

The Ice Line in Pre-Solar Protoplanetary Disks

Sanford S Davis

Space Science & Astrobiology Division

NASA Ames Research Center

Moffett Field, CA 94035

sanford.s.davis@nasa.gov

650-604-4197

Abstract

Protoplanetary disks contain abundant quantities of water molecules in both gas and solid phases. The distribution of these two phases in an evolving protoplanetary disk will have important consequences regarding water sequestration in planetary embryos. The boundary between gaseous and solid water is the “ice line” or “snow line.” A simplified model that captures the complicated two-branched structure of the ice line is developed and compared with recent investigations. The effect of an evolving Sun is also included for the first time. This latter parameter could have important consequences regarding the thermodynamic state and the surface reaction environment for the time-dependent chemical reactions occurring during the 1- to 10-million-year lifetime of the pre-solar disk.

1. Introduction

Water is both an essential solvent and a critical factor for life-sustaining chemistry. In particular, terrestrial water has the unique ability to maintain itself in gas, liquid and solid phase states. Water molecules are also abundant in the protoplanetary disk but occur only as gas and ice. The spatial distribution of these two phases, and the dynamics of their phase transformation by sublimation in an evolving protoplanetary disk, will have important consequences regarding water sequestration in growing planets. Subsequent planetary evolution will generate an atmosphere that will enable water to coexist in all three phases.

The question of water ice in the early solar nebula has been the subject of many studies. The simplest analysis assumes that the ice line occurs at the radial position where the temperature drops to a predetermined freeze point as proposed by Hayashi (1981). Stevenson and Lunine (1988) considered the migration of water vapor/ice in the region of Jupiter. Although they mention the presence of “snow clouds” above the midplane, the analysis is essentially one-dimensional. Cassen (1994) discussed the issue of competing radiative and viscous heating sources, and appears to be the first to directly address the issue of water ice condensing above the disk midplane. Thus, both viscous heating and outer disk radiative heating can compete with one another to induce an ice-cloud effect. D’Alessio et al. (1999) considered a two-dimensional T Tauri model that included viscous and direct stellar radiative heating, and showed temperature inversions at some radial stations, implying that there could be a phase change in the vertical direction over some portion of the disk. Both viscous and radiative heating were considered by Sasselov and Lecar (2000) but not fully explored because of the complex and multidisciplinary physics involved. Kamp and Dullemond (2004) and Woitke et al. (2009), using variants of the code presented in Dullemond et al. (2002), also showed non-monotonic behavior of the temperature distributions, but they did not specifically consider the ice line. Meijerink et al. (2009) explicitly points out that the snow line is actually a curve that becomes almost parallel to the disk surface over a portion of the disk similar to earlier arguments by Cassen (1994). Thus, the “horizontal cold finger” (radial convection and/or diffusion) discussed by Stevenson and Lunine is augmented by diffusion and/or convection in the vertical direction. These physical processes in protoplanetary disks can be directly affected by the presence of the ice cloud that is the subject of this paper.

A sublimation curve for water ice, coupled with the basic disk formulation of Dullemond et al. (2002), was investigated by Davis (2005a) for an evolving solar-nebula model where a cusped time-dependent ice line (also called the snow line) was computed. In that case, the gas/solid interface used a phase-diagram approach to predict the local sublimation temperature, and the partial pressure of water vapor was approximated as a simple fraction of the computed hydrogen number density and local temperature. A more refined calculation (Davis, 2007) used a chemical network to augment the disk dynamics used in the former paper to directly compute the water partial pressure. This calculation also indicated a cusp in the sublimation curve for a specific steady-state disk model.

