

Root Raised Cosine (RRC) Filters and Pulse Shaping in Communication Systems

Erkin Cubukcu

Abstract

This presentation briefly discusses application of the Root Raised Cosine (RRC) pulse shaping in the space telecommunication. Use of the RRC filtering (i.e., pulse shaping) is adopted in commercial communications, such as cellular technology, and used extensively. However, its use in space communication is still relatively new. This will possibly change as the crowding of the frequency spectrum used in the space communication becomes a problem. The two conflicting requirements in telecommunication are the demand for high data rates per channel (or user) and need for more channels, i.e., more users. Theoretically as the channel bandwidth is increased to provide higher data rates the number of channels allocated in a fixed spectrum must be reduced. Tackling these two conflicting requirements at the same time led to the development of the RRC filters. More channels with wider bandwidth might be tightly packed in the frequency spectrum achieving the desired goals. A link model with the RRC filters has been developed and simulated. Using 90% power Bandwidth (BW) measurement definition showed that the RRC filtering might improve spectrum efficiency by more than 75%. Furthermore using the matching RRC filters both in the transmitter and receiver provides the improved Bit Error Rate (BER) performance.

In this presentation the theory of three related concepts, namely pulse shaping, Inter Symbol Interference (ISI), and Bandwidth (BW) will be touched upon. Additionally the concept of the RRC filtering and some facts about the RRC filters will be presented.



Root Raised Cosine Filters & Pulse Shaping in Communication Systems

Erkin Cubukcu
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Outline



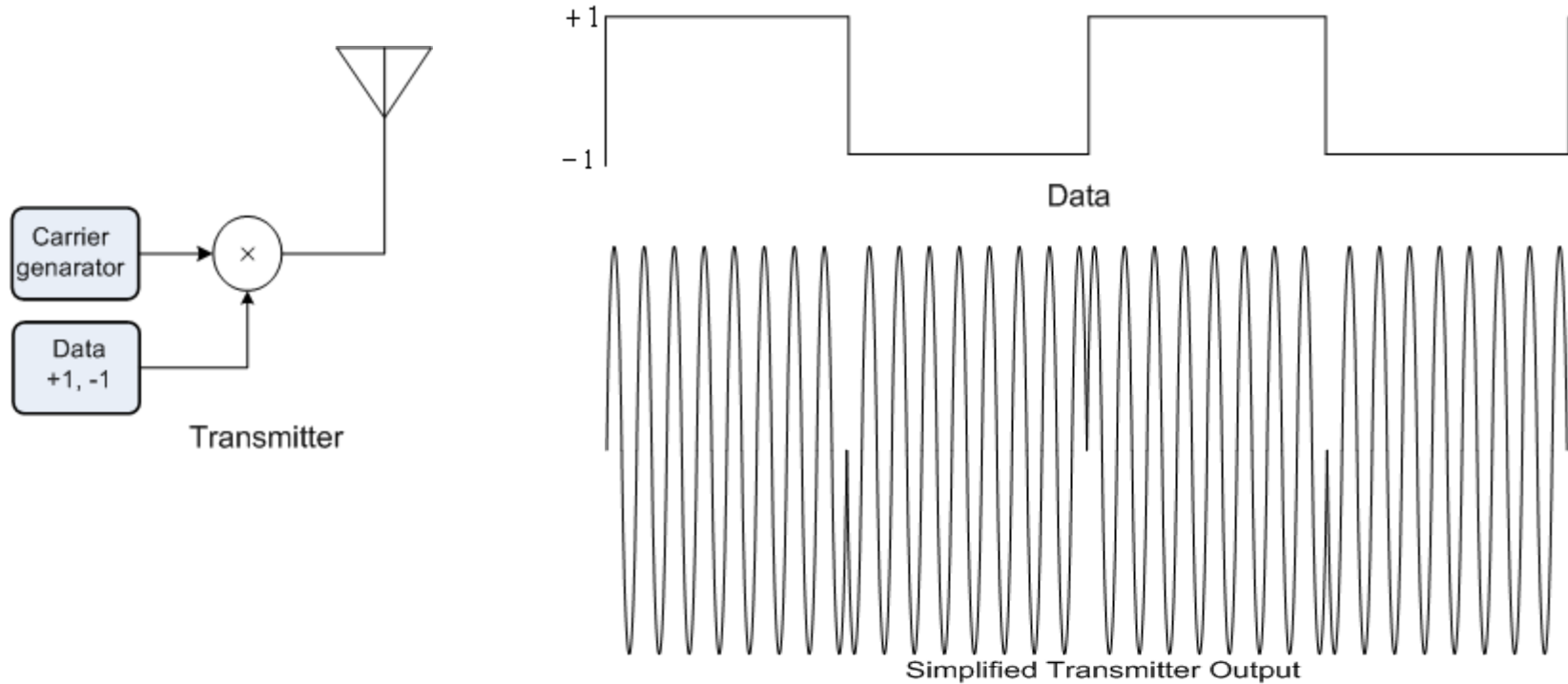
- **Pulse Shaping, Intersymbol Interference (ISI), and Bandwidth**
- **Ideal Low Pass Filter (LPF)**
- **Raised Cosine (RC)**
- **Root Raised Cosine (RRC)**
- **Facts about RRC**
- **Link Modeling with the RRC Filters**
- **Signal Spectra**
- **BER Plots**
- **Conclusions**



Pulse Shaping, Intersymbol Interference (ISI), and Bandwidth



Digital Modulation

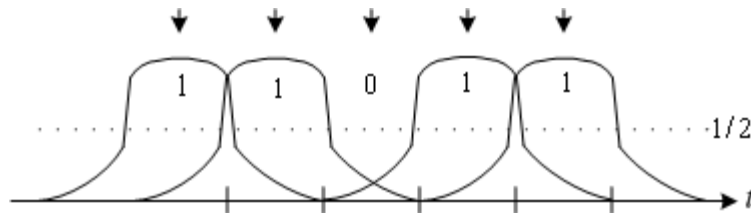




Pulse Shaping, Intersymbol Interference (ISI), and Bandwidth (con't)



- Theory



- Two conflicting requirements!:
 - Demand for High data rates (more information).
 - Need for Narrow Bandwidth (more users, more channels, less noise).
- If channels are too narrow the symbols will be too wide, hence. At sampling points (in time) there will be signal (tails) of the previous and next symbols. This is called Inter-symbol Interference (ISI)
- One possible solution is to use an Ideal Low Pass Filter (ILPF) (rectangular in frequency).

