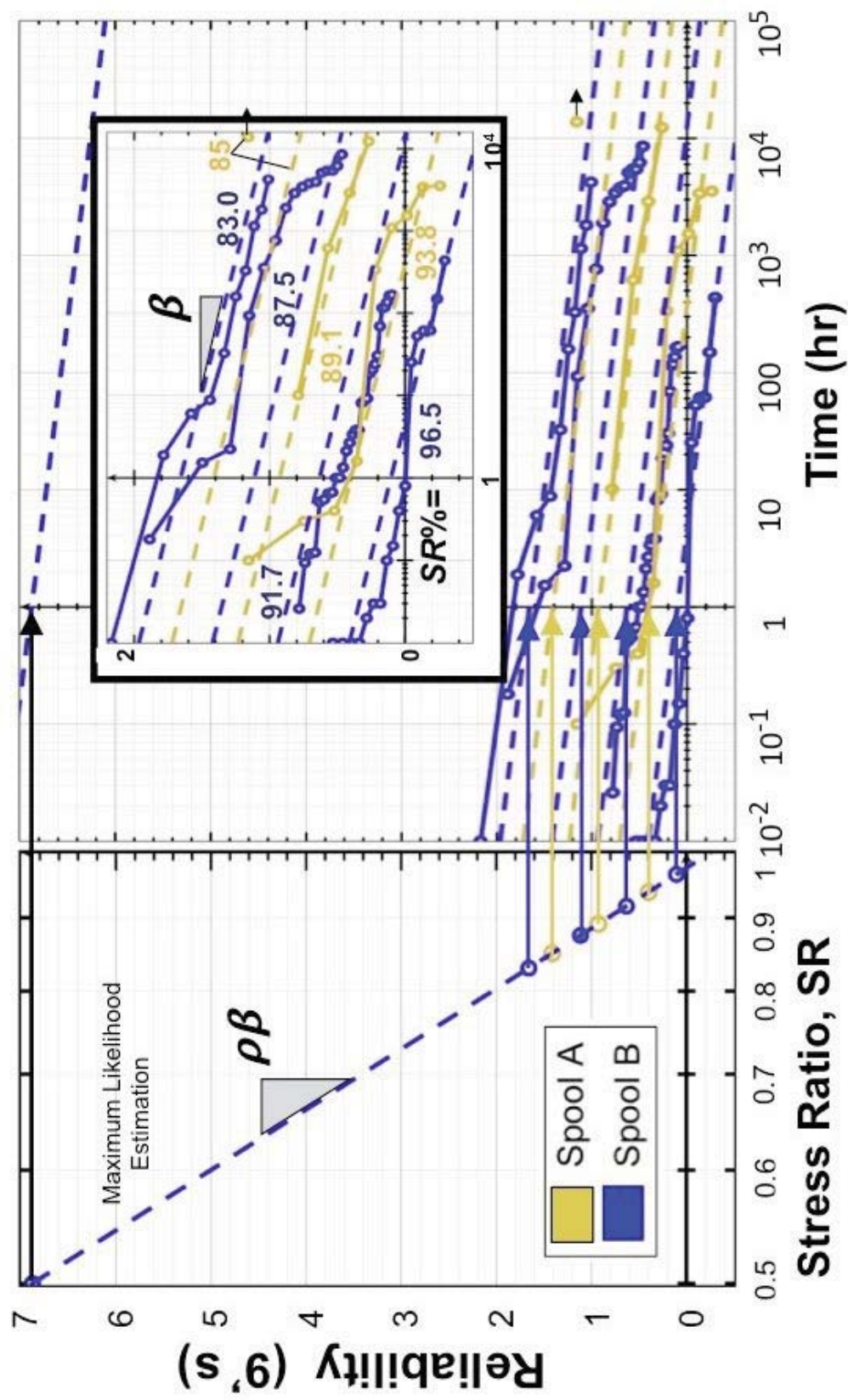
Figure 3. IM6/epoxy composite stress rupture data fit with a classic model[3].

Reliability predictions from the classic model are shown as dashed lines in the right-side plot of Figure 3. While the right-side plot indicates how reliability decreases with time at various stress levels, the left-side plot shows how the reliability curves shift with *SR*. The reliability at 1 hr is used to define the shift in the reliability curve with stress ratio as indicated by the arrows linking the left and right side plots. The special "Reliability in 9's" scale allows a Weibull distribution as assumed by the classic model to appear linear when plotted against log time. Notice that the right-side plot also includes an insert region showing an expanded scale in the range where data were collected.

The parameters of the classic model can then be chosen to fit the model to the data as shown in Figure 3. Note that the slopes of the classic model lines in the two plots are defined by the model parameters, ρ and β. A number of fitting routines can be used to select model parameters, including a simple least squares linear fit to the data. Here, a routine was used which maximizes the probability of the model explaining the observed response. This fitting routine is called the Maximum Likelihood Estimate (MLE) [4]. The classic model is therefore purely empirical, and as shown in Figure 3, there is significant extrapolation from the reliabilities where data are collected to the reliability and stress levels (50%) generally used in COPV design. When the classic model was fit to the IM6 data, the model parameters were determined to be a β=0.154, ρ=154, and t_{ref}=0.02 hrs.

It has been observed that when specimens that have not failed during a stress rupture test program are then tested for strength, no discernable reduction in strength is measured[5]. This seems inconsistent with the fact that for a stress rupture specimen to fail, the strength must drop to the applied stress level. A possible explanation for these inconsistent observations is that a small amount of strength reduction may occur early in life, but it is not enough to be observed over the normal scatter in strength values. The majority of strength loss then occurs near the end of life.

In addition, acoustic emission data that have been collected on specimens that are tested for stress rupture have, at times, measured an increase in activity close to the end of life[6]. Assuming that the acoustic activity indicates damage that would reduce the strength of the test specimens, these signals would indicate that most of the reduction in strength occurs late in life.

In the following sections, a new model for a stress rupture prediction will be proposed based on strength decay. This new strength decay model will help explain why no discernable strength loss has been observed in specimens that survive stress rupture testing. For constant stress tests, the strength decay model will also predict stress rupture failure rates very similar to the classic model.

COPVs in service are required to survive short periods of elevated loading as a proof test. Recently, concerns developed that this short period of high stress may damage COPVs and significantly reduce the reliability of these structures in service. The form of the classic model does not allow this possible drop in reliability to be modeled. A fiber breakage model was proposed to account for damage due to the proof cycle [7] and indicated that significant reductions in reliability could be expected. Careful examination of the assumptions underlying the model revealed significant flaws which cause the credibility of the model as originally

presented to be highly questionable [8], but concern that proof loading could damage COPVs still exists.

The strength decay model introduced in this paper models a reduced life of any given vessel due to the proof cycle. The model will also predict that the post proof reliability will increase significantly by eliminating weak vessels, which would tend to fail early. In fact, the model predicts periods of safety when no failures are predicted following a proof test. The strength decay model will be shown to predict reliabilities following a proof load that are higher than the classic model predictions. If the new model should be validated to be correct through testing, the improved understanding of higher reliability following proof loading could lead to certification and safe operation of COPVs with higher levels of stress, making the required structure lighter and more structurally efficient. The new model also helps explain why so few problems with COPVs in service have been reported. In addition, suggestions will be made for how to test to determine if the proposed model will predict composite stress rupture reliability after a proof test.

