Reporting Recommended Patch Density from Vehicle Panel Vibration Convergence Studies using both DAF and TBL Fits of the Spatial Correlation Function

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Introduction/Agenda

- Motivation
- Evaluating Patch Approximation of Spatial Correlation Sinc Function for Diffuse Acoustic Field (DAF)
  - *Qualitative Comparison: How does patch approximation resemble the continuous function?*
  - *Quantitative Comparison: Comparison of Lobe by Lobe Enclosed Area.*
- Report Convergence Trials
  - *Relate to Test Article Modes of Interest*
  - *Present the Patch Assumptions and Center to Center Distances*
- Can the same be said for Corcos Correlation Function and TBLs?
  - *Qualitative/ Quantitative Comparison included in Backup Charts*
- Conclusion Summary and Recommendations
  - *Relate patch size to fluid wavelength*
  - *Recommendation*
- References

Motivation

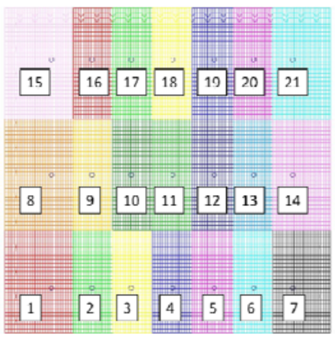
- Using the patch method to represent the continuous spatial correlation function of a phased pressure field over a structural surface is an approximation.
- The approximation approaches the continuous function as patches become smaller.
- Plotting comparisons of the approximation vs the continuous function may provide insight revealing:
  - *For what patch size/density should the approximation be very good?*
  - *What the approximation looks like when it begins to break down?*
  - *What the approximation looks like when the patch size is grossly too large.*
- Following these observations with a convergence study using one FEM may allow us to see the importance of patch density.
- We may develop insights that help us to predict sufficient patch density to provide adequate convergence for the intended purpose frequency range of interest.

Motivation

How is the Spatial Correlation Function Used?

Pressure Excitation of Panel (The Patch Method)

Cross correlation may be calculated between the pressures any pair of patches (exhibiting a non-zero cross-spectral density between them). The random pressure field is represented as a Hermitian matrix of spectral densities of dimension N_p, the total number of pressure patches. The pressure autospectra occur on the diagonal of the matrix. The cross-spectra appear on off diagonals:



$$P(\omega) = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N_p} \\ P_{21} & P_{22} & \cdots & P_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N_p1} & P_{N_p2} & \cdots & P_{N_pN_p} \end{bmatrix}, \qquad (8)$$

where $P_{jk} = P^*_{kj}$. If spatial functions $\gamma(\omega, r)$ are defined to relate the autospectra to the cross-spectra, Eq. (8) may be written as :

$$P(\omega) = \begin{bmatrix} \gamma_{11}\hat{P}_{11} & \gamma_{12}\hat{P}_{12} & \cdots & \gamma_{1N_p}\hat{P}_{1N_p} \\ \gamma_{21}\hat{P}_{12} & \gamma_{22}\hat{P}_{22} & \cdots & \gamma_{2N_p}\hat{P}_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N_p1}\hat{P}_{1N_p} & \gamma_{N_p2}\hat{P}_{2N_p} & \cdots & \gamma_{N_pN_p}\hat{P}_{N_pN_p} \end{bmatrix}, \qquad (9)$$

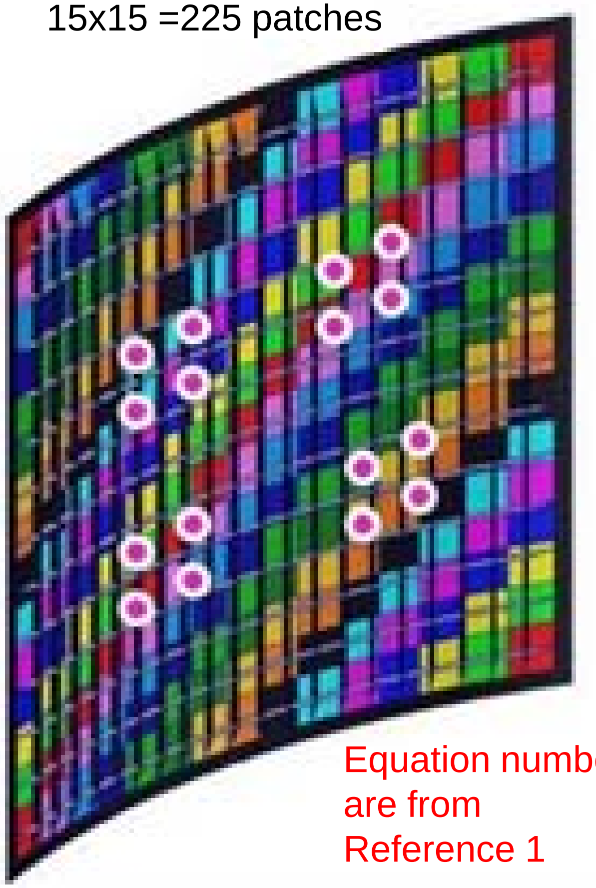
Equation numbers are from Reference 1

Motivation

How is the Spatial Correlation Function Used?

Pressure Excitation of Panel (The Patch Method)

15x15 =225 patches



Equation numbers are from Reference 1

When j = k the spatial functions coincide with the patch autospectra and the gamma approaches unity by L'Hopital's Rule.

where $\hat{P}_{jk} = \sqrt{P_{jj}P_{kk}}$. As will be discussed shortly, in the case of a DAF,

$\gamma_{jk}(\omega, r_{jk}) = 1$ along the diagonals;

however, the γ terms have been included in the diagonal terms in Eq. (9) for generalization. The expression for $\hat{P}_{jk}$ satisfies an inequality requirement on the coherence which states that

$$0 \le \frac{|P_{jk}(\omega)|^2}{P_{jj}(\omega)P_{kk}(\omega)} \le 1.0. \qquad (10)$$

For a DAF, the spatial functions may be expressed as:

$$\boxed{\gamma_{jk}(\omega, r_{jk}) = \frac{\sin(kr_{jk})}{kr_{jk}}, \qquad (11)}$$

17

Pressure Excitation of Panel (The Patch Method)

Finally, the components of the pressure matrix may be expressed as products of frequency-dependent scaling functions, W_{jk} and an arbitrary reference autospectrum pressure, P_{ref} :

$$P_{jk}(\omega) = \gamma_{jk} W_{jk}(\omega) P_{ref}(\omega) \qquad (12)$$

and:

$$P(\omega) = \begin{bmatrix} W_{11} & \gamma_{12} W_{12} & \cdots & \gamma_{1N_p} W_{1N_p} \\ \gamma_{12} W_{12} & W_{22} & \cdots & \gamma_{2N_p} W_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1N_p} W_{1N_p} & \gamma_{2N_p} W_{2N_p} & \cdots & W_{N_p N_p} \end{bmatrix} P_{ref}, \qquad (13)$$

or more compactly:

$$\boxed{P(\omega) = [W]^{\frac{1}{2}} [\Gamma] [W]^{\frac{1}{2}} P_{ref}, \qquad (14)}$$

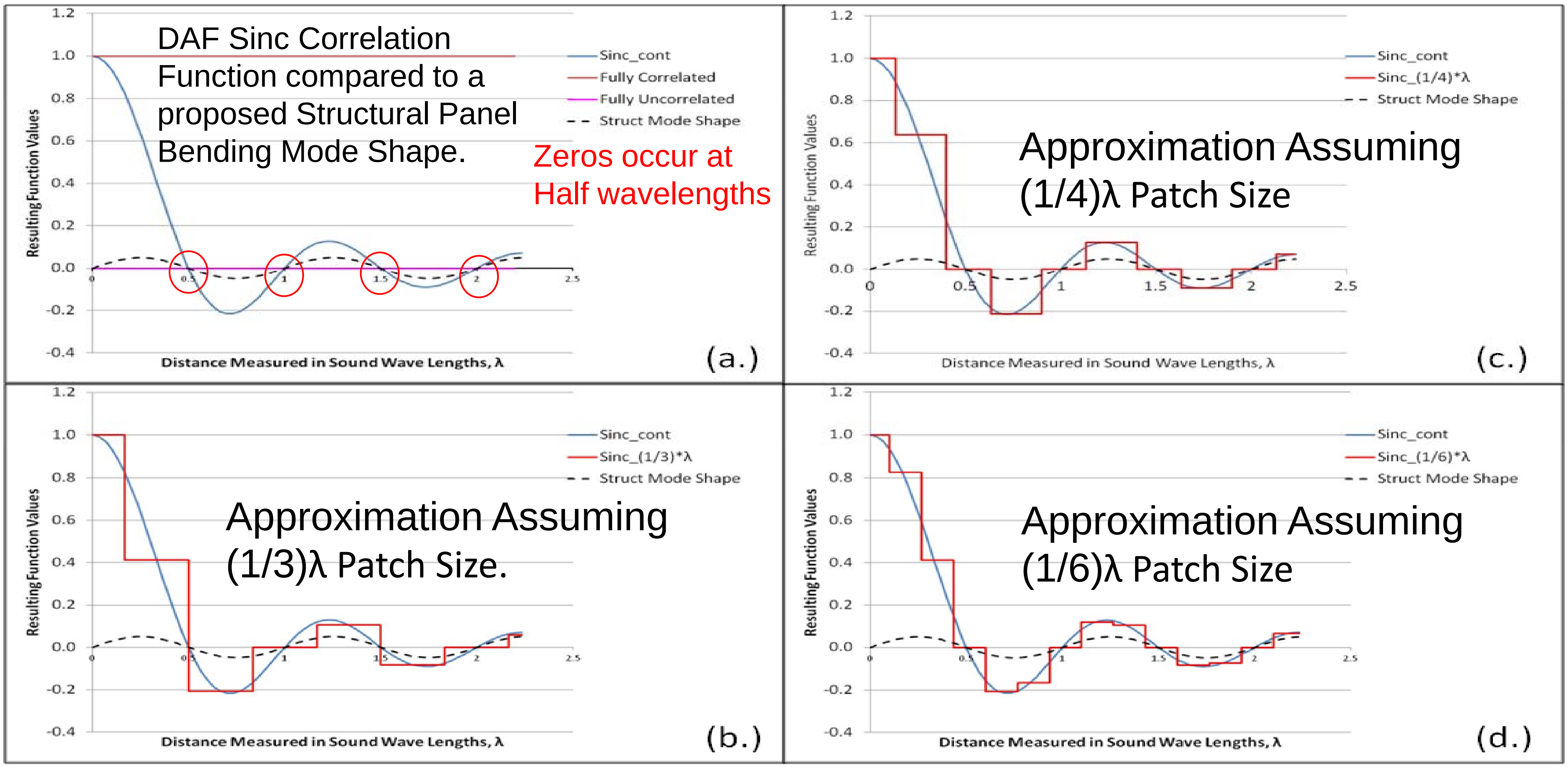
and for a DAF:

$$\Gamma(\omega) = \begin{bmatrix} 1 & \frac{\sin(kr_{12})}{kr_{12}} & \cdots & \frac{\sin(kr_{1N_p})}{kr_{1N_p}} \\ & 1 & \cdots & \frac{\sin(kr_{2N_p})}{kr_{2N_p}} \\ \text{SYM} & & \ddots & \vdots \\ & & & 1 \end{bmatrix}. \qquad (15)$$

Equation numbers are from Reference 1

Evaluating Patch Approximation of Spatial Correlation Function

Selecting Adequate Patch Density Resolution
For Largest Frequency of Interest



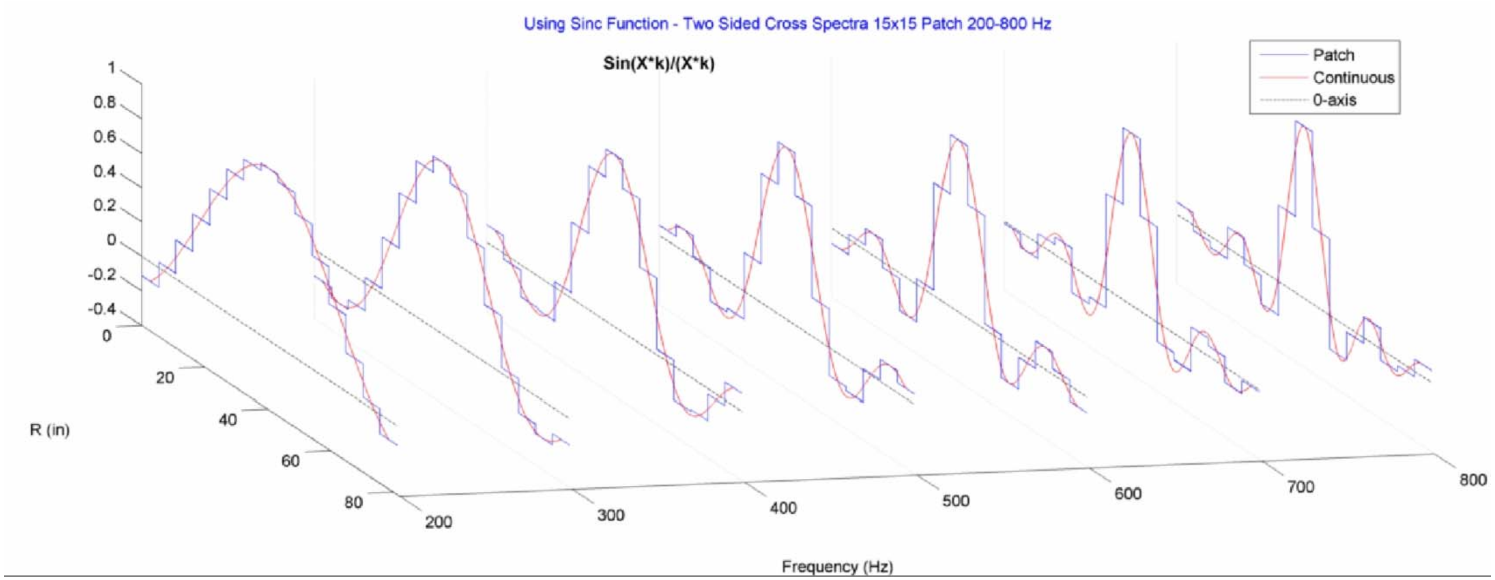
Qualitative Observation may indicate that the
Choice for Patch-Size/Patch-Density should lie between 1/3 and 1/6 of the fluid wavelength.