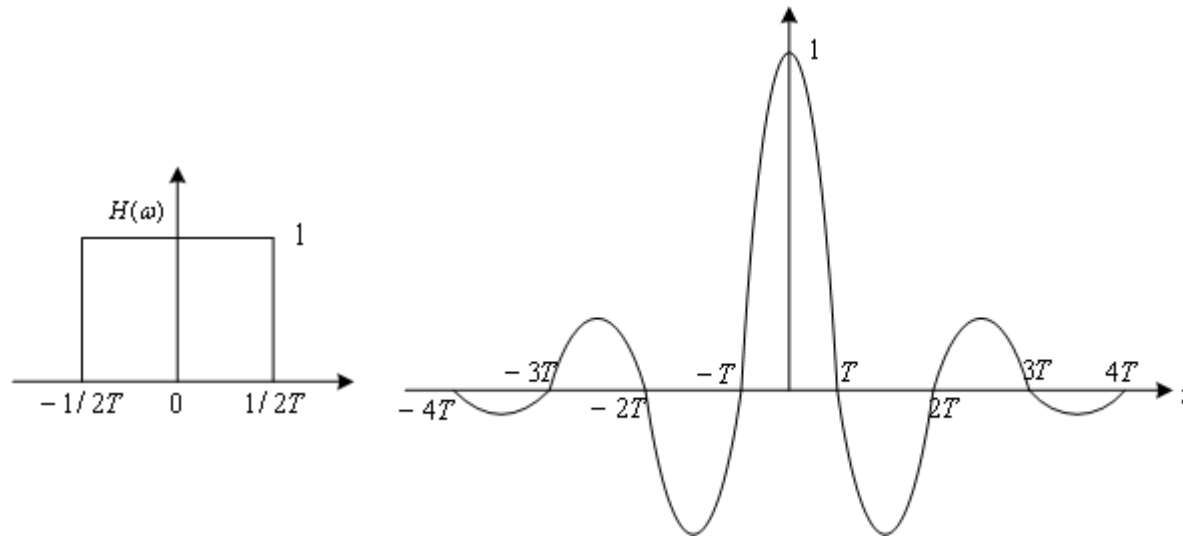


Ideal Low Pass Filter (ILPF)



- No ISI
- Narrow bandwidth channel

$$\frac{\sin \pi t / T}{\pi t / T}$$



- Issues
 - ▶ Physically unrealizable and difficult to approximate.
 - ▶ If attainable, require extreme precise synchronization, synchronization is a problem (jitter in the system might be detrimental).



Solution : Raised Cosine (RC) Filters



- It is shown by Nyquist that
 - If the frequency characteristic has odd symmetry at the cutoff frequency, the impulse response will have zeros at uniformly spaced intervals.
- Much simpler to attain
- Effects of jitter may be minimized

$$H(f) = \begin{cases} T & \left(0 \leq |f| \leq \frac{1-\beta}{2T} \right) \\ \frac{T}{2} \left\{ 1 + \cos \left[\frac{\pi T}{\beta} \left(|f| - \frac{1-\beta}{2T} \right) \right] \right\} & \left(\frac{1-\beta}{2T} \leq |f| \leq \frac{1+\beta}{2T} \right) \\ 0 & \left(|f| > \frac{1+\beta}{2T} \right) \end{cases} \quad (\text{Eq. 1})$$

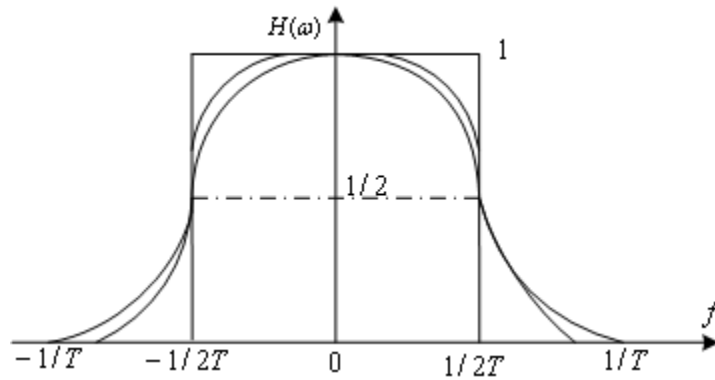
$$h(t) = \frac{\sin \pi t / T}{\pi t / T} \frac{\cos \pi \beta t / T}{1 - (4\beta^2 t^2 / T^2)} \quad (\text{Eq. 2})$$



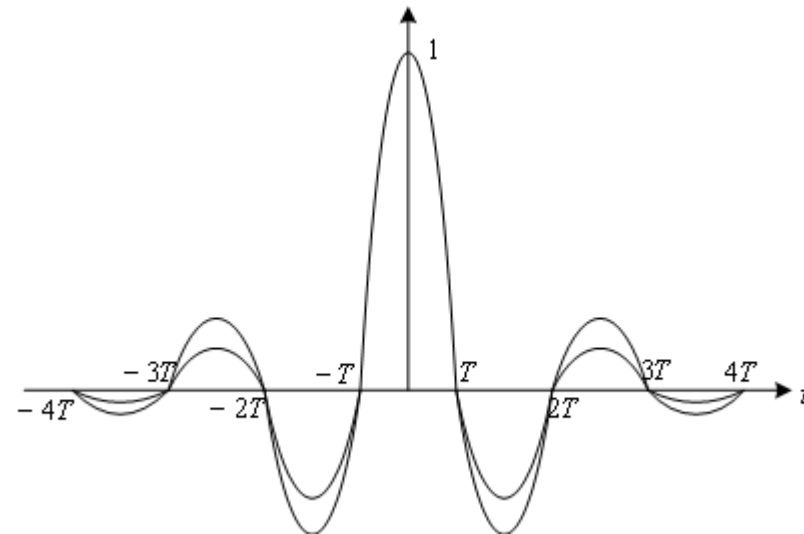
Plot of Raised Cosine (RC) filter



- Frequency Response



- Impulse Response





Raised Cosine Filter (Cont)



- Impulse response now has a sinc term that ensures that it has zero crossings as like ideal low pass filter.

$$\frac{\sin \pi t / T}{\pi t / T}$$

- In addition, it has another term

$$\frac{\cos \pi \beta t / T}{1 - (4 \beta^2 t^2 / T^2)}$$

- That decays in time hence reduces the tails reducing the impact of jitter.



Bandwidth of Raised Cosine (RC) Filter



$$\frac{\sin \pi t / T}{\pi t / T} \quad \text{Where the sampling time is } T$$

$$\omega_c t = \pi t / T \rightarrow \omega_c T = \pi T / T \rightarrow \omega_c T = \pi \rightarrow T = \pi / \omega_c = \pi / (2\pi f_c) = 1 / 2 f_c$$

- If ideal LPF were used the baseband bandwidth would be

$$B = f_c = 1 / 2T \quad \text{Nyquist Bandwidth}$$

- Since $H(f) = 0 \left(|f| > \frac{1 + \beta}{2T} \right)$

Bandwidth for a realizable RC filter

$$B = (1 + \beta) / 2T \rightarrow \text{Nyquist Bandwidth times } 1 + \beta$$

- So the baseband transmission bandwidth

$$B = (1 + \beta) f_c$$



Root Raised Cosine (RRC) Filter



- The overall channel transfer function must be Raised Cosine (RC) as discussed above.
- One way of achieving it is to take square root of the raised cosine filter in frequency domain and use this new filter in the Tx and Rx. This is the so called Root Raised Cosine filter.

$$H_{rrc}(\omega) = \sqrt{H(\omega)_{rc}} = \sqrt{\frac{1}{2}(1 + \cos \pi\omega / 2\omega_c)} \quad |\omega| < 2\omega_c$$

- When the transmitter and receiver filters are cascaded one gets raised cosine filter transfer characteristic

$$H_{rc}(\omega) = H_{rrc,tx}(\omega)H_{rrc,rx}(\omega)$$

- Or

$$H_{rc}(\omega) = \sqrt{H_{rc}(\omega)}\sqrt{H_{rc}(\omega)}$$



Root Raised Cosine Filter (RRC) (con't)



- By taking square root of RC filter frequency response, one gets.

$$H(f) = \begin{cases} \sqrt{T} & \left(0 \leq |f| \leq \frac{1-\beta}{2T} \right) \\ \sqrt{\frac{T}{2} \left\{ 1 + \cos \left[\frac{\pi T}{\beta} \left(|f| - \frac{1-\beta}{2T} \right) \right] \right\}} & \left(\frac{1-\beta}{2T} \leq |f| \leq \frac{1+\beta}{2T} \right) \\ 0 & \left(|f| > \frac{1+\beta}{2T} \right) \end{cases}$$

- Finding its impulse Response is a little bit tricky.

$$h(t) = \frac{2\beta}{\pi\sqrt{T}} \frac{\cos[(1+\beta)\pi t/T] + \frac{\sin[(1-\beta)\pi t/T]}{4\beta t/T}}{1 - (4\beta t/T)^2}$$

- Impulse response can also be generated numerically using IFFT.