Strength Decay Model

Previous reliability models have only used time to failure data and normalized the results by original strength test data. No assumption is made about how the material changes with time from the initiation of loading until the specimen is failed. In the strength decay model, the strength of a given vessel will initially have a strength, s_o, which is unknown. It is unknown because a single vessel cannot be broken to determine a strength and broken at a lower stress level to determine a time to failure in stress rupture. This inability to know both the initial strength of a given specimen and the time to failure in stress rupture complicates the understanding of the stress rupture process. So the initial strength of a vessel under load will be unknown, and over time, the strength of the vessel, $s(t)$, will decay. Eventually the strength may drop to the level of the applied stress, σ, at which time the vessel fails. Instantaneous strength (the strength at any point in time) will be treated as an internal state variable by the strength decay model. Although the internal state variable may not be directly measurable, it is assumed to control the observed failure phenomenon.

Figure 4 shows how the strength might decay as a function of the time before failure $(t\text{-}t_f)$ assuming a power law relation for instantaneous strength, $s(t)$, as given by eq. 7.
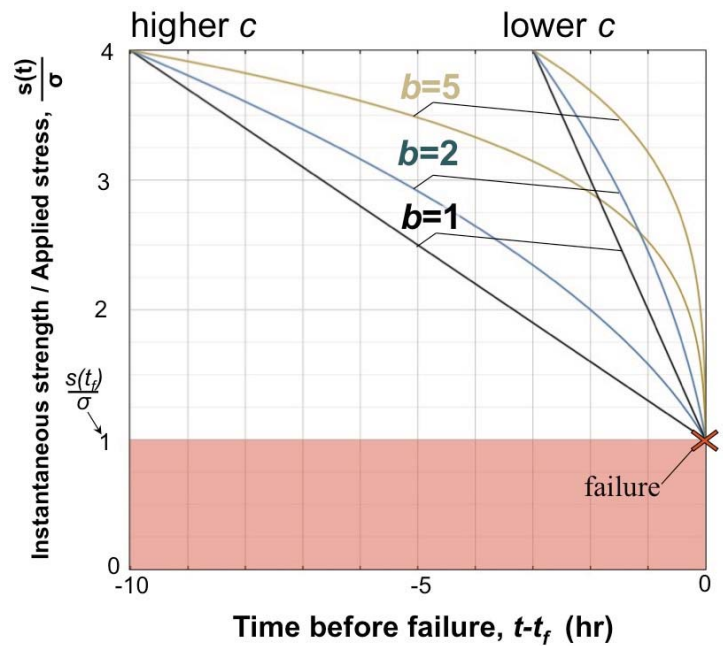

Figure 4. Strength decay to $s(t_f)/\sigma = 1$

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$$\frac{s(t)}{\sigma} = \left(1 - \frac{t - t_f}{c}\right)^{\frac{1}{b}} \tag{7}$$

so at failure $\sigma = s(t_f) \equiv s_f$ and $s(t_f)\big/\sigma = 1$. The parameter, b, determines the shape of the strength degradation curve with a b of one causing a linear decrease in strength with time. When $b>1$, the strength decay accelerates as the strength approaches the applied stress. The c parameter scales the rate of the strength decrease for a given value of b. When c is doubled, the time to failure from a given initial strength, s_o, is doubled. For given input parameters b and c, the time to failure is only a function of how high the initial strength is compared to the applied stress.

The curves can be shifted in time, as shown in Figure 5, so that all curves start at the same initial strength which shows how the time to failure would change as the model parameters b and c changed. The strength decay in Figure 5 was also normalized by the initial strength s_o. The reduction in strength and the time to failure is expressed as a function of the initial strength normalized by the applied stress in eq. 8 and 9.

$$\frac{s(t)}{\sigma} = \left[\left(\frac{s_o}{\sigma}\right)^b - \frac{t}{c}\right]^{1/b} \tag{8}$$

Figure 5. Strength decay for various parameter values

Failure will occur when $s(t_f)=\sigma$ or $s(t_f)\big/\sigma = 1$. Therefore, for a constant applied stress:

$$t_f = c\left(\left(\frac{s_o}{\sigma}\right)^b - 1\right) \quad \text{or} \quad \frac{\sigma}{s_o} = \frac{1}{\left(1 + \dfrac{t_f}{c}\right)^{\frac{1}{b}}} \quad \text{or} \quad s_o = \sigma\left(1 + \frac{t_f}{c}\right)^{\frac{1}{b}} \tag{9}$$

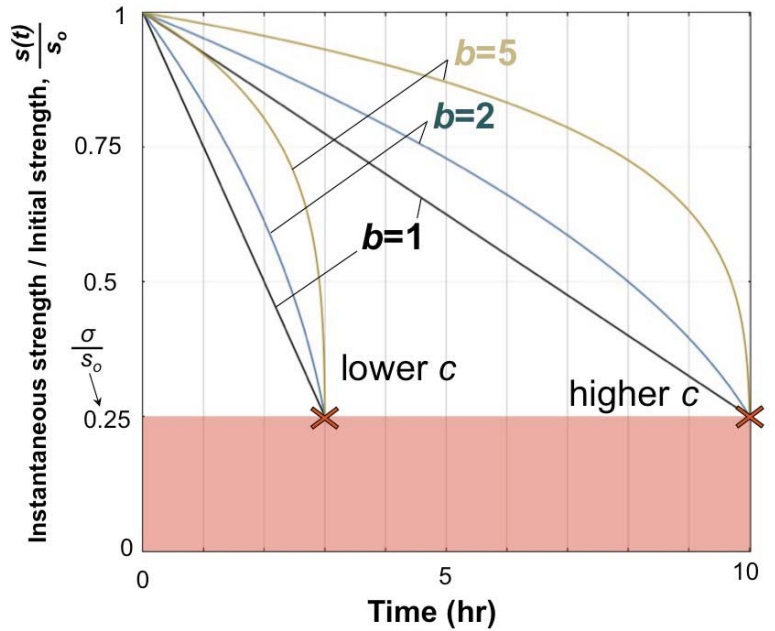Figure 6 shows s(t)/s$_o$ (eq. 8) for several values of applied stress $\sigma\big/s_o$. Notice all the solid lines start near 1 at small values of time and end where $s(t)/s_o = \sigma/s_o$. These points can be connected by the heavy dashed curve which is defined by eq. 9. In this example, values of $b=5$ and $c=1$ were used, and to include a longer period of time, the results are plotted against a log time scale. Notice that the strength is constantly decreasing, but at lower applied stress levels, the decrease in strength early in the life is imperceptibly small. The deterministic strength decay model suggested in this section was introduced without a clear motivation for precise mathematical form chosen. In the next section, this form of strength decay will be shown to produce stress rupture reliability response very similar to that shown in Figure 3 once the scatter in initial strength is considered.

Statistical Model

In the last section, a strength decay model was formulated based on the initial strength, s_o. However, when applying a stress rupture model, the initial strength of a given composite part will not be known. Instead, the strength distribution, **S**, of ostensibly similar specimens can be found through testing. To account for the scatter in initial strength, a statistical model is required.

Figure 7 shows how the time to failure might change assuming a collection of specimens that have a distribution of strengths, **S**, described by equation 3. Notice that the strength decay is now normalized by the scale parameter of the distribution of initial strengths, $\hat{s}_o$. Figure 7 shows that for selected values of b and c, a specimen loaded at 80% of the nominal value would have a time to failure of 2 hrs, if the specimen had the nominal strength $s_o/\hat{s}_o = 1$. However, if an individual specimen in the current example were 10% weaker than the nominal value, the time to failure would drop to 0.8 hrs while an abnormally strong specimen might fail in 6 hrs.