Dodson-Robinson et al. (2009) considered the formation and evolution of the ice line as part of their study of condensed matter in an evolving solar system. An important part of that study was the computation of the evolving surface density distribution using the disk evolution equations. Surface densities along with appropriate chemical adsorption-desorption models were used to compute the condensation of a number of ices including water. Unfortunately, the movement of the water-ice line through the solar nebula was investigated only along the disk centerline. Min et al. (2011) conducted a detailed investigation of the two-dimensional water-ice condensation front

using an unsteady model of the protosolar nebula. Their focus was on ice-line evolution when both radiative transfer and viscous dissipation are both important. The time-dependent surface densities extracted from Min et al. are used to compute the ice line using a new, simplified calculation and an analytical unsteady surface density distribution with an evolutionary Sun. All of the previous work assumes that the stellar luminosity is that of the current-day Sun.

Rather than a simple, smooth curve, the ice line, as predicted by Davis (2005a), is actually a two-branched curve meeting at a cusp located somewhat closer to the central star than where the ice line meets the midplane. The lower branch is due to viscous heating and the upper to radiative heating. The predicted ice line contains an ice “canopy” over the expected terrestrial planet regions. This canopy may contribute to the capture of water ice by precipitation or other vertical transport mechanisms. The presence of water ice in the disk above a planet-forming region would enable multiple mechanisms for water uptake as the planets grow. This ice cloud could enable a number of vertical and horizontal transport mechanisms as described, for example, by Ciesla and Cuzzi (2006). The time scale for settling of grains and ice particles relative to the time scales for the formation of the ice cloud would be an important consideration. Nakagawa et al. (1986) consider in some detail the settling of grains (ice particles) in a minimum-mass solar nebula.

This study extends previous calculations by: (1) using a surface density that is consistent with the solar-type steady-state minimum mass protoplanetary disk as developed by Davis (2005b); (2) using the fact that the luminosity of the Sun evolves during the disk phase of planet formation; and (3) simplifying the criterion for water-ice condensation. With respect to the third point, the sublimation of water vapor in a disk can be expressed in a number of ways. In Davis (2005a) vapor pressure curves were used to explore the ice-line location. In this paper a simplified method is used. Hayashi (1981) recommends that the criterion should be $T = 170$ K for the solar nebula. The phase diagram for water at nebula pressures shows that the partial pressure of water vapor varies from 10^{-14} to 10^{-9} Pa over a temperature range 100–170 K. The upper end is relevant to the denser centerline region but is too high in outer disk. Rather than 170 K, a value of 130 K was adopted because it is an average temperature in the range of vapor pressures predicted in the disk.

In Section 2 a method for computing the ice line is described. The two-dimensional disk model includes both radiant and viscous heating, and predicts the thermodynamic state of the protoplanetary nebula at any instant. A simple approach using a temperature criterion ($T = 130$ K) is used to compute the ice-line geometry. This criterion is justified by the fact that the water sublimation curve (partial pressure of H_2O vs. T) is essentially independent of the temperature at the low pressure levels in a disk (Barshay and Lewis, 1976; Davis, 2007). This avoids the complicated chemical computation of water-gas/ice species but succeeds in capturing the complex vertical structure in a straightforward way.

An important issue is the effect of grain opacity and surface density on the ice-cloud geometry. Temperature variations and opacity parameters are investigated in Section 3. The last section compares the published ice-line geometry using surface densities extracted from Min et al. (2011) with the analytical surface densities used here. Finally, the effect of including an evolving Sun along with an evolving surface density is investigated.

The results show that a relatively simple model is sufficient to predict global characteristics of an evolving ice line that compares favorably with previous work. The main effect of disk evolution with a maturing Sun compared to a static star is to increase the region where water is evaporated in the early solar system and affecting the dynamics of the ice-line evolution. An inwardly moving ice front is a key feature of this calculation and could have important ramifications regarding the relative extent of gaseous and ice-particle-laden regions.

