Uncertainty analysis of sonic boom levels measured in a simulator at NASA Langley

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A sonic boom simulator has been constructed at NASA Langley Research Center for testing the human response to sonic booms heard indoors. Like all measured quantities, sonic boom levels in the simulator are subject to systematic and random errors. To quantify these errors, and their net influence on the measurement result, a formal uncertainty analysis is conducted. Knowledge of the measurement uncertainty, or range of values attributable to the quantity being measured, enables reliable comparisons among measurements at different locations in the simulator as well as comparisons with field data or laboratory data from other simulators. The analysis reported here accounts for acoustic excitation from two sets of loudspeakers: one loudspeaker set at the facility exterior that reproduces the exterior sonic boom waveform and a second set of interior loudspeakers for reproducing indoor rattle sounds. The analysis also addresses the effect of pressure fluctuations generated when exterior doors of the building housing the simulator are opened. An uncertainty budget is assembled to document each uncertainty component, its sensitivity coefficient, and the combined standard uncertainty. The latter quantity will be reported alongside measurement results in future research reports to indicate data reliability.

1 INTRODUCTION

Measured data form the basis for many engineering decisions. Well-informed decisions can be made only if the supporting data are reliable. However, the reliability of published data is not always easy to assess. Often, reliability must be inferred from a detailed description of the data collection technique or the reputation of the person collecting data. These indicators are, at best, qualitative. In recent decades, formal uncertainty analysis techniques have become standardized[1,2] as objective, *quantitative* assessments of data reliability. These techniques enable data comparison within the same laboratory, among different laboratories, or between laboratory

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data and field data. The practice of reporting laboratory data without an associated uncertainty leaves readers unable to quantify reliability and is increasingly viewed as incomplete.

An uncertainty analysis has been conducted on sound pressure levels measured at the exterior and interior of the Interior Effects Room (IER) at NASA Langley Research Center. The results of this analysis are uncertainty ranges for one-third-octave band levels for sonic boom signals measured at the facility exterior, and for one-third-octave band levels of boom signals alone and boom plus rattle signals at the facility interior. The combined standard uncertainty will be reported alongside measurement results as an indication of data reliability when documenting research studies. In the short term, the uncertainty in measured interior levels will be used to determine whether interior sound levels across proposed listener locations can be considered uniform for a given excitation signal.

2 THEORY

2.1 Measurement Equation

The first step in an uncertainty analysis is to create a measurement model. Often Y, the quantity of interest, is not measured directly, but calculated as a function of N input quantities[1]:

$$Y = f(X_1, X_2, \dots, X_N) \tag{1}$$

The uncertainty in the quantity of interest, $u(Y)$, can then be calculated by propagating the uncertainty of each input quantity X_i through the measurement function Eq. (1) via Eq. (2). Equation (2) assumes no correlation among input quantities, which is assumed but untested for all quantities in this study.

$$u(Y) = \sqrt{\left[\sum_{i=1}^{N} \frac{\partial f}{\partial X_i} u(X_i)\right]^2} \tag{2}$$

When the measurand, or quantity being measured[1], is measured directly, the measurement equation takes a different form, as shown in Eq. (3). The true, unknowable value of a measurand is expressed as a function of the indicated value, Y', and a number of corrections. Each correction corresponds to a possible source of measurement error.

$$Y = f(Y', K_1, K_2, \dots, K_N) = Y' - K_1 - K_2 - \cdots - K_N \tag{3}$$

In Eq. (3), Y' is the indicated value, K_i are corrections, and Y is the true value of the measurand.

Measurement error has two components: systematic error and random error. Systematic error is present in all measurements in the form of an offset. To the extent that sources of systematic error can be identified, they can be corrected. Random error occurs due to random fluctuations in the input quantities. Random errors, by contrast, cannot be corrected for; they can only be quantified through repeated measurements.

The notation used in Eq. (3) is efficient because each term represents both systematic and random error. The value of a correction, K_i, corresponds to the systematic error, and the uncertainty in the correction, $u(K_i)$, corresponds to the random error. Often the value of the correction is zero, but the uncertainty is non-zero.