Wavelength of sound in fluid at the highest frequency of interest = λ

Evaluating Patch Approximation of Spatial Correlation Function

- Using the patch method to represent the continuous spatial correlation function of a phased pressure field over a structural surface is an approximation.
- The approximation approaches the continuous function as patches become smaller.
- Determining what size patch is adequately small to represent the correlation depends on several parameters
  - *Since the continuous function depends on wave number/wave length the answer is frequency dependent.*
    - The higher the frequency of interest for analysis the smaller the size of the patch needed to represent the pressure forcing function. (implies a gross limit)
  - *Since the approximation keys off the distance between patch centers and assigns a constant value to all the nodes within a patch, the calculated value at the center of patch is important.*
    - Since the continuous function has regularly spaced zeros, one should avoid a Patch “center to center distance” nearly or exactly equal to ½ the fluid pressure wavelength for the frequency of interest.(very nearly the uncorrelated case.)
  - *Is the vehicle panel response sensitive to spatial correlation? Does an uncorrelated solution or fully correlated analysis ever provide an adequate solution?*

Evaluating Patch Approximation of Spatial Correlation Function



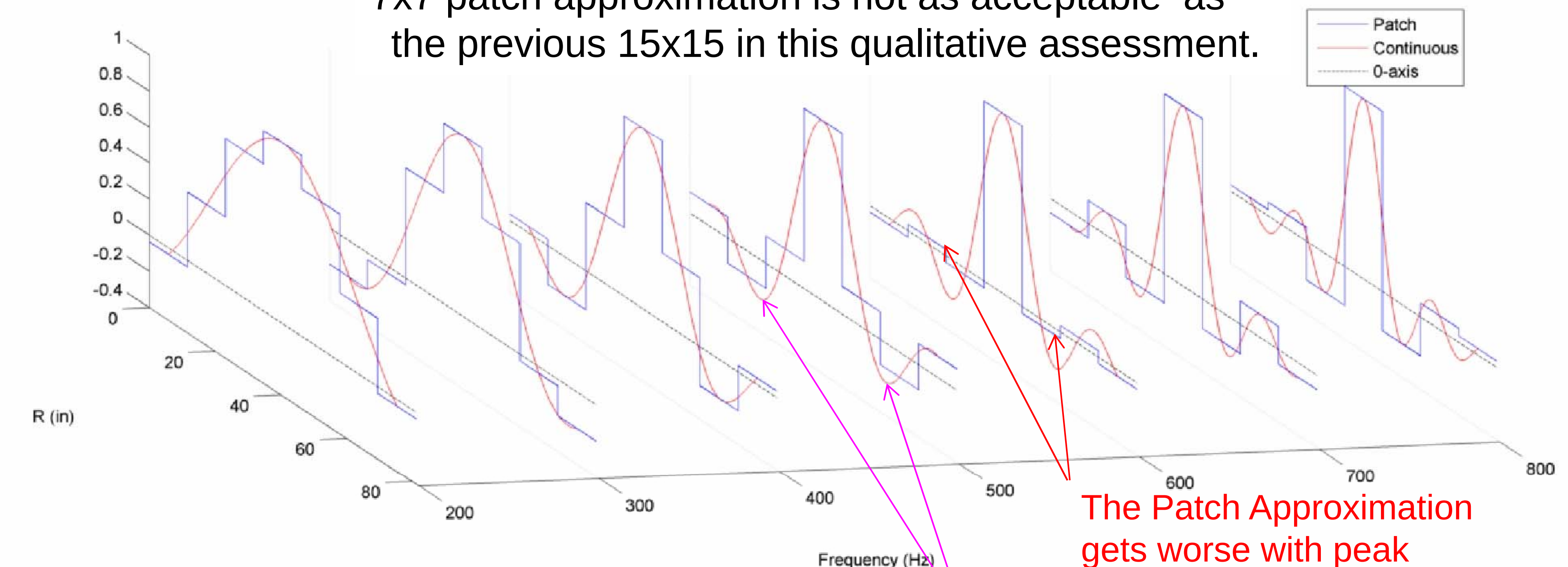
Illustrating the Frequency Dependence of the Sinc Spatial Correlation Function

- Plot presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function
- The Sinc function is a function of wave-number and thus changes with frequency.
- A Smaller Patch Size/Higher Patch Density is required to assess higher frequencies
- Choice of patch density should depend on the highest frequency of interest.

- 15x15 patch approximation of the continuous sinc function seems acceptable in this qualitative assessment for all the frequencies up to 800 Hz.

Evaluating Patch Approximation of Spatial Correlation Function

7x7 patch approximation is not as acceptable as the previous 15x15 in this qualitative assessment.

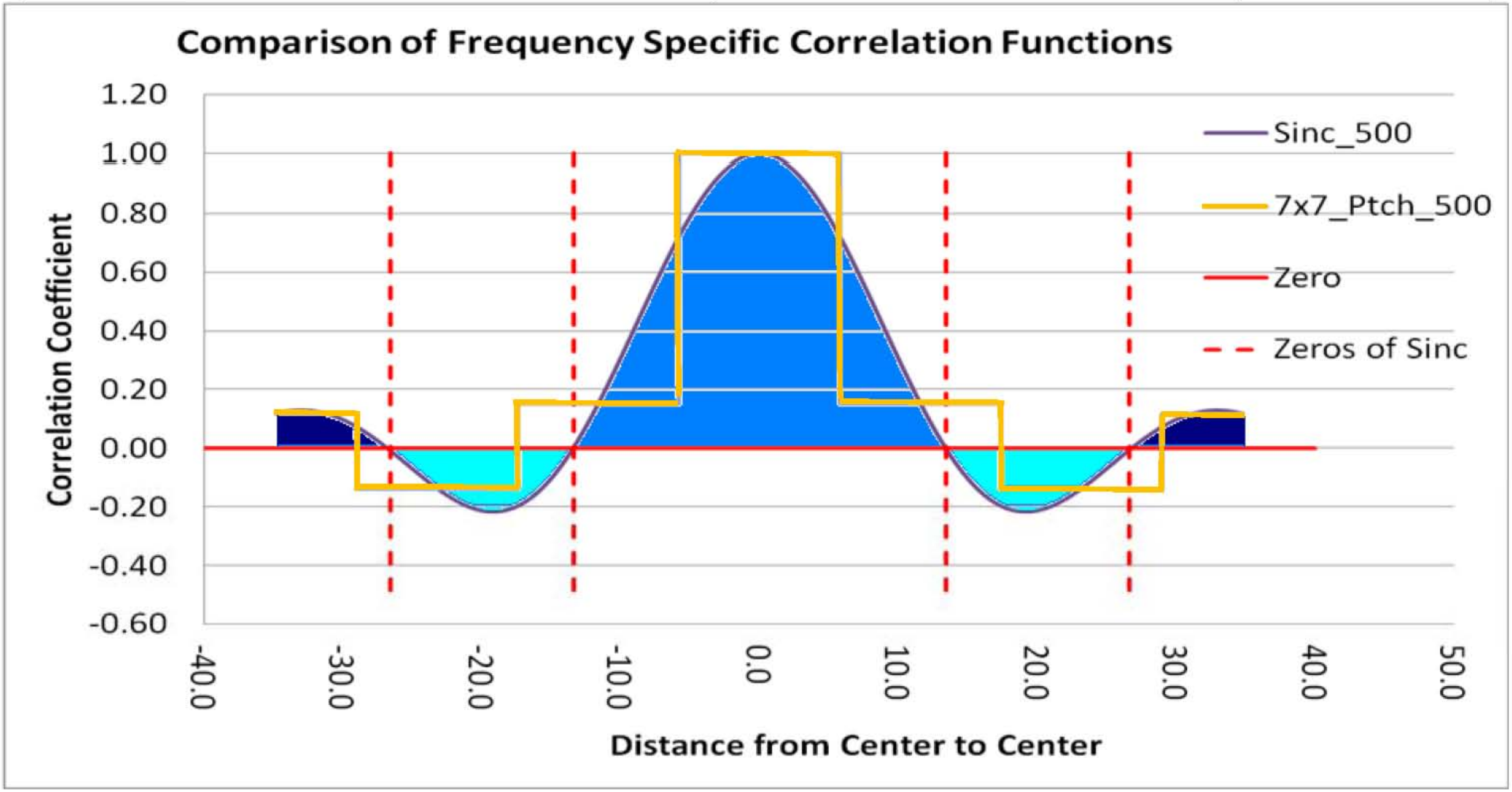


The Patch Approximation gets worse with peak values approaching zero.

The "First Local Minimum" of the Correlation Function is missed by the Patch Approximation.

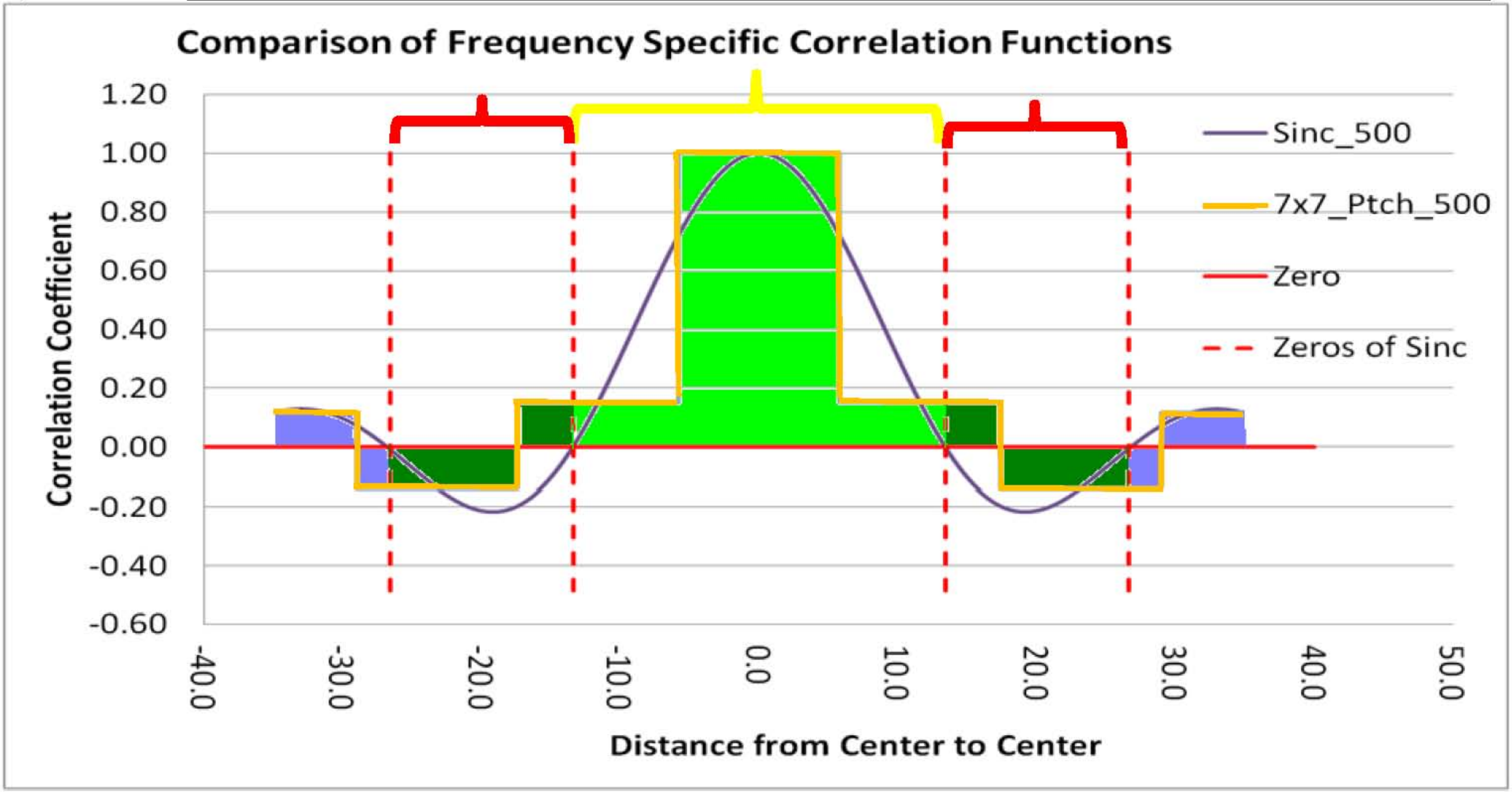
- Where does the Patch assumption seem to break down?
- Can I recognize it without running a convergence study?
- Will a convergence study confirm my qualitative observations?
- **Need Quantitative Comparison of the Correlation Functions – "Enclosed Area"**