2. Approach

Predicting the ice line is not a simple process because it requires combining disk chemistry and dynamics. In general, the chemical network governing the relative abundances of the trace nebular species must be analyzed numerically, and the local water-vapor partial pressure extracted from this calculation. This partial pressure must be tested with an “absorption-desorption” model (Brown and Charnley, 1990; Dodson-Robinson et al., 2009) or a phase diagram approach (Davis, 2007) to assess the local physical state of the water. Guided by previous work, a simple temperature-based Hayashi sublimation criterion is used. This study also ignores the latent heat effect and assumes that the gas and dust are at the same temperature throughout the disk and, as mentioned previously, only the local temperature determines the ice-line location. The problem now reduces from a combined chemical/thermo/gas-dynamic system to the thermo/gas-dynamic problem of predicting the temperature, pressure, and density distribution in a protoplanetary disk.

A passive disk is heated by either stellar radiation or energy from nearby stars. Following a basic two-layer grazing angle model for an illuminated disk (Chiang and Goldreich, 1997), more refined radiative-transfer procedures for passive disks were developed (Dullemond and Natta, 2003). The radiative-transfer method used by Dullemond et al. (2002) forms the basis of the passive (nonviscous) portion of the model used in this paper. The model consists of the following components: (1) The stellar radiation flux interacts with a flared disk as a simple nonscattering exponential extinction function. (2) The radiative transfer calculation requires dust density and opacity. (3) The dust density is assumed to be 0.01 percent of the gas-mass density. (4) The grain opacity is obtained from the multicomponent frequency-dependent data in Henning and Stognienko (1996). (5) Once the primary radiation is obtained, the local temperature is found from a one-dimensional Eddington “slab model” radiative-transfer calculation.

The local viscous internal heating per unit volume is $q_v = \frac{9}{4} \nu \rho \Omega^2$ and is most active near the centerline in the inner region of the disk. It is proportional to the turbulent viscosity $\nu = \alpha c H$ as defined by Shakura and Sunyaev (1973). This “alpha” viscosity model consists of three terms: the empirical coefficient α , the isothermal speed of sound c , and the disk semi-height H . A value of $\alpha = 0.01$ representing a typical accretion disk is used herein.

The total heat-source consists of both radiative heating and viscous dissipation. The final step is to invoke vertical hydrostatic equilibrium to compute the equilibrium thermodynamic state. These three steps (radiative transfer, viscous heating, and hydrostatic equilibrium) are iterated following Dullemond’s prescription until convergence. The outer edge and far disk are most affected by passive stellar radiation, and active viscous internal heating is most intense in the inner midplane region of the disk. The final result is a relatively complex temperature field with temperature inversions at specific disk radii. The combined effect of these heating sources produces a two-dimensional temperature distribution in the r, z plane from which the water sublimation boundary is obtained directly.

Calculations show typical variations in the condensation curve are similar in certain respects to previous calculations using more complex condensation criteria (Davis, 2005a, 2007). Although the detailed geometry may change, the qualitative features consisting of a cloud of ice over some regions in the vicinity of the habitable zone for a particular choice of luminosity and disk structure is a consistent feature.

3. Disk Parameters

Three essential determinants of the ice line are stellar luminosity, surface-density distribution, and disk opacity. The first parameter governs outer disk heating by direct solar radiation, and the latter two govern the intensity and extent of global heating. The stellar luminosity has not been considered as thoroughly as the other two parameters to date, but is shown to be an important governing factor in the early evolution of the ice line during the planet-forming epoch.

The solar nebula is illuminated by a primitive Sun. The evolution of the Sun on the Hertzsprung–Russell (HR) diagram is indicated in Figure 1. These trajectories are adopted from the data given in Kusaka et al. (1970) and attributed to original work by Ezer and Cameron (1965). This track was further validated using the extensive graphical and tabular data in D’Antona and Mazzitelli (1994).

The disk is active during its lifetime of about 1–10 million years, and the relevant stellar state is shown at the indicated times. The luminosity of the early Sun varies from about $500 L_{\odot}$ to slightly less than $1 L_{\odot}$. The Sun evolves from the Stellar Giants region of the HR diagram to cooler regions (relative to the current-day Sun) of the Main Sequence. This evolution is mainly one of decreasing luminosity at a reduced temperature relative to its current value. Subsequently,

the track follows a mostly horizontal path to the Main Sequence. As is shown, the decreasing luminosity of the shrinking Sun in the critical protoplanetary disk phase has an important effect on the water-ice condensation front.