2.2 Uncertainty as a Standard Deviation

The uncertainty indicates the range of variation that can be attributed to the measurand. The probability of obtaining a certain measurement result is governed by a statistical distribution, which must be known to quantify the uncertainty. There are two analysis methods for determining the statistical distribution. A 'Type A' analysis consists of repeating measurements until the distribution of the underlying population can be identified. The uncertainty is then assessed as a single standard deviation (1σ) of this distribution. A 'Type B' analysis uses previous knowledge to assess the distribution of an input quantity, for example by means of an instrument's tolerance as reported by a manufacturer. The tolerance is converted to an uncertainty and propagated through the measurement equation to yield an uncertainty in the measurand. For the current study, both Type A and Type B analyses were used. For all Type A analyses, the distributions were tested using a χ^2 test for normality[3], and nearly all distributions for each component were found to be normal. All Type B analyses are assumed to have a uniform distribution, which is recommended when there is no specific knowledge about a distribution[1]. A table of correction components, distribution characteristics, analysis types, and uncertainties is given in Table 1. The table is divided into Environmental Factors, Measurement Equipment, and Measurement Setup. This grouping of uncertainty components is illustrated schematically in Fig. 2 and is the organizing principle for the analysis in subsections 3.1 – 3.3 and the results presented in Figs. 4 – 6.

3 UNCERTAINTY COMPONENTS

3.1 Environmental Factors: Meteorological conditions and background noise

The error in sound pressure level introduced by environmental factors is expressed as:

$$L_{\mathrm{p}} = L'_{\mathrm{p}} - K_{\mathrm{backg.}} - K_{\mathrm{met}} \text{ [dB]} \tag{4}$$

Where L_{p} is the corrected signal level, L'_{P} is the indicated signal level, $K_{\mathrm{backg.}}$ is the correction for background noise, and K_{met} is the correction for meteorological conditions: ambient temperature, pressure, and humidity. These environmental factors affect the characteristic impedance of air, which determines the sound radiated for a given loudspeaker diaphragm velocity. Plane wave propagation is assumed.

Eq. (5) shows how variations in ambient temperature and pressure translate to changes in measured sound level. The derivation of Eq. (5) is omitted here for brevity, although it is reproduced in Starnberg[4]:

$$K_{\mathrm{met}} = 20 * \log_{10}\left(\frac{B_{\mathrm{meas}}}{B_{\mathrm{ref}}}\right) - 10 * \log_{10}\left(\frac{T_{\mathrm{meas}}}{T_{\mathrm{ref}}}\right) \text{ [dB]} \tag{5}$$

In Eq. (5), B_{meas} is the ambient pressure at the time of measurement and B_{ref} is the reference ambient pressure, both in pascals. T_{meas} is the ambient temperature at the time of measurement, and T_{ref} is the reference ambient temperature, both in kelvin. The correction associated with both factors is quite small. Even if the temperature varied, according to a uniform distribution, between 12 °C and 32 °C, it would only produce a variation in level of 0.09 dB. The ambient pressure distribution in Hampton, VA over the past two years[6] is normally distributed with a mean of 1013.25 hPa and a standard deviation of 8.07 hPa, which corresponds to an uncertainty of only 0.08 dB. The ambient humidity could be uniformly distributed between 0% and 100% at 22 °C and would only correspond to an uncertainty of 0.015 dB, as calculated by Eq. (2) in Wong[5]. The meteorological conditions also affect the microphone characteristics in a minor way. The microphone meteorological sensitivity coefficients are shown in Table 1 for completeness. Due to the small magnitude of the meteorological corrections, the authors do not intend to apply them to future measurements. In support of this decision, an ISO standard[2] suggests that normalization to reference atmospheric conditions is not necessary at altitudes below 500 m above sea level and within a temperature range of -20° C to 40° C.

Background noise is another environmental factor that can contribute to the uncertainty in signal levels. If the signal is 15 to 20 dB higher than the background noise, the background noise contributes almost no uncertainty to the measured levels. As the signal to noise ratio decreases, it becomes increasingly unclear whether the measured levels represent the signal or the noise. The correction for background noise[2] is expressed as:

$$K_{\mathrm{backg.}} = -10 * \log_{10}\left[1 - 10^{\frac{-(\Delta L)}{10}}\right] \,[\mathrm{dB}] \tag{6}$$

where $\Delta L = L'_P - L_B$ [dB]. L'_P is the apparent signal level measured in the presence of background noise, and L_B is the measured background noise level, both in dB. The uncertainty in the correction in Eq. (6) is found by applying Eq. (2) to Eq. (6).

$$u\left(K_{\mathrm{backg.}}\right) = \sqrt{\left[\frac{\partial K_{\mathrm{backg.}}}{\partial L'_{\mathrm{P}}} u\left(L'_{\mathrm{P}}\right)\right]^2 + \left[\frac{\partial K_{\mathrm{backg.}}}{\partial L_{\mathrm{B}}} u\left(L_{\mathrm{B}}\right)\right]^2} \,[\mathrm{dB}] \tag{7}$$

Evaluating Eq. (7) yields

$$u\left(K_{\mathrm{backg.}}\right) = \frac{10^{-0.1(\Delta L)}}{1-10^{-0.1(\Delta L)}} \sqrt{u^2(L'_{\mathrm{P}}) + u^2(L_{\mathrm{B}})} \,[\mathrm{dB}] \tag{8}$$