Facts about RRC



- RRC theoretically has infinite number of taps so it has infinite attenuation in the stop band. However, in implementation its length should be reduced to a finite value.
- Decreasing the number of samples (filter delay) reduces the stop band attenuation.
- The rolloff factor is a measure of the excess bandwidth of the filter, i.e., the bandwidth occupied beyond the Nyquist bandwidth of $1/2T$, where $1/T$ is symbol rate.
- As rolloff increases eye in the eye diagram opens up. This means that if there were no bandwidth restrictions it would be easier on the receiver if one used a large rolloff. (However, for bandwidth efficiency rolloff should be smaller.)
- Smaller rolloff gives narrower bandwidth. However, its side lobes increases so attenuation in stop band is reduced.
- RRC filters are implemented in the base band as a digital filter. Since implementing narrow (high Q) filters in the RF bands is difficult.



Facts about RRC (cont.)



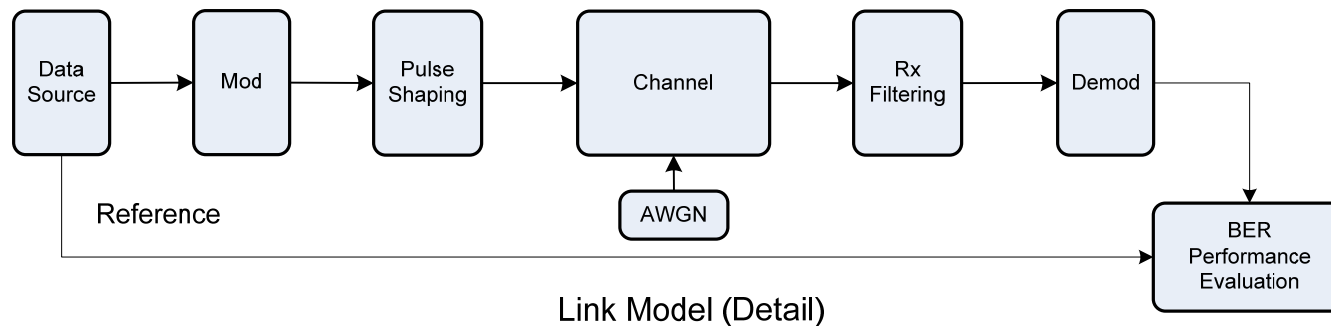
- Another issue to consider in pulse shaping is the Peak-to-Average Power Ratio (PAPR).
- PAPR is determined by combination of
 - Modulation
 - Constellation of the signal
 - Pulse shaping
- High PAPR reduces power amplifier efficiency since it must operate with large back off (higher PAPR requires higher back off and/or more linearity)
- PAPR of an RRC will increase with
 - reduced excess bandwidth⁵
 - increased filter length⁵



Link Modeling with the RRC Filters



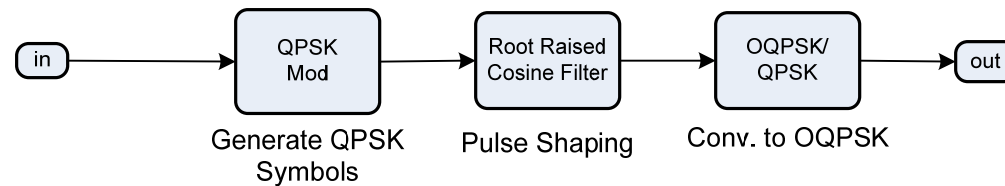
A SIMULINK Model was developed and simulated to study the spectral efficiency and Bit Error Rate (BER) performance of the RRC filters. Below is a high level block diagram of this model.





Link Modeling with the RRC Filters (Cont.)

Tx Model with RRC Pulse Shaping

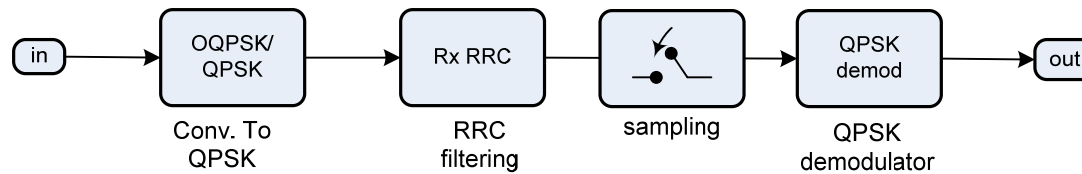


The modulator generates one symbol for each pair of data bits. The symbols generated by the modulator is up sampled and pulse shaped (filtered) to comply with the channel bandwidth restrictions. Typically, the pulse shaping is the last stage of transmitter before (DAC and) PA.



Link Modeling with the RRC Filters (Cont.)