Turning now to the surface density, the disk evolution equation (Pringle, 1981) is:

$$\frac{\partial \Sigma(r,t)}{\partial t} - \frac{3}{r} \frac{\partial}{\partial r} (r^{1/2} \frac{\partial \nu \Sigma(r,t) r^{1/2}}{\partial r}) = 0 \quad (1)$$

which is a diffusion equation for the decreasing surface density as a result of stellar mass accretion. Here $\Sigma(r,t)$ is the space-time surface density, and the kinematic viscosity plays the role of a diffusion parameter. A common choice for the viscosity is the alpha viscosity approach described previously ($\nu = \alpha cH$). Unfortunately, this formulation is nonlinear and altitude dependent (through the sound speed), and an appropriate average value of c is difficult to predict.

The approach used here is to solve Equation (1) using the “beta” viscosity model described in Davis (2003) and introduced by Hure et al. (2001). In this case, $\nu = \beta \sqrt{GM} r^{1/2}$ where the viscosity depends on the radial distance alone, and the preceding linear diffusion equation allows an exact solution as an initial-value problem. The exact solution from Davis (2003) is

$$\Sigma(r,t) = \frac{81(GM)^{3/2} M_0^4 \Gamma_{7/3}^3}{256\pi J_0^3 \sqrt{r} (1 - 3\dot{M}_0 t / M_0)^{4/3}} \exp\left(-\frac{27(GM)^{3/2} M_0^3 \Gamma_{7/3}^3 r^{3/2}}{64J_0^3 (1 - 3\dot{M}_0 t / M_0)}\right) \quad (2)$$

where GM is the product of the solar mass and the universal gravitational constant (in astronomical units GM is $4\pi^2$); the initial disk mass is M_0 ; the initial mass accretion rate is \dot{M}_0 (a negative quantity); and the initial angular momentum is J_0 . Note that the surface density decay constant depends on initial conditions through the ratio M_0 / \dot{M}_0 and reflects the overall linear nature of the equation. The auxiliary function Γ is the complete Gamma function of the indicated order. β is an empirical constant constrained by observational data regarding the lifetimes of protoplanetary disks; β is of the order 10^{-6} to 10^{-5} . This corresponds to $\alpha \sim 0.001$ to 0.01 following the discussion in Davis (2003).

Using the relation between \dot{M}_0 and β from the original paper, the final surface density can be written in terms of β instead of \dot{M}_0 :

$$\Sigma(r,t) = \frac{81(GM)^{3/2} M_0^4 \Gamma_{7/3}^3}{256\pi J_0^3 \sqrt{r} \left(1 + \frac{729GM^2 M_0^3 \Gamma_{7/3}^3}{256J_0^3} \beta t\right)^{4/3}} \exp\left(-\frac{27(GM)^{3/2} M_0^3 \Gamma_{7/3}^3 r^{3/2}}{64J_0^3 \left(1 + \frac{729GM^2 M_0^3 \Gamma_{7/3}^3}{256J_0^3} \beta t\right)}\right) \quad (3)$$

Note that the time evolution in Equation (3) appears only as a βt product if this factor is large compared to unity. This is the similarity solution of Equation (1) (Pringle, 1981) and was applied to T Tauri disks by Hartmann et al. (1998). In this case the evolution time now scales inversely to β , and further simplifications are possible.

