Correlation Between Analog Noise Measurements and the Expected Bit Error Rate of a Digital Signal Propagating Through Passive Components

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Summary

A method of determining the bit error rate (BER) of a digital circuit from the measurement of the analog S-parameters of the circuit has been developed. The method is based on the measurement of the noise and the standard deviation of the noise in the S-parameters. Once the standard deviation and the mean of the S-parameters are known, the BER of the circuit can be calculated using the normal Gaussian function.

Introduction

It is possible to estimate the contribution to the bit error rate (BER) of passive components of a digital circuit by measuring the S-parameters of those components. This involves measuring the S-parameters many times so that an estimate of the standard deviation and average of the S-parameters can be determined. It is this noise in the S-parameters of the passive components that contributes to the BER of the circuit.

This effort was motivated by the need to relate the noise generated in very long Quadrax cables for use on NASA's Orion project. The cables were to connect communication components in the service module with those in the command module—a length of 25 m. It became necessary to estimate the amount of noise generated through the cable during launch. To make this estimate, the cables would be shaken at 32g (root-mean-square). Accordingly, performance network analyzers (PNAs) were chosen to measure the noise in the S-parameters of the cables before, during, and after shaking. However, the noise data alone were not very useful in determining the expected BER from the cables being shaken. Consequently, the development of a methodology to relate the noise measured by the PNA to the BER was necessary. This report explains how to relate analog noise measurements to the expected BER of a digital signal going through a passive component.

Noise

The type of random noise in a communication system that determines the BER of a circuit is thermal noise. Thermal noise can be described by an average white noise (AWN) across a narrow frequency band of interest. Although it is possible to have other types of noise from interfering signals combine with thermal noise in the final BER, the type of noise addressed in this report is purely Gaussian noise.

Because of the assumption of AWN, each noise source can be described by a single temperature. These temperatures are additive so that the total noise energy in the system is given by $k_B T_{\text{total}}$, where k_B is Boltzmann's constant and T_{total} is the system noise temperature in Kelvin. Note T_{total} is the sum of all the individual noise temperatures in the system that is normalized by the previous gains in the system (Ref. 1).

Because the measurements are assumed to be on the long wavelength side of the peak of the blackbody radiation, the appropriate distribution function to be used for the measurement is the Gaussian distribution. The Gaussian distribution gives the probability of detecting an observed value, given the average of the measurement (i.e., its expected value) and the standard deviation of the measurement.

In Figure 1, the y-axis is the value of the Gaussian function, and the x-axis is the number of standard deviations. The average of the Gaussian function µ has been set to 0.

For binary digital circuits there is a measurement threshold. Below that threshold, the value of the measurement is usually referred to as a digital 0, and above that threshold it is a digital 1. In a real circuit a signal is sent out with an amplitude or phase of one value to represent a 0 and a different value to represent a 1. For the discussion of noise in this report, amplitude variation is to be used.

For noise analysis purposes assume that the signal at the output of a transmitter has no noise, similar to the signal depicted in Figure 2; it is then reasonable to ask what the signal at the input of the receiver looks like with the Gaussian noise it has picked up between the transmitter and the receiver. What is then the probability of the receiver has misinterpreted a 1 for a 0 or a 0 for a 1. This misinterpretation will only occur when the noise increases the signal level for a 0 above the threshold or reduces the signal level below the threshold for a 1.

The probability of detecting the wrong value is dependent on the amplitude of the signal, how close the signal is to the threshold value, and the standard deviation in the signal amplitude due to noise. As can be seen in Figure 3, the larger the standard deviation, the smaller the number of standard deviations that need to be added to the average of the signal for the signal to cross the threshold.