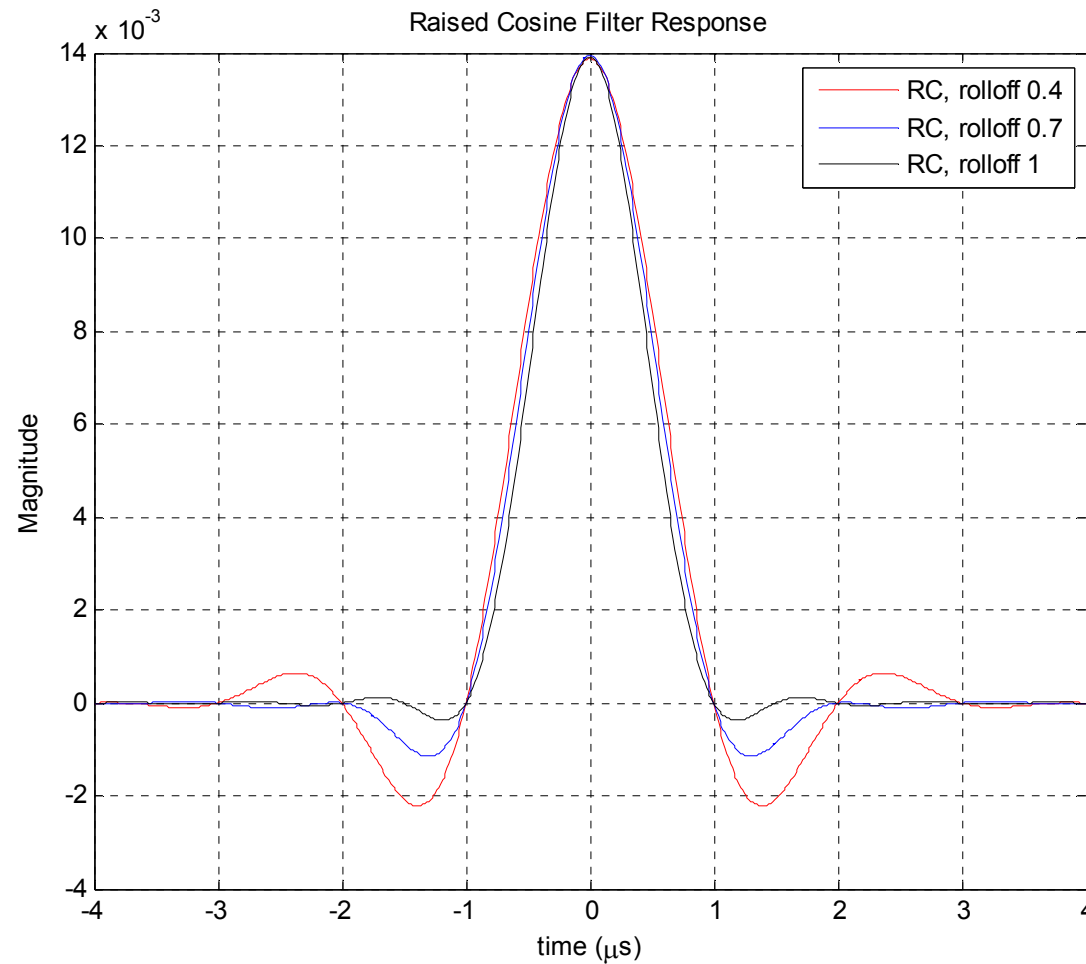
Rx Model with RRC Filter



RRC filter filters out the signal (i.e., equalizes to give a nearly zero Intersymbol Interference, ISI, if Tx is also using RRC filter pulse shaping). The output is sampled at the optimum points (i.e., down-sampled) to give one sample per constellation symbol. Demodulator finds out which quadrant the received sample falls and based on that decision generates a pair of bits.