From eq. 9, the time to failure is controlled by the initial strength of a given specimen, and eq. 3 describes the distribution of initial strengths of a collection of specimens, **S**. The distribution in times to failure can be found by substituting eq. 9 into eq. 3. More precisely eq. 10 gives the probability of a specimen from the population having a time to failure, t_f, exceeding some time t.

Figure 6. Strength decay with time.
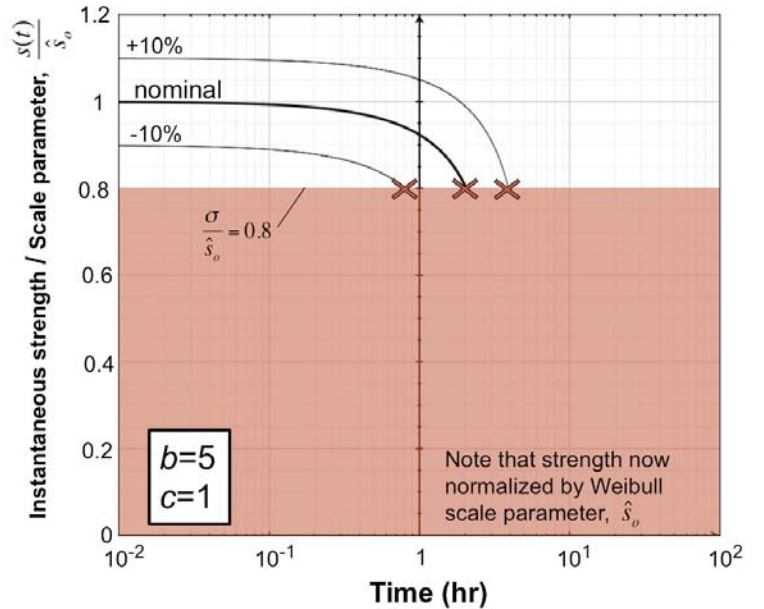
Figure 7. Time to failure accounting for variation in initial strength.

$$R(t) = P(t_f > t) = e^{-\left(\frac{\sigma}{\hat{s}_o}\left(1+\frac{t}{c}\right)^{\frac{1}{b}}\right)^{\alpha}} = e^{-\left(SR\left(1+\frac{t}{c}\right)^{\frac{1}{b}}\right)^{\alpha}} \tag{10}$$
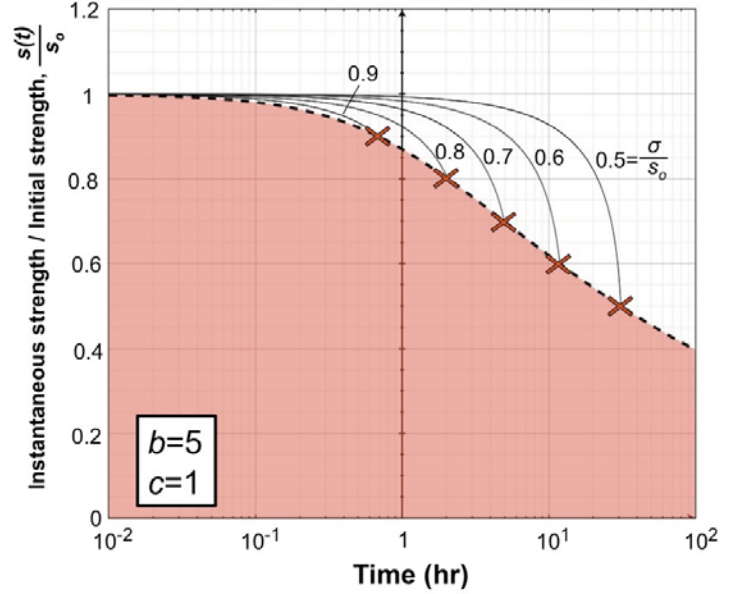
This is a reliability prediction similar to that of the classic model given by eq. 4. The ratio of applied stress to the scale parameter in strength has already been defined as the stress ratio (*SR*) which leads to the second equality of eq. 10. The strength decay reliability model is plotted with the classic model in Figure 8 along with IM6/Epoxy data presented earlier in Figure 3. The strength decay model is so similar to the classic model that it can be difficult to discern that they are not the same, but the strength decay model does have a small nonlinearity in the predicted reliabilities shown in the right side graph at less than 10^{-1} hr. This agreement was achieved by comparing eq. 4 to eq. 10 which leads to setting $\alpha = \rho\beta = 0.154 \times 154 = 23.7$ and $b = \rho = 154$, and $c = t_{ref} = 0.02$ hr. The models diverge slightly at short time periods, but data reported at very short time periods are questionable because the time to failure would be influenced by the time spent loading the specimen up to full load. Therefore, by carefully choosing model parameters, close agreement between the two models is achieved for all times where the data quality is high. The agreement between the models in Figure 8 is artificially high, however, because the strength decay model parameters were chosen to fit the classic model. This ignores the fact that the α parameter has physical meaning as the Weibull shape parameter (scatter) in initial strength. This initial strength was measured in the original study [1], and the Weibull strength parameter was determined to be 22.3 [3] which is only slightly different from the value that was used in Figure 8. Using $\alpha = 22.3$ and fitting the strength decay model to the IM6 data, the strength decay model shown in Figure 9 is created ($b = 147$, $c = 0.04$ hr). The agreement between the classic model and the strength decay model is not quite as precise as it was in Figure 8, but it is still quite close considering that the α parameter was determined independent of the plotted data. Note that the high value of b indicates that the strength decay is extremely slow until very late in life, when the
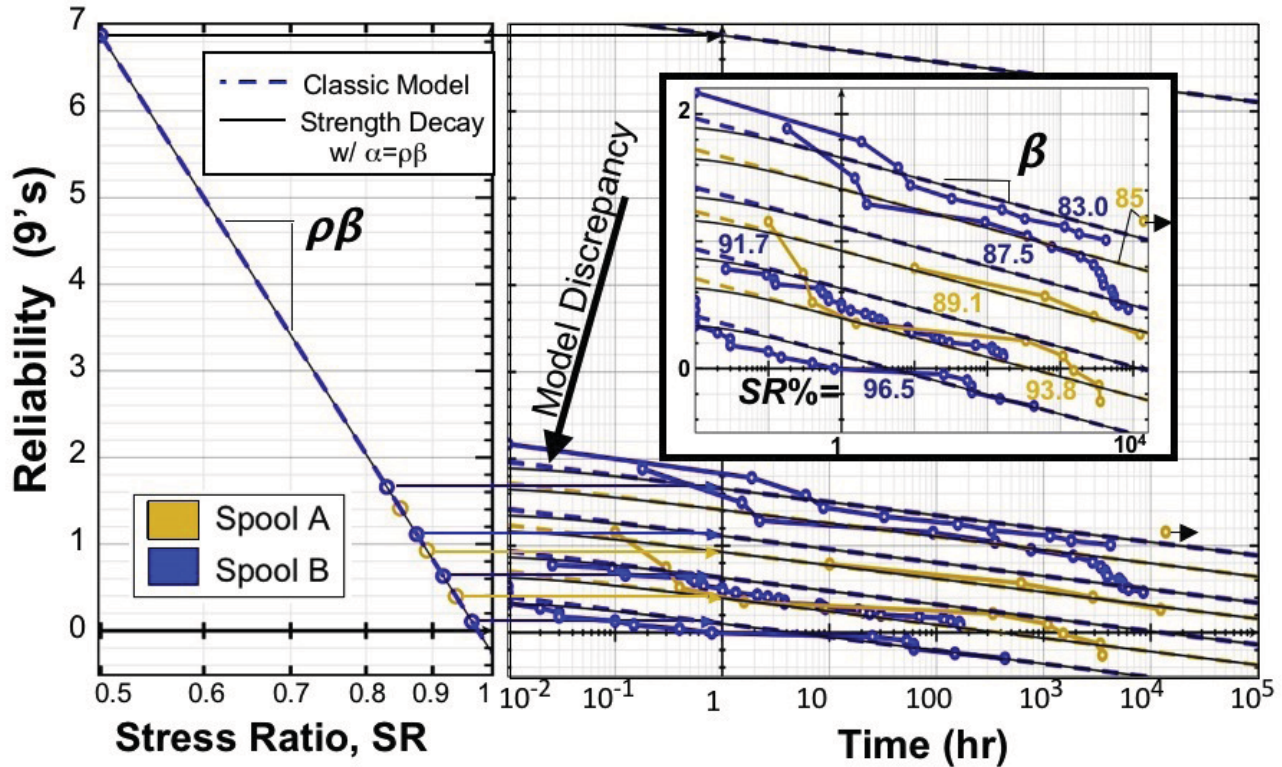