Across most of the bandwidth, signals are well above the background noise levels. At high frequencies, where sonic booms have much less energy, particularly after transmitting through building walls, the signal to noise ratio decreases, which increases the background noise correction. A sample signal measurement at the facility interior, along with the background noise and corrected signal level, is shown in Fig. 3. Acquired levels of boom signals alone are truncated above 5000 Hz where no boom signal is produced. The maximum uncertainty in $K_{\mathrm{backg.}}$ at each one-third-octave band across multiple measurement locations is shown in Fig. 4. The uncertainty is shown separately for microphones at the facility exterior and interior. The interior signals are further divided into boom signals alone, transmitted by the exterior arrays, and boom signals accompanied by rattle signals, transmitted by the interior satellite loudspeakers. Adding rattle noises increases high frequency signal levels, which decreases uncertainty between 2 and 5 kHz, as shown in Fig. 4.

It is recommended that the background noise correction be calculated for each signal individually. The signal to noise ratio, and thus the uncertainty associated with the background noise, depends on the playback level of the signals and whether interior loudspeakers are used or not. A typical boom and rattle level were used to produce the data shown here. For this combination, the boom governs measured levels at and below 500 Hz, and the rattle sound governs above 500 Hz. Software has been written to facilitate determining the background noise correction and resulting uncertainty in signal level.

The pressure fluctuations generated by opening exterior doors to the building housing the facility were found to have no effect in the bandwidth of interest (6 Hz – 20 kHz). The measurement results that led to these conclusions are excluded for brevity.

3.2 Measurement Equipment

Assuming that any errors associated with the cables or cable junctions in the measurement chain are negligible, the equation for the measurement chain is

$$L_{\mathrm{p}} = L'_{\mathrm{p}} - K_{\mathrm{mic+preamp}} - K_{\mathrm{sig.\ cond.}} - K_{\mathrm{analog/digital}} - K_{\mathrm{acquisition\ software}}\ [\mathrm{dB}] \qquad (9)$$

The quantity $u(K_{\mathrm{mic+preamp}})$ is the uncertainty associated with the microphone and preamplifier frequency response. It is determined from a Type A analysis of the electrostatic actuator calibration[7]. The quantity $u(K_{\mathrm{sig.\ cond.}})$ is the uncertainty associated with the signal conditioning amplifier determined by a Type B analysis[8]. The quantity $u(K_{\mathrm{analog/digital}})$ is the uncertainty associated with the analog to digital conversion by the data acquisition card and is determined by a Type B analysis[9]. The quantity $u(K_{\mathrm{acquisition\ software}})$ is the uncertainty associated with the acquisition software. In this case, the acquisition software only applies the microphone sensitivity acquired from an in-situ calibration procedure, so the uncertainty in the correction is the uncertainty in the microphone sensitivity.

The in-situ calibration procedure presents a known sound pressure level to a microphone and records the resulting voltage. The measurement equation is:

$$S = \frac{V\ [\mathrm{mV}]}{P[\mathrm{Pa}]} \qquad (10)$$

where S is the microphone sensitivity, P is the sound pressure produced by the pistonphone, and V is the resulting voltage acquired. Applying Eq. (2) for propagation of error yields

$$u(S) = \sqrt{\left(\frac{1}{P}\right)^2 u^2(V) + \left(\frac{-V}{P^2}\right)^2 u^2(P)}\ \left[\frac{\mathrm{mV}}{\mathrm{Pa}}\right] \qquad (11)$$

The governing term in Eq. (11) is $u^2(P)$. According to the manufacturer, the uncertainty (1σ) in level of a Type 4228 pistonphone is 0.15 dB (0.55 Pa) after adjusting for the ambient pressure with the included Class 1L barometer[10]. An additional correction is included in Eq. (12) to quantify the uncertainty associated with attaching the pistonphone. The uncertainty in the microphone sensitivity must be converted to decibels prior to insertion in Eq. (12).

$$u(K_{\mathrm{acquisition\ software}}) = \sqrt{u^2(S) + u^2\left(K_{\mathrm{attaching\ pistonphone}}\right)}\ [\mathrm{dB}] \qquad (12)$$

A Type A analysis was undertaken to determine the uncertainty associated with attaching the pistonphone. The result is 0.01 dB, as shown in Table 1. When repeated measurements are made with the pistonphone left attached to the microphone, the random error is 0.007 dB. For a conscientious user, therefore, the process of attaching the pistonphone is highly repeatable.