Evaluating Patch Approximation of Spatial Correlation Function



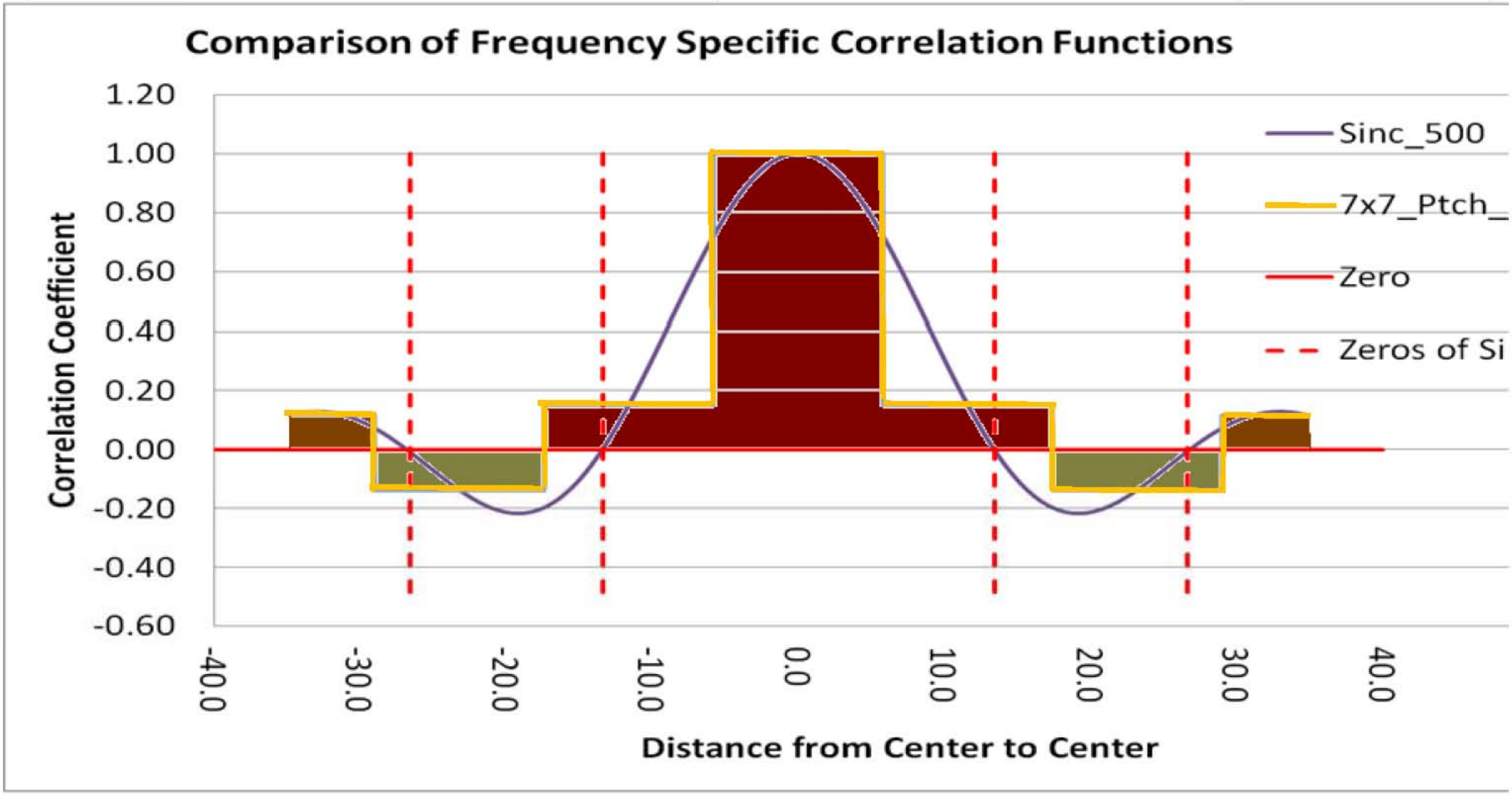
- Sinc Function Evaluated at 500 Hz
- Patch approximation of the same function at 500 Hz
- Enclosed area of the continuous Sinc correlation function is presented in shaded lobes above and below zero
- **500 Hz example shows that 7x7 patch approximation is not Ideal at this frequency**.

Evaluating Patch Approximation of Spatial Correlation Function



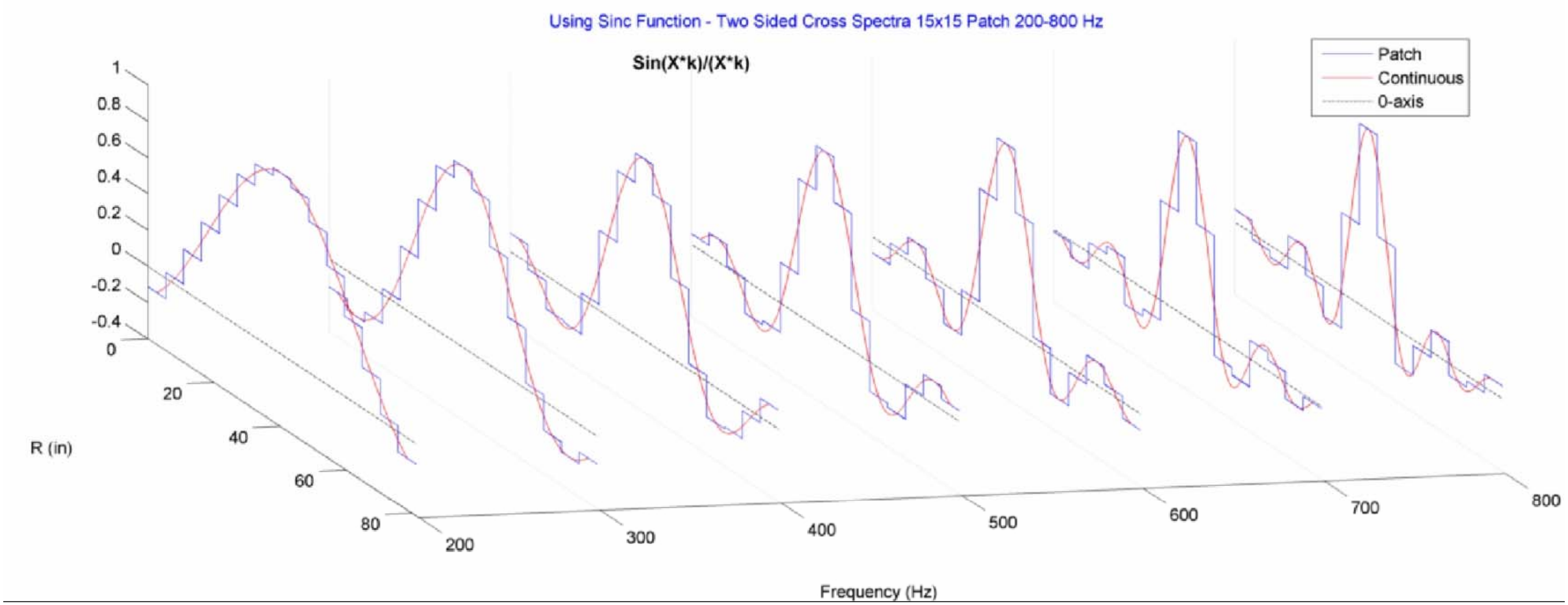
- Between the yellow bracketed Zeros the Function should be positive with positive enclosed area.
- Between the red bracketed Zeros the Function should be negative with negative enclosed area.
- **The approximation is not perfect by this evaluation criteria**
- The net enclosed area in red is negative (but smaller negative)

Evaluating Patch Approximation of Spatial Correlation Function



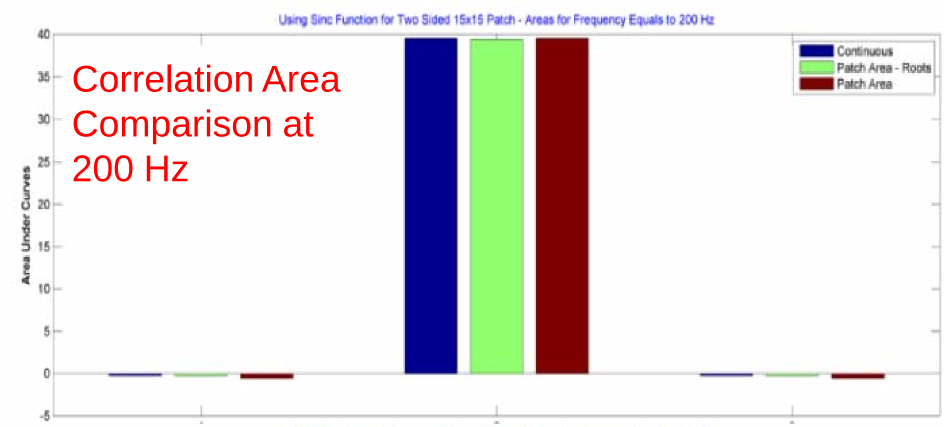
- The actual enclosed area of the 7x7 approximation is spatially distributed differently than the Sinc
- The size of its realized positive and negative lobes was also evaluated without regard to the zeros of the sinc function.

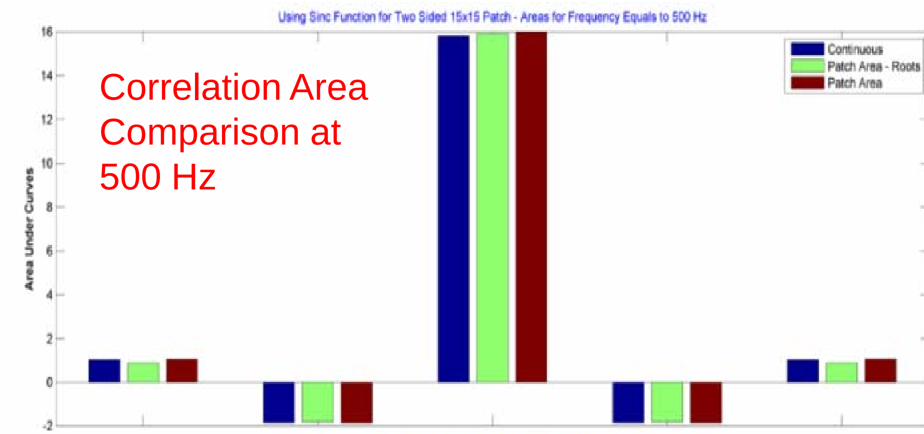
Evaluating Patch Approximation of Spatial Correlation Function

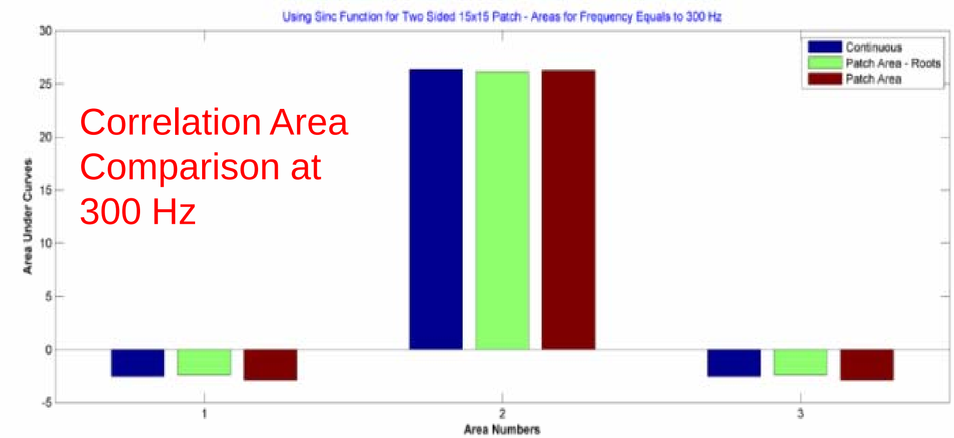


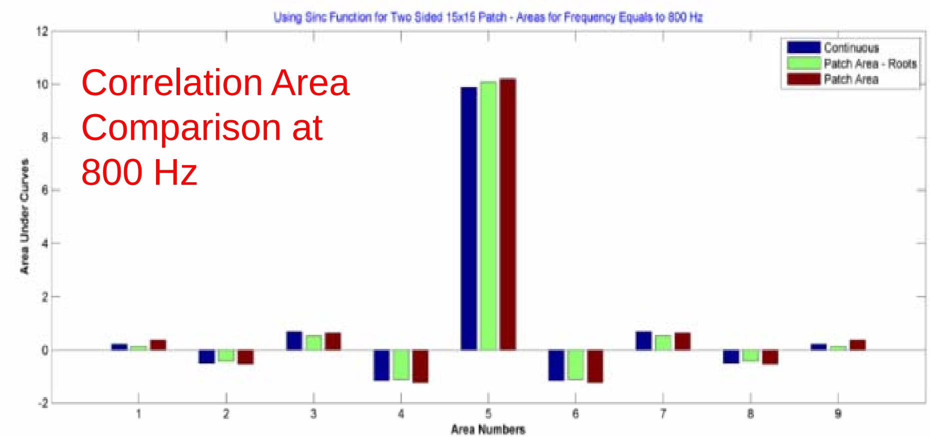
- The plot presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function **providing a good approximations in the range form 200 to 800 Hz.**
- The Sinc function is a function of wave-number and thus changes with frequency.
- A Smaller Patch Size/Higher Patch Density is required to assess higher frequencies
- Choice of patch density should depend on the highest frequency of interest.