RC Filter Impulse Response, rolloff 0.4, 0.7, and 1 Symbol rate 1 Msp



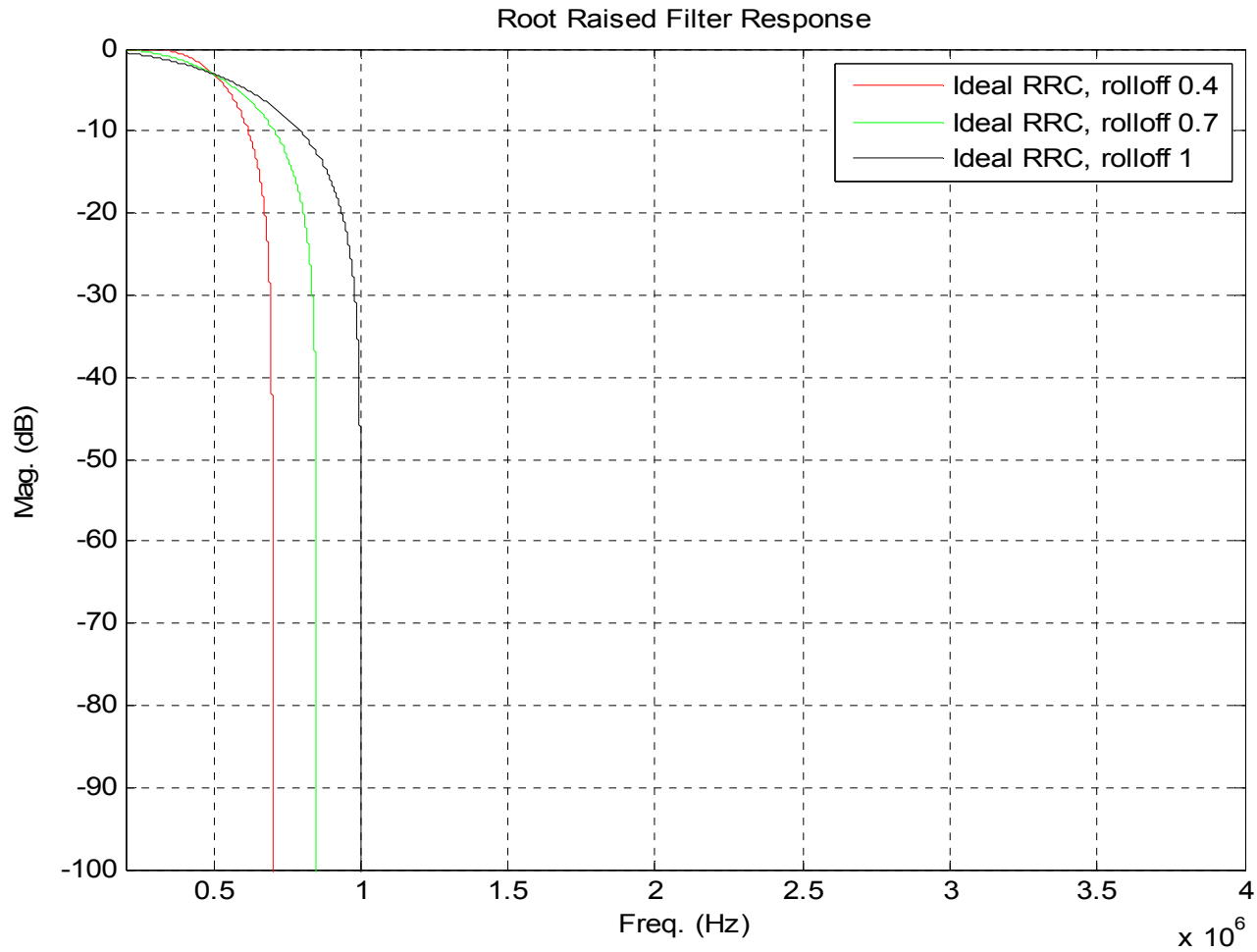


Ideal RRC Frequency Response with rolloff 0.4, 0.7, and 1.0



Just the plot of the equation

Symbol rate 1 Mbps

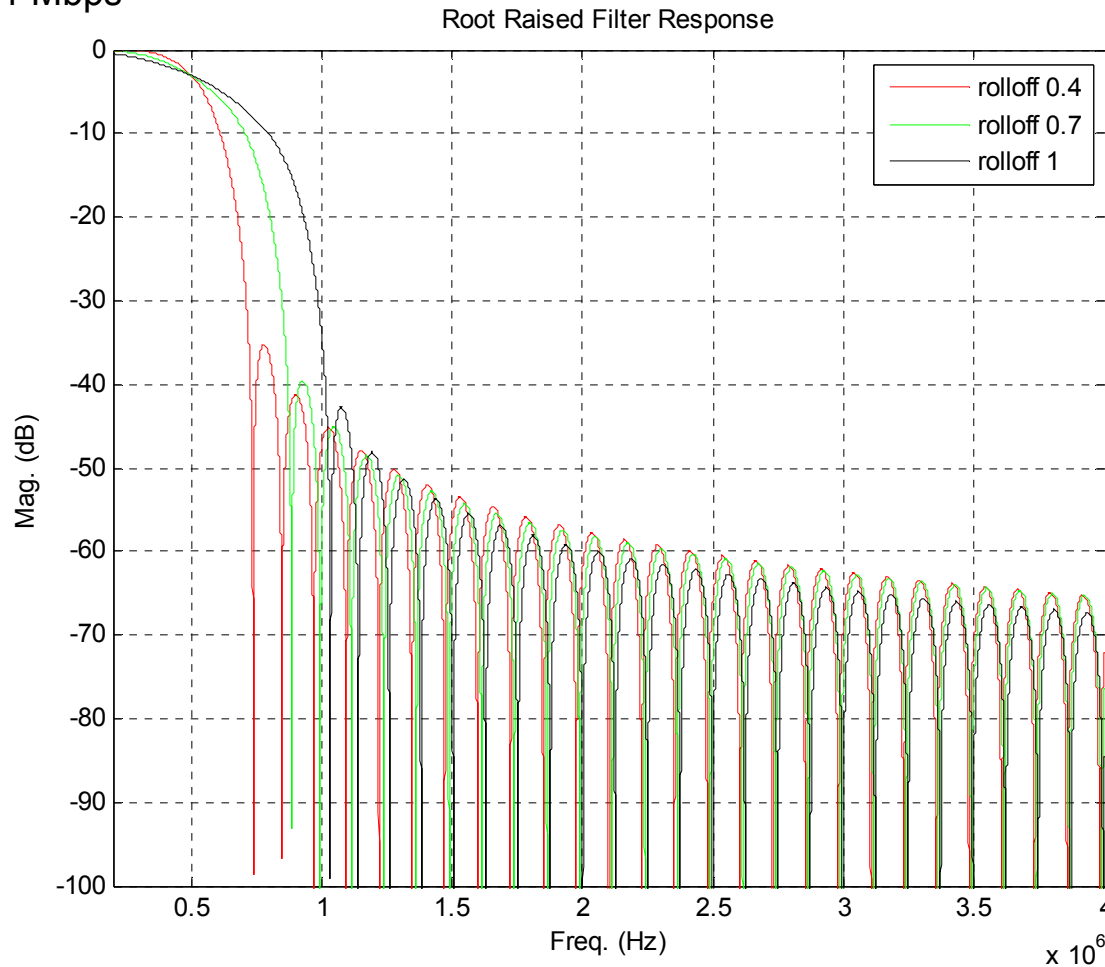




Frequency Response of Realizable Root Raised Cosine Filter with Rolloff 0.4, 0.7, and 1



Symbol rate 1 Mbps

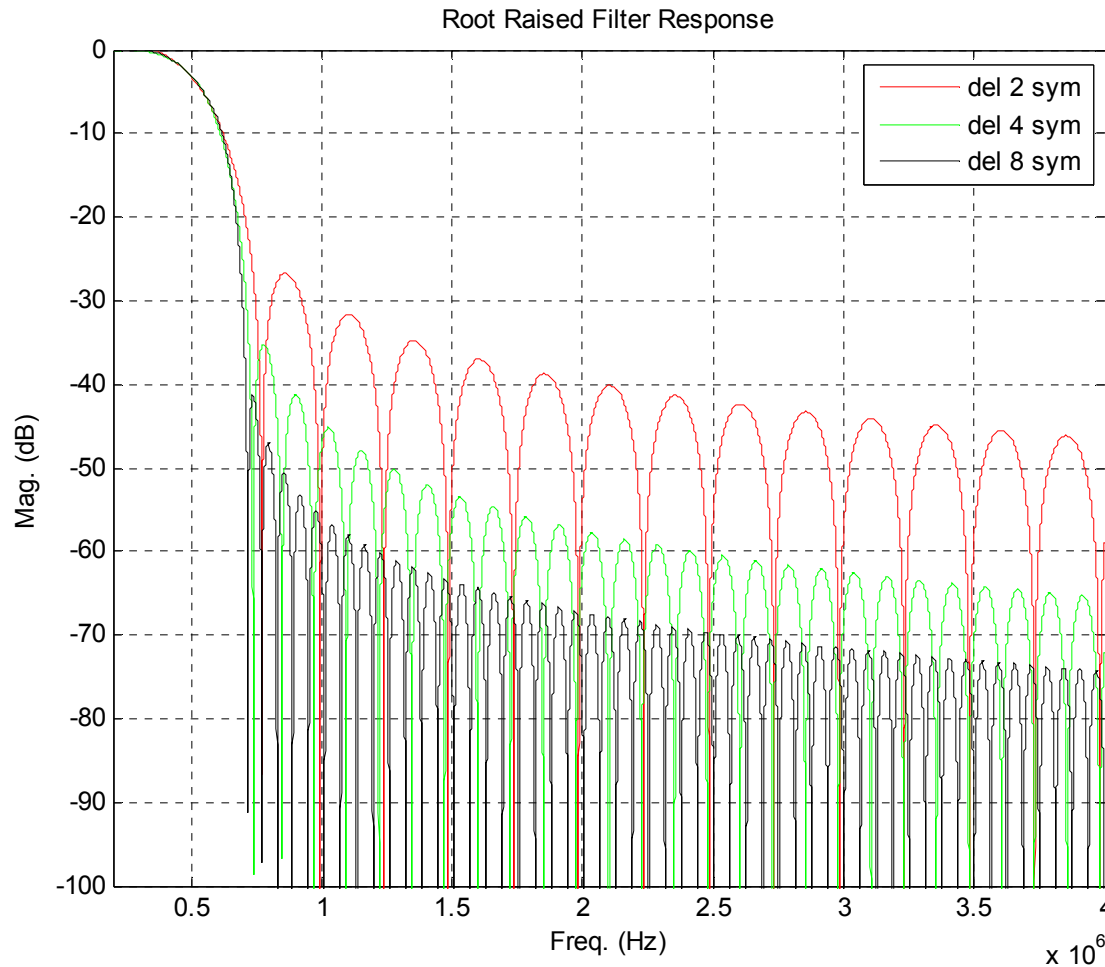




Effect of the length of the Filter (Realizable) delay 2, 4, and 8 sym (rolloff 0.4)