Figure 8. IM6/epoxy stress rupture data fit with the classic model and the strength decay model.

strength approaches the applied stress. This would explain why no perceptible loss in strength has been observed in specimens that survived a stress rupture test. The difference in reliability predictions at SR= 0.5 is an indication about how sensitive these extrapolated predictions are to small model changes. The two models in Figure 9 are quite close in the region where data are collected yet when the models are extrapolated out to SR=0.5, a difference in the models is perceived. The strength decay model directly incorporates an influence of initial strength scatter into the prediction of stress rupture failures. Because the classic model and the strength decay model give similar results for the constant stress load case, there would be little advantage of one over the other, if this were the load case of primary interest. However, all COPVs put in service receive a proof load, and the reliability predicted by the two models will be shown to be quite different after proof loading. Therefore, a validated strength decay model could result in improved predictions of reliability for the COPV's as they are actually used in service.

Classic Model Conditional Reliability and Reliability Following Proof Loading

Conditional reliability in stress rupture is simply the reliability at some point in time given that it survived to some shorter time, t_1. Equation 11 gives the equation for conditional reliability in terms of the reliability that specimen survived to t_1 *and* then to some longer time t, $R(t>t_1)$, which is then divided by the reliability at time t_1, $R(t_1)$.

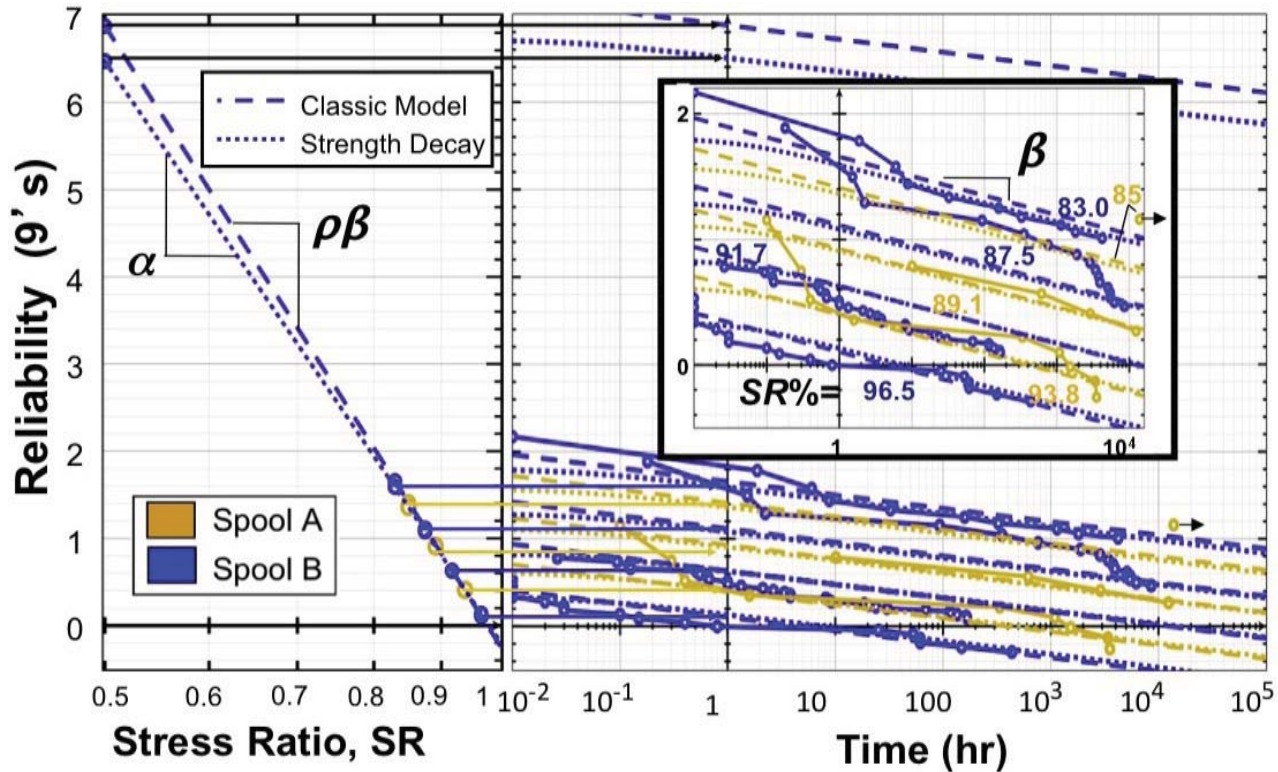$$R_c(t) = R(conditional\ on\ surviving\ t_1) = \frac{R(t > t_1)}{R(t_1)} \tag{11}$$

Figure 9. Strength decay model compared to classic model without enforcing $\alpha = \rho\beta$.

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So, for the classic model at a constant stress level, the conditional reliability is

$$R_c(t > t_1) = \frac{e^{-\left(\frac{t}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta}}}{e^{-\left(\frac{t_1}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta}}} = e^{\left(\frac{t_1}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta} - \left(\frac{t}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta}} \tag{12}$$

Figure 10 shows $R(t)$ and $R_c(t)$ for this case when $\mathrm{Pr} \equiv \sigma_{proof}/\sigma_{service} = 1$. However, for a proof test to be meaningful, the proof ratio should be higher than 1, and to certify COPVs, a proof load of between $Pr=1.25$ and 1.5 is generally required. Eq. 13 and 14 give the classic model reliabilities when the proof stress level is different from the normal operating level. The influence of short periods (e.g. 4 sec.) of stress overload are also shown in Figure 10.

$$R(t_1) = e^{-\left(\frac{t_1}{(Pr*SR)^{-\rho}\,t_{ref}}\right)^{\beta}} \quad \text{and} \quad R(t > t_1) = e^{-\left(\frac{t_1}{(Pr*SR)^{-\rho}\,t_{ref}} + \frac{t-t_1}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta}} \tag{13}$$

$$R_c(t > t_1) = e^{\left(\frac{t_1}{(Pr*SR)^{-\rho}\,t_{ref}}\right)^{\beta} - \left(\frac{t_1}{(Pr*SR)^{-\rho}\,t_{ref}} + \frac{t-t_1}{(SR)^{-\rho}\,t_{ref}}\right)^{\beta}} \tag{14}$$