The uncertainty components corresponding to the measurement chain are plotted in Fig. 5. Below 1000 Hz, the uncertainty is governed by the uncertainty in pistonphone excitation level, which appears in the acquisition software component. Between 10 kHz and 20 kHz, the uncertainty in the microphone and preamplifier frequency response governs. It is noted that the electrostatic actuator calibration of the microphone does not account for the effect of the microphone grid cap.

3.3 Measurement Setup

The measurement equation relating to the measurement setup is:

$$L_{\mathrm{p}} = L'_{\mathrm{p}} - K_{\text{sound reproduction}} - K_{\text{latching}} - K_{\text{position+incidence angle}}\ [\mathrm{dB}] \tag{13}$$

The quantity $u(K_{\text{sound reproduction}})$ is the uncertainty associated with the repeatability of playback over the sound reproduction system. The uncertainty is assessed by a Type A analysis using 20 repeats of a 30-second pink noise signal. Pink noise is used rather than a sonic boom so the result is not subject to the narrow band troughs present in a sonic boom spectrum. It is assumed, but unverified, that the uncertainty in level across repeated acquisitions of pink noise is equivalent to the uncertainty in level across repeated sonic boom acquisitions. The distribution of the repeated measurements is normal, and the uncertainty is less than 0.045 dB at all one-third-octave bands.

The quantity $u(K_{\text{latching}})$ is examined to determine whether failing to tighten the turnbuckles introduces uncertainty into the measurement procedure. The turnbuckles, shown in Fig. 1, are the four horizontal rods in the center of the picture that hold the array flush with the structure. A Type A analysis is used to assess the uncertainty across repeated acquisitions of a sonic boom signal. Twenty repeats of the signal are acquired with the turnbuckles fully tightened, and then twenty repeats are acquired with the turnbuckles fully loosened. The difference, in quadrature, in uncertainty across these two conditions is the uncertainty associated with latching, as shown in Eq. (14). The resulting uncertainty reaches a maximum value of 0.08 dB at 800 Hz for exterior microphones and 0.03 dB at 1600 Hz for interior microphones.

$$u(K_{\text{latching}}) = \sqrt{u^2(K_{\text{unlatched}}) - u^2(K_{\text{sound reproduction}})}\ [\mathrm{dB}] \tag{14}$$

The uncertainty $u(K_{\text{position+incidence angle}})$ is associated with the inability to place the microphone in the exact same position and with the exact same orientation as reported. It is the uncertainty anticipated if a different researcher were to place the microphones at specified positions indicated in a report. Three interior microphones were moved and relocated to their measurement position between each of 20 repeats of the 30-second pink noise signal for this Type A analysis. At each one-third-octave band, the average variation across the three typical listener locations is used as the typical value. The uncertainty is evaluated via Eq. (15). It is observed that this component governs the combined uncertainty associated with the measurement setup for interior microphones. Exterior microphone locations, by contrast, are fixed and well defined. A simple linear envelope is generated as a conservative estimate of the uncertainty. Both the envelope and the raw data are shown in Fig. 6. It is noteworthy that the repositioning uncertainty is approximately the same whether the excitation comes from the exterior array or the interior satellite loudspeakers.

$$u(K_{\text{position+incidence angle}}) = \sqrt{u^2(K_{\text{moved}}) - u^2(K_{\text{sound reproduction}})}\ [\text{dB}] \qquad (15)$$

3.4 Combined Standard Uncertainty

The overall measurement equation including all corrections is:

$$L_{\text{p}} = L'_{\text{p}} - K_{\text{met}} - K_{\text{backg.}} - K_{\text{mic+preamp}} - K_{\text{sig. cond.}} - K_{\text{A/D}} - K_{\text{acquisition software}} - K_{\text{sound reproduction}} - K_{\text{latching}} - K_{\text{position+incidence angle}}\ [\text{dB}] \qquad (16)$$

Therefore the combined standard uncertainty is calculated as the sum, in quadrature, of the uncertainty in each correction. This combined standard uncertainty is plotted in Fig. 7. The main components contributing to uncertainty are identified as follows. Across the entire spectrum, the combined uncertainty for exterior microphones is governed by the uncertainty in the measurement chain, with a small contribution from the latching above 500 Hz. The combined uncertainty for interior microphones is governed by the measurement chain below 10 Hz. Between 10 Hz and 10 kHz, the combined uncertainty on the interior microphones is governed by the microphone positioning, except for the range from 3 – 5 kHz for the boom alone, which is governed by both microphone positioning and background noise. The background noise affects the combined uncertainty for boom and rattle signals only above 10 kHz.