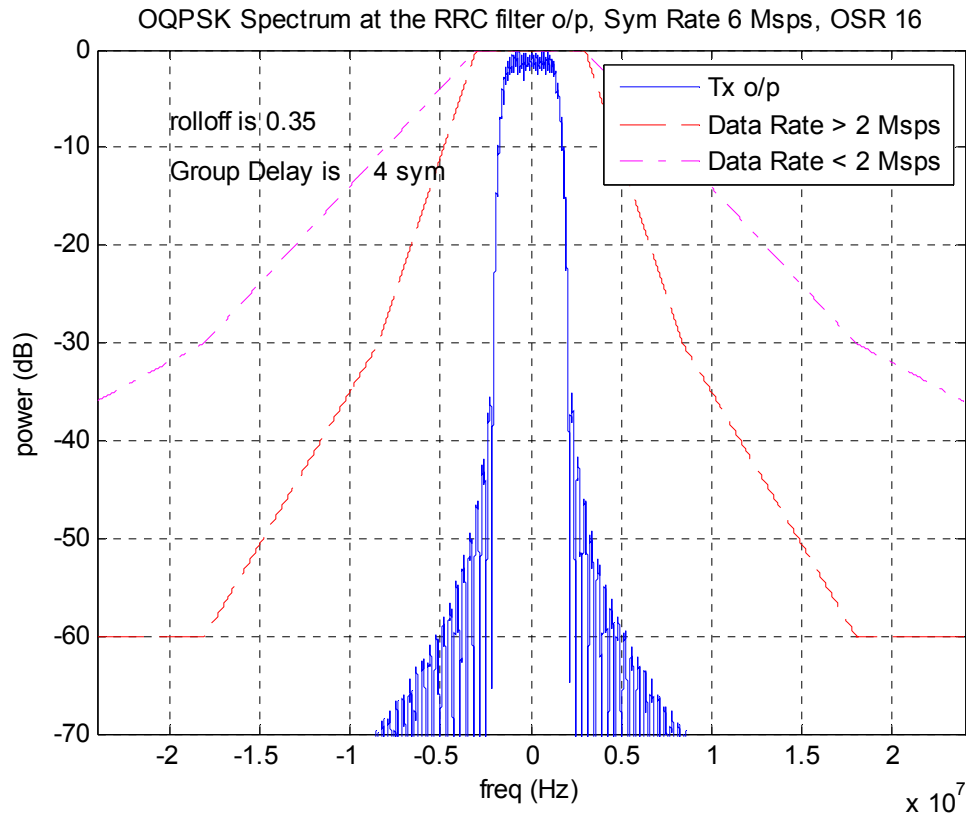


Delay is half of the filter length in time. Here is given as the symbol length





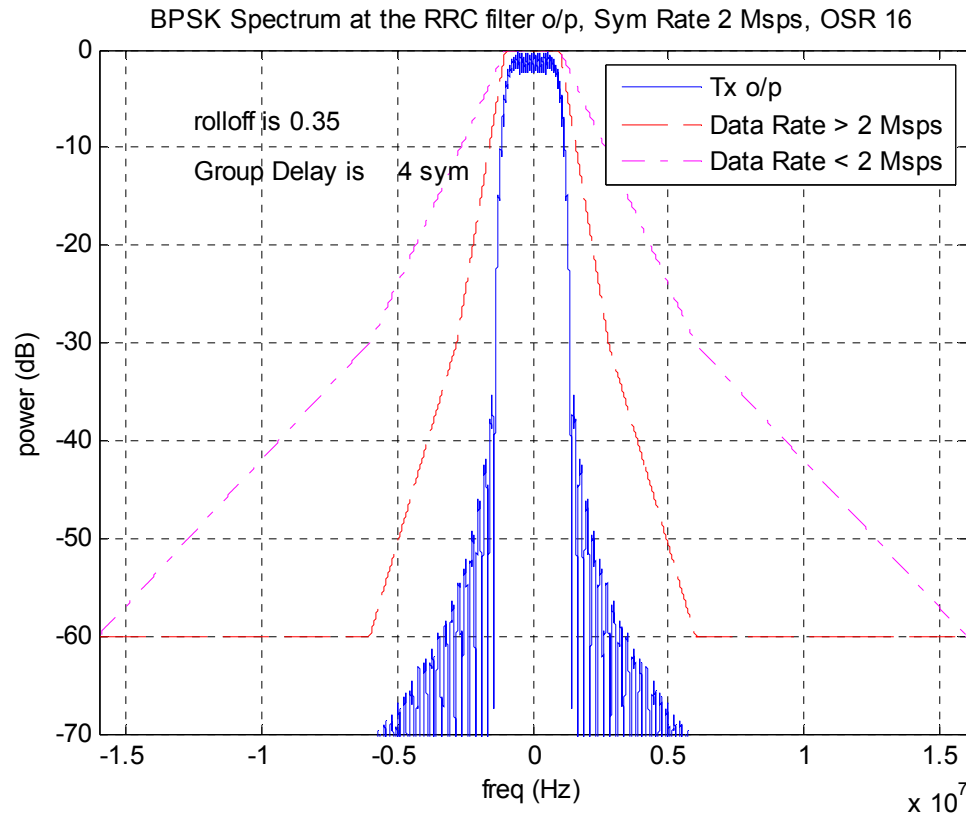
OQPSK Tx RRC Output Spectrum for a Symbol Rate of 6 Msps w/ Category A Masks



- Rolloff 0.35, Group delay 4, 2-sided BW = $3 \times 1.35 = 4$ MHz.
- Measured 99% 2-sided BW = 3.48 MHz
- Measured 90% 2-sided BW = 2.78 MHz



BPSK Tx RRC Output Spectrum for a Sym Rate of 2 Msps w/ Category A Masks



Group delay 4 sym, rolloff 0.35

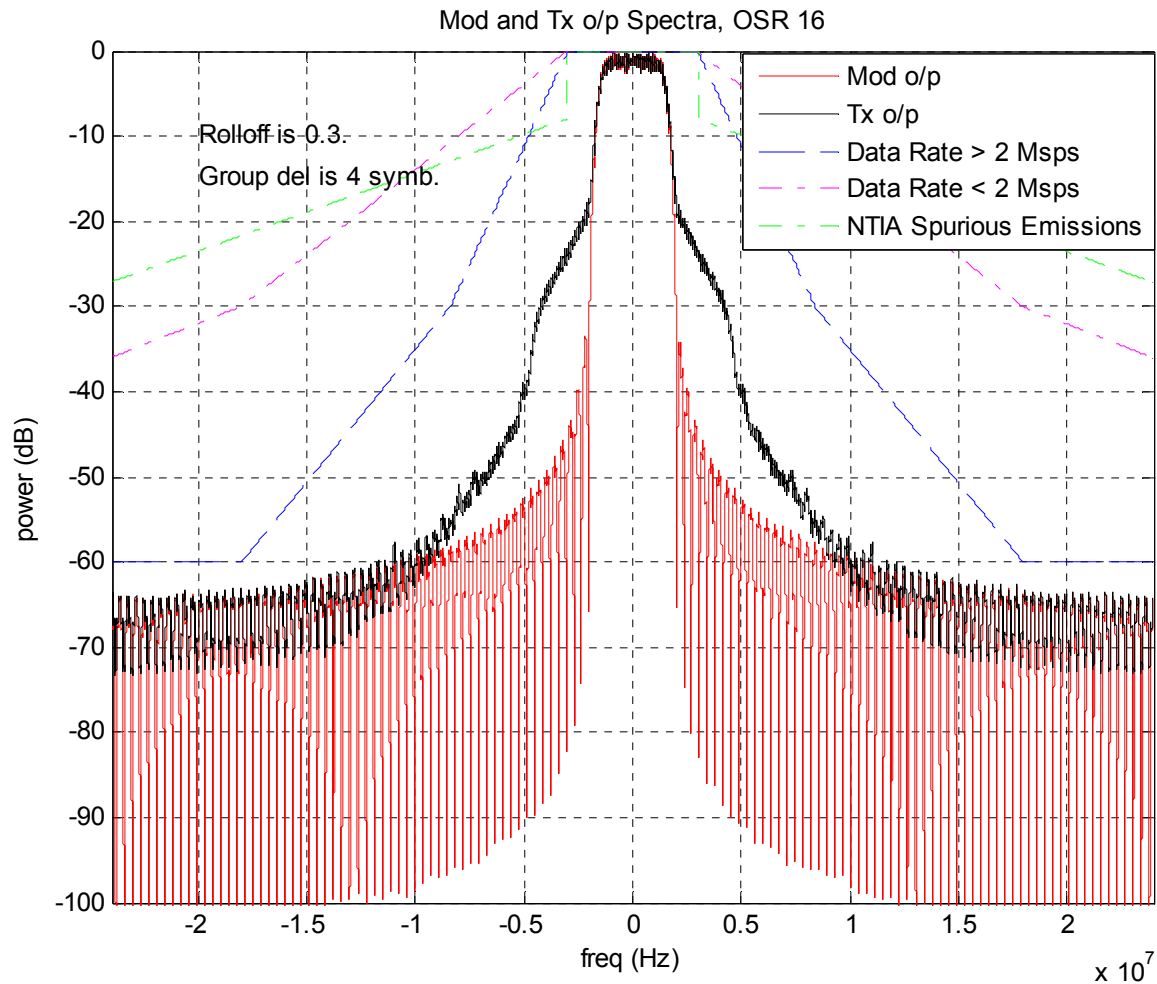
2-sided BW = $2 \times 1.35 = 2.7$ MHz.