To know the probability of a signal being misinterpreted by the system, one only needs to know the number of standard deviations that is added or subtracted from the average signal level so that the signal will cross the threshold value. Then this number of standard deviations is related to the probability of an error occurring, using the Gaussian function.
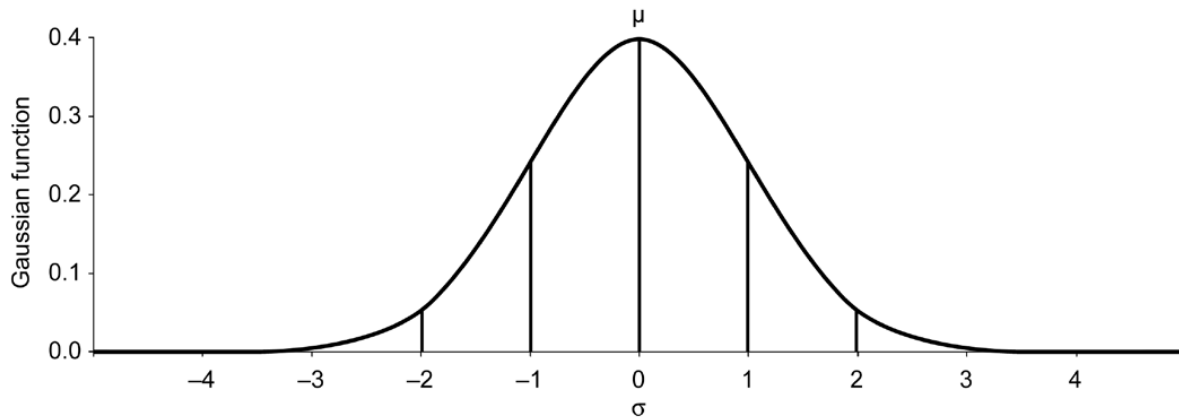

Figure 1.—Normalized Gaussian function plotted versus standard deviations with µ as the average value of the measurement. σ is the number of standard deviations away from the mean, µ.
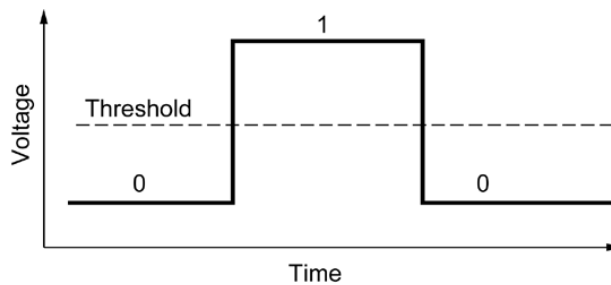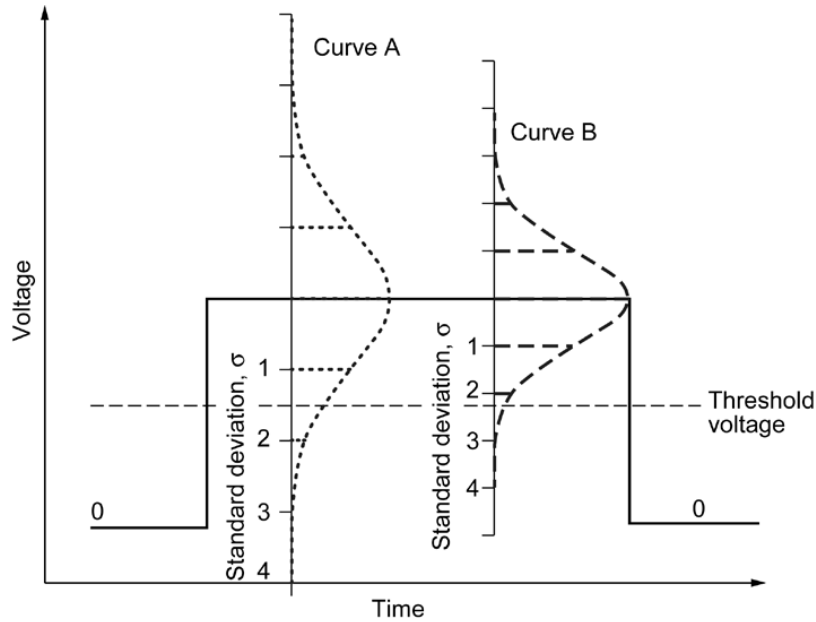

Figure 2.—Digital binary signal.

Figure 3.—Curves A and B represent two bits with the same mean voltage, but curve A's standard deviation σ in the voltage is 60 percent larger than curve B's. The threshold voltage is the voltage at which an error occurs, and it determines the number of standard deviations for each curve.

Relationship Between Noise and BER

The relationship of the probability of an error occurring and the number of standard deviations can be found in many books on digital communications. One useful book on this topic is Digital Modulation Techniques, Second Edition, by Fuqin Xiong (Ref. 2). Chapter 2 and Appendix B in this book discuss how to determine the BER for various modulation and coding techniques.