4 DISCUSSION AND CONCLUSIONS

The primary result of this work is the uncertainty associated with sonic boom levels measured in the Interior Effects Room at NASA Langley. According to the results, the uncertainties are below 1 dB across most of the one-third-octave band center frequencies of interest. This magnitude agrees well with the uncertainty of airborne sound measurements on automobile components determined by Starnberg[4]. In the present study, fluctuations were observed in the facility's background noise levels. Because the background noise correction depends on the signal to noise ratio, it is recommended that the correction be calculated separately for each signal. For completeness, the ambient meteorological conditions will be recorded at the time of measurement, even if they will not be corrected for.

The uncertainties reported here do not indicate the uncertainty of what test subjects in the facility hear. What a test subject hears depends on additional factors, including the orientation and position of a subject's head, a subject's individual head-related transfer function, and the unknowable incidental noises from other test subjects that may occur. The uncertainties reported here are intended for use only when reporting objective measurements in the facility.

The uncertainties have been determined here at one-third-octave bands. The next step planned for this work is to propagate these uncertainties to yield an uncertainty in noise and sound quality metrics calculated for each signal.

5 REFERENCES

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Fig. 1 – One of two arrays at exterior of Interior Effects Room. Turnbuckles, four of which can be seen within the yellow oval, are used to secure array to the exterior facility wall.

Table 1 – Uncertainty Budget.

	Correction Component	Distribution	Analysis Type	Uncertainty (1σ) For Type 4193 Microphone[A]
Environmental Factors				
Sound Radiation	Temperature	Uniform $\bar{x}$ = 22 °C σ = 5.8 °C, estimated	B	0.09 dB [11]
Sound Radiation	Pressure	Normal $\bar{x}$ = 1013.25 hPa σ =8.07 hPa[6]	B	0.08 dB [11]
Sound Radiation	Humidity	Uniform $\bar{x}$ = 50% σ =29%, estimated	B	0.015 dB [11]
	Background	Normal, varies with frequency	A	depends on signal to noise ratio
	Door	Normal	A	(Not applicable)
Measurement Equipment				
Microphone	Frequency Response actuator calibration, including preamplifier	Normal, varies with frequency	A	0 - 0.185 dB [7] before interpolation
Microphone	Temperature	(see above)	B	(Sensitivity coefficient 0.002 dB/°C [11]) 0.012 dB
Microphone	Pressure	(see above)	B	(Sensitivity coefficient -0.005 dB/kPa [11]) 0.004 dB
Microphone	Humidity	(see above)	B	Negligible [11]
	Signal Conditioning Amp	Uniform	B	0.050 dB [8]
	Analog/Digital Converter	Uniform	B	0.0035 dB [9]
	Attaching Pistonphone	Normal	A	0.01 dB
	Pistonphone (and Acquisition Software)	Normal Uniform	A B	Random error ~ 0.007 dB Systematic error ~ 0.15 dB [10]
Measurement Setup				
	Sound Reproduction	Normal, varies with frequency	A	≤ 0.045 dB
	Latching	Normal, varies with frequency	A	≤ 0.18 dB (interior) ≤ 0.08 dB (exterior)
	Position and Incidence Angle (interior only)	Normal, varies with frequency	A	≤ 0.69 dB (envelope)

[A] The original intent was to report results separately for Brüel and Kjær Type 4193 and G.R.A.S. Type 40AQ microphones, as both types are used in the facility. However, many technical details available for the Type 4193 microphone[11] were not available for the Type 40AQ, so only uncertainty results for the Type 4193 are reported.