Evaluating Patch Approximation of Spatial Correlation Function



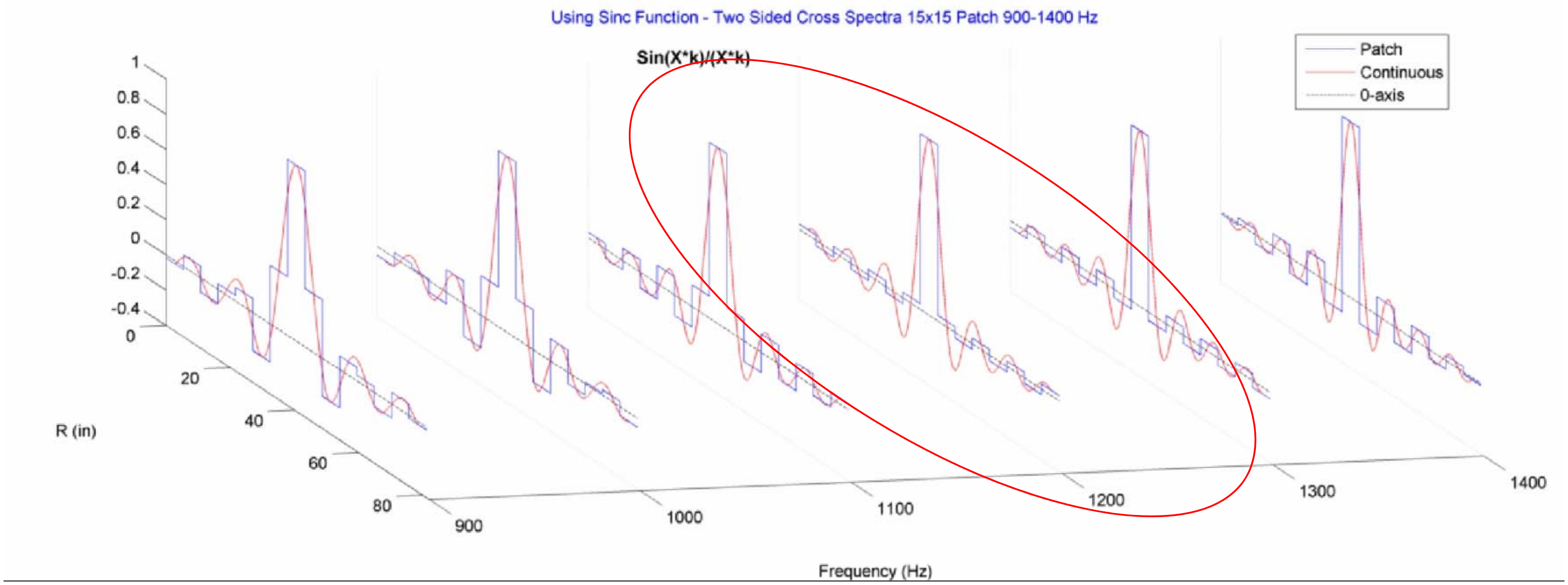






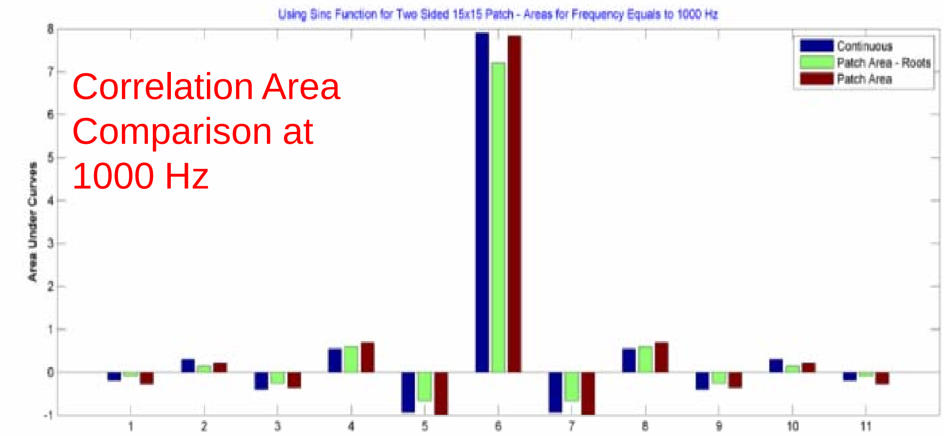
- **The areas compare well for this 5.4 inch patch size from 200-800 Hz**

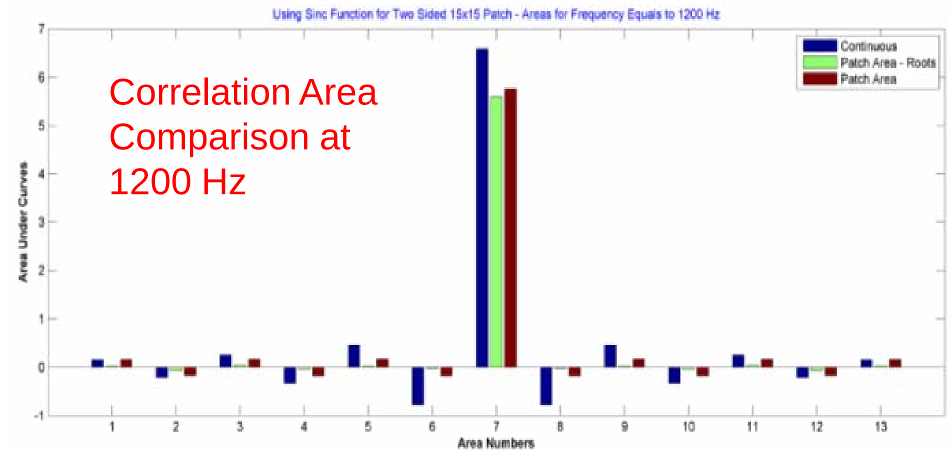
Evaluating Patch Approximation of Spatial Correlation Function

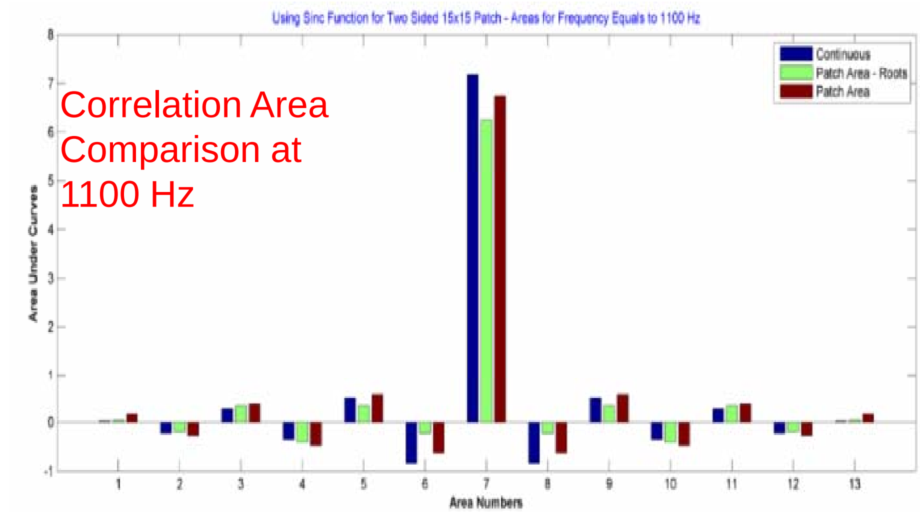


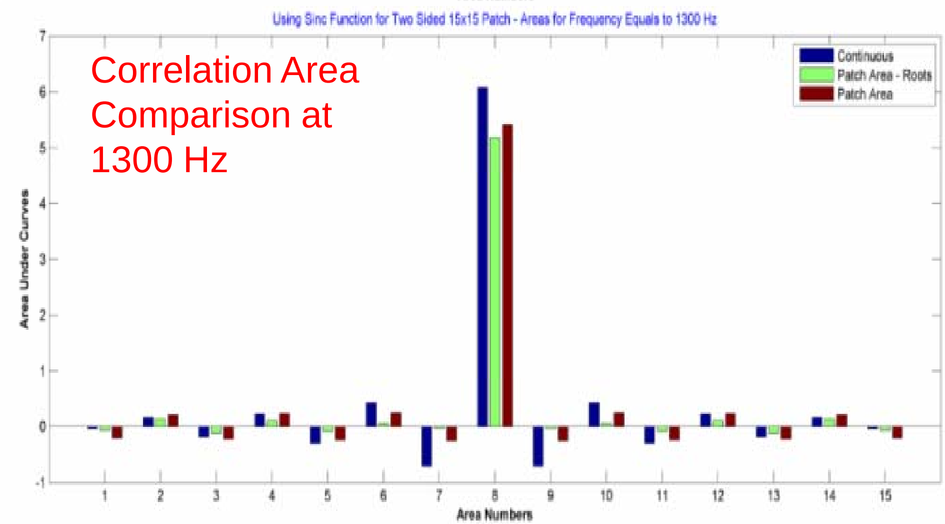
- The plot above presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function for frequencies spanning the range from 900 –1400 Hz.
- Because the 5.4 in patch size is too coarse, it is approximately equal to ½ the fluid wavelength in the range near 1200 Hz.
- The result is that the continuous function details are not well approximated spatially, in fact the patch approximation magnitude approaches zero for all the side lobes of the function. A condition approximating the field similar to **uncorrelated – "rain on the roof".**

Evaluating Patch Approximation of Spatial Correlation Function



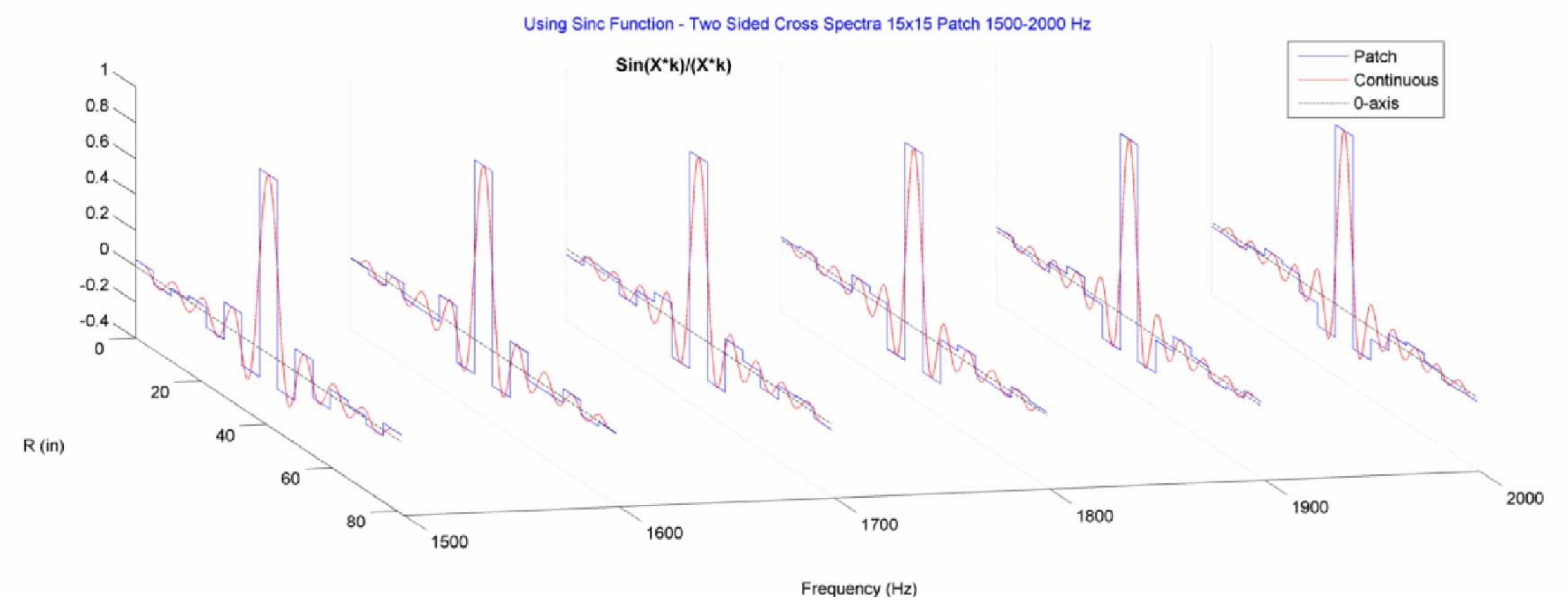






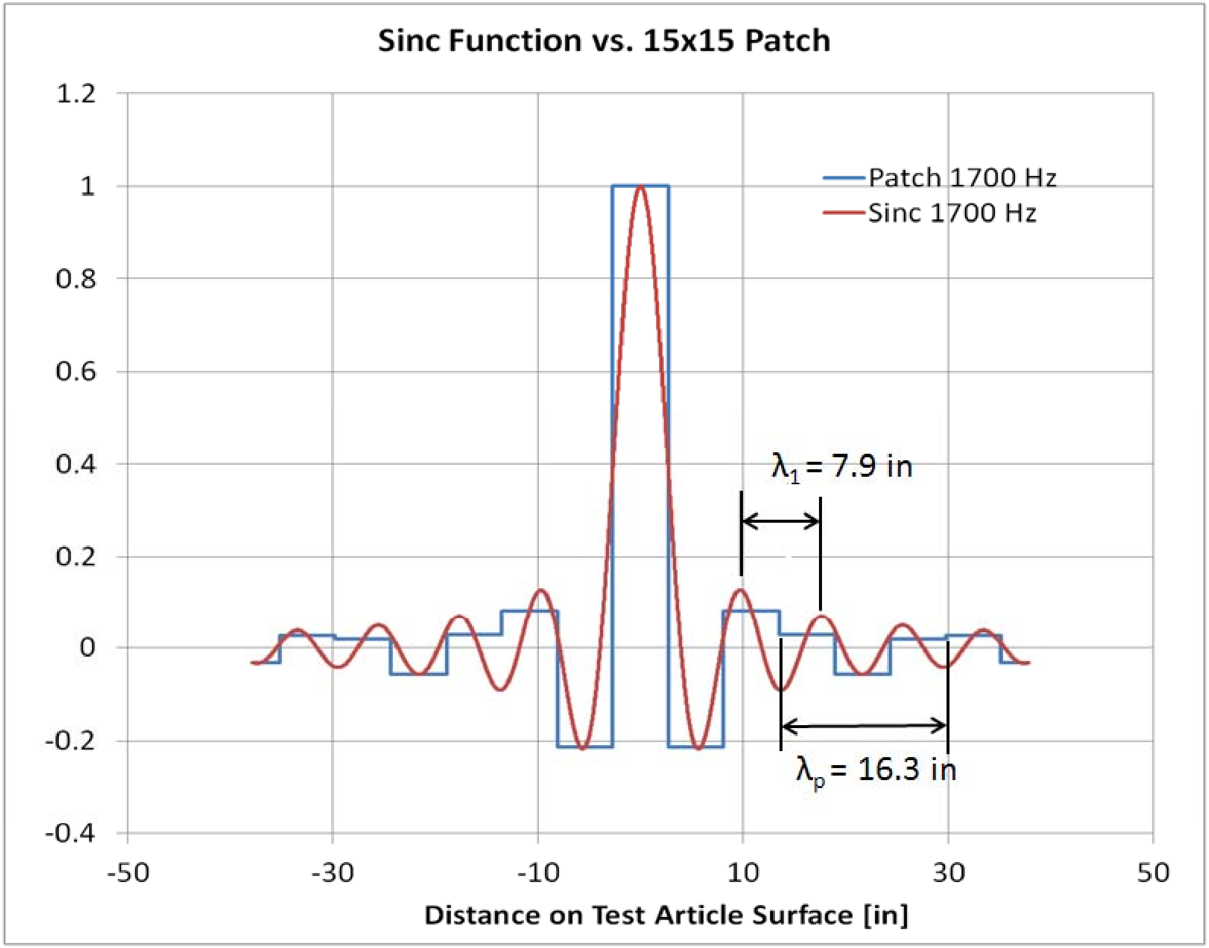
- The patch approximation areas diminish from 1000-1200 & 1300 Hz
- If the Cross correlation Values are represented with near zero values this approximates an Uncorrelated Pressure Field Trial.
- **Note that the dark blue continuous function has not gone to zero**

Evaluating Patch Approximation of Spatial Correlation Function



- The plot above presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function for frequencies spanning the range from 1500 – 2000 Hz.
- Because the 5.4 in patch size is too coarse, the continuous function details are not well approximated spatially.
- The detailed plot a 1700 Hz is examined more closely on the next slide.

