Improved Finite-Volume Method for Radiative Hydrodynamics

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Abstract: Fully coupled simulations of hydrodynamics and radiative transfer are essential to a number of fields ranging from astrophysics to engineering applications. Of particular interest in this work are hypersonic atmospheric entries and associated experimental apparatus, e.g., shock tubes and high enthalpy testing facilities. The radiative transfer calculations must supply to the CFD a heating term in the energy equation in the form of the divergence of the radiative heat flux and the radiative heat fluxes to bounding surfaces. It is most efficient to solve the radiative transfer equation on the same grid as the CFD solution, and this work presents an algorithm with improved accuracy for such simulations on structured and unstructured grids compared to more conventional approaches. Results will be shown for shock radiation during hypersonic reentry. Issues of parallelization within a radiation sweep will also be discussed.

Keywords: Numerical Algorithms, Computational Fluid Dynamics, Radiative Transfer

1 Introduction

The radiative transfer equation (RTE) for the radiative intensity I in direction Ω in a non-scattering medium is given by

$$\Omega \cdot \nabla I(\mathbf{x},\Omega) = \chi(\mathbf{x})\big(S(\mathbf{x}) - I(\mathbf{x},\Omega)\big) \tag{1}$$

where S is the source function and χ is the opacity. Frequency dependence has been suppressed for simplicity of notation. Upon integration over a computational cell and discretizing, approximating the cell volume integral with the values at the center, one obtains

$$\sum_{\text{faces } k} I(\mathbf{x}_k,\Omega)\Omega \cdot \Delta\mathbf{S}_k = \chi(\mathbf{x}_c)\big(S(\mathbf{x}_c) - I(\mathbf{x}_c,\Omega)\big)V \tag{2}$$

where $\mathbf{x}_c$ is the position of the cell center, $\mathbf{x}_k$ is the center of face k, V is the cell volume, and $\Delta\mathbf{S}_k$ is the surface area vector of face k of the cell. Splitting the surface summation into incoming and outgoing parts and using cell-center values for the outgoing facial intensities in each cell, one gets the standard finite-volume method for radiative transfer [1]:

$$I(\mathbf{x}_c,\Omega) = \frac{\displaystyle\sum_{k,\Omega\cdot\Delta\mathbf{S}_k<0} I(\mathbf{x}_k,\Omega)\left|\Omega \cdot \Delta\mathbf{S}_k\right| + \chi(\mathbf{x}_c)S(\mathbf{x}_c)V}{\displaystyle\sum_{k,\Omega\cdot\Delta\mathbf{S}_k>0} \Omega \cdot \Delta\mathbf{S}_k + \chi(\mathbf{x}_c)V} \tag{3}$$

2 Improved formulation

The relatively inaccurate volume-integral approximation in (2) and the approximation of using cell-center values of I at the outgoing faces in (3) can both be eliminated by using the 1-d exact radiative transfer solution for constant χ and S along direction Ω. To determine the effective cross-cell propagation distance d in direction Ω, we divide the volume V by the total area of the incoming (or, equivalently, outgoing) faces projected on that direction:

$$d \approx \frac{V}{\displaystyle\sum_{i,\Omega\cdot\Delta S_i<0} \left|\Omega \cdot \Delta\mathbf{S}_i\right|} \tag{4}$$

Then the intensity on each outgoing face o is given by

$$I(\mathbf{x}_o,\Omega)=\frac{\sum_{i,\Omega\cdot\Delta\mathbf{S}_i<0} I(\mathbf{x}_i,\Omega)\left|\Omega\cdot\Delta\mathbf{S}_i\right|}{\sum_{i,\Omega\cdot\Delta\mathbf{S}_i<0}\left|\Omega\cdot\Delta\mathbf{S}_i\right|}e^{-\chi(\mathbf{x}_c)d}+(1-e^{-\chi(\mathbf{x}_c)d})S(x_c) \tag{5}$$

where $\mathbf{x}_o$ indicates an outgoing face center and $\mathbf{x}_i$ an incoming face center. This method accounts more accurately for emission and absorption acting between the incoming and outgoing faces of a computational cell, essentially by treating these processes with sub-cell accuracy using an analytic form. Eqs. (5) and (3) are the same to first order in the optical thickness of the cell, $\chi(\mathbf{x}_c)d$. Values of I are only required at the cell faces. The total radiative heating in a cell, needed for the CFD energy equation, is easily computed as

$$\int_{\text{cell}}\nabla\cdot\mathbf{q}\,dV=\int_{\text{surface}}\mathbf{q}\cdot d\mathbf{S}\approx\int_{\Omega}\sum_{\text{faces }k} I(\mathbf{x}_k,\Omega)\boldsymbol{\Omega}\cdot\Delta\mathbf{S}_k\,d\Omega \tag{6}$$