One can calculate the probability (P) of an error occurring:

$$P = A * Q(x) \tag{1}$$

where A is a constant dependent on the modulation and coding techniques used, x is the number of standard deviation of the detection level away from the mean signal level, and $Q(x)$ is called the Q-function, which is simply related to the error function (erf) by

$$Q(x) \equiv \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right] \tag{2}$$

$Q(x)$ is only the tail of the error function on either the low or high side. The definition of the error function is

$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \tag{3}$$

From Fuqin Xiong's book (Ref. 2), the signal energy per bit divided by $E_b/N0$, the average noise power within the frequency band of measurement time (the time period of the bit), is a function of x. Here, E_b is

the energy in one bit and $N0$ is the noise energy derived from the noise power and the bit period. The quantity $E_b/N0$ can be expressed as

$$\frac{E_b}{N0} = B * x^2$$

(4)

where B is a constant that depends on the modulation and coding schemes used.

For the simplest modulation scheme and no coding ($A = B = 1$), therefore

$$P = Q\left[\left(\frac{E_b}{N0}\right)^{\frac{1}{2}}\right]$$

(5)

It is this equation that is used in analog measurements of the S-parameters to determine the effect of vibration on the BER of the system.

Relating P to the S-Parameters

Forces (voltages) are what is normally measured in electronic systems, and then those measurements are related to current or power through calibration. Looking at the argument of the Q-function, E_b and $N0$ are both energies; therefore, the square root of the ratio of the two energies is a ratio of voltages that will be used later to determine the BER. Also, note the S-parameters from the PNA are ratios of energies.

For a digital circuit, the link budget equation of interest can be written as

$$\frac{E_b}{N0} = \frac{TransmitPower * TransmissionCoefficient}{RNP + IP}$$

(6)

where RNP is the random noise power and IP is the power associated with any interfering signal like reflections or outside signals.

For an internal digital circuit, the outside interfering signals can be neglected. Also, power due to reflections will be ignored for clarity. For most well-designed passive circuits the reflected power is a fraction of the noise power and usually can be ignored unless the communications system is operating at cryogenic temperatures.

By multiplying the numerator and the denominator by the time period of 1 bit, Equation (6) becomes

$$\frac{E_b}{N0} = \frac{TransmitEnergy * TransmissionCoefficient}{N0}$$

(7)

It is now clear that the transmission coefficient is the measured S-parameter of the circuit, S_{21}, because E_b is the received power.

$$\frac{E_b}{N0} = \frac{TransmitEnergy * S_{21}}{N0} = \frac{E_t * S_{21}}{N0}$$

(8)

where E_t is the transmitted energy. The next procedure relates $N0$ to the standard deviations of S_{21}.

The signal energy at the detector within the bit period with noise can be written as

$$SE = SE_0 + N0$$

(9)

where $N0$ is the noise energy in 1 bit, SE is the instantaneous signal energy, and SE_0 is the signal energy without noise.

The transmission coefficient S_{21} can be written as

$$S_{21} = \langle S_{21} \rangle + \sigma_{21}$$

(10)

where σ_{21} is the standard deviation of S_{21} and $\langle S_{21} \rangle$ is the average of S_{21}.

Now multiplying Equation (10) by the initial transmit energy E_t results in

$$E_t * S_{21} = E_t * \langle S_{21} \rangle + E_t * \sigma_{21}$$

(11)

It is easy to see that $SE = E_t * S_{21}$ and $SE_0 = E_t * \langle S_{21} \rangle$

$$\therefore N0 = E_t * \sigma_{21}$$

(12)

Substituting Equation (12) into Equation (8) one obtains

$$\frac{E_b}{N0} = \frac{S_{21}}{\sigma_{21}}$$

(13)

Therefore, by measuring the *S*-parameters and their standard deviations of the link one can obtain the $E_b/N0$ of the link.

Conclusion

It has been shown that one can use analog measurements to determine the energy per bit divided by the noise energy density $E_b/N0$ of a circuit if one can measure the analog *S*-parameters of the circuit and their standard deviations. For this technique to be useful, the power input in the analog circuit must be the same as in the digital circuit. When this condition exists, $E_b/N0 = S_{21}/\sigma_{21}$ is the correct value to be considered for the link in the circuit, where S_{21} is the transmission coefficient and T_{21} is its standard deviation. This is because the noise in the circuit is independent of the input power into the circuit.

References

1. Sklar, Bernard: Digital Communications, Fundamentals and Applications. Prentice-Hall, Englewood Cliffs, NJ, 1988, p. 219.
2. Xiong, Fuqin: Digital Modulation Techniques. Second ed., Artech House, Inc., Boston, MA, 2006.

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