Evaluating Patch Approximation of Spatial Correlation Function



The continuous Sinc Function:

$$\gamma = \frac{\sin(kx)}{kx}$$

The apparent but not physical speed of sound implied by the poor patch approximation:

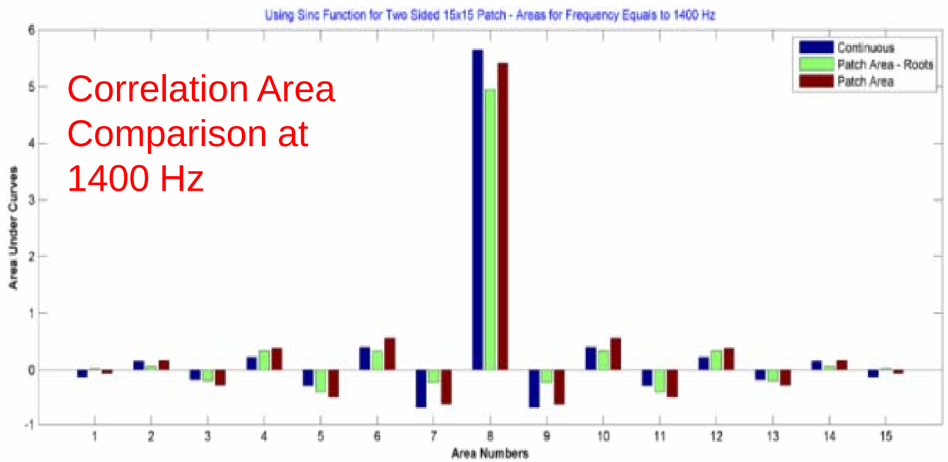
$$c_{apparent} = f * \lambda_p$$

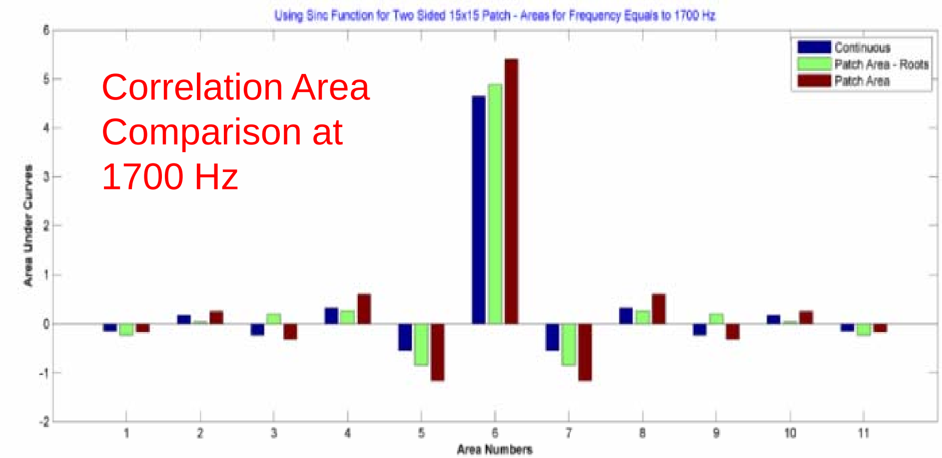
$$c_{apparent} = 1700 * 16.3$$

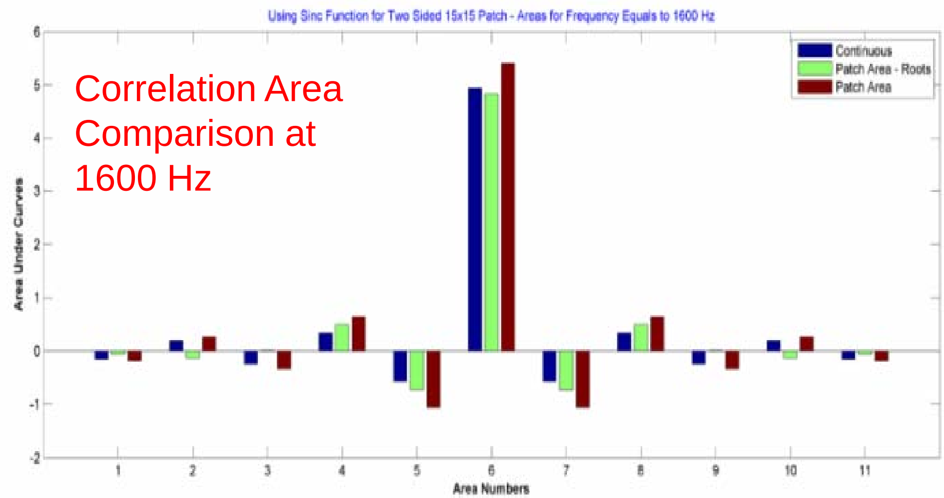
$$c_{apparent} = 27710 [in / \sec]$$

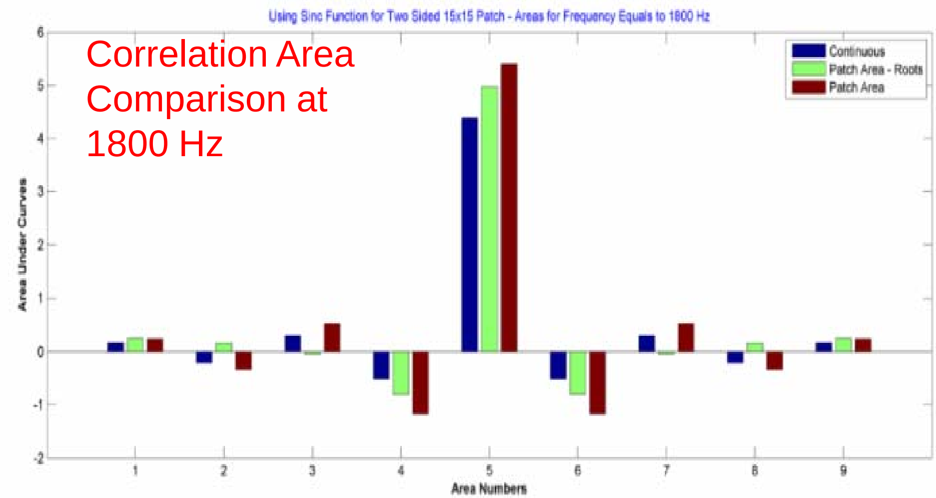
- The details of how the continuous function varies spatially are lost plot at 1700 Hz because the patch size is too large to approximate the function well.
- The poor approximation resembles a frequency aliasing affect. Could produce a pseudo coincidence frequency for some designs.

Evaluating Patch Approximation of Spatial Correlation Function









- Patch approximated areas **increase from 1400-1800Hz- exceeding those for Sinc**
- If the Cross Correlation Values represented by this coarse approximation suggest a different Speed of Sound than is the Physical Reality (The gross approximation is too efficient at exciting the structure). **An unacceptable Simulation?**

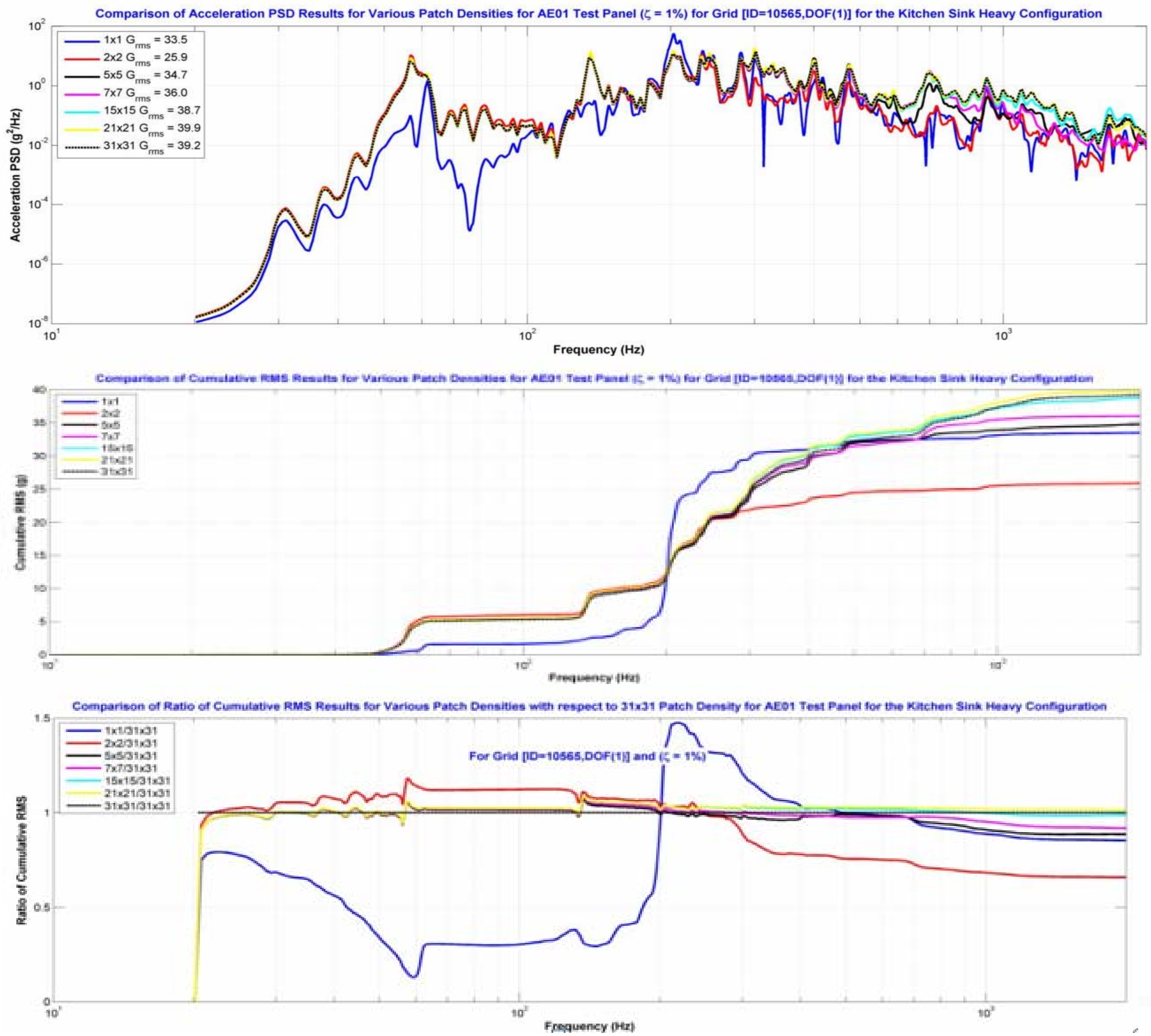
Report Convergence Trials

Loaded Vehicle Panel Responding to Fluctuating Acoustic Pressure Excitation

- What difference does Patch Density make?
  - *When Patch density is grossly too coarse such as in the case of a fully correlated pressure phasing over the entire surface (Example 1x1=1):*
    - The solution can over estimate the more fundamental response mode of the panel system (Odd function such as first drum head, trampoline mode)
    - The solution can fail to excite even modes efficiently.
  - *Coarse Patch Density trials other than Fully Correlated can also widely diverge from the actual Response (Example 2x2=4).*
  - *Plots on the following pages compare response from 1x1, 2x2,5x5,7x7, 15x15, and 21x21 to a 31x31 which is treated as the truth model for a vehicle panel section approximately 81 inches on each side.*

Report Convergence Trials

Show Convergence by Comparison



1X1 Patch approximation:
- Under predicted Even Mode near 56 Hz
- over-predicted the panel response near 200 Hz. Was this an Odd mode? A trampoline mode?