No differences are needed to compute this quantity, only a summation over the cell faces. The other quantity typically needed in a rad-hydro calculation is the heat flux at a body surface. Assuming that face k of a cell lies on this surface, the heat flux through that surface is simply:

$$\mathbf{q}_k\approx\int_{\Omega}\boldsymbol{\Omega}\,I(\mathbf{x}_k,\Omega)\,d\Omega \tag{7}$$

3 Verification and Comparison

Figure 1 below shows a comparison between a Cartesian-mesh, short-characteristic-based RTE solution (leftmost), the same problem on a pyramidal mesh using the method described here (middle), and a solution using the classical method (eq. (3)) on the same pyramidal mesh (rightmost). Contours of cell-average div($\mathbf{q}$) are shown in the midplane for a hyperboloidal radiative source emulating a reentry shock around a bluff body; x is streamwise and y is cross-stream. The rightmost plot shows a loss of heating in the classical method compared to the other two, presumably due to approximations in eq. (3) that have been improved using the current method (eq. (5)). The characteristic method result shows it to be slightly more diffusive, as expected, compared to both finite-volume methods, but the approximate agreement serves to verify the implementations of the latter. The final paper will make extensive comparisons for media of various degrees of optical thickness to show the advantages of the sub-cell treatment of emission and absorption.

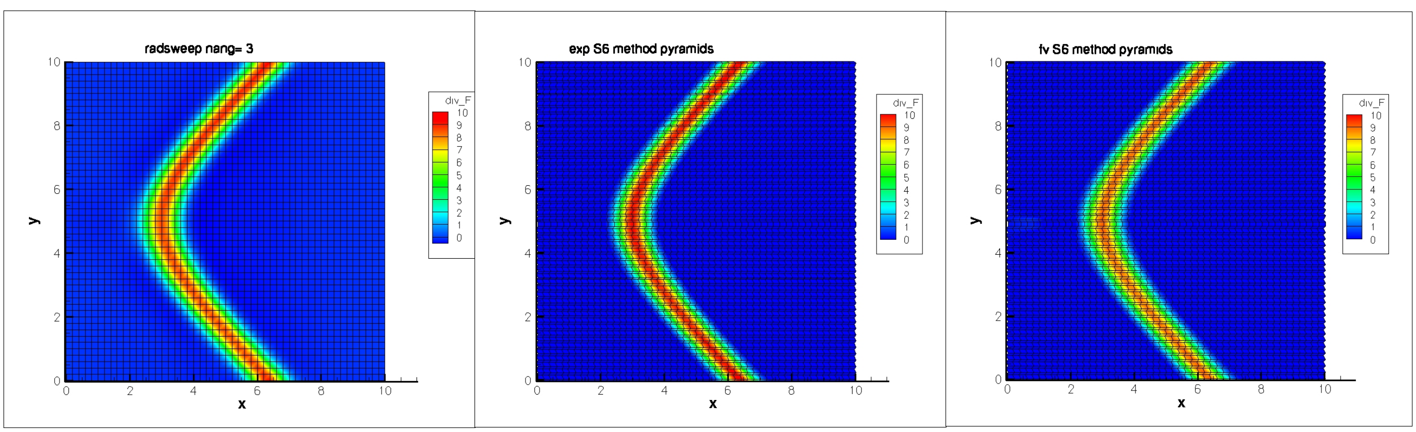

Fig.1: Comparison of div($\mathbf{q}$) for Cartesian short-characteristic, and current and classical finite volume methods.

4 References

[1] Modest, M., *Radiative Heat Transfer, 2nd ed.*, Academic Press, p. 525, 2003.