A better analytical approximation to the steady solar nebula than a 3/2-power law was developed by Davis (2005b) using Equation (3) to fit existing planetary mass data augmented by nebular gas. Using the empirical modeling parameters M_0 , \dot{M}_0 , and J_0 as $10^{-6} M_\odot/\text{yr}$, $0.0682 M_\odot$, and $0.379 M_\odot \text{ AU}^2/\text{yr}$ respectively, the instantaneous surface density is reduced to the numerical form:

$$\Sigma(r,t) = \frac{10^{-4} \exp\left(-\frac{2r^{3/2}}{27\pi\beta t}\right)}{\sqrt{r}(\beta t)^{4/3}} \quad (4)$$

The value of β varies from approximately 10^{-6} to 10^{-5} and from Equation (4) only affects the timescale of the evolutionary disk. That is, using the smaller value relative to the larger one would only change the time for given surface density by a factor of 10. A graphical representation of Equation (4) is indicated in Figure 2 during disk evolution using the smaller value of β . The decrease in mass due to solar accretion and the increased disk size is apparent from the figure. The steady-state surface density from Davis (2005b) is indicated by the short dashed line along with the commonly used Hayashi minimum mass model. Both steady-state models use planetary masses and augmented gas modeling. Although quite useful in many applications, the Hayashi minimum mass model has been criticized for not having enough mass in the region where the giant planets form. The current steady-state model alleviates this problem, and the unsteady model, with the chosen value of β , passes through this distribution at about 1 million years.

The total mass and accretion rates at any instant are easily found by integrating the surface density over the disk area and are respectively:

$$\begin{aligned} M(t) &= \frac{0.0682}{(1+0.000043988t)^{1/3}} M_\odot \\ \dot{M}(t) &= -\frac{10^{-6}}{(1+0.000043988t)^{4/3}} M_\odot / \text{yr} \end{aligned} \quad (5)$$

The instantaneous disk mass (M_\odot) and accretion rate (M_\odot/yr) are shown in Figure 3. The disk starts at a mass slightly less than $0.1 M_\odot$ and its mass slowly accretes into the Sun until about 10^4 years when the mass decreases quicker and fully dissipates at about 10 million years. The accretion rate decreases as $t^{-1.25}$ at later epochs.

Finally, the ice line may be sensitive to disk opacity, and this was an important consideration in the work of Min et al. (2011). The opacity associated with astronomical silicates (Draine and Lee, 1984) was implemented by Dullemond et al. (2002). Here, this basic composition is augmented with disk-specific Rossland mean opacities from more representative aggregations (Henning and Stognienko, 1996). Two grain compositions from that paper are considered. They are those associated with a variety of dust species (iron, silicates, volatile organics, and water ice) and a reduced set with the volatile organics and water ice removed. The influence of these opacity models on the ice-line location is indicated in Figure 4(a).

This figure also illustrates the prototypical ice-line geometry. It consists of two branches: an upper part driven by the radiative heating and a lower segment due to viscous heating in the orbital plane. The viscous heating in particular is constrained to a pocket (to about 8 AU in this example). The two curves meet at a cusp at about 4 AU. This example corresponds to a hypothetical star whose luminosity is about $7 L_{\odot}$ with an effective temperature of 4,500 K. This corresponds to a point in the evolutionary curve at about 200,000 years. The solid line in Figure 4(a) is the location of the ice line using all of the grain components, and the dashed line is that if volatile organics and water-ice components are eliminated. (Note that both of these models are applied throughout the disk.) The outer branch (radiative-dominated) is not affected, and the lower (viscous) branch moves outward with the full opacity model. In each case qualitative characteristics of the ice line are unchanged. This example indicates that although opacity is important, the particular model used does not drastically affect the calculated ice-line geometry. In the following, the opacity model indicated by the solid line will be used. In addition, the calculation is streamlined by assuming a ratio of solid to gas density of 0.01 by mass following original work by Dullemond et al. (2002).

In Figure 4(b) the effect of using constant temperature criteria to define the ice line is illustrated with calculations for two temperatures compared with computations using the full-phase-diagram approach described in Davis (2005a). In this case the surface density is taken as the Hayashi minimum mass model, the luminosity is $1 L_{\odot}$, and the ice line intersects the midplane at 3.7 AU. The solid and dashed lines correspond to ice lines using temperatures of 130 and 170 K respectively. The symbols are calculations using the method in the cited reference. It clearly shows that 130 K is the appropriate criteria for the outer branch, and 170 K is a better match in the viscous region. This reflects the fact that the sublimation phase diagram is a mildly variable function of temperature. In subsequent calculations the temperature criterion of 130 K is adopted because the error (about 0.5 AU) in the centerline ice location is about the same as differences in opacity, and using the temperature criterion speeds up the calculations for parametric studies.