It is clear from the plot that the predicted conditional reliability following proof loading is much higher than the reliability without proof loading. The model predicts that the larger the proof loading, the higher the increase in reliability. However, this increased reliability will eventually decrease, and at very long times, the reliability with and without proof loading will be the same. In an effort to be conservative, this predicted increase in reliability due to proof loading is normally not acknowledged during certification because of a fear that it might be artificial.
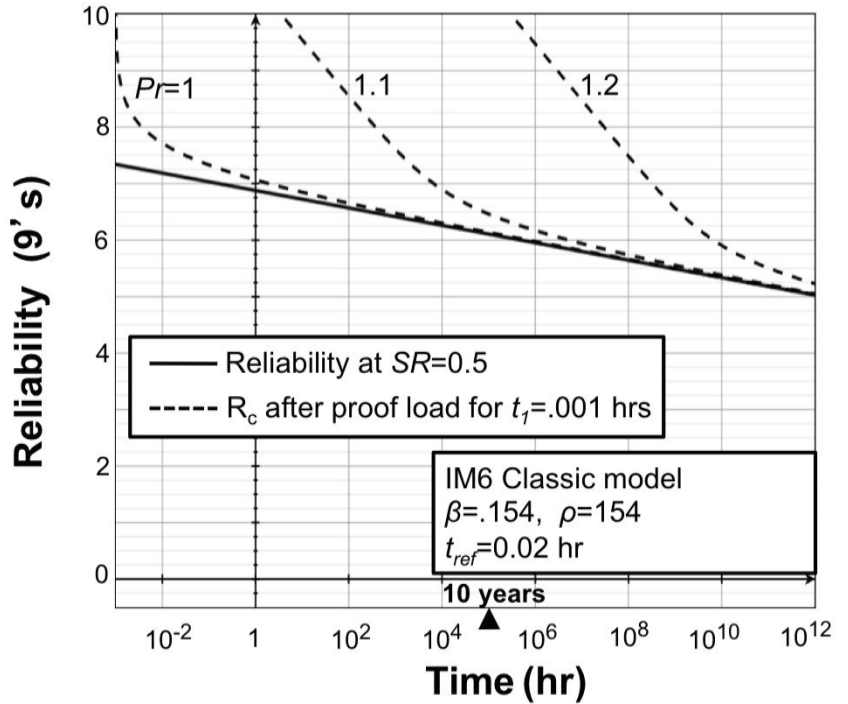

Figure 10. Increased reliability predicted by classic model proof due to loading.

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Strength Decay Model Conditional Reliability Following Proof Loading

In order to predict the reliability following proof loading, it is helpful to first predict how the strength is decreased due to the proof load. Using eq. 8, the strength at the end of the proof load for a given specimen with initial strength s_o can be expressed as

$$\frac{s(t_1)}{Pr\,\sigma} = \left[\left(\frac{s_o}{Pr\,\sigma}\right)^b - \frac{t_1}{c}\right]^{1/b} \tag{15}$$

With the strength decay model, the remaining time to failure is only a function of the instantaneous strength and the applied stress level. This was shown in Figure 4. Starting with the strength at the end of proof loading, the strength decrease under normal loading can then be derived by substituting eq. 15 back into eq. 8 as follows

$$\frac{s(t > t_1)}{\sigma} = \left[\left(\frac{s(t_1)}{\sigma}\right)^b - \frac{t - t_1}{c}\right]^{\frac{1}{b}} = \left[\left(\frac{Pr\,\sigma\left[\left(\frac{s_o}{Pr\,\sigma}\right)^b - \frac{t_1}{c}\right]^{1/b}}{\sigma}\right)^b - \frac{t - t_1}{c}\right]^{\frac{1}{b}} \tag{16}$$

$$= \left[\left(\frac{s_o}{\sigma}\right)^b - \frac{t}{c} + \frac{t_1}{c}\left(1 - \left(Pr\right)^b\right)\right]^{\frac{1}{b}}$$

Figure 11 shows the strength decay similar to Figure 6, but with the effect of changing the stress from $Pr\,\sigma/s_o = 0.8$ to some other ratio after 1 hr. Note that again the example model parameters of $b=5$ and $c=1$ are used here. In this example, the initial hour of stress at $\sigma = 0.8\,s_o$, decreased the strength to $s(t_1) = 0.935 s_o$. If after 1 hr, the applied stress is changed, the rate of strength degradation will change as indicated by the change in slope of the strength degradation curves. The black dashed curve gives the time to failure if the specimen was held at a constant load for the
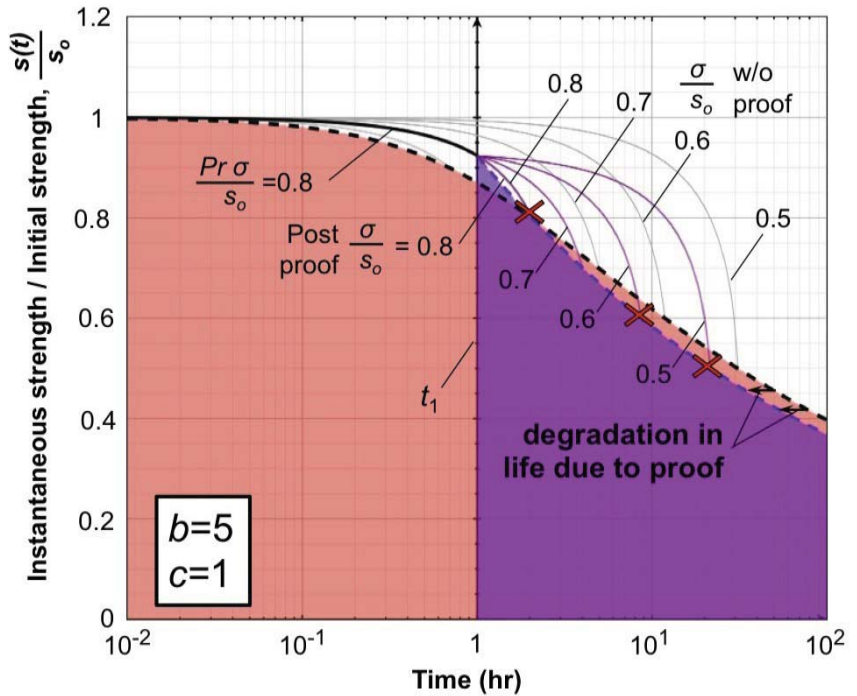

Figure 11. Strength decay after 1 hr of applied stress at s/s₀=0.8.