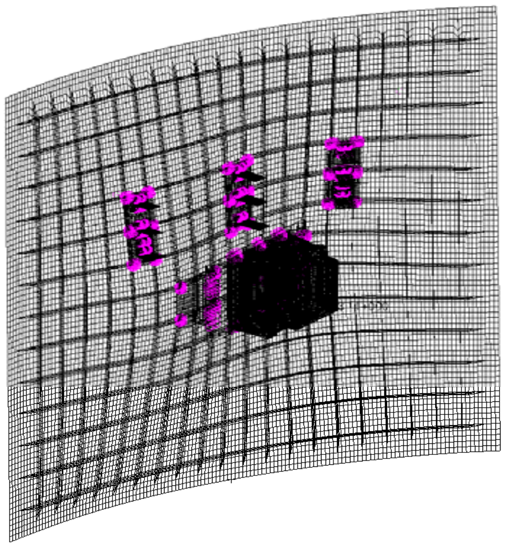
2X2 Patch approximation
- Over predicted Even Mode near 56 Hz
- Under predicted >200 Hz

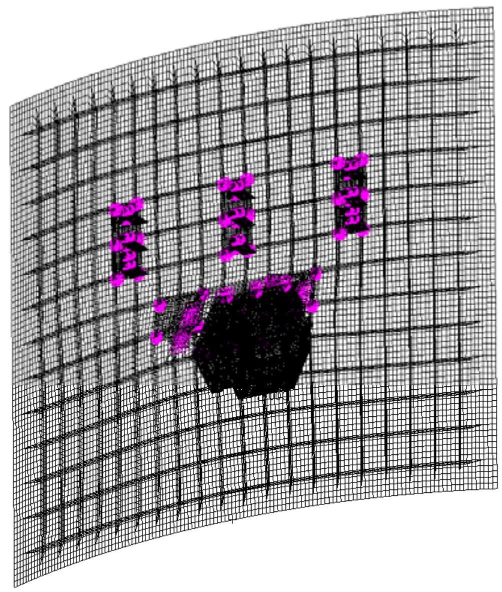
Both 1X1 and 2X2 display large dropouts at higher frequency. **View dropouts as an indication that Patch density is inadequate**

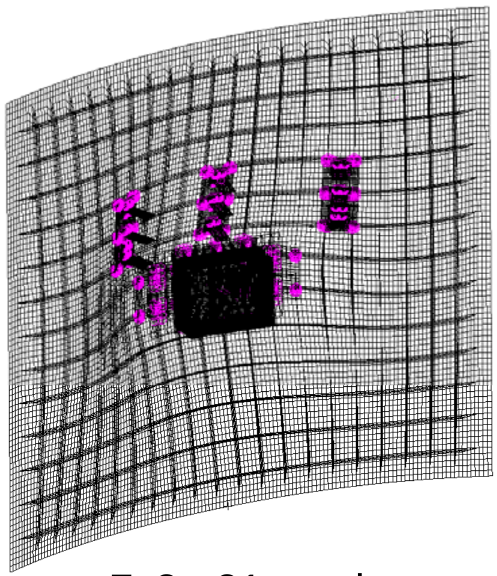
15x15 over-predicted response from 1500-1800 Hz

Report Convergence Trials

Loaded Vehicle Panel Responding to Fluctuating Acoustic Pressure Excitation







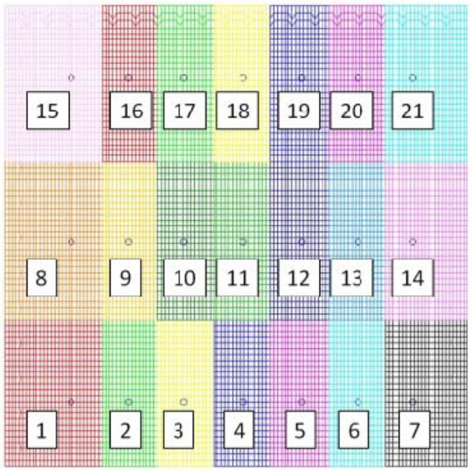
15x15 =225 patches

At the top are the first three **system mode shapes** of the test article (57.0, 59.5 and 61.5 Hz respectively) Hundreds of modes are used across the frequency range for Vibroacoustics.

On the left - A patch density assumption of **15x15. Center to center ~ 5.4 in**

On the right - A patch density assumption of 7x3 Center to center ~ 10 x 27 in for this 7x3 **Center to center ~ 11.6 in for a 7x7**

7x3 =21 patches



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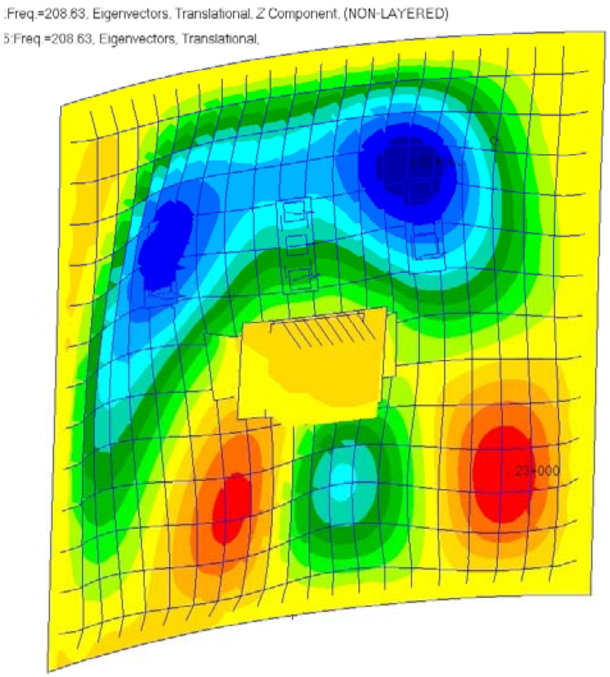
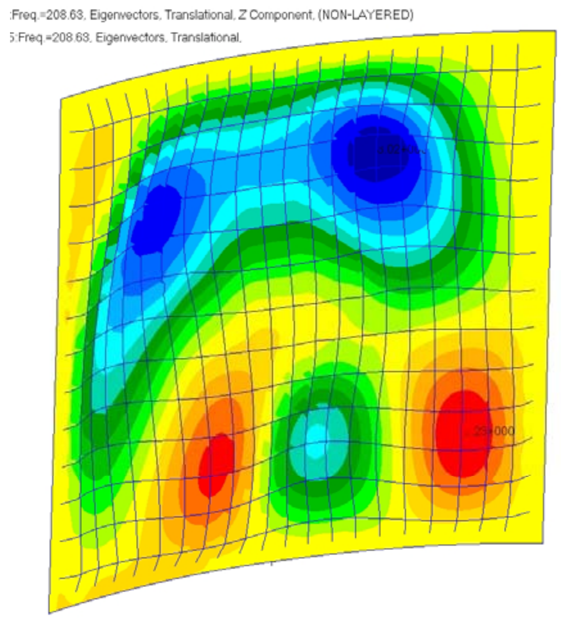
Report Convergence Trials

Loaded Vehicle Panel Responding to Fluctuating Acoustic Pressure Excitation

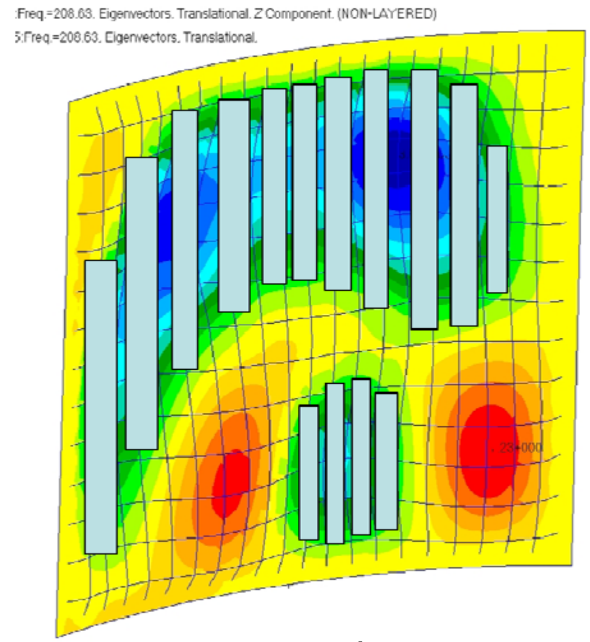
- Fully correlated oscillating field has advantage for Odd Modes, **those that have greater displacement areas in one direction than the other**. Example 209 Hz.



209 Hz Mode With Entire Test Article



209 Hz Mode With Equipment Hidden



209 Hz Mode
With Equipment Hidden
Strips highlight portion of panel

- **Fully correlated is not the correct physics for sound field in the Reverberant room at 209 Hz, but excites the mode more efficiently than a better approximation.**

Conclusions/Recommendations

- Spatial correlation function approximation using the patch method was illustrated across the frequency range for several different patch assumptions. Ranges where the approximation is not ideal were identified by qualitative/quantitative comparison.
- A Convergence study illustrating sensitivity to patch density was presented.
- **A summary relating rectangular "Patch Size" to "Fluid Acoustic Wavelength" is provided below:**

For a Vehicle Panel of Length 81.125 in				Number of Patches	Patch Size [in]	Fraction of Wavelength								
Patch Denisty Trials						67 in 200 Hz	26.8 in 500 Hz	19.1 in 700 Hz	14.9 in 900 Hz	12.2 in 1100 Hz	10.3 in 1300 Hz	8.9 in 1500 Hz	6.4 in 2100 Hz	4.3 in 3100 Hz
1	X	1	=	1	81.1	1.21	3.03	4.24	5.45	6.66	7.87	9.08	12.71	18.77
2	X	2	=	4	40.6	0.61	1.51	2.12	2.72	3.33	3.94	4.54	6.36	9.38
5	X	5	=	25	16.2	0.24	0.61	0.85	1.09	1.33	1.57	1.82	2.54	3.75
7	X	7	=	49	11.6	0.17	0.43	0.61	0.78	0.95	1.12	1.30	1.82	2.68
9	X	9	=	81	9.0	0.13	0.34	0.47	0.61	0.74	0.87	1.01	1.41	2.09
11	X	11	=	121	7.4	0.11	0.28	0.39	0.50	0.61	0.72	0.83	1.16	1.71
13	X	13	=	169	6.2	0.09	0.23	0.33	0.42	0.51	0.61	0.70	0.98	1.44
15	X	15	=	225	5.4	0.08	0.20	0.28	0.36	0.44	0.52	0.61	0.85	1.25
21	X	21	=	441	3.9	0.06	0.14	0.20	0.26	0.32	0.37	0.43	0.61	0.89
31	X	31	=	961	2.6	0.04	0.10	0.14	0.18	0.21	0.25	0.29	0.41	0.61

Recommend Patch Size be based on the highest frequency range of interest. Maintain "Patch center to center distance" less than 1/2 the fluid wavelength at highest frequency of interest. Note 1/3 or 1/4 of fluid wavelength is preferred.

References

[1]Smith, A., Davis, R.B., LaVerde, B., Hunt, R., Fulcher, C., Jones, D., Band, J., "Calculation of Coupled Vibroacoustics Response Estimates from a Library of Available Uncoupled Transfer Function Sets," AIAA SDM April 2012.

[2]Rafaely, B., "Spatial-temporal correlation of a diffuse sound field," Institute of Sound and Vibration Research, University of Southampton, J. Acoust. Soc. Am. Vol. 107, No. 6, June 2000, pp. 3254-3258.

[3]Corcos, G.M., "Resolution of Pressure in Turbulence," J. Acoust. Soc. Am. Vol 35, No. 2, (6), 1963, pp. 192-199.

[4]Peck, J., Smith, A., Fulcher, C.., LaVerde B., Hunt, R.., "Development of Component Interface Loads on a Cylindrical Orthogrid Vehicle Section from Test-Correlated Models of a Curved Panel," Proceedings of 2011 Spacecraft and Launch Vehicle Dynamic Environments Workshop, June 2011.

[5]Frady, G., Duvall, L.., Fulcher, C.., LaVerde B., Hunt, R.., "Test-Anchored Vibration Response Predictions For An Acoustically Energized Curved Orthogrid Panel With Mounted Components," Proceedings of JANNAF's 8th Modeling and Simulation Subcommittee (MSS), December 2011.