99% measured 2-sided BW = 2.32 MHz.

90% measured 2-sided BW = 1.87 MHz.



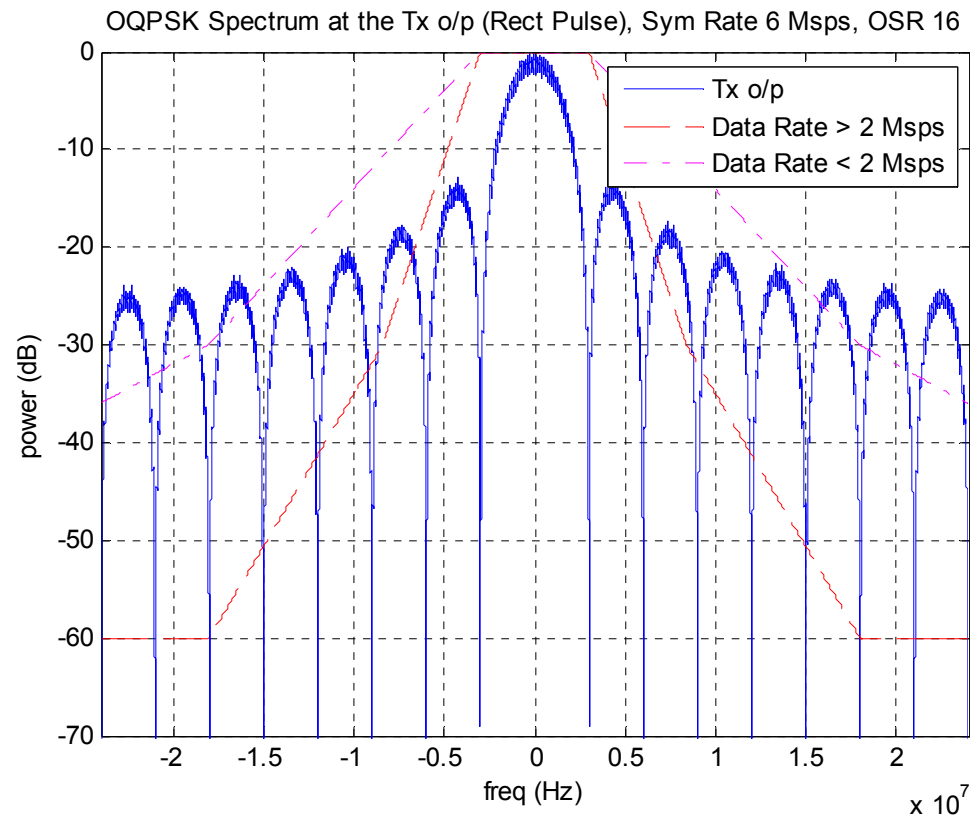
Power Amplifier (PA) Output Spectrum for 6 Msp/s OQPSK



The spectrum of the Modulator output is also shown in the plot.



OQPSK Tx Output Spectrum for a Rect Pulse at a Sym Rate of 6 Msps w/ Category A Masks

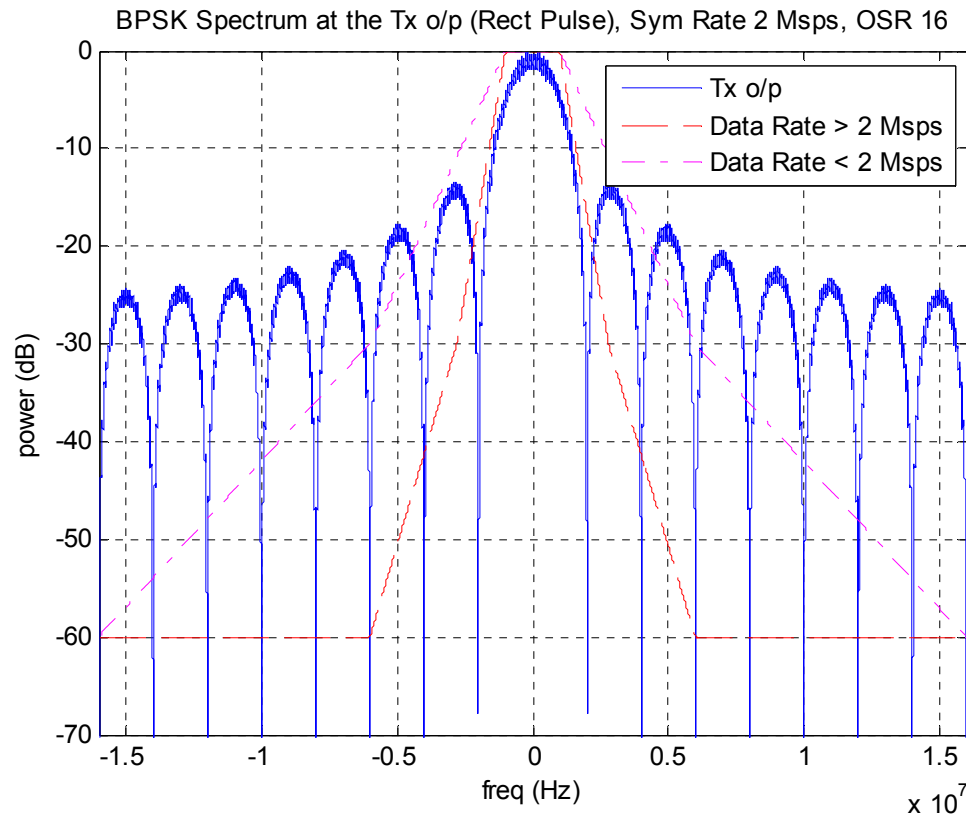


Rectangular pulse shaping does not comply with Category A spectral mask requirements.

90% measured 2-sided BW = 4.96 MHz.



BPSK Tx Output Spectrum for a Rectangular Pulse, for a Symbol Rate of 2 Msps w/ Category A Masks