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entire time. The purple dashed curve, which starts at 1 hr and soon drops below the black dashed curved, shows the time to failure when the first hour of loading is at $Pr\,\sigma/s_o = 0.8$. So, in this example, a specimen with a constant stress level of $0.5\,s_o$ would fail in 30 hrs. If the specimen received a proof load for 1 hr at $Pr\,\sigma/s_o = 0.8$ before being loaded at $0.5\,s_o$, then the total time to failure would be reduced to 20 hrs. Therefore, the strength decay model does address reduced life due to the elevated stress.
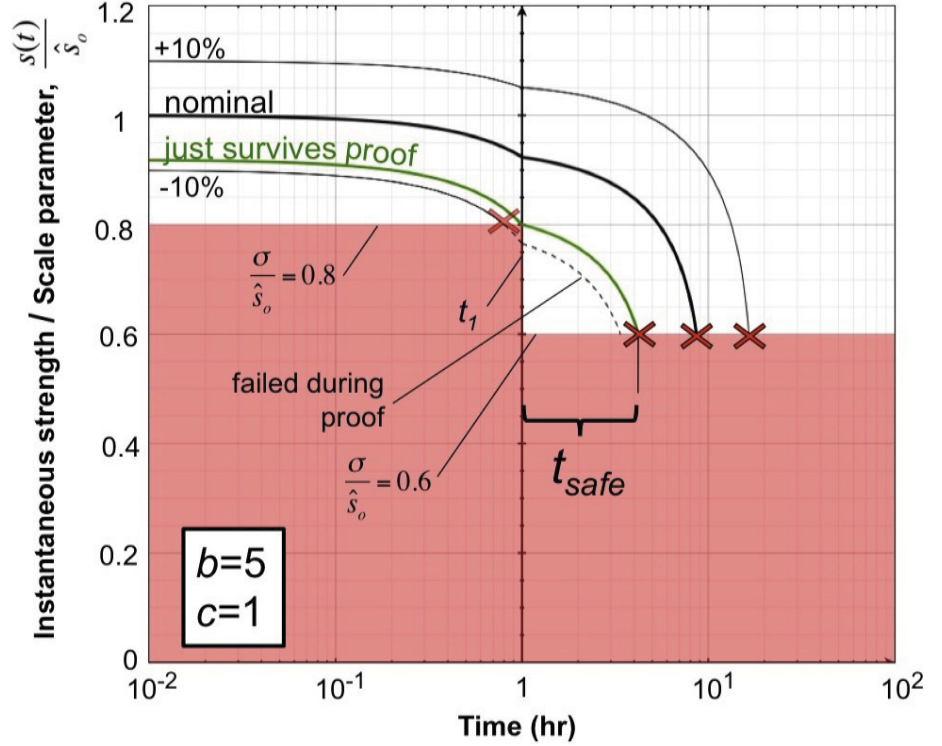

Figure 12. Strength decay with proof loading.

The reliability of a group of specimens surviving proof and going on to survive some additional time to t_f can be derived by first solving the deterministic strength decay model given by eq. 16 for s_o when $s(t_f)/\sigma = 1$

$$s_o = \sigma\left(1 + \frac{t_f}{c} - \frac{t_1}{c}\left(1 - Pr^b\right)\right)^{1/b} \qquad \text{for } t_f > t_1. \tag{17}$$

Substituting eq. 17 into eq. 3 translates the variation in initial strengths into a variation in times to failure resulting in the reliability equation given by eq. 18.

$$R(t) = P(t_f > t) = e^{-\left(SR\left(1 + \frac{t}{c} - \frac{t_1}{c}\left(1 - Pr^b\right)\right)^{\frac{1}{b}}\right)^{\alpha}} \tag{18}$$

Figure 12 shows how the instantaneous strength degrades as the initial strength varies about a nominal value, $\hat{s}_o$, similar to Figure 7, but this time with a 1 hr proof loading at $Pr\,\sigma/\hat{s}_o = 0.8$ followed by nominal loading at $\sigma/\hat{s}_o = 0.6$. It is clear from the figure that the time to failure is significantly influenced by the scatter about the nominal value. If the initial strength is too low, the specimen will fail during proof loading. To survive proof loading, the strength at t_1 must be at least $Pr\,\sigma$. From eq. 9, the original strength must be
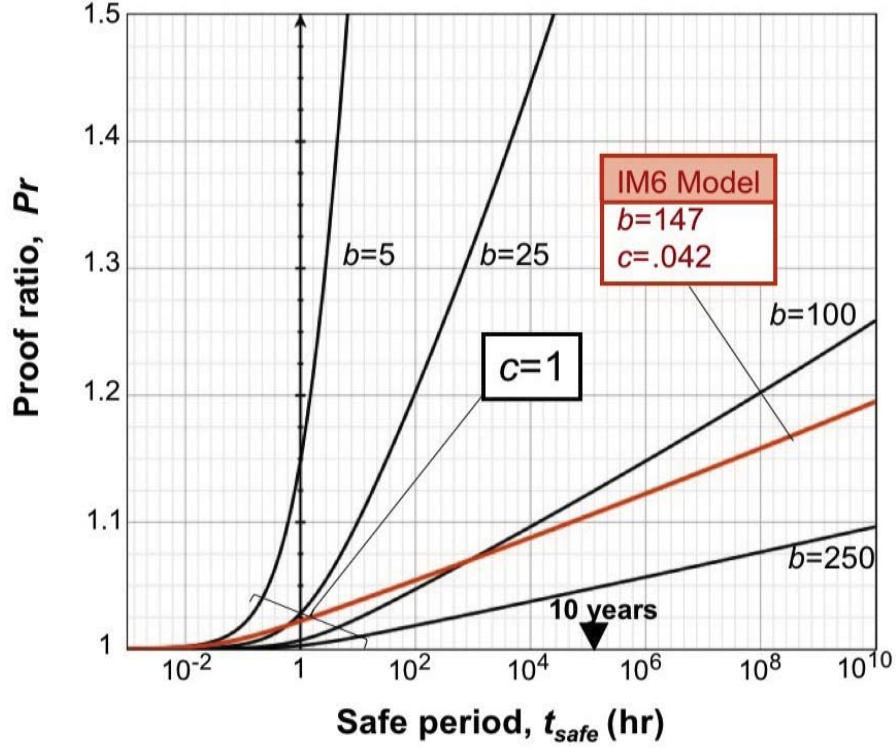
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Figure 13. Safety period, t_{safe}, predicted for the strength decay model for various values of b.

$$s_o > Pr \ \sigma \left(1 + \frac{t_1}{c}\right)^{\frac{1}{b}} \tag{19}$$

Following proof loading, there is a period where no failures are predicted because time is required to decrease the strength from $Pr \ \sigma$ to σ. Using eq. 8, the time required for this strength decrease can be defined as t_{safe}, a safe period, where

$$1 = \frac{s(t)}{\sigma} = \left[\left(\frac{Pr \ \sigma}{\sigma}\right)^b - \frac{t_{safe}}{c}\right]^{1/b} \qquad \text{or} \qquad t_{safe} = c\left(Pr^b - 1\right) \tag{20}$$

Figure 13 shows how t_{safe} is affected by the model parameters b and c. The c parameter controls the approximate time scale when the safe period initiates as the proof ratio is increased above 1. This can be seen in the family of curves with $c=1$ on Figure 13. The b parameter controls how quickly the t_{safe} grows as the proof ratio is further increased as can be seen from the curves where b increases from 5 to 250. Changing c would shift the group of curves either left or right, but since the time axis is logarithmic, an order of magnitude change in c would be needed to make much of a difference. Also shown in Figure 13 are the predicted t_{safe} periods for the IM6 model parameters determined from fitting the data presented in Figure 9. The plots show that to attain a safe period of greater than 10 years, a proof level just over $Pr=1.1$ would be required.

The reliability for a group of specimens surviving t_1 can be found by substituting eq. 19 into eq. 3 as

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$$R(t_1) = e^{-\left(Pr\ SR\left(1+\frac{t_1}{c}\right)^{\frac{1}{b}}\right)^{\alpha}}$$

(21)

and conditional reliability from eq. 11 becomes

$$R_c(t) = e^{\left(Pr\ SR\left(1+\frac{t_1}{c}\right)^{\frac{1}{b}}\right)^{\alpha} - \left(SR\left(1+\frac{t}{c}-\frac{t_1}{c}\left(1-Pr^b\right)\right)^{\frac{1}{b}}\right)^{\alpha}}$$

(22)

The conditional reliability for IM6/Epoxy is plotted in Figure 14. Notice that the conditional reliability from the strength decay model is even higher than that from the classic model. Also, notice that for the classic model as time is decreased, the conditional reliability curves become linear. In contrast, as time is decreased for the strength decay model, conditional reliability curves each appear to approach infinity at some point (the curves become vertical). The point at which each curve approaches infinity is $t_1 + t_{safe}$. At times less than $t_1 + t_{safe}$, no failures are expected, and the reliability is infinite. As shown in Figure 13, the level of proof loading can have a dramatic effect on the length of the safe period following proof. Also note, from eq. 20, that the safe period is not a function of SR.