Using the evolutionary track in Figure 1, unsteady surface densities from Equation (4), the indicated opacity model, and the simplified temperature criterion, the ice-line evolution is predicted during a number of epochs during the lifetime of the disk.

4. Disk Evolution

Two recent modeling studies of water ice in the solar nebula are relevant to this study. Dodson-Robinson et al. (2009) investigated evolving solar nebula dynamics and chemistry. However, only limited results regarding the water-ice line are given. The second study (Min et al., 2011) presents the full ice-line geometry as a two-branched cusped curve in the context of improved radiative transfer modeling. Both studies consider the stellar source as the constant current-day Sun.

The latter paper investigates the location of the ice line in a protoplanetary nebula with disk structure determined by stellar parameters and accretion dynamics. A new element is the use of a sophisticated three-dimensional (3D) radiation transfer algorithm. Predictions are compared with Davis (2005a) who used a simplified flared disk radiation model. Min et al. (2011) also adopt instantaneous accretion rate, rather than physical time, to record the age of the disk. Calculations are compared for the case when $\alpha = 0.01$ is the viscosity parameter using the “alpha” dynamic viscosity model.

The global disk mass and accretion rates were extracted from surface density profiles (Min’s Figure 4) and are indicated in Figure 5 herein. The square symbols correspond to the four surface density profiles in Min et al. (2011). Also indicated in the figure by a solid line is the trajectory from Equation (5) with these four accretion rates highlighted as triangles. These specific mass accretion rates relate to the four physical times of 0, 4×10^5 , 7×10^5 , and 4×10^6 years. These curves indicate that the analytical surface density covers a smaller range of disk mass than the Min model while maintaining similar accretion rates. Part of the differences may be attributed to the viscosity models used in the calculations. The disk is also very tenuous at the smallest accretion rate shown in Figure 2. The accretion rate is the primary driver behind viscous heating (Pringle, 1981), so it is an appropriate parameter to use as a comparison standard.

Surface densities from Equation (4) were used to compute the ice-line geometry and are compared in Figure 6 with data extracted from Figure 1 in Min et al. (2011). The computed ice lines are in reasonable agreement except for the largest accretion rate (earliest time) that, from Figure 5, corresponds to a much heavier initial disk. Notwithstanding differences in disk modeling, the 3D radiative transfer, and a more refined approach to opacity modeling, the resulting ice lines are similar. The viscous branch and cusp locations are in reasonable agreement considering these widely differing models. Both of these cases are computed for a current-day Sun with an effective temperature of 5,770 K. Next, the stellar model is recalibrated to an evolutionary Sun, and its effect on the ice line is highlighted.

In Figure 7(a) ice lines for the evolving (solid) and current-day (dash) Sun are indicated to 3×10^5 years and in Figure 7(b) from 6×10^5 to 2×10^6 years. At each indicated time, the surface density follows Equation (4), and the solid line tracks the local luminosity (Figure 1). An

expanding and accreting disk, along with a variable illumination, affects both radiative and viscous heating.

At 100 years the early Sun is so luminous that it is the primary heating source. The 100-year ice line (not shown) is at about 90 AU which is consistent with simple effective temperature calculations from the Stefan–Boltzmann law for this luminosity. The current Sun, however, is not as luminous, and a pocket of ice is predicted between 4 and 8 AU at an altitude less than 1 AU. With increasing time the outer disk ice line moves inward, reflecting reduced solar luminosity. The centerline viscosity-induced region moves outward from 6 to 9 AU and then inward after about 500,000 years. On the other hand, the constant-luminosity Sun, as expected, induce minimal changes in the outer disk region. The inner viscosity-dominated region slowly moves inward as the surface density decreases. After about 10^6 years both models predict similar ice-line shapes, but the evolving solar luminosity actually drops below its current value (c.f. Figure 1) before reaching its current-day value. The ice line regresses as the disk ages to about the radius of Jupiter.

The behavior of the evolutionary Sun shows that the disk has larger regions of vaporized water than predicted using a constant luminosity model. This could have important consequences regarding the chemical evolution of certain species such as those that depend on photoprocesses in the outer reaches of the disk and may be exposed to the higher temperatures of the early Sun, which can enhance their reaction rates.

5. Conclusions

The evolution of the ice line is a dynamic process whose position changes as the disk evolves and eventually disappears when the disk disperses. At this point much of the water is embedded in planetary embryos and other solid matter in the “debris disk.” However, the fact that there is viable ice line during the disk lifetime and the process of water sequestration is continuous indicates that some sort of ice cloud over part of the nebula probably exists and may be an important factor in developing terrestrial planets. This could also be a mechanism for the migration of planetary silicates and organics because ice grains can easily capture a variety of molecular species at predetermined radial and vertical locations in the disk and physically transport them inwards. Because certain molecules are preferentially forms at specific temperatures and densities, this could be an efficient transport mechanism that should be further studied.

Fixing the many important parameters regarding the ice-line location must await more detailed observations of protoplanetary disks associated with growing planets. However, it is clear from this investigation that ice-line movements are much more dynamic when variable luminosity is included. In any event, more refined models concerning the evolution of these disks and the

transport of materials in these dynamic systems need to be considered. These effects should be assessed and contrasted with widely accepted planet-migration modeling to determine the relative importance of these processes. One important open question regarding the planet-building process is the epoch during which the ice line sweeps over the inner-radial region of the disk and its effect on generating the gas giants.

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Figure Captions

Figure 1: Hertzsprung–Russell diagram showing evolution of the luminosity of the Sun during the lifetime of the protoplanetary disk to 5–10 million years and its approach to the main sequence.

Figure 2: Radial distribution of the surface density (solid lines) during accretion to the central star. The steady-state model of Davis (2005b) is shown dashed and the minimum mass model ($\Sigma = 1700 r^{-3/2}$) from Hayashi (1992) is the straight line with slope $-3/2$.

Figure 3: Time histories of disk mass and accretion rate from Equation (5). The initial parameters from Davis (2005b) are $M_0 = 0.0682 M_\odot$, $\dot{M}_0 = 10^{-6} M_\odot/\text{yr}$, and $J_0 = 0.379 M_\odot \text{AU}^2/\text{yr}$.

Figure 4: Parametric effects on the location of the ice line. (a) Opacity models at 200,000 years. Dashed line: aggregates consisting of iron and silicates. Solid line: aggregates consisting of iron, silicates, volatile organics, and water ice. Opacity models from Henning and Stognienko (1996). (b) Temperature models for the current-day Sun. Dashed line: ice line at 170 K. Solid line: ice line at 130 K. Dotted line: ice line from phase diagram calculation.

Figure 5: Variation of accretion rate with disk mass. Dashed line: data extracted from Min et al. (2011) for three solar disk models using the “alpha” viscosity model. Solid line: Eq. (5). Symbols represent accretion rates of 10^{-6} , 10^{-7} , 10^{-8} , and 10^{-9} from Equation (5) using the “beta” viscosity model.

Figure 6: Comparison of predicted ice lines from Min et al. (2011) (dashed) and current calculation (solid). The accretion rate is indicated above each panel, and the central star has solar luminosity ($\alpha = 0.01$).

Figure 7(a). Evolution of the ice line for the younger disk. Solid line: Solar luminosity according to the evolutionary track in Figure 1. Dashed line: Computation using luminosity of the current Sun.

Figure 7(b). Same as Figure 7(a) but at later times showing close agreement between the two models.