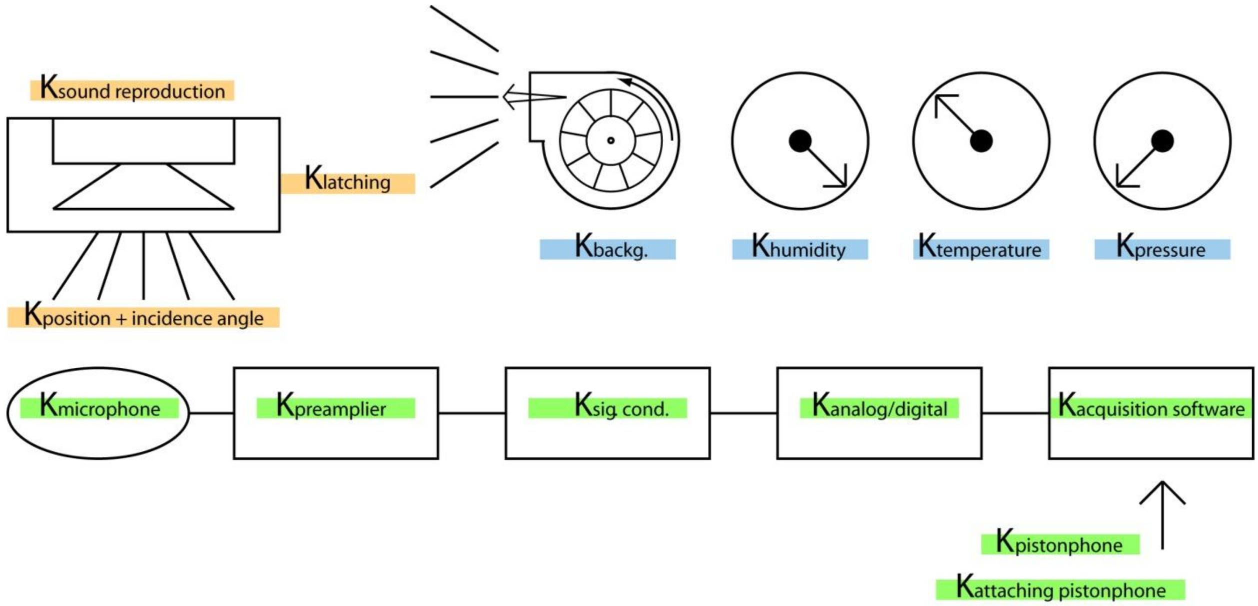

Fig. 2 – Schematic illustration of uncertainty sources in the IER. Sources are grouped into Environmental Factors (blue), Measurement Equipment (Green), and Measurement Setup (orange). The same groupings are used in Section 3 of the paper, and in Table 1.

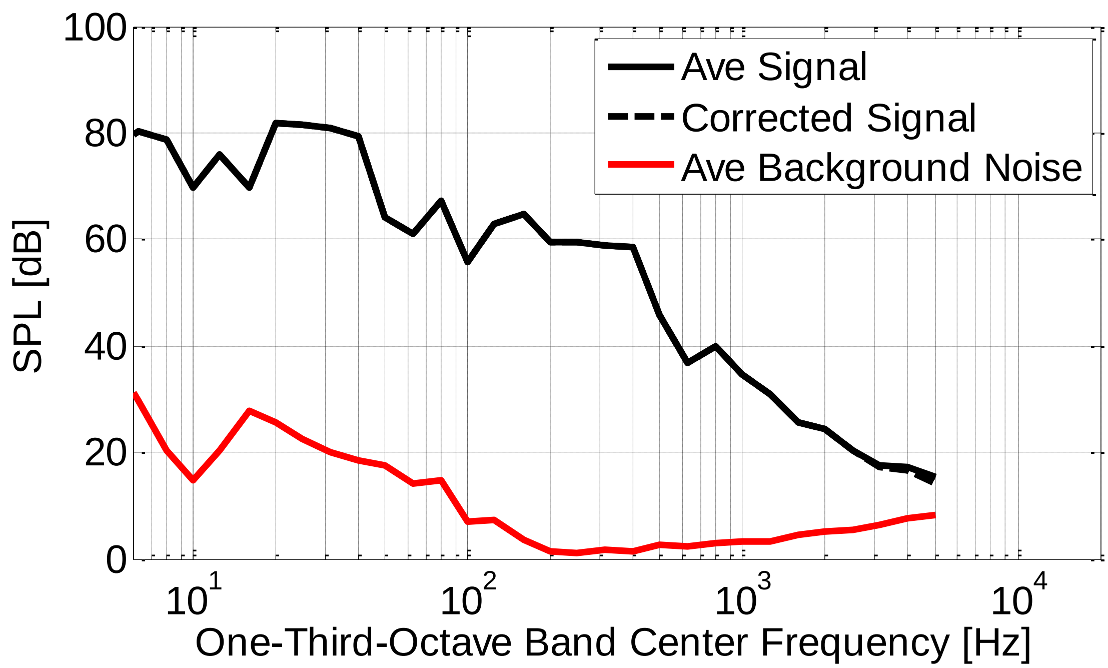

Fig. 3 – Average sonic boom levels across 20 repeats for boom alone (no rattle noise) and average background noise. Both quantities measured at facility interior.