Because of the large b parameter, only a small proof ratio is needed to create very long safe periods, and these safe periods are not affected by the operating SR. This suggests that if the strength decay model is validated to be correct, COPVs and similar structures can have safe operation for very long periods of time with fairly low proof loads, even if an increase in the
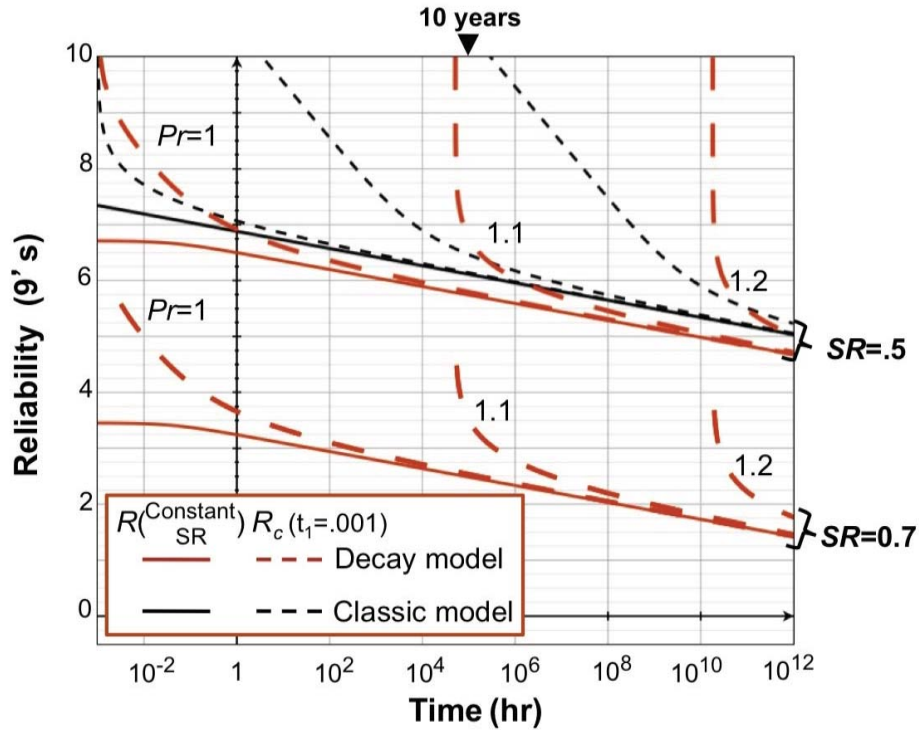

Figure 14. Conditional reliability from the strength decay model.

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operational stress were allowed from a normal *SR*=0.5 to perhaps *SR*=0.7. Although the model indicates the *SR* could be raised to any level without affecting the safe period, setting the *SR* too high would cause problems. For instance, an operating *SR*=0.8 with a *Pr* of 1.3 would create a proof stress ratio of *SR$_P$*=1.04 where most of the specimens would fail during the proof cycle.

Suggested validation study for the strength decay model

As shown in the last section, if the strength decay model can be validated, COPVs and other structures can be operated reliably at much higher stress levels. This will improve the structural efficiency of these critical structures. When testing stress rupture theories, it has proven difficult to test enough specimens for long enough periods of time to observe sufficient failures. Numerous failures are needed to show clear trends in a phenomenon that exhibits significant scatter. This is further complicated by the sharply decreasing failure rate, which means that the longer a specimen is tested, the less likely a failure is to occur. The problem with creep rupture testing is clearly seen by considering an example where predictions are desired. Normal operating conditions might be at a *SR*=0.5 after surviving a proof loading of *Pr*=1.25. From Figure 14, a proof load between 1.2 and 1.3 would have six 9's of reliability at 10^8 hrs. Clearly, no one would be willing to test 1 million specimens for 10,000 years to observe just one failure. Even if this type of testing could be performed, comparing a model to just one experimentally measured failure would not provide much of a model validation because the phenomenon exhibits such large scatter.

Figure 15, shows an "Observable Region" which is the region on the reliability plot where data might be collected to validate a model. The region does not extend above 1.25 9's, which is about 1 failure in 20. Because several failures are desired at a condition to show clear trends in a
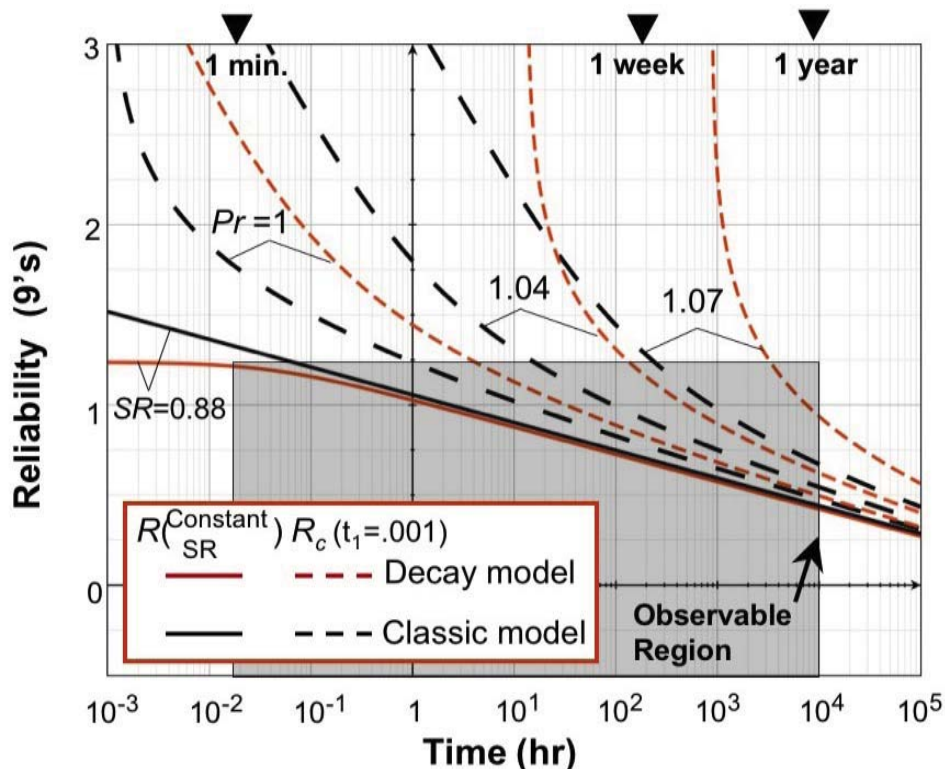

Figure 15. Testing the strength decay model.