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Backup Charts

Because we Crawl - Walk - Run,
Corcos Fit of the Turbulent Boundary Layer Pressure Field was Simplified to one Dimension of Correlation:

$$\gamma = e^{-\left(C_x k' x\right)} \cos\left(k_x x\right)$$

The Convection Velocity: $U_c = 0.7U_\infty$

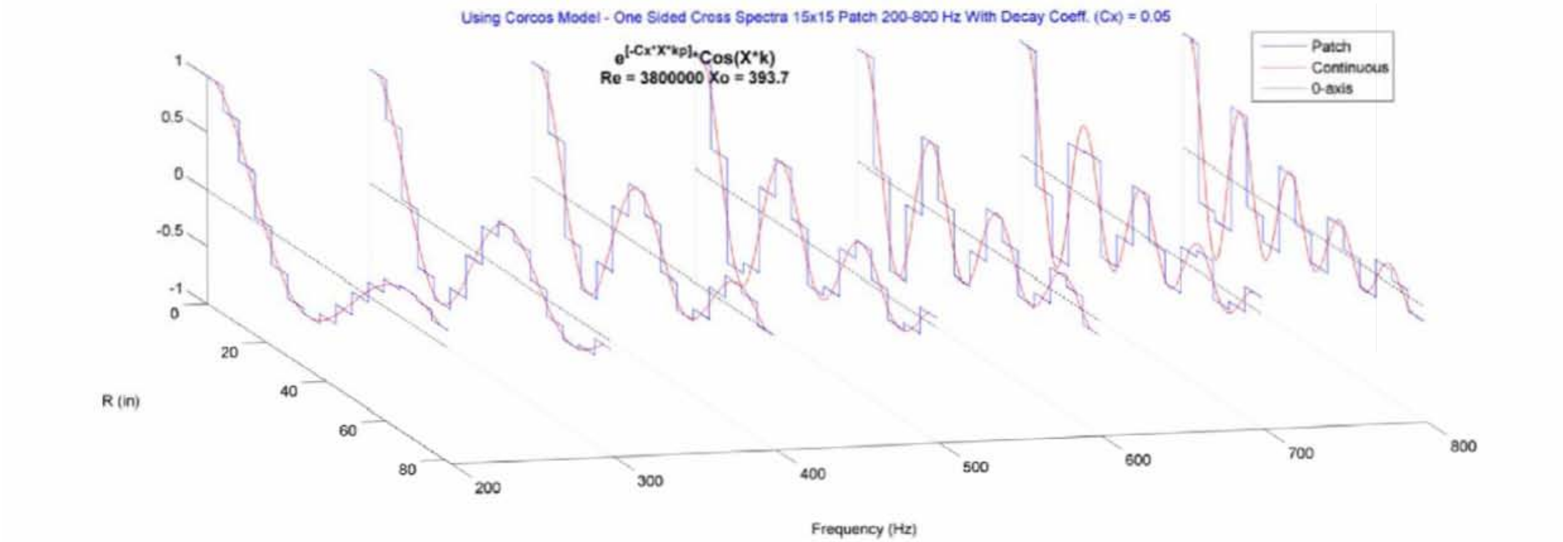
Wave-number in the flow direction: $k_x = \dfrac{\omega}{U_c}$

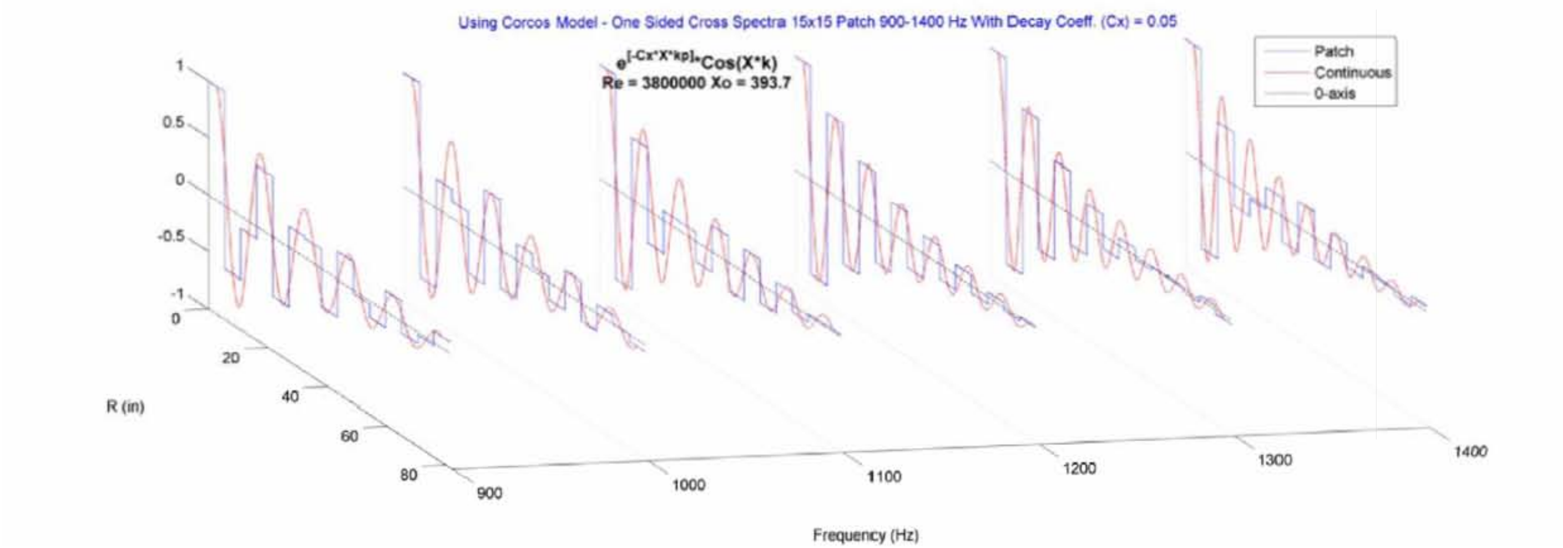
Modified Wave-number for decay: $k' = \sqrt{\left(\dfrac{\omega}{U_c}\right) + \dfrac{1}{9\delta_*^{\,2}}}$

Turbulent Boundary Layer thickness: $\delta = 0.37\left(\dfrac{X_0}{\mathrm{Re}^{1/5}}\right)$

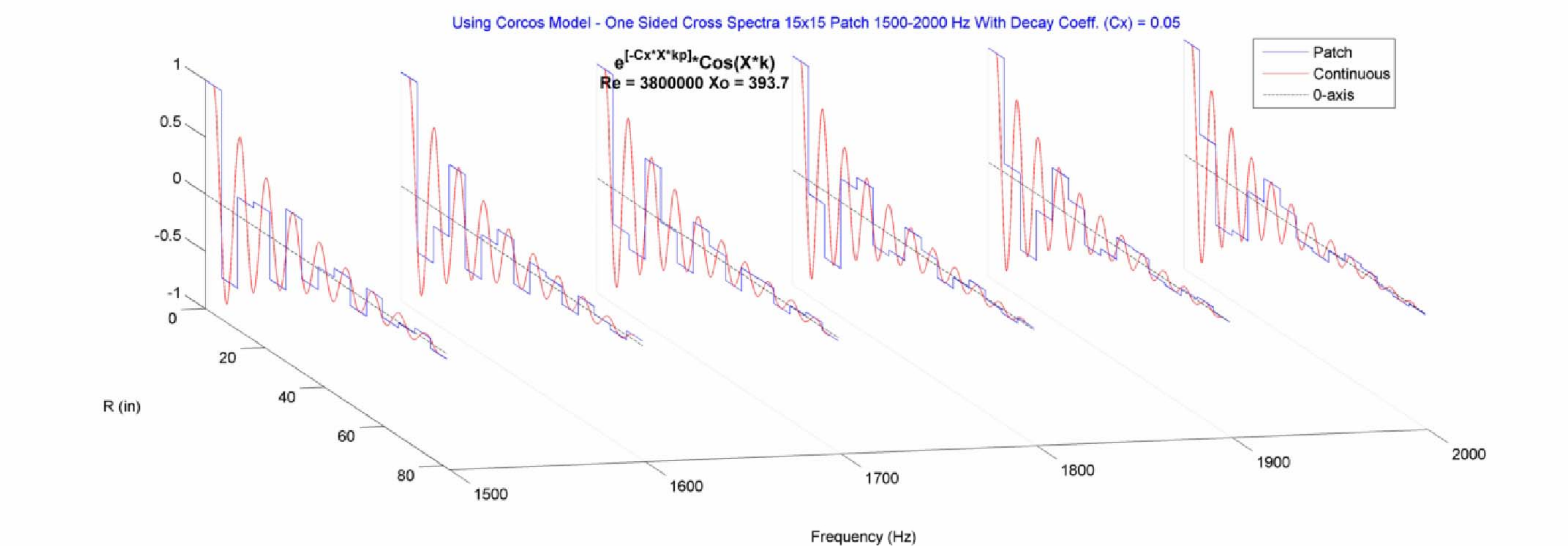
Displacement thickness: $\delta_* = \delta/8$

Evaluating Patch Approximation of Corcos Model (15x15 Patch)



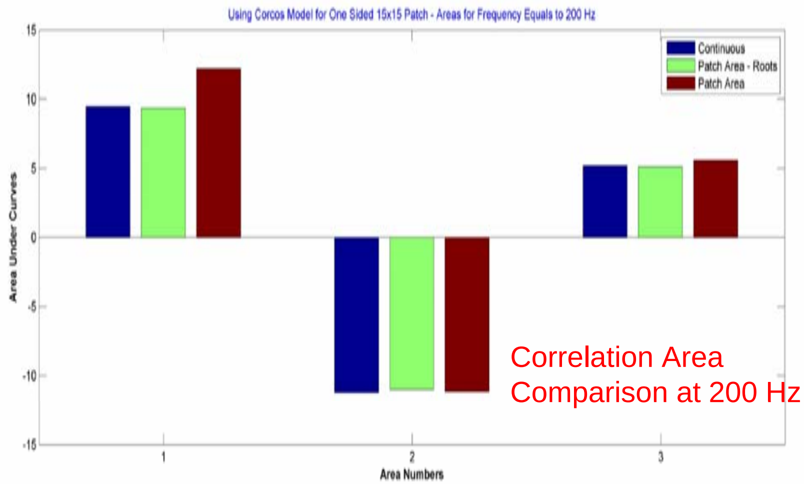


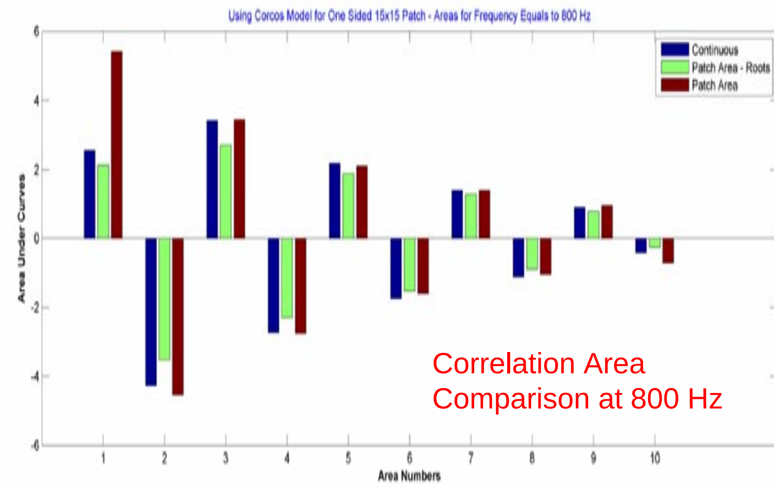
Evaluating Patch Approximation of Corcos Model (15x15 Patch)

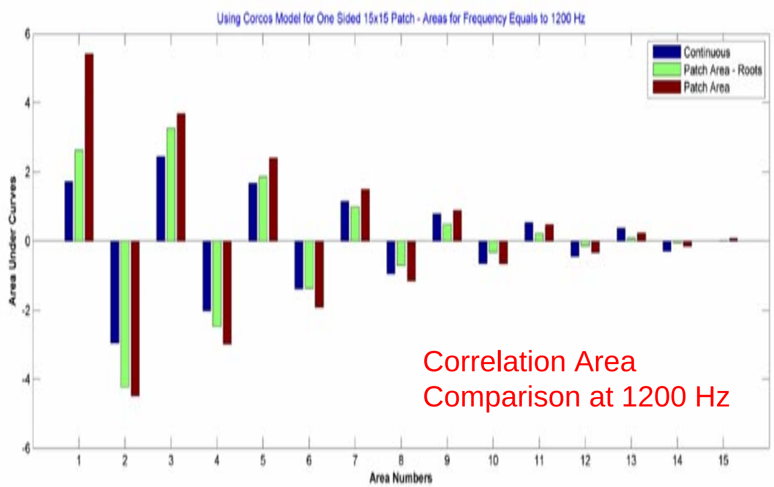


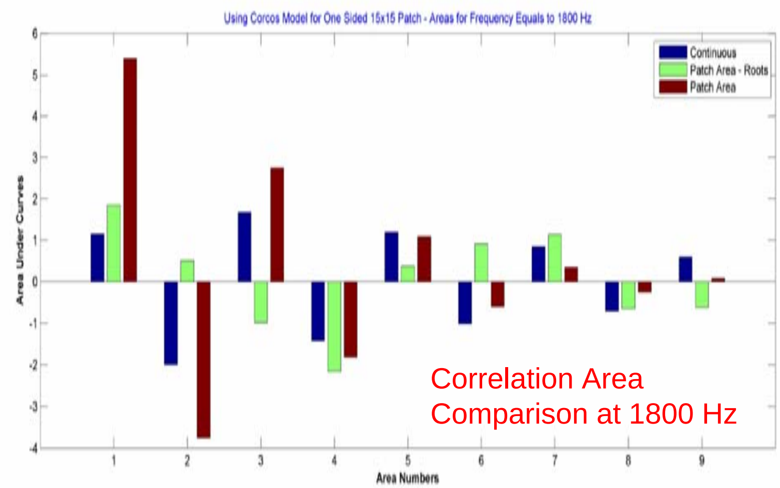
- Plot presents a single row of 15 patches from a 15x15 approximation of a Corcos Model correlation function
- The Corcos model is a function of wave-number, Reynolds number and distance from leading edge of TBL and thus varies with frequency
- The 15x15 patch density qualitative measurement begins to break down at 800 Hz
- A Smaller Patch Size/Higher Patch Density is required to assess higher frequencies
- Choice of patch density should depend on the highest frequency of interest.