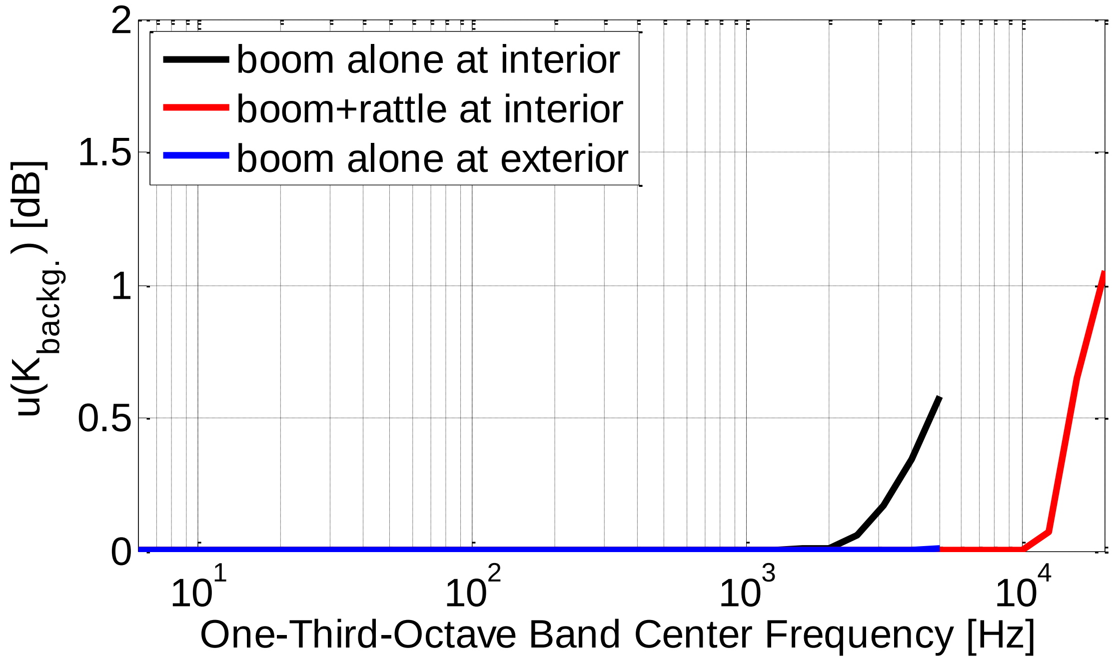

Fig. 4 – Uncertainty associated with background noise, u(K_backg.).

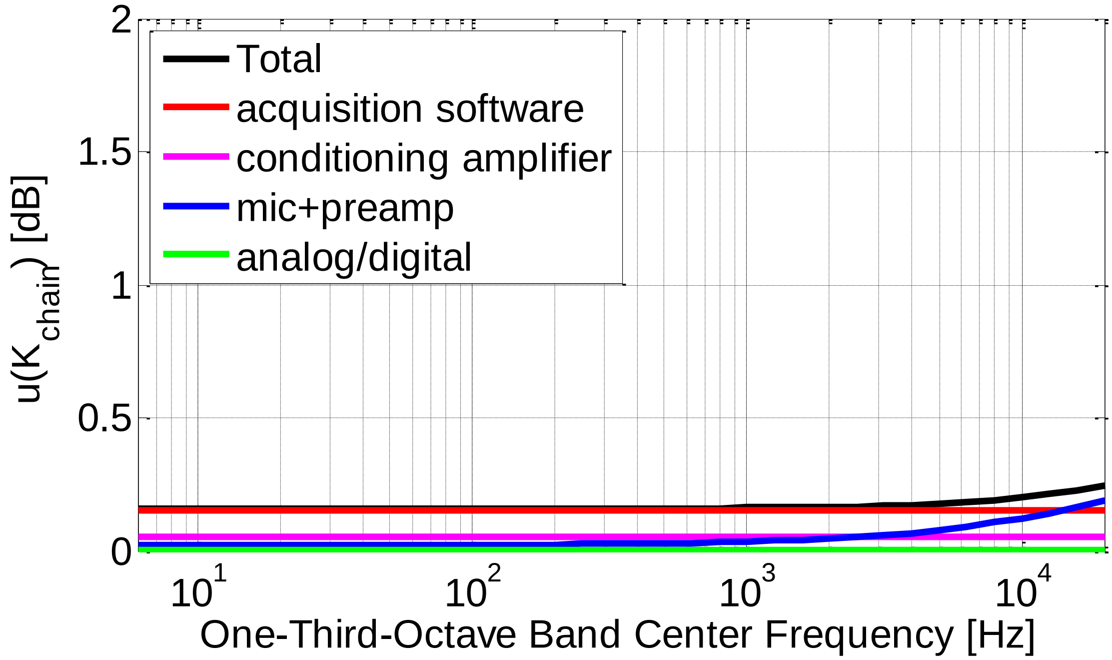

Fig. 5 – Uncertainty associated with measurement chain, u(K_chain).