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<p align="center">**Table 1. Suggested strength decay study.**</p>

Proof Level	Operating Stress	Proof Ratio	Number of Specimens				Time, hrs		
SR_P	SR	Pr	Initial	survive proof	on test	survive testing	t_1+t_{safe}	$t_{first\ failure}$	end
Strength Tests									
	Strength		30						
Stress Rupture Tests									
88%	88%	1	21	20	5	15	0.001	1	10,000
91.5%	88%	1.04	23	20	4	16	13.4	60	10,000
94.2%	88%	1.07	26	20	2	18	876	2024	10,000

phenomenon that exhibits significant scatter, testing in this range would mean testing between 60 and 100 specimen to observe 3 to 5 failures, respectively. This is the number of specimens at just one condition. Reliability testing must also be completed in a reasonable amount of time. For this reason, the observable region does not extend above 10^4 hrs or about a year. Testing for very short periods of time is also problematic because these very early results will be influenced by the time spent loading the specimen to full load. For this reason, the observable region does not extend below 0.0015 hrs or about a minute. Although the boundaries of the observable region are not absolute, time is plotted on a log scale and reliability is plotted on effectively a log scale. Therefore, large changes in the number of specimens tested or in the length of time tested would be needed to significantly change the size of the "observable region." Designing a test program to show an effect inside the shaded observable region becomes the objective. To obtain failures in a reasonable test time, the proof ratio level must be kept fairly low while keeping the operations stress level fairly high as demonstrated later in the example. As the proof load level $SR \cdot Pr$ approaches 1, the majority of the specimens will fail in the proofing process, which should also be avoided.

Also shown in Figure 15 are reliability curves for a proposed test plan. This test plan is outlined in Table 1 to provide some validation of the strength decay model following proof loading. Thirty specimens would be used to determine the initial strength distribution of the specimens. Three different proof levels would be tested ($Pr= 1$, 1.04 and 1.07) with the proof lasting 4 seconds. A high operating stress ratio of $SR=0.88$ is chosen to reduce the reliability to a level where several failures would be expected within one year. The table shows that with the high operating stress ratio and a proof ratio of 1.07, the stress ratio during proof loading is 0.994. At this load level, approximately 26 specimens would need to be proof loaded to obtain 20 that survived. Fewer initial specimens would be required at the lower proof levels. Starting stress rupture testing with 20 specimens surviving proof loading at each level, 5 specimens with $Pr =1$ would be expected to fail within 10,000 hours at $SR=0.88$, similarly 4 failures with $Pr=1.04$, and 2 failures with $Pr=1.07$. This number of failures is minimally sufficient to show trends in the response because significant variability in time to failure might be expected from any one test. Just as important as the number of failed specimens is the time when the specimens failed. Because of the predicted safe period, the strength decay model predicts that the number of failures will occur much later in the test period than would be predicted by the classic model. The reliability predictions for this proposed test plan are plotted in Figure 15 to show how portions of the reliability curves pass through the observable region. Testing a lower SR or

higher *Pr* would tend to increase the reliability predictions so that they fall outside the observable region. Testing at a higher *SR* would increase the proof load level so high that a large majority of specimens would fail during proof loading.

Tight control of the load level will be particularly important in the proposed study because all of the load levels are close together and close to the nominal strength. It would also be wise to avoid removal of the specimen from the test fixture between proof loading and stress rupture testing so that changes in gripping do not influence the results. Following proof loading, specimens should be visually inspected, and any damage that would normally cause a specimen to be rejected should be noted. It is recommended that all surviving specimens be tested, but the data should be analyzed with and without the specimens that have obvious damage. The strength decay model should be evaluated by first fitting the strength data to obtain α and $\hat{s}_o$, and then the stress rupture data without proof loading (or proof level *Pr*=1) should be used to fit the *b* and *c* parameters. The updated model can then be used to make new conditional reliability predictions. Adjustments to the proof levels can be made, if required, to obtain adequate numbers of predicted failures within one year. The proofed stress rupture tests can be performed, and the results compared to the updated model. Random scatter would allow some failures prior to the predicted first failure, but no failures would be expected before the end of the t_{safe} period.

A total of 100 test specimens would be required for this study with 10 expected to fail during proof loading. This test program could be run on strands or COPV vessels, but because strands are relatively inexpensive compared to vessels, it is recommended that strand testing be performed before vessel testing is considered.

Summary and Conclusions

A new reliability model has been proposed for stress rupture of COPVs and similar composite structures. The reliability model is based on assumed strength degradation over time. The strength decay model can predict a similar response to the "classic model" for stress rupture under constant stress. Moreover, the model predicts that even though there is strength degradation during a proof load, there is significant increase in reliability gained by eliminating any weak vessels. The model suggests that most strength decay does not occur until the decreasing strength approaches the applied stress level late in life, and this behavior helps explain why strength tests following prolonged stress rupture testing have not exhibited a measurable reduction in strength. The model also is consistent with increased levels of acoustic activity late in the life of a COPV. The model predicts that there should be significant periods of safe life following a proof loading because of the time required for the strength to decay from the proof load level to the subsequent loading level. Validating reliability stress rupture models is very difficult because of the high degree of scatter exhibited in observed times to failure. Suggestions for testing the strength decay reliability model have been made which allows observations with a reasonable number of test specimens and within a reasonable time period. Should the reliability model be validated to be accurate, COPVs may be safely designed to carry higher levels of stress than is currently allowed, which will enable the production of lighter structures.

References

1. Shaffer, J.T., "Stress Rupture of Carbon Fiber Composite Materials," in 18th International SAMPE Conference, 1986.
2. Cameron, K.D. et al., "Shelf Life Phenomenon and Stress Rupture Life of Carbon/Epoxy Composite Overwrapped Pressure Vessels," NASA Engineering and Safety Center, RP-06-83, 2006.
3. Grimes-Ledesma, L., et al., "Composite Pressure Vessel Working Group (CPVWG) Task 3: Stress Rupture Test Approach," NASA Engineering and Safety Center, RP-09-00537, 2012.
4. Abernethy, R.B., *The New Weibull Handbook, 5th Edition*, 2006.
5. Moore, R.L., M.A. Hamstad, and T.T. Chiao, "Stress Rupture Behavior of Graphite Fiber/Epoxy Strands," *Composite Materials and Structures*, 3(1): p. 19-23, 1974.
6. Hamstad, M.A., "Local Characterization of Fiber Composites by Acoustic Emission," in *A Critical Review: Techniques for the Characterization of Composite Materials*, Office of Naval Research: Cambridge, MA, 1981.
7. Phoenix, S.L. and L.N.P. Murthy, "Pro's and Cons of Proof Testing Carbon Composite Overwrapped Pressure Vessels: A Comparison of Two Mathematical Models," in 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2007.
8. Reeder, J.R., "A Critique of a Phenomenological Fiber Breakage Model for Stress Rupture of Composites Materials," NASA Langley Research Center, NASA TM-2010-216721, 2010.

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14. ABSTRACT

A model is proposed to estimate reliability for stress rupture of composite overwrap pressure vessels (COPVs) and similar composite structures. This new reliability model is generated by assuming a strength degradation (or decay) over time. The model suggests that most of the strength decay occurs late in life. The strength decay model will be shown to predict a response similar to that predicted by a traditional reliability model for stress rupture based on tests at a single stress level. In addition, the model predicts that even though there is strength decay due to proof loading, a significant overall increase in reliability is gained by eliminating any weak vessels, which would fail early. The model predicts that there should be significant periods of safe life following proof loading, because time is required for the strength to decay from the proof stress level to the subsequent loading level. Suggestions for testing the strength decay reliability model have been made. If the strength decay reliability model predictions are shown through testing to be accurate, COPVs may be designed to carry a higher level of stress than is currently allowed, which will enable the production of lighter structures.

15. SUBJECT TERMS

COPV; Composite Overwrap Pressure Vessel; Reliability Model; Stress Rupture

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