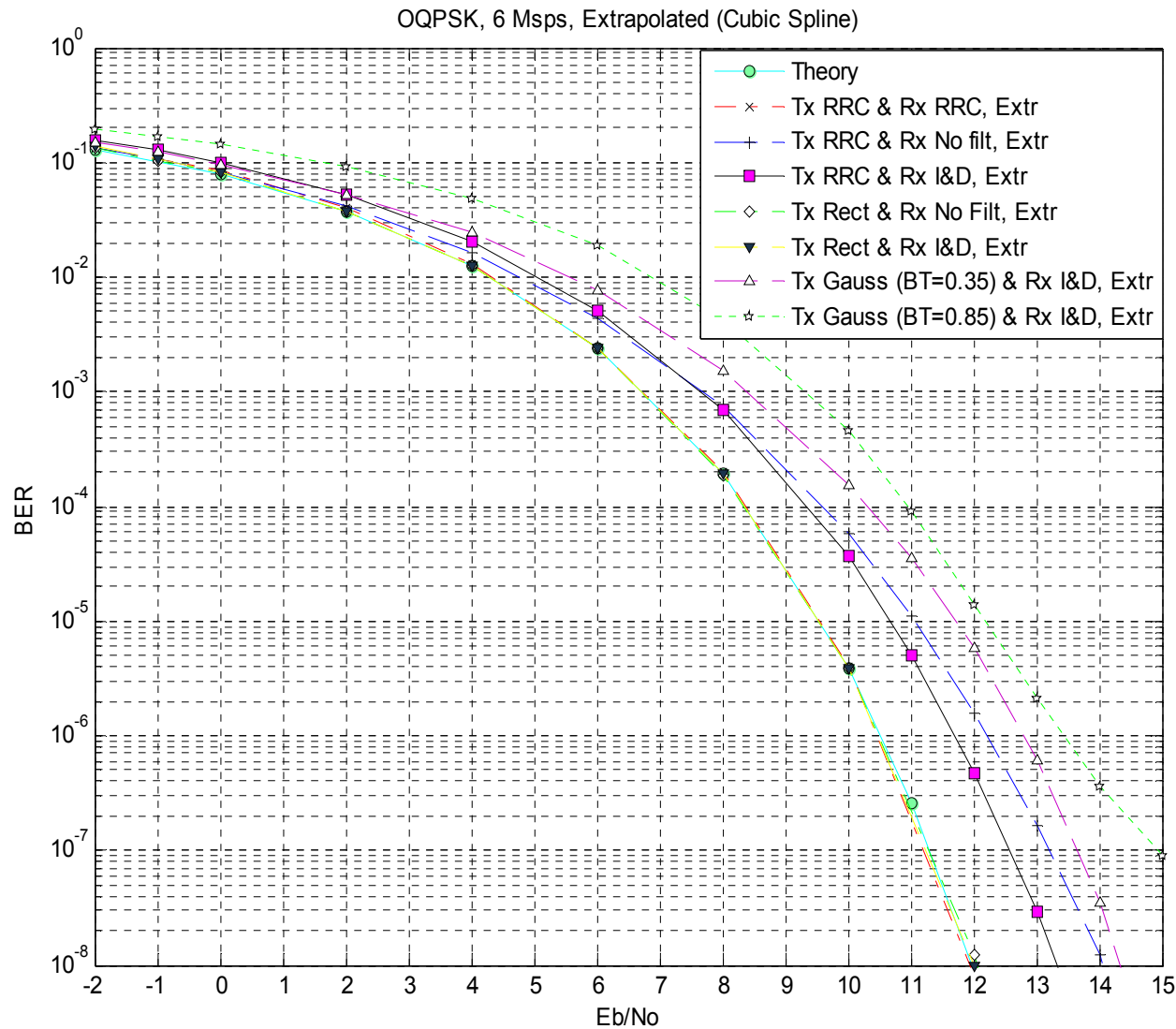
Rectangular pulse shaping does not comply with Category A spectral mask requirements ⁶.

99% measured 2-sided BW = 2.22 MHz.

90% measured 2-sided BW = 3.34 MHz.



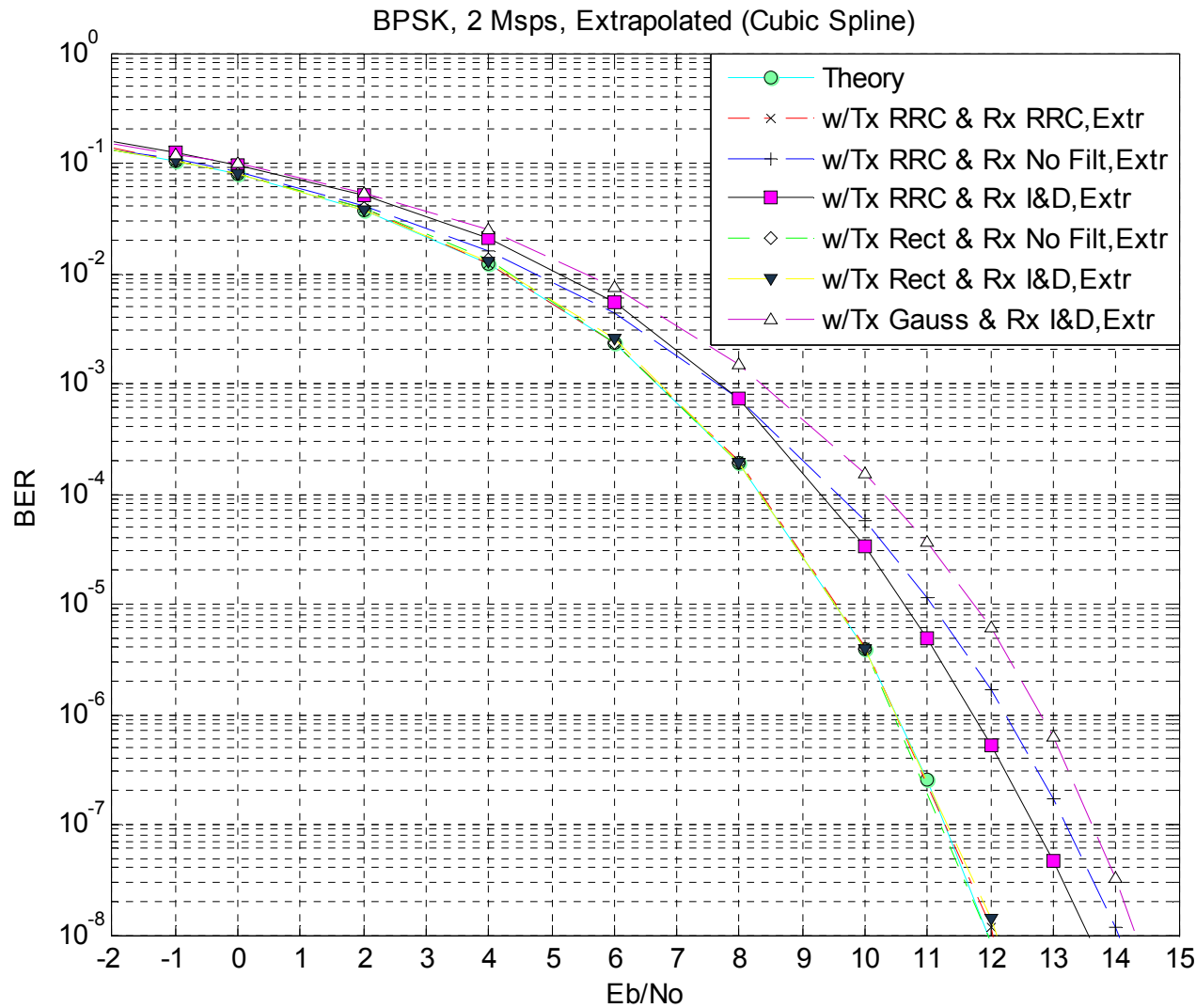
OQPSK, 6 Msp



Please Note that the curves are (Cubic Spline) extrapolated after 10-11 dB EbNo and Simulations are ran for +1000 error bits unless stated otherwise.



BPSK, 2 Msps



*Please Note that the curves are (Cubic Spline) extrapolated after 10-11 dB EbNo and Simulations are ran for +1000 error bits unless stated otherwise.



Conclusions



- RRC is more bandwidth efficient than NRZ (i.e., no pulse shaping, rectangular waveform).
- Matching TX and RX filters achieves optimum Bit Error Rate (BER) performance.
- This study provides insight and guidance in the system design for spectral efficiency and Bit Error Rate (BER) performance.



References



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2. Proakis, J. (1995). *Digital Communications* (3rd ed.). McGraw-Hill Inc. ISBN 0-07-113814-5.
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 - 3.4..2.1 Spectral Emissions Mask for Spurious Emissions (NTIA), p 42
 - 3.4.2.2 Spectral Emissions Mask for Category A Missions, p 43