Evaluating Patch Approximation of Corcos Model (15x15 Patch)

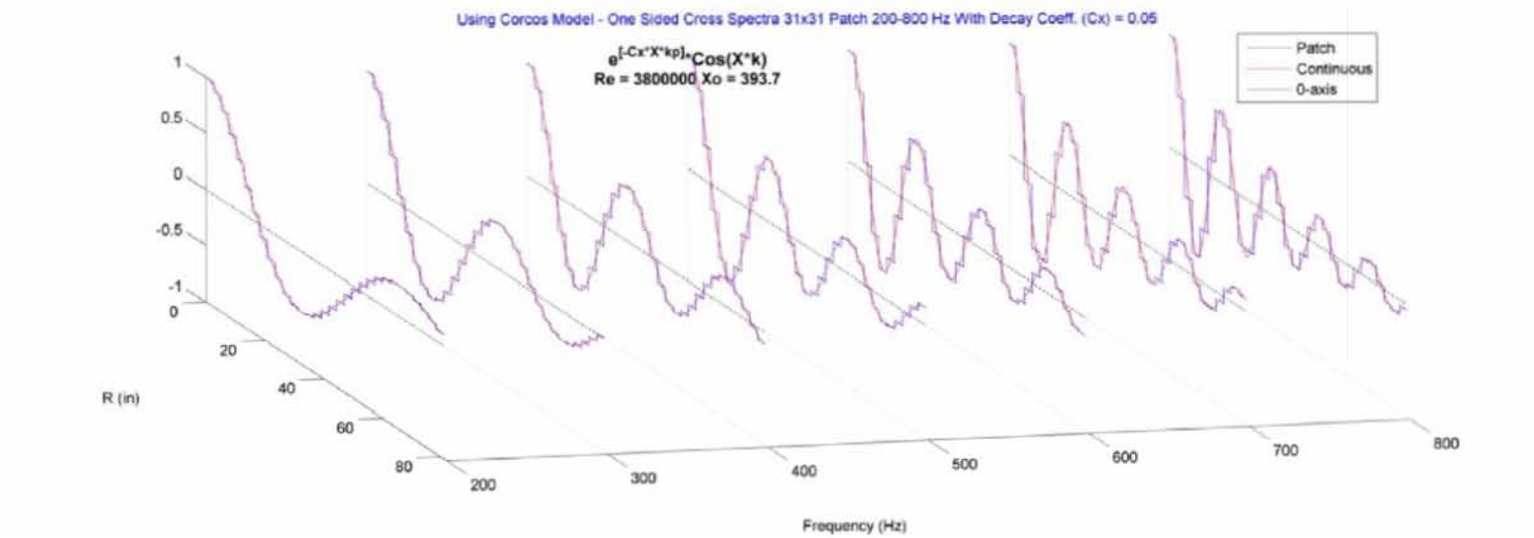


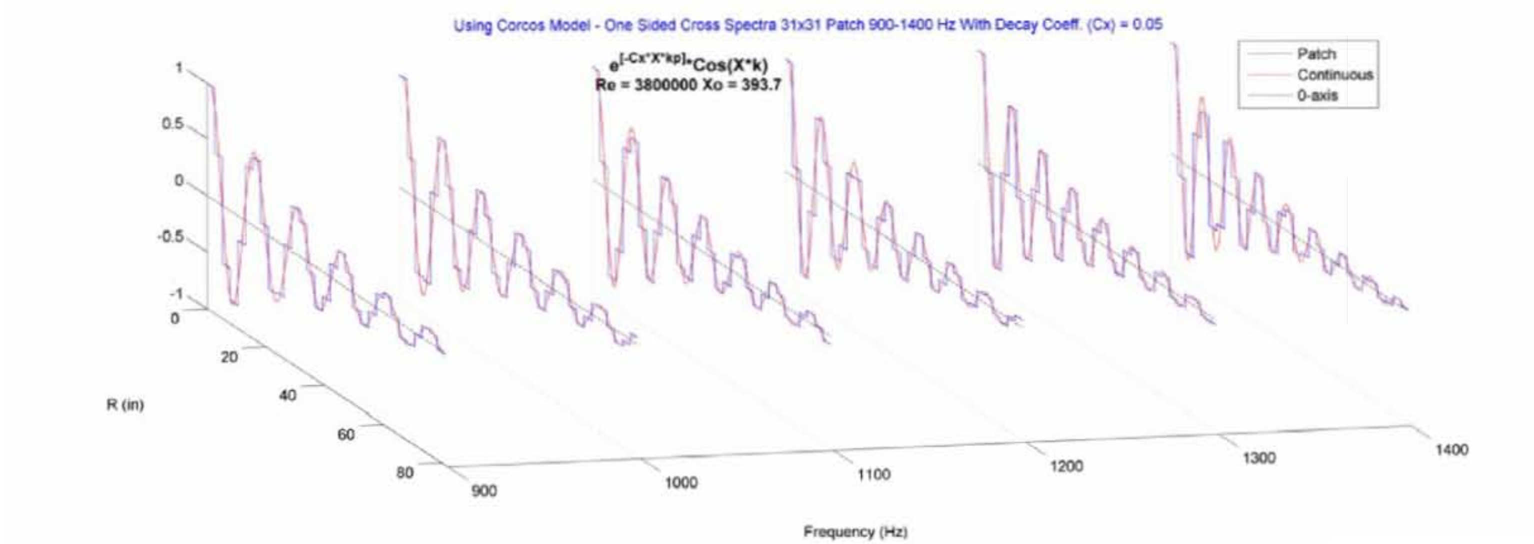




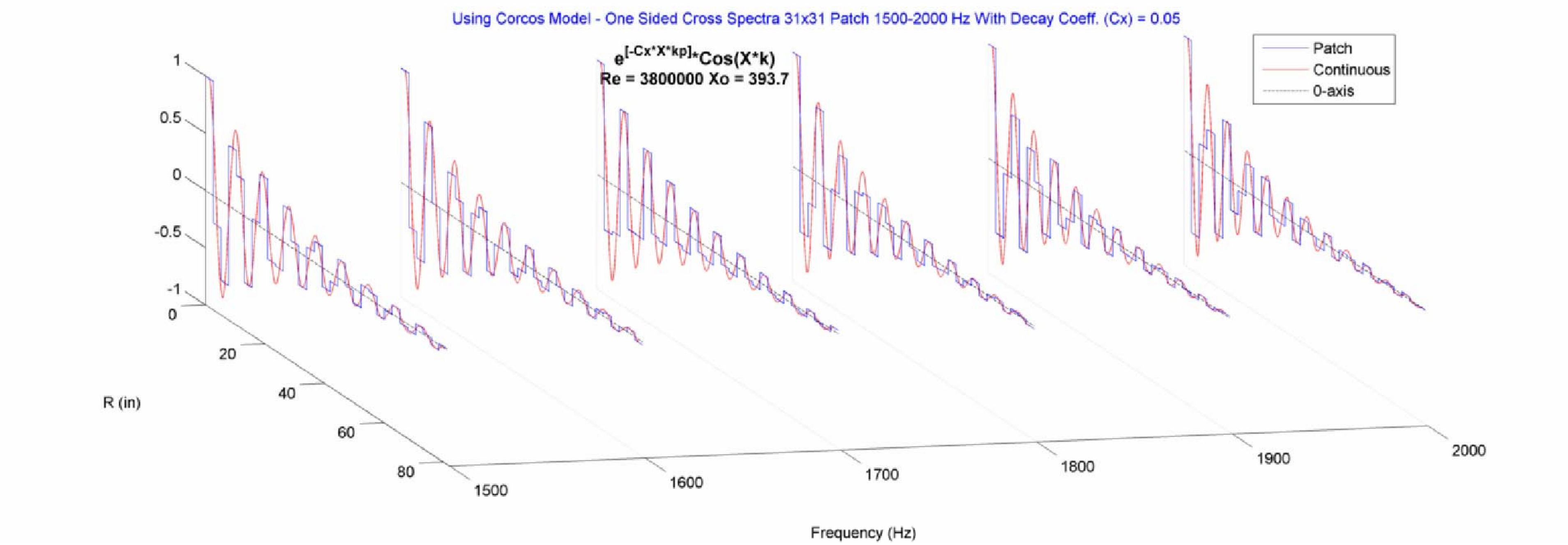


Evaluating Patch Approximation of Corcos Model (31x31 Patch)



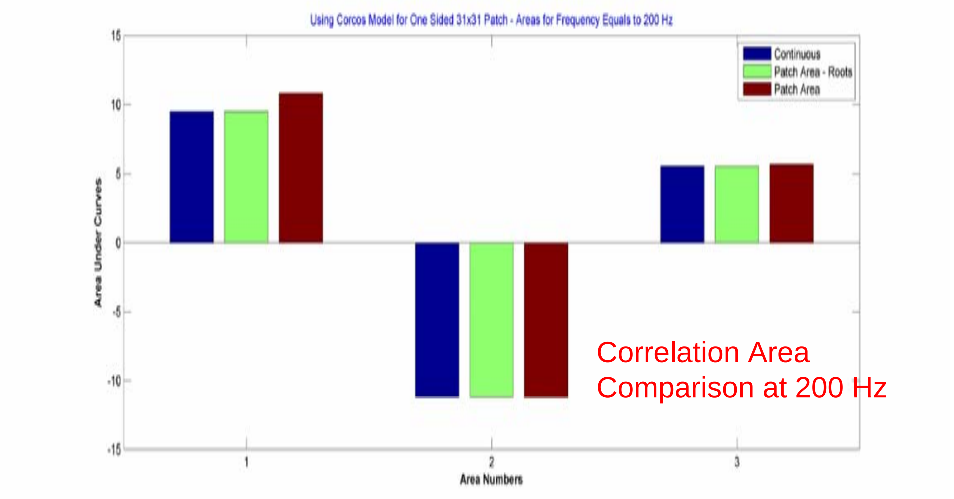


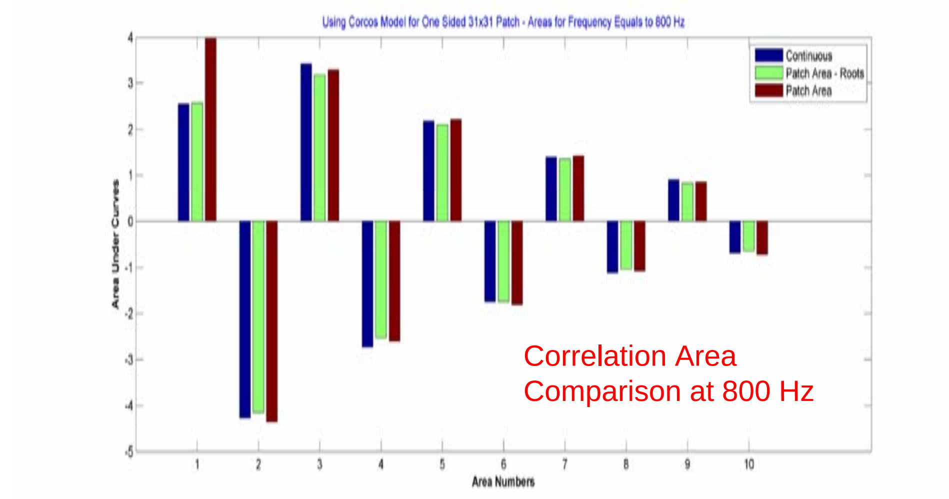
Evaluating Patch Approximation of Corcos Model (31x31 Patch)

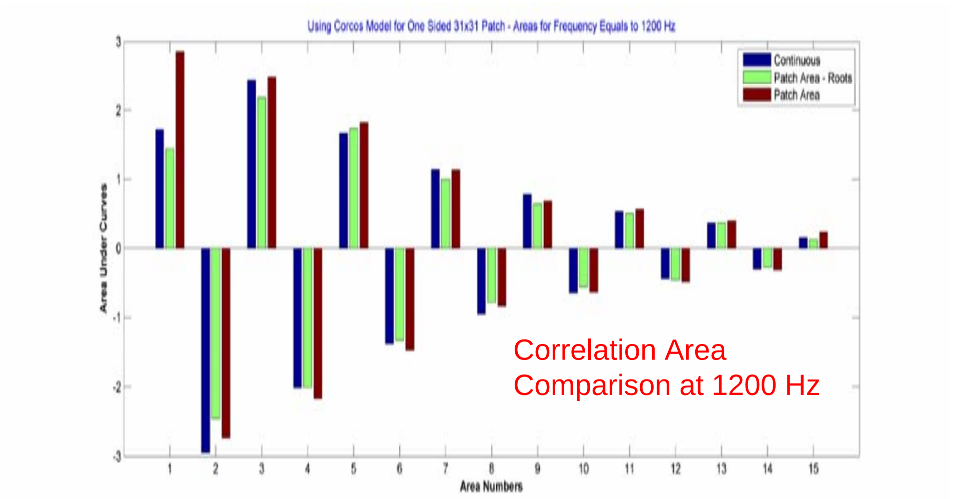


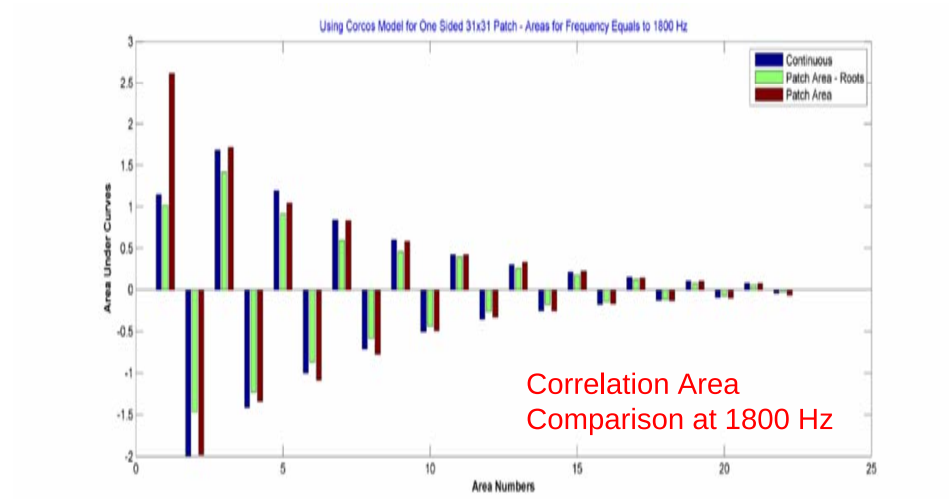
- Plot presents a single row of 31 patches from a 31x31 approximation of a Corcos Model correlation function
- The Corcos model is a function of wave-number, Reynolds number and distance from leading edge of TBL and thus varies with frequency
- The 31x31 patch density qualitative measurement begins to break down at 1600 Hz
- A Smaller Patch Size/Higher Patch Density is required to assess higher frequencies
- Choice of patch density should depend on the highest frequency of interest.

Evaluating Patch Approximation of Corcos Model (31x31 Patch)









Reporting Recommended Patch Density from Vehicle Panel Vibration Convergence Studies using both DAF and TBL Fits of the Spatial Correlation Function

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