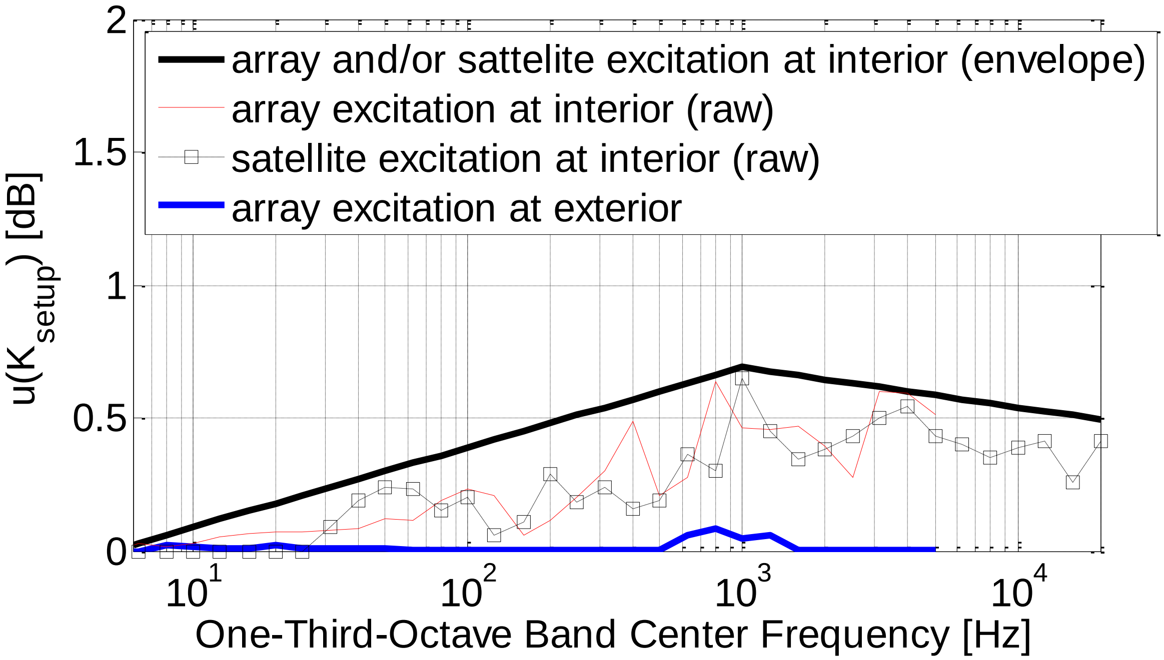

Fig. 6 – Uncertainty associated with measurement setup, u(K_setup).

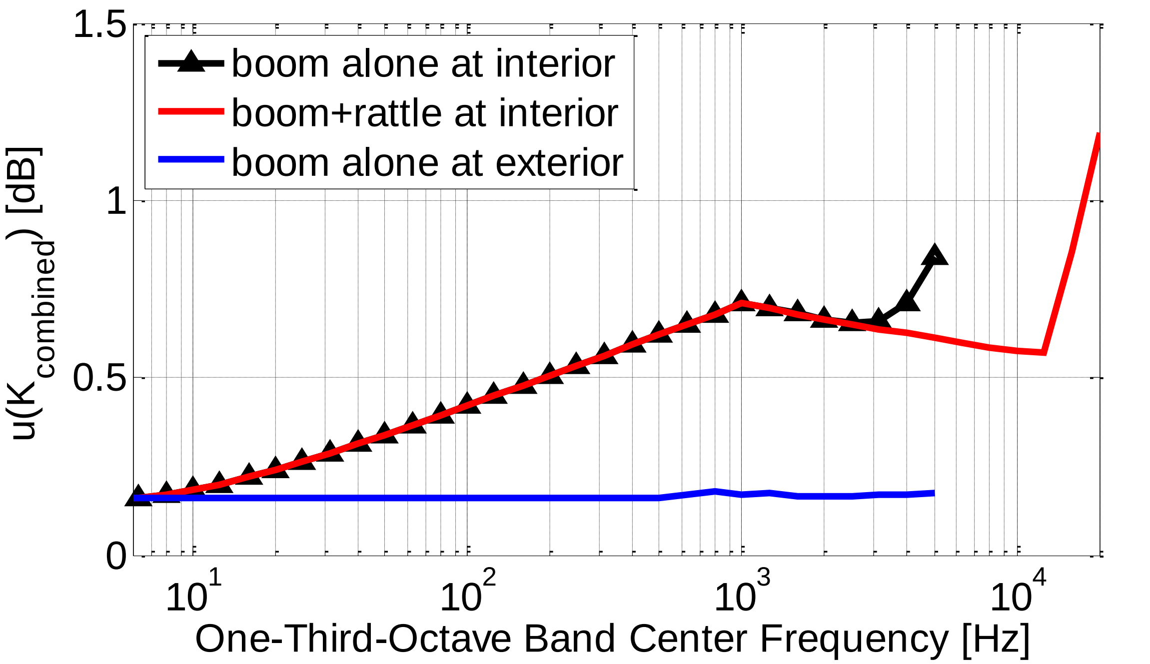

Fig. 7 – Combined standard uncertainty, u